THEORY OF THE NEUTRON ELECTRIC DIPOLE MOMENT

John Ellis

CERN -- Geneva

Invited talk at the
Workshop on Fundamental Physics with Slow Neutrons
Institut Laue-Langevin, Grenoble
March 8.10, 1989
1. WHAT WE KNOW ABOUT CP VIOLATION

The first CP-violating observable measured [1] in the laboratory was the mass mixing parameter in the \( K^0 - \bar{K}^0 \) system [2]:

\[
|\varepsilon_K| = (2.26 \pm 0.02) \times 10^{-3}
\]  

(1)

which can be accommodated in the Standard Model as we discuss later. More recently, direct CP violation has been measured [3] in \( K^0 \to 2\pi \) decay amplitudes:

\[
\frac{\varepsilon'}{\varepsilon_K} = (3.3 \pm 1.1) \times 10^{-3}
\]  

(2)

which can also be accommodated in the Standard Model [4]. Another piece of circumstantial evidence for CP violation is the baryon asymmetry of the Universe, corresponding to a present baryon-to-photon ratio

\[
\frac{n_B}{n_\gamma} \approx 10^{-11} \text{ to } 10^{-8}
\]  

(3a)

inferred [5] from direct observation, or

\[
\frac{n_B}{n_\gamma} \approx (3 \text{ to } 5) \times 10^{-10}
\]  

(3b)

inferred [5] from the success of Big Bang nucleosynthesis calculations [6]. It seems very likely that explaining \( n_B/n_\gamma \) requires physics beyond the Standard Model. The limits on the neutron electric dipole moment \( d_n \) quoted at this meeting were

\[
d_n = (-1.4 \pm 0.6) \times 10^{-25} \text{ e.cm} \quad \text{(Ref. [7])}
\]

\[
= (-0.7 \pm 0.4) \times 10^{-25} \text{ e.cm} \quad \text{(Ref. [8])}
\]

(4a)

corresponding to
\[ d_n < 2.6 \times 10^{-25} \text{ e.cm} \]  

which we will later compare with predictions from various models of CP violation.

2. - **APPROACHES TO CALCULATING** $d_n$ [9]

2.1 **Naïve Quark Model**

In this approach, one first calculates CP-violating interactions at the quark level, and then uses the non-relativistic quark model to calculate hadronic matrix elements of the quark operators. For example, one might calculate the electric dipole moments $d_{u,d}$ of the $u$ and $d$ quarks and then use a non-relativistic SU(6) wave function

\[ d_n^{(e)} = \frac{1}{3} (4d_d - d_u) \]  

Or one might calculate the colour electric dipole moments $f_{u,d}$ of the quarks and use the same SU(6) assumption to obtain

\[ d_n^{(c)} = \frac{1}{3} e \left( \frac{4}{3} f_d + \frac{2}{3} f_u \right) \]  

However, the successes of chiral symmetry in meson dynamics tell us that $m_{u,d} \ll \Lambda_{QCD}$ so that the $u$ and $d$ quarks are unlikely to be non-relativistic. Moreover, measurements of the $\pi N$ $\sigma$-term [10] and of the spin structure of the proton [11] tell us that the proton and neutron contain $\bar{s}s$ pairs. Therefore the naïve quark model with its non-relativistic SU(6) wave functions containing just valence quarks may not always be very reliable.
2.2 Effective Lagrangian

In this approach one uses a phenomenological Lagrangian for baryons and pseudoscalar mesons, including strong couplings

\[ \mathcal{L}_S = -f Z e B^i \bar{\chi}_s \gamma_i B' P + (h.c.) \]  

(6a)

weak couplings

\[ \mathcal{L}_W = f Z e B^i \bar{\chi}_s \gamma_i B' P + (h.c.) \] 

(6b)

and meson kinetic terms

\[ \mathcal{L}_\pi = \lambda e^{i \theta} D^\mu \pi^* D_\mu \pi + (h.c.) \] 

(6c)

The magnitudes of the couplings are fixed using phenomenology and/or some particular weak interaction model. Loop diagrams calculated with this effective Lagrangian are the leading contributions as \( m_p^2 \to 0 \) (\( m_u, d, s \to 0 \)). There are two important classes of diagrams, those with one weak vertex as in Fig. 1a, and those with two as in Fig. 1b. The generic result of calculating the one-weak-vertex diagrams is [9]

\[ d^{(1)}_n \propto \frac{e g f}{4 \pi^2} \sin(-\phi) \left[ \frac{1}{m_B} f^{(2)}(m_M/m_B) \right] \] 

(7a)

where the first line in (7a) comes from using the charge coupling of the photon \( e \gamma^\mu \), and the second from using the magnetic moment coupling \( e (k_B / m_B) \sigma_{\mu \nu} q^\nu \). The generic result of calculating the two-weak-vertex diagrams in Fig. 1b is [9]

\[ d^{(2)}_n \propto \frac{e g h f}{4 \pi^2} \sin(\theta - \phi) \left[ \frac{1}{m_B} f^{(2)}(m_M/m_B) \right] \] 

(7b)
where again the first line in (7b) is from the charge coupling of the photon, and the second line from the magnetic moment coupling. Because of the logarithms in (7) and the phenomenological enhancement of some non-leptonic weak couplings due to the \( \Delta I = \frac{1}{2} \) rule, the contributions (7) to \( d_n \) can be larger than the naïve quark model contributions (6) in a given model.

3. - MODEL PREDICTIONS FOR \( d_n \)

3.1 Standard Model

In the Standard Model, CP violation is due to a single complex phase \( \delta \) in the Cabibbo-Kobayashi-Maskawa matrix \( U_{KM} \) [12]:

\[
\mathcal{L}_{SM} = \bar{u}_L \gamma_\mu U_{KM} d_L W^{\mu} + \text{(h.c.)} \tag{8}
\]

where \( U_{KM} \) may be parametrized by

\[
U_{KM} = \begin{pmatrix}
C_1 & -S_1 C_3 & -S_1 S_3 \\
S_1 C_2 & C_1 C_2 C_3 - S_2 S_3 e^{i\delta} & C_1 C_2 S_3 + S_2 C_3 e^{i\delta} \\
S_1 S_2 & C_1 S_2 C_3 + C_2 S_3 e^{i\delta} & C_1 S_2 S_3 - C_2 C_3 e^{i\delta}
\end{pmatrix} \tag{9}
\]

where \( c_i (s_i) \equiv \cos \theta_i (\sin \theta_i) \) (i=1,2,3). Note that there is a five-fold ambiguity in the phase convention: \( q_L \rightarrow e^{i\alpha} q_L \), and that all physical quantities are independent of the choice of convention. All CP-violating quantities are proportional to the combination \( s_2 s_3 \sin \delta \), which can in principle be determined from \( \epsilon_K \) [4,9]:

\[
\epsilon_K \propto \left| \frac{M_{\pi K_1 K_2}}{M_{K_1} - M_{K_2}} \right| = \chi_K s_2 s_3 s_8 \tag{10}
\]

The hadronic matrix element factor is only poorly known, although it is in principle calculable once the top quark mass \( m_t \) is known.
Figure 2a shows the value of $s_2$ and the allowed range of $s_3$ for a plausible set of values of hadronic matrix elements, and Fig. 2b shows the corresponding range of $\sin \delta$, as functions of $m_t$ [4]. One finds that

$$2 \times 10^{-4} \lesssim s_2 \lesssim 3 \times 10^{-3}$$

(11)

where the lower limit comes from $m_t \lesssim 200$ GeV as indicated by precision neutral current experiments [13], and the upper limit comes from $m_t \gtrsim 41$ GeV as indicated by the published UA1 search [14] at the CERN pp collider. Figure 3 shows that the value of $\epsilon'/\epsilon_K$ predicted [4] in the Standard Model on the basis of Eqs. (10) and (11) agrees well with the experimental value (2) [3]. The precision of the agreement is not high, but could be improved with time.

It was soon realized [15,16] that Standard Model diagrams containing only two $W^\pm$ vertices could not give any CP violation, as they were proportional to $(U^+_{MK})_i (U_{KM})_j$, which is real. It is possible to obtain CP violation with four $W^\pm$ vertices, since $(U^+_{KM})_j (U_{KM})_k (U^+_{KM'})_g (U_{KM'})_l$ is complex in general. However, all the internal quark masses must be different, and one must beware of symmetrization that can cancel out the CP violation. For example, the sum of diagrams with just one internal quark line and no gluons is real [17]. One needs at least two quark lines and/or several strong interaction corrections. The resulting contribution to $d_n$ is small

$$d_n \approx \left(10^{-33} \text{ to } 10^{-34}\right) \text{cm}$$

(12)

when calculated in the naïve quark model.

The fact that one needs four $W^\pm$ vertices means that in the effective Lagrangian approach one needs diagrams with at least two weak vertices such as $f$, $h$ in (7b), (7c). Experimental results on $\Lambda$ and $\Sigma$ decays tell us that [9]
\( |s| \gtrsim 1 \times 10^{-7} \) \hspace{1cm} (13a)

whilst a theoretical calculation indicates that

\[ \Phi \gtrsim 0.05 s_2 s_3 \sin \delta \] \hspace{1cm} (13b)

The rate of \( K^0 \rightarrow 2\pi \) decays tells us that

\( |h| \gtrsim 1.5 \times 10^{-7} \) \hspace{1cm} (14a)

whilst another theoretical calculation indicates that

\[ \Theta \gtrsim 0.3 s_2 s_3 \sin \delta \] \hspace{1cm} (14b)

It should be noted that both \( |f| \) and \( |h| \) are much larger than would be estimated in the naive quark model, because of the \( \Delta I = \frac{1}{2} \) rule. Using (13) and (14) in the generic two-weak-vertex formula (7b) one finds

\[ d_n \propto \frac{e g_{1f}^4}{4 \pi^2 m_B^2} \sin (\Theta - \phi) f^{(2)} \left( \frac{m_N}{m_B} \right) \]

\[ \propto \left( 10^{-29} \text{ to } 10^{-28} \right) s_2 s_3 \sin \delta \quad \text{e.cm} \] \hspace{1cm} (15)

After substituting the range (11) for \( s_2 s_3 \sin \delta \) into (15), one finds [9]

\[ d_n \propto \left( 10^{-33} \text{ to } 2 \times 10^{-31} \right) \text{ e.cm} \] \hspace{1cm} (16)

which is somewhat larger than the naive quark model estimate (12), but still far below the present experimental sensitivity.
3.2 QCD $\theta$ parameter

In the presence of non-perturbative effects in QCD, one must allow [18] for a term

$$\mathcal{L}_\theta = -\theta_{\alpha\beta\gamma} \frac{g^2}{32\pi^2} G_{\mu\nu} \varepsilon_{\mu\nu\rho} \cdot \varepsilon_\gamma = \varepsilon_{\mu\nu\rho} \mathcal{C}_{\alpha\beta\gamma}$$  \hspace{1cm} (17)

that is C-even (because it has two gluon field strengths $G_{\mu\nu}$) and P-odd (because of the $\varepsilon_{\mu\nu\rho}$) and hence violates CP. By making an anomalous global $U(1)$ transformation on the quark fields, one can replace the $\theta$-term (17) by a phase in the quark mass matrix:

$$\mathcal{L}_m \rightarrow i \theta_{\alpha\beta\gamma} \frac{m_u m_d m_s (\bar{u} x_u + \bar{d} x_d + \bar{s} x_s)}{(m_u m_d m_s + m_u m_s + m_d m_s)}$$  \hspace{1cm} (18)

The contribution this violation of CP makes to $d_n$ has been calculated in the bag model [19]:

$$d_n \approx 2.7 \times 10^{-16} \theta_{\alpha\beta\gamma} \text{ e.c.m.}$$  \hspace{1cm} (19a)

and using pion-nucleon loops [20]:

$$d_n \approx -3.8 \times 10^{-16} \theta_{\alpha\beta\gamma} \text{ e.c.m.}$$  \hspace{1cm} (19b)

Including all pseudoscalar meson-baryon loops and both $\gamma$ and $\sigma$ couplings of the proton, it has been estimated that [9]

$$2 \times 10^{-16} < \left| \frac{d_n}{\theta_{\alpha\beta\gamma} \text{ e.c.m.}} \right| < 5 \times 10^{-16}$$  \hspace{1cm} (19c)

The experimental upper limit (4b) therefore tells us that

$$|\theta_{\alpha\beta\gamma}| \leq 10^{-9}$$  \hspace{1cm} (20)

and the big question is: why?
In the Standard Model there is a complex renormalization of the quark mass matrix at the two-loop level which has been estimated [21] to yield

\[ |\delta \Theta_{QCD}| \approx 10^{-16} \]  

(21)

and hence according to (19c)

\[ |d_m| \approx \text{few} \times 10^{-32} \text{ e.cm} \]  

(22)

which is comparable to the previous Standard Model contributions (12) and (16). However, other models can easily give $\delta \Theta_{QCD} = 0$ in one-loop order. In a Grand Unified Theory (GUT), the dominant source of the baryon asymmetry (3) is often believed to be GUT Higgs decay [22], via the generic diagrams in Fig. 4a. In the minimal SU(5) GUT the baryon asymmetry only arises in three-loop order, and is far too small [22]. However, it may well be that any model giving a large enough baryon asymmetry will also give a large contribution to $\Theta_{QCD}$ [23]. This is because there is a renormalization of the quark mass matrix which is proportional to a Higgs coupling (see Fig. 4b) analogous to that appearing in the baryon asymmetry diagram of Fig. 4a. This possible connection suggests that [23]

\[ |\delta \Theta_{QCD}| \approx 10 \left( \frac{n_B}{n_\gamma} \right) \left( \frac{\mathcal{G}_{H}}{\mathcal{G}_{QCD}} \right)^2 \]  

(23)

Putting in (23) the estimate (3b) of $n_B/n_\gamma$ and either the Higgs coupling to the b quark ($m_b \sim 5$ GeV) or to the t quark ($m_t \sim 100$ GeV) gives the range

\[ |d_m| \approx \left( 6 \times 10^{-28} \right) \text{ to } 2 \times 10^{-25} \text{ e.cm} \]  

(24)

which approaches the experimentally accessible range.
One reason why $\theta_{QCD}$ may be very small is that it is actually zero, thanks to the Peccei-Quinn mechanism [24]. According to this, there is an approximate $U(1)$ global symmetry realized with an almost massless pseudoscalar boson, the axion. Its mass and couplings

$$m_a \sim \left( \frac{m_f^2 \Lambda^3_{QCD}}{f_a} \right)^{1/2}, \quad g_{a\gamma\gamma} \sim \frac{m_f}{f_a}$$

are inversely scaled by a large axion decay constant $f_a$ that is often related to a large Higgs v.e.v.. Astrophysics, including most stringently the supernova 1987a, and cosmology [26] tell us that

$$10^{10} \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$$

leaving a narrow window in which the axion could provide an interesting and observable amount of dark matter. Detection of this dark matter would presumably rule out $\theta_{QCD}$ as a source of a large $d_n$.

3.3 Two Higgs doublet model

The requirement of natural flavour conservation imposes that one Higgs doublet give masses to all the charge +2/3 quarks, and one to all the charge -1/3 quarks [27]. This means that there is no spontaneous CP violation, and that the only source of CP violation is the familiar phase $\delta$ in the Kobayashi-Maskawa matrix. The only difference from the Standard Model calculation of $d_n$ is in the extra $h^\pm$ exchange diagrams. Since we in any case took the values of the weak vertices from $\Lambda$, $\Sigma$ and $K$ decays, their only possible effect could be to alter the theoretical predictions for the phases $\phi$, $\theta$ (13b) and (14b). However, since Higgs couplings to light quarks are small, this effect is so small, and one estimates [9,28]

$$d_n \sim \left(10^{-33} \text{ to } 2 \times 10^{-31}\right) \text{ e.m.}$$

as in the Standard Model.
3.4 Supersymmetry

There are two new sources of CP violation in a minimal supersymmetric extension of the Standard Model, namely phases in the gluino mass [29]

\[ m_\tilde{g} = |m_\tilde{g}| e^{-2i\phi} \]  
(28)

and in the squark mass matrix [30], which may be parametrized as

\[
\begin{pmatrix}
\tilde{t}_L^2 + C_d M_d^+ \tilde{d}_L + C_u M_u^+ \\
\tilde{u}_d \tilde{u}_d^+ \\
\tilde{u}_d \tilde{d}_d^+ + C_d M_d^+ \tilde{d}_d + \tilde{u}_u \tilde{u}_u^+ \\
\end{pmatrix}
\begin{pmatrix}
\tilde{t}_L \\
\tilde{u}_L \\
\tilde{d}_L \\
\end{pmatrix}
\]

for the charge -1/3 squarks, and similarly for the charge +2/3 squarks. We can diagonalize the conventional quark mass matrices \( M_d \) and \( M_u \) by the Kobayashi-Maskawa matrix \( U_{KM} \), but the squark mass matrix (29) will in general require an extra diagonalization by a matrix \( \tilde{U} \) that can introduce additional flavour and CP violation into the \( \tilde{q}\tilde{q} \) couplings.

The latter we can factorize out as a diagonal phase matrix \( e^{i\tilde{\phi}} \): \( \tilde{U} \equiv \tilde{U} e^{i\tilde{\phi}} \) (det \( \tilde{U} \) real) in the couplings so that

\[
\mathcal{L} \sum_{\tilde{q}} \bar{\tilde{d}} \frac{\tilde{d}}{2} \tilde{\psi}_s \tilde{\psi}_s^{\dagger} \left[ \tilde{U} e^{i\tilde{\phi}} - \frac{1}{2} \lambda \tilde{\psi}_s \tilde{\psi}_s^{\dagger} \right] \tilde{d}
\]

(30a)

where

\[
\phi \equiv \phi_\tilde{g} - \tilde{\phi}
\]

(30b)

is the observable relative phase, which is present even if there are only two generations.

There are then one-loop diagrams [31] like those in Fig. 5 which contribute to the quark electric dipole moments
\[ d_d = -\frac{2}{9} \frac{\alpha_s}{\pi} \frac{\alpha^2}{m_{\tilde{g}}} (2\phi) f(\text{mass ratios}), \text{ etc.} \] (31)

and similarly to the colour dipole moments. These can be used directly in the naïve quark model formulae (5) to obtain an estimate of \( d_u \).

Alternatively, one can estimate the CP-violating part of the \( \bar{B}B'P \) vertex and use it in the effective Lagrangian formula (7a). One can estimate the order of magnitude by assuming

\[ m_{\tilde{l}}^2 \times m_{\tilde{r}}^2 \approx m_{\tilde{g}}^2 \approx m_W^2 \] (32)

in which case [32]

\[ d_u \approx 10^{-22} \phi \text{ e.cm} \] (33)

It should be emphasized that the estimate (33) is sensitive to the assumption (32) on the sparticle masses, but taken at face value it suggests when combined with the upper limit (4b) that

\[ \phi \lesssim O(10^{-3}) \approx O\left(\frac{\alpha}{\pi}\right) \] (34)

This is not necessarily a problem for realistic supersymmetric models, where \( \phi \) often appears at the one-loop level.

3.5 Weinberg Multi-Higgs Model

If there are three or more Higgs doublets, their v.e.v.'s may be relatively complex: \( \langle v_j \rangle = 0 \), which is a possible source of spontaneous CP violation even if \( U_{\text{KM}} \) is real [33]. The observed value of \( \epsilon_K \) (1) can be used to constrain the model parameters, which can then be used to calculate quark electric dipole moments, colour dipole moments and inputs to the hadron-loop calculations. In the latter case, the largest contribution involves the \( K\bar{\epsilon}n \) coupling, and is [34]

\[ d_u \approx 10^{-24} \text{ e.cm} \] (35)
This looks too large, but the calculation could be wrong by some factor, so it might be premature to reject the Weinberg model completely. There is also the possibility of a cancellation between different diagrams, so we conclude that Weinberg is almost dead, but...

3.6 Left-Right Supersymmetric Model

In a model with gauge group $SU(2)_L \times SU(2)_R \times U(1)$ there are two generalized Kobayashi-Maskawa matrices $U_{KM}^L$ and $U_{KM}^R$. The number of independent parameters, and in particular the number of phases, depends on the degree of left-right symmetry assumed, but in general there are more phases than the Standard Model [35]. For example, if the model has "pseudo-manifest" symmetry, so that $\theta_L^i = \theta_R^i$ and $\theta_L^i = \theta_R^i$ (i=1,2,3), there are still $(n-1)(N-2)/2 + (2n-1) = n(n+1)/2$ phases. In this case, one does need three generations to obtain CP violation. The dominant contributions to $d_n$ are from one-loop diagrams involving left-right mixing as in Fig. 6. Since the model has several phases, they are not completely constrained by $\epsilon_K$ (1) and $\epsilon'$ (2), and there is a relatively large possible range for $d_n$ [36]:

$$|d_n| \sim (10^{-25} \text{ to } 10^{-26})\text{ cm}$$ (36)

However, it should be said that getting $|d_n|$ as large as $10^{-25}$ requires quite a special choice of parameters.

4. WHAT IF ...?

What would be the implications for the various models discussed in Section 3 if $d_n$ were to turn out to be $\sim 10^{-25}$ cm? The Table summarizes the predictions of the various models, and gives a verdict on them in this hypothetical case. The only clear conclusion is that a value of $d_n$ in this range would be unambiguous evidence for some physics beyond the Standard Model. Which physics is a different matter, and largely a matter of taste at this point. My own favourite explanations would be
the QCD $\theta$ parameter or supersymmetry. However, we should not fantasize too freely: $d_n$ might well be many orders of magnitude below the present experimental sensitivity.

**TABLE - Summary of Model Predictions**

<table>
<thead>
<tr>
<th>Model</th>
<th>Prediction for $d_n$ (e. cm)</th>
<th>What if ...?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Model</td>
<td>$(10^{-33} \text{ to } 2 \times 10^{-31})$</td>
<td>x</td>
</tr>
<tr>
<td>QCD $\theta$ Parameter</td>
<td>$3 \times 10^{-16} \theta_{\text{QCD}}$</td>
<td>$\checkmark (\theta \sim 10^{-9})$</td>
</tr>
<tr>
<td>Two-Higgs Model</td>
<td>$(10^{-33} \text{ to } 2 \times 10^{-31})$</td>
<td>x</td>
</tr>
<tr>
<td>Supersymmetry</td>
<td>$10^{-22} \phi$</td>
<td>$\checkmark (\phi \sim 10^{-3})$</td>
</tr>
<tr>
<td>Weinberg Model</td>
<td>$10^{-24}$</td>
<td>(x)</td>
</tr>
<tr>
<td>Left-Right Symmetric</td>
<td>$10^{-27} \text{ to } 10^{-25}$</td>
<td>$\checkmark$</td>
</tr>
</tbody>
</table>


L. Abbott and P. Sikivie - Phys.Lett. 120B (1983) 133;


FIGURE CAPTIONS

Fig. 1 Classes of loop diagrams contributing to $d_n$: (a) with one weak vertex, (b) with two weak vertices.

Fig. 2 (a) The value of $s_2$ and the range of $s_3$, and (b) the value of $\sin\theta$ extracted from a Standard Model analysis [4] of $B\bar{B}$ mixing for a range of $m_t$.

Fig. 3 Standard Model prediction [4] for $\varepsilon'/\varepsilon_K$ compared with experiment [3].

Fig. 4 The possible connection [23] between $\theta_{QCD}$ and the baryon number of the Universe: (a) GUT Higgs decay diagram, and (b) contribution to $\theta_{QCD}$.

Fig. 5 One-loop diagrams contributing to $d_n$ in a supersymmetric model.

Fig. 6 One-loop diagrams contributing to $d_n$ in a left-right symmetric model.
- Fig. 1 -
Fig. 2 -
- Fig. 3 -

- Fig. 4 -
REFERENCES


