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Spontaneous CP Violation and Neutral Flavour Conservation*

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Abstract

The conditions for one-loop stability of neutral flavour conservation in $SU(2)_L \times U(1)$ models with spontaneous CP violation are analysed. In addition to previously known cases there is an essentially unique two-generation model with two Higgs doublets where neutral flavour conservation is guaranteed to all orders by a non-standard CP symmetry.

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There are two distinct ways of implementing CP violation in gauge theories of electroweak interactions. In the first mechanism, as realized in the standard model [1] with the number of fermion generations $n_G \geq 3$, the Lagrangian itself violates CP. Since the gauge interactions of fermions are CP conserving, CP violation resides in the scalar sector, in particular in arbitrary phases of Yukawa couplings.

In the second scenario of spontaneous CP violation (SCPV) the total Lagrangian conserves CP which is broken by the ground state along with the gauge symmetry. With a low-energy gauge group $SU(2)_L \times U(1)$ two Higgs doublets ($n_H = 2$) are in principle sufficient for SCPV [2], but neutral flavour conservation (NFC) is in general violated by neutral Higgs exchange. Without resorting to fine-tuning [3], there are two possibilities to account for the observed suppression of flavour violating interactions in a natural way:

- i) The masses of NFC violating neutral Higgs bosons are at least of $O(10 \text{ TeV})$. Such a scale is difficult to understand in a low-energy gauge theory based on $SU(2)_L \times U(1)$, but it may be perfectly natural in extended gauge theories. The minimal left-right symmetric model [4] is an attractive example in this class of models with SCPV [5].
- ii) NFC is promoted to natural flavour conservation by some symmetry ensuring stability under radiative corrections. It is generally taken for granted that such a symmetry is incompatible with SCPV for the minimal case $n_H = 2$.

The purpose of this letter is to reconsider the arguments which seem to require $n_H > 2$ leading in particular to the Weinberg model of SCPV [6]. It is known that NFC together with SCPV cannot be enforced by a horizontal symmetry for $n_H = 2$ and arbitrary n_G [6,7,8]. Here, we shall employ a more general strategy by investigating the conditions that NFC is stable at least at the one-loop level.

Such an analysis serves two purposes:

- i) If the one-loop stability conditions cannot be fulfilled for given n_H and n_G , there can be no symmetry whatsoever that would guarantee NFC and allow for SCPV.
- ii) One may envisage the more general scenario of a (one-loop) effective symmetry [9]. Imagine an extended theory with additional degrees of freedom where NFC is ensured to all orders of perturbation theory by a certain symmetry. This symmetry may be spontaneously broken at some large scale where at the same time some fields acquire large masses. If these heavy degrees of freedom are integrated out, NFC will no longer hold in the truncated sector of light particles. If divergences requiring NFC violating counterterms appear first at the $(n+1)$ -loop level we may speak of an n -loop effective symmetry. In models with n -loop effective symmetries it makes sense to calculate amplitudes in the light sector up to n -loop accuracy without knowing the full theory. In the present context, we shall explore the possibility of one-loop effective symmetries for NFC compatible with SCPV for $n_G \leq 3$.

The most general Yukawa interaction of n_G generations of quarks with n_H Higgs doublets Φ_α in $SU(2)_L \times U(1)$ is given by

$$-\mathcal{L}_Y = \sum_{\alpha=1}^{n_H} \{ \bar{q}_L \Gamma_\alpha \Phi_\alpha n_R + \bar{q}_L \Delta_\alpha \tilde{\Phi}_\alpha p_R \} + h.c. \quad (1)$$

in the usual notation with n_G -dimensional Yukawa matrices $\Gamma_\alpha, \Delta_\alpha$.

We assume

- i) real $\Gamma_\alpha, \Delta_\alpha$ so that $\mathcal{L}_Y$ conserves CP;
- ii) the matrices Γ_α (and, separately, Δ_α) can be simultaneously diagonalized (NFC at tree level):

$$U_L^{n\dagger} \Gamma_\alpha U_R^n = \hat{\Gamma}_\alpha, \quad U_L^{p\dagger} \Delta_\alpha U_R^p = \hat{\Delta}_\alpha \quad (2)$$

with diagonal $\hat{\Gamma}_\alpha, \hat{\Delta}_\alpha$. The weak mixing matrix is then

$$V = U_L^{p\dagger} U_L^n \quad (3)$$

and it can always be chosen real for $n_G \leq 3$ [8,10].

Let us now examine the divergent one-loop contributions to $\Gamma_\alpha, \Delta_\alpha$ which could upset NFC. The self-energy corrections can easily be dismissed [9]. Of the one-particle irreducible diagrams, the neutral gauge boson and the neutral Higgs loop contributions conserve NFC whereas the W loop is finite. The only relevant divergent one-loop diagrams involve charged Higgs fields and are shown in Fig. 1. In the basis of weak eigenstates, the one-loop corrected Yukawa couplings take the form

$$\Gamma_\alpha^{(1)} = \Gamma_\alpha + \lambda \sum_{\beta=1}^{n_H} \Delta_\beta \Delta_\alpha^\dagger \Gamma_\beta, \quad \Delta_\alpha^{(1)} = \Delta_\alpha + \lambda \sum_{\beta=1}^{n_H} \Gamma_\beta \Gamma_\alpha^\dagger \Delta_\beta \quad (4)$$

where λ is a divergent constant of $O(\hbar)$ and we have omitted all terms in (4) which are either finite or which respect NFC trivially.

The problem of one-loop stability of NFC is equivalent to the problem of simultaneous bidiagonalizability of the $\Gamma_\alpha^{(1)}$ and, separately, of the $\Delta_\alpha^{(1)}$ to $O(\hbar)$. The necessary and sufficient conditions for the simultaneous diagonalization of a set of square matrices are by now well established [11]. Before presenting the general conditions, we discuss two special cases.

- i) The two sets of Higgs fields coupling to either p - or n -quarks have no common element implying

$$\Delta_\beta \Delta_\alpha^\dagger \Gamma_\beta = \Gamma_\beta \Gamma_\alpha^\dagger \Delta_\beta \equiv 0 \quad (5)$$

for each α, β . This is the case of one-loop stability of NFC considered by Frère and Yao [9]. Except for special cases such as the Weinberg model [6] there is no guarantee that NFC extends to higher orders.

- ii) The coupling matrices are proportional to each other:

$$\Gamma_\alpha = a_\alpha \Gamma, \quad \Delta_\alpha = b_\alpha \Delta \quad (a_\alpha, b_\alpha \in \mathbf{R}). \quad (6)$$

By a real orthogonal basis transformation on the Higgs fields we can always achieve

$$\begin{aligned} a_\alpha &= 0 & (\alpha > 1) \\ b_\alpha &= 0 & (\alpha > 2) \end{aligned} \quad (7)$$

so that at most two Higgs doublets couple to quarks. In this basis, NFC to one-loop order can only be satisfied if either

- a) $b_1 = 0$ yielding the Weinberg model where NFC is actually stable to all orders due to a horizontal symmetry [12], or
- b) $b_2 = 0$ leaving a single Higgs doublet coupling to quarks as in the standard model. The Yukawa interaction conserves CP even after spontaneous symmetry breaking.

Excluding from now on both cases (5) and (6) and restricting the further discussion to $n_H = 2$ and $n_G \leq 3$, the necessary and sufficient conditions for the simultaneous bidiagonalizability of $\Gamma_\alpha^{(1)}$ and $\Delta_\alpha^{(1)}$ to $O(\hbar)$ are

$$\begin{aligned} \hat{\Gamma}_1^2 + \hat{\Gamma}_2^2 &= \sigma \mathbf{1}, & \hat{\Delta}_1^2 + \hat{\Delta}_2^2 &= \rho \mathbf{1} & (\sigma > 0, \rho > 0) \\ [\hat{\Gamma}_1 \hat{\Gamma}_2, V^T \hat{\Delta}_1^2 V] &= [\hat{\Gamma}_1^2, V^T \hat{\Delta}_1 \hat{\Delta}_2 V], \end{aligned} \quad (8)$$

where $\hat{\Gamma}_\alpha$, $\hat{\Delta}_\alpha$ and V were defined in Eqs. (2) and (3). We omit the straightforward, but somewhat tedious proof. Note that the possibility that Eq. (6) holds only in one charge sector, but not in the other, can be excluded.

Counting the number of free parameters for $n_G = 3$, solutions of the conditions (8) can only be expected for specific relations between the six quark masses and the three mixing angles¹. In fact, a careful numerical and approximate analytical analysis shows that there are no solutions at all for $n_G = 3$ for realistic $m_t > m_b$. As a consequence, there is no non-trivial (other than Eq. (5)) one-loop effective symmetry for NFC and $n_G = 3$, provided that the structure of V is essentially given already at tree level, i.e., provided radiative corrections to the weak mixing angles are not important.

There are several reasons to investigate the NFC stability conditions (8) also for $n_G = 2$:

- i) the mixing angles to the third generation are small;
- ii) the strongest experimental constraints on flavour violating transitions exist for $\Delta S \neq 0$ related to the Cabibbo angle Θ_C ;
- iii) the b - and t -quarks may be considered as heavy degrees of freedom in the previous sense of effective symmetries.

For $n_G = 2$, there is a unique solution of the conditions (8) up to basis transformations:

$$\begin{aligned} \hat{\Gamma}_1 &= g_1 \mathbf{1}, & \hat{\Gamma}_2 &= g_2 \sigma_3 \\ \hat{\Delta}_1 &= d_1 \mathbf{1}, & \hat{\Delta}_2 &= d_2 \sigma_3 \end{aligned} \quad g_\alpha, d_\alpha \in \mathbf{R} \quad (9)$$

and Θ_C is arbitrary. This model exhibits SCPV and NFC is one-loop stable. Can we make any statements about stability in higher orders?

¹Remember that the mixing matrix V is real.

The solution (9) turns out to be a special case of a model with so-called generalized CP invariance [8,13]

$$\begin{aligned}
q_L(x) &\rightarrow V_L C q_L^*(\tilde{x}) \\
p_R(x) &\rightarrow V_R^p C p_R^*(\tilde{x}) \\
n_R(x) &\rightarrow V_R^n C n_R^*(\tilde{x}) \\
\Phi_\alpha(x) &\rightarrow V_{H,\alpha\beta} \Phi_\beta^*(\tilde{x})
\end{aligned}
\quad \tilde{x} = (x^0, -\vec{x}) \quad (10)$$

where V_L , V_R^p , V_R^n and V_H are two-dimensional unitary matrices ($n_G = n_H = 2$). Invariance of the Yukawa Lagrangian (1) with respect to the generalized CP transformations (10) for

$$V_L = V_R^p = V_R^n = i\sigma_2, \quad V_H = \mathbf{1} \quad (11)$$

implies [8]

$$\Gamma_\alpha = g_\alpha X_\alpha, \quad \Delta_\alpha = d_\alpha Y_\alpha, \quad g_\alpha, d_\alpha \in \mathbf{R}^+, \quad X_\alpha, Y_\alpha \in SU(2) \quad (\alpha = 1, 2). \quad (12)$$

The matrices Γ_α and Δ_α are simultaneously diagonalizable and can always be brought to the form

$$\begin{aligned}
\hat{\Gamma}_1 &= g_1 \mathbf{1}, & \hat{\Gamma}_2 &= g_2 \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix} \\
\hat{\Delta}_1 &= d_1 \mathbf{1}, & \hat{\Delta}_2 &= d_2 \begin{pmatrix} e^{i\psi} & 0 \\ 0 & e^{-i\psi} \end{pmatrix}.
\end{aligned} \quad (13)$$

For $\varphi = -\psi = \pi/2$ and after a phase transformation on the Higgs field Φ_2 we recover the real solution (9). Since NFC is enforced by a symmetry, albeit *not* by a horizontal symmetry, the general model (13) and the special solution (9) are actually NFC stable to all orders.

Lagrangians with non-standard CP invariance exhibit an interesting feature: in the limit of strict CP invariance, i.e. with a CP invariant ground state, the fermion masses are pairwise degenerate. This is a stronger correlation between mass degeneracy and CP invariance than usually encountered. Recall that in the standard model for $n_G = 3$ degeneracy of any two quarks of the same charge implies CP invariance, but certainly not vice versa. In models of the type (13) CP can only be strictly conserved if any two fermions connected by a non-standard CP transformation are degenerate.

Let us sketch an indirect proof of this remarkable correlation. CP invariance of the Yukawa Lagrangian (1) amounts to the relations

$$\Gamma_\beta^* = V_{H,\alpha\beta} V_L^\dagger \Gamma_\alpha V_R^n, \quad \Delta_\beta^* = V_{H,\alpha\beta}^* V_L^\dagger \Delta_\alpha V_R^p. \quad (14)$$

If the vacuum also conserves CP, i.e.

$$v_\alpha = V_{H,\alpha\beta} v_\beta^*, \quad v_\alpha = \langle \Phi_\alpha^0 \rangle_{vac}, \quad (15)$$

the mass matrices M_n, M_p defined by

$$M_n = \sum_{\alpha=1}^{n_H} v_\alpha \Gamma_\alpha, \quad M_p = \sum_{\alpha=1}^{n_H} v_\alpha^* \Delta_\alpha \quad (16)$$

obey the conditions

$$V_L^\dagger M_n V_R^n = M_n^*, \quad V_L^\dagger M_p V_R^p = M_p^*. \quad (17)$$

Using these equations and assuming non-degeneracy of M_n and M_p , one can show that there exists a basis of weak eigenstates where

$$V_L = V_R^n = V_R^p = 1 \quad (18)$$

and the mixing matrix V is real. On the other hand, the generic form of a generalized CP transformation V_L, V_R^n, V_R^p, V_H after a suitable basis transformation is [14]

$$V_{CP} = \begin{pmatrix} O_1 & & & \\ & O_2 & & \\ & & \ddots & \\ & & & O_m \\ & & & & 1 \end{pmatrix} \quad O_i = \begin{pmatrix} \cos \Theta_i & \sin \Theta_i \\ -\sin \Theta_i & \cos \Theta_i \end{pmatrix} \quad 0 < \Theta_i \leq \frac{\pi}{2}. \quad (19)$$

However, submatrices with $\Theta_i > 0$ as in Eq. (11) ($\Theta = \pi/2$) cannot be brought to diagonal form in disagreement with Eq. (18). Therefore, the corresponding fermions must be degenerate in the limit of a CP invariant ground state.

The general model (13) possesses an unusual property for a model with SCPV and only two Higgs doublets: it depends on three independent phases φ, ψ and $\arg(v_1 v_2^*)$. Of course, the observed quark mass hierarchy constrains these phases and also requires

$$|g_1 v_1| \simeq |g_2 v_2|, \quad |d_1 v_1| \simeq |d_2 v_2|. \quad (20)$$

Is the model compatible with the CP violation phenomenology for light quarks?

The main problem is that the $\Delta S = 1$ Hamiltonian induced by charged Higgs exchange² for the light quarks u, d, s violates CP in general and is not suppressed by m_u, m_d as in the Weinberg model. Although our knowledge of hadronic matrix elements of four-quark operators is still rudimentary, the $\Delta S = 1$ induced CP violation will in general be too large compared to $\Delta S = 2$ generating a too big ε'/ε . However, we have checked that the parameters in (13) can be chosen such that the $\Delta S = 1$ tree level Higgs exchange (without charm quarks) conserves CP. In this case, the dominant CP violating mechanism for $\Delta S = 1$ is due to the scalar penguin contribution [15] as in the Weinberg model. At the same time, the scalar penguin generates ε via long-distance contributions. According to present wisdom [16], the chiral suppression of the scalar penguin operator in $K \rightarrow 2\pi$ guarantees a value of ε'/ε compatible with experiment [17], although the short-distance contributions to ε are very small. As in the Weinberg model, the model (13) gives rise to

²There is a single physical charged Higgs boson for $n_H = 2$.

a neutron electric dipole moment of the order of its present experimental upper bound [18] $|d_n| \leq 2.6 \cdot 10^{-25}$ e.cm. A more detailed phenomenological analysis seems premature in view of the hadronic uncertainties and in view of the a priori arbitrary embedding into a three-generation model.

We have investigated the conditions for one-loop stability of NFC for CP conserving Yukawa couplings. If radiative corrections for the weak mixing angles are negligible, there is no solution of the stability conditions for $n_G = 3$ other than the case where a given Higgs doublet couples to either up- or down-quarks, but not to both. The Weinberg model of CP violation falls into this class. For $n_G = 2$ the stability conditions allow for a non-trivial solution where two Higgs doublets couple to both quark charge sectors. This solution turns out to be a special case of a more general model where a non-standard CP symmetry enforces NFC which is therefore stable to all orders of perturbation theory. In models with non-standard CP invariance, a CP conserving ground state implies degeneracy of quark masses. The $n_G = 2$ model of Eq. (13) is compatible with CP violation phenomenology in the light quark sector for a certain domain in parameter space.

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Figure Caption

Fig. 1: Potentially NFC violating one-loop contributions to Yukawa couplings.