Phenomenological Lagrangian Approach to the Symmetry Breaking Sector of the Standard Model

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ABSTRACT

A chiral Lagrangian expansion is used in order to parametrize the unknown symmetry breaking dynamics of the Standard Model, in the hypothetical case of a strongly interacting symmetry breaking sector. To lowest order, $O(s)$, the model is completely determined by the experimental fact that $\rho \simeq 1$. The dependence on the underlying theory shows up to order s^2 where consistency requires one-loop effects to be included through two renormalized parameters. The unitarity properties and the domain of applicability of the model are studied. Some experimental prospects are also commented at the end.

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Although the Standard Model of electroweak interactions provides a successful description of the particle physics phenomenology, it has to be remarked that almost nothing is known about the origin of the symmetry breaking mechanism which is responsible for the masses of the W^+ , W^- and Z gauge bosons. We do not really know the quanta of the symmetry breaking sector (SBS) and their interactions. Within the assumed framework of a spontaneously broken $SU(2) \times U(1)$ gauge symmetry, we know that at least three Goldstone bosons must appear in the SBS to become the longitudinally polarized modes of the gauge bosons. As is made precise by the Equivalence Theorem [1], at large energies compared to the gauge boson mass, the longitudinal modes can be identified with the underlying Goldstone bosons from the SBS, so, in this sense, we could say that three particles of the SBS have already been discovered. We also know one energy scale related to the onset of the symmetry breaking mechanism, the vacuum expectation value, which is fixed by the Fermi constant to be $v = 246 \text{ GeV}$.

For the rest, all that can be said is that there must exist a physical system that induces an $SU(2) \times U(1)$ asymmetric ground state while leaving the $U(1)_{em}$ symmetry unbroken. Several scenarios proposed in the literature for this physical system include: the minimal Higgs boson model or minimal standard model with just one doublet of scalar fields, non-minimal Higgs boson models with more than one doublet, as in supersymmetric theories, and dynamical models such as technicolour, composite models etc., where there is no fundamental scalar Higgs boson. All of these models and any other that one can imagine may be roughly divided into two classes by the strength of their interactions. From a phenomenological point of view, strongly interacting SBS's are characterized by the absence of light quanta or resonances while weakly interacting SBS's would contain typically a few light modes in the low energy spectrum (say, much below the TeV scale). The minimal standard model with one light elementary Higgs boson, and the supersymmetric models, are typical examples of theories with weakly interacting SBS's whereas dynamical models or the minimal standard model with one heavy Higgs boson have typically strongly interacting SBS's [2]. Since the weakly interacting scenarios will be the most sensitive to be tested in the next generation of accelerators, we will concentrate in the rest of this note on the strongly interacting case. As stated above, this means that we will be concerned with the case where the only physical states belonging to the SBS below the TeV scale are the longitudinal components of the weak bosons (LWB).

The main purpose of this note is to provide a phenomenological viable model for the scattering amplitudes of the LWB that could be applied at energies below the mass of the first resonances in the SBS. Under the hypothesis of a strongly interacting SBS, the relative low mass of the gauge bosons may be understood on the basis of a certain approximate global symmetry of the unknown dynamics of the SBS which is spontaneously broken. The LWB may be interpreted in this case as the Goldstone bosons associated to this symmetry breaking, and this is the reason why they are light when compared to the TeV scale. On the other hand, within the above scheme, the low energy interactions of the LWB are completely determined at order s , as was shown by Chanowitz, Georgi and Golden [3] (the precise meaning of this statement will become clear later). Here we will present the $O(s^2)$ corrections to these low energy predictions or Low Energy Theorems (LET). These corrections improve the bad unitarity properties of the LET predictions and, as far as

they are model dependent, they will allow us in a certain sense to parametrize our lack of knowledge of the underlying SBS dynamics. The main applications of our results are the following: firstly, to provide a useful tool to study our ability in testing the strongly interacting SBS hypothesis generically within the next generation colliders, and secondly, to study the viability of distinguishing among the patterns of different strongly interacting dynamics in the future experiments.

In the task of computing the $O(s^2)$ corrections to the LET predictions for the LWB scattering amplitudes we will follow closely the phenomenological Lagrangian approach proposed by Weinberg in [4]. This method has proved to be very useful in describing the scattering of pions which can be considered as the Goldstone bosons associated to the spontaneously symmetry breaking of the chiral symmetry [5]. In the case considered here we will assume that the LWB are the Goldstone bosons appearing because of the breaking of a global approximate symmetry group G into a subgroup H . Although we do not know which groups are G and H , local gauge invariance restricts them to those satisfying $G \supset SU(2)_L \times U(1)_Y$ and $H \supset U(1)_{em}$. Since we have assumed that there are no other Goldstone or pseudo-Goldstone bosons in the low energy spectrum (compared with the first resonance mass), then it follows that we have two possibilities: 1) $G = SU(2)_L \times SU(2)_R$, $H = SU(2)_{L+R}$, and 2) $G = SU(2)_L \times U(1)_{em}$, and $H = U(1)_{em}$. The residual group $SU(2)_{L+R}$ in the first case is called the custodial group because it has the important property of guaranteeing the value of the ratio $\rho = m_W^2/m_Z^2 \cos^2 \theta_W$ to be one which is in agreement with the experimental value $\rho_{exp} = 1.01 \pm 0.1$ [6]. In the following we will adopt this first pattern $SU(2)_L \times SU(2)_R \longrightarrow SU(2)_{L+R}$ and will neglect electroweak interactions, i.e. we will not consider the coupling of the LWB to the transverse components of the weak bosons nor the photon. Since our interest is in the first instance to study the dominant corrections to the low energy scattering amplitudes for the LWB, we expect the results to be insensitive to this assumption.

In order to obtain a G -invariant Lagrangian we will consider a matrix field $U(x)$ belonging to the quotient space $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$. This matrix field can be parametrized as

$$U(x) = \exp\left(\frac{i}{v} w^a(x) \sigma^a\right)$$

with $w^a, a = 1, 2, 3$, being the Goldstone bosons fields, σ^a the Pauli matrices and $v = 246 \text{ GeV}$. Under a $SU(2)_L \times SU(2)_R$ transformation the $U(x)$ transforms as

$$U(x) \longrightarrow V_L U(x) V_R^\dagger$$

$$(V_L, V_R) \in SU(2)_L \times SU(2)_R$$

The most general Lagrangian invariant under the above transformation is a sum of infinite terms with increasing (even) number of derivatives in the $U(x)$ and the $U(x)^\dagger$ fields and with an infinite number of arbitrary parameters. Different underlying theories will predict different values of these parameters and ultimately we should be able to obtain them as predictions of the unknown real theory. The effective theory in that generic form is renormalizable in the sense that counterterms needed to remove infinities from the Green

functions are already present in the original Lagrangian. The whole point of an expansion in terms of an increasing number of derivatives, or equivalently in terms of increasing powers of the energy, is that at low energy only a finite number of terms are important. Therefore keeping just a few terms in the Lagrangian makes sense and the model can be used to parametrize in a consistent way the low energy scattering amplitudes as a function of a finite number of parameters. Working at the lowest order, i.e. order s (or, equivalently, E^2 in powers of the energy), there is only one relevant term in the expansion which is given by

$$\mathcal{L}_0 = \frac{v^2}{4} \text{tr}(\partial_\mu U \partial^\mu U^\dagger) \quad (1)$$

whereas, to next order, that is to order s^2 (or E^4), two additional terms have to be included which can be written as

$$\mathcal{L}_1 = M \text{tr}(\partial_\mu U \partial^\mu U^\dagger) \text{tr}(\partial_\nu U \partial^\nu U^\dagger) + N \text{tr}(\partial_\mu U \partial_\nu U^\dagger) \text{tr}(\partial^\mu U \partial^\nu U^\dagger) \quad (2)$$

with M and N being two dimensionless parameters. To lowest order, the predictions for the Goldstone boson interactions are just those of $\mathcal{L}_0$, which are unique and lead to the LET results mentioned above. For next order predictions one must use $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ instead of $\mathcal{L}_0$, and the dependence on the underlying theory will show up in the scattering amplitudes as a dependence in the dimensionless parameters M and N . Different choices for the magnitude and sign of M and N would correspond to different choices for the underlying unknown theory. Furthermore, for the purpose of computing scattering amplitudes to order s^2 one must also include one-loop corrections to $\mathcal{L}_0$ since they are also of order s^2 . The main effect of including loop corrections is the introduction of a scale dependence into the renormalized parameters M_R and N_R and the addition of new logarithmic terms in the amplitudes derived from $\mathcal{L}$, in such a way that the amplitudes do not depend on that scale. At this stage, we can figure out the range of energies where the approach considered here is expected to work. In the first instance, the energy has to be large enough to allow us to neglect the weak bosons' mass. On the other hand, the energy has to be less than the masses of the first resonances and also less than the effective scale parameter that controls the derivative expansion which is $4\pi v$ ($\sim 3 \text{ TeV}$). This last restriction will ensure that the $O(s^3)$ corrections can be safely neglected. Further constraints on the range of applicability of the model based on unitarity requirements will be discussed below.

In the following we will present the results for the on-shell four-point Green functions of the Goldstone bosons valid to order s^2 that are obtained from $\mathcal{L}$ and which include the one-loop corrections coming from $\mathcal{L}_0$. In order to regularize the divergences appearing in the integrals we have used the dimensional regularization scheme. As was shown some time ago, this method allows us to use naively the Feynman rules as extracted directly from $\mathcal{L}$, without introducing any non-invariant terms into the on-mass shell amplitudes in the one-loop approximation [7]. The general expressions for the renormalized four-point amplitudes have the following structure

$$T = T^{LET} + T^{(4)} \quad (3)$$

where T^{LET} can be obtained directly from eq.(1) and corresponds to the LET predictions [3]. The dependence on the underlying dynamics appears in $T^{(4)}$ (here, 4 stands for E^4).

contributions) which includes the contribution from eq.(2) and the one-loop corrections to eq.(1). By the use of crossing symmetry, any other amplitude can be related to the three independent ones T_{+-+} , T_{0000} and T_{+-00} , where we have used the standard basis for the Goldstone bosons $w^\pm = 1/\sqrt{2}(w_1 \mp iw_2)$, $z^0 = w^3$. The explicit formulae obtained for these amplitudes are

$$\begin{aligned}
T_{+-+}^{LET} &= -\frac{u}{v^2} \\
T_{0000}^{LET} &= 0 \\
T_{+-00}^{LET} &= \frac{s}{v^2} \\
T_{+-+}^{(4)} &= \frac{4}{v^4} [2M_R(\nu)(s^2 + t^2) + N_R(\nu)(s^2 + t^2 + 2u^2)] \\
&+ \frac{1}{(4\pi)^2 v^4} \left[\frac{-1}{12} (9s^2 + u^2 - t^2) \log \frac{-s}{\nu^2} - \frac{1}{12} (9t^2 + u^2 - s^2) \log \frac{-t}{\nu^2} - \frac{1}{2} u^2 \log \frac{-u}{\nu^2} \right] \\
T_{0000}^{(4)} &= \frac{8}{v^4} [M_R(\nu) + N_R(\nu)](s^2 + t^2 + u^2) \\
&+ \frac{1}{(4\pi)^2 v^4} \left[-s^2 \log \frac{-s}{\nu^2} - t^2 \log \frac{-t}{\nu^2} - u^2 \log \frac{-u}{\nu^2} \right] \\
T_{+-00}^{(4)} &= \frac{4}{v^4} [2M_R(\nu)s^2 + N_R(\nu)(t^2 + u^2)] \\
&+ \frac{1}{(4\pi)^2 v^4} \left[-\frac{1}{12} (3t^2 + u^2 - s^2) \log \frac{-t}{\nu^2} - \frac{1}{12} (3u^2 + t^2 - s^2) \log \frac{-u}{\nu^2} - \frac{1}{2} s^2 \log \frac{-s}{\nu^2} \right] \quad (4)
\end{aligned}$$

where s, t and u are the usual Mandelstam variables and ν is an arbitrary energy scale. The renormalized coupling constants M_R and N_R are defined from the bare ones N and M through

$$\begin{aligned}
M_R &= M + \frac{1}{24(4\pi)^2} \left(\frac{2}{\epsilon} - \gamma + \log 4\pi \right) + C_M \\
N_R &= N + \frac{1}{12(4\pi)^2} \left(\frac{2}{\epsilon} - \gamma + \log 4\pi \right) + C_N \quad (5)
\end{aligned}$$

Here $\epsilon = 4 - d$ and C_M and C_N are finite constants irrelevant for our discussion. The renormalized constants M_R and N_R are in fact functions of the renormalization scale ν . The dependence of M_R and N_R on ν can be easily obtained by demanding that the physical amplitudes $T^{(4)}$ be independent on that scale parameter, i.e. $dT^{(4)}/d\nu = 0$. The result is

$$\begin{aligned}
M_R(\nu) &= M_R(\nu_0) - \frac{1}{12(4\pi)^2} \log \frac{\nu}{\nu_0} \\
N_R(\nu) &= N_R(\nu_0) - \frac{1}{6(4\pi)^2} \log \frac{\nu}{\nu_0} \quad (6)
\end{aligned}$$

where ν_0 is again a reference scale which cannot be set to zero. Formulae (4) and (6) are in agreement with the ones presented in [4,5] to describe the low-energy pion dynamics. The

amplitudes $T^{(4)}$ can be easily interpreted as follows. The polynomial terms proportional to M_R and N_R are the same that would have been obtained at the tree level from $\mathcal{L}_1$ with the substitution of the renormalized constants by the bare ones. Besides, as we have anticipated above, the effect of the one-loop corrections to $\mathcal{L}_0$ is explicitly shown in the amplitudes by the appearance of new logarithmic terms and a scale dependence in the coupling constants. The model in this form is completely meaningful and depends on two unknown parameters M_R and N_R which contain the information relative to the underlying dynamics. The only thing to bear in mind is that the eventual measure of these parameters will have to be accompanied by the value of the scale ν at which they have been defined. The value of the constants at any other scale ν' , if required, can be obtained by the use of eq.(6) and, of course, the amplitudes remain ν -invariant. Note that this approach is rather different to that of reference [8]. There, only the one-loop corrections to $\mathcal{L}_0$ are considered and, therefore, there are no "a priori" free parameters to fit the underlying dynamics, neither renormalizability to order s^2 .

One important advantage of including one-loop contributions, from the point of view of the consistency of the model, is the improvement in the unitarity behaviour of the amplitudes. This is ruled by the appearance of an imaginary part in $T^{(4)}$ coming from the $\log(-s)$ factors that with our prescription read $\log(s) - i\pi$. In order to study the effect of the imaginary terms in $T^{(4)}$, we have computed the partial wave amplitudes a_{IJ} defined in the standard fashion as

$$a_{IJ}(s) = \frac{1}{64\pi} \sum_{\alpha\beta\gamma\delta} \int_{-1}^1 dx P_J(x) T_{\alpha\beta\gamma\delta}(s, x) \Pi_{\alpha\beta\gamma\delta}^I \quad (7)$$

with $x = 1 + 2t/s$ and the isospin projectors given by

$$\Pi_{\alpha\beta\gamma\delta}^I = \frac{1}{2I+1} \sum_{I_z} \langle \alpha\beta | II_z \rangle \langle II_z | \gamma\delta \rangle \quad (8)$$

The result for the partial wave amplitudes corresponding to the lowest values of J are the following

$$\begin{aligned} a_{00}(s) &= \frac{s}{16\pi v^2} + \frac{s^2}{64\pi v^4} \left[\frac{1}{3} (176M_R(\nu) + 112N_R(\nu)) \right. \\ &\quad \left. + \frac{1}{(4\pi)^2} \left(\frac{11}{27} - \frac{50}{9} \log \frac{s}{\nu^2} + 4i\pi \right) \right] \\ a_{20}(s) &= \frac{-s}{32\pi v^2} + \frac{s^2}{64\pi v^4} \left[\frac{32}{3} (M_R(\nu) + 2N_R(\nu)) \right. \\ &\quad \left. + \frac{1}{(4\pi)^2} \left(\frac{25}{54} - \frac{20}{9} \log \frac{s}{\nu^2} + i\pi \right) \right] \\ a_{11}(s) &= \frac{s}{96\pi v^2} + \frac{s^2}{96\pi v^4} \left[4(-2M_R(\nu) + N_R(\nu)) \right. \\ &\quad \left. + \frac{1}{(4\pi)^2} \left(-\frac{1}{18} + \frac{i\pi}{6} \right) \right] \end{aligned}$$

$$a_{01}(s) = a_{21}(s) = 0 \quad (9)$$

As is well known, the requirement for elastic unitarity in terms of the partial wave amplitudes reads

$$|a_{IJ}|^2 = \text{Im } a_{IJ}$$

Obviously, this condition cannot be fulfilled in the case of a real amplitude such as those obtained from $\mathcal{L}_0$ at tree level, i.e. the LET predictions. In the case of the partial wave amplitudes in eq.(9) one can immediately check that

$$|a_{IJ}^{LET}|^2 = \text{Im } a_{IJ}^{(4)} \quad (10)$$

where we have used the same notation as in eq.(3). This result is already an indication of the announced improvement in the unitarity behaviour of the amplitudes when going to order $O(s^2)$. In order to see this in a more transparent way, let us parametrize the amplitudes in terms of the phase shifts δ_{IJ} and the inelasticity parameter η_{IJ}

$$a_{IJ}(s) = \frac{\eta_{IJ}(s) \exp(i2\delta_{IJ}(s)) - 1}{2i} \quad (11)$$

so that the requirement for elastic unitarity is $\eta_{IJ}^2 = 1$ and it is expected to hold below the resonant threshold. For the amplitudes in eq.(9), given in a compact form by

$$a(s) = Ks + s^2(A + iB - C \log s),$$

it is a simple exercise to get

$$\eta(s) = 1 - 4s^2(B - K^2) + 8Ks^3(A - C \log s) + 4s^4(B^2 + (A - C \log s)^2) \quad (12)$$

Written in this form, eq.(10) is expressed as $B = K^2$ which in turn means that the terms proportional to s^2 in eq.(12) cancel out, hence, the first terms that violate elastic unitarity are of order s^3 and $s^3 \log s$. This has to be compared with the amplitudes obtained from the LET (in our notation it corresponds to setting $A=B=C=0$) where not only elastic unitarity but the more basic requirement of unitarity $\eta^2 \leq 1$ is violated to order s^2 .

As stressed above the main aim of the model considered here is to provide a consistent framework for the description of the LWB scattering in a way that it is independent of the particular underlying SBS dynamics. Different choices of M_R and N_R at an arbitrary fixed scale ν will describe different physical situations in the unknown sector. As an example of how this works, we will show next how the model can mimic two typical examples, namely, the minimal standard model and a QCD rescaled model in the spirit of technicolour models. In the first case we start with the result given recently in ref. [9] for the real part of the amplitude $T(W_L^+ W_L^- \longrightarrow W_L^+ W_L^-)$ to one loop in the minimal standard model which, in the limit $s \ll m_H^2$, can be written in the form

$$\text{Re } T(W_L^+ W_L^- \longrightarrow W_L^+ W_L^-) = -\frac{u}{v^2} + \frac{1}{(4\pi)^2 v^4} \left[-\frac{1}{12} (9s^2 + u^2 - t^2) \log \frac{s}{m_H^2} \right]$$

$$\begin{aligned}
& -\frac{1}{12}(9t^2 + u^2 - s^2) \log \frac{-t}{m_H^2} - \frac{1}{2}u^2 \log \frac{-u}{m_H^2} \\
& + \frac{1}{(4\pi)^2 v^4} \left[\left(\frac{9\pi}{2\sqrt{3}} - \frac{76}{9} \right) (s^2 + t^2) - \frac{4}{9}u^2 \right]
\end{aligned} \tag{13}$$

One can immediately convince oneself that this formula can be reproduced by T_{+--+} in eq.(3) and eq.(4) provided one defines the renormalized coupling constants $M_R(\nu)$ and $N_R(\nu)$ in terms of m_H as

$$\begin{aligned}
M_R(\nu) &= \frac{1}{8} \left(c_1 - \frac{c_2}{2} \right) + \frac{1}{12(4\pi)^2} \log \frac{m_H}{\nu} \\
N_R(\nu) &= \frac{c_2}{8} + \frac{1}{6(4\pi)^2} \log \frac{m_H}{\nu}
\end{aligned} \tag{14}$$

with

$$\begin{aligned}
c_1 &= \frac{1}{(4\pi)^2} \left(\frac{9\pi}{2\sqrt{3}} - \frac{76}{9} \right) \\
c_2 &= -\frac{4}{9} \frac{1}{(4\pi)^2}
\end{aligned} \tag{15}$$

Note that, as pointed out in [9], the different approach followed in [8] does not account for the non-logarithmic terms appearing in eq.(13) whereas the model which is considered here does.

For the rescaled-QCD case we start with the values of the parameters corresponding to our M_R and N_R in the chiral lagrangian for low energy QCD that are obtained from the fits to $\pi - \pi$ scattering data. In particular, we will consider the values given in [10] which are defined at $\nu = 1 \text{ GeV}$. We will define rescaled-QCD as a model with the same numerical values for the parameters M_R and N_R but referred to a scale $\nu = \alpha \times 1 \text{ GeV}$. More precisely

$$\begin{aligned}
M_R(\alpha \times 1 \text{ GeV}) &=^{def} M_R^{QCD}(1 \text{ GeV}) =^{exp} -0.011/4 \\
N_R(\alpha \times 1 \text{ GeV}) &=^{def} N_R^{QCD}(1 \text{ GeV}) =^{exp} 0.0046/4
\end{aligned}$$

where α is defined as

$$\alpha = \frac{v}{f_\pi} \sqrt{\frac{3}{N_{TC}}}$$

Here $f_\pi \simeq 93 \text{ MeV}$ is the pion decay constant and N_{TC} is the number of colours, or more properly technicolours, of the rescaled-QCD model. This last definition can be understood in the context of the $1/N_{TC}$ expansion.

Fig.1 is a representation of the parameter space where in principle each point would correspond to a different underlying theory. Here, we have fixed the renormalization scale to the value of 1 TeV and we have plotted different points corresponding to different choices of m_H and N_{TC} for the two models mentioned above. As can be seen, the minimal standard model (corresponding to the solid line in the figure) typically belongs to the upper region

of the map, while the QCD-like models belong to the lowest one. The dependence on m_H and N_{TC} is such that both $M_R(1 \text{ TeV})$ and $N_R(1 \text{ TeV})$ increase as m_H increases, whereas both decrease as N_{TC} increases. Furthermore, from the figure the qualitative differences between the two models are clear. In particular, the sign of M_R shows up as an indicator of the two types of models, that is, Higgs-like or QCD-like models. Note that these two cases are typical representative examples of scalar dominated and vector dominated theories (vector dominated models for the strongly interacting SBS of the Standard Model have also been considered in the literature within the framework of the Hidden Symmetry approach [11]). This last fact suggests that the experimental determination of M_R and N_R could provide some insight into the rough features of the resonant spectrum of the underlying theory [12].

In order to study the different behaviour of the model corresponding to these two examples, we have plotted in Fig. 2 the values of η^2 , δ , and $|a|$ for the channels $I = 0, J = 0$ and $I = 1, J = 1$ which correspond to the quantum numbers of the Higgs and the ρ -boson respectively. We have considered the minimal standard model for different values of the Higgs boson mass and rescaled QCD for $N_{TC} = 3$. For comparison we have also plotted the LET predictions in the figure. The first question we would like to address is what is the real range of applicability of the model discussed here. Apart from the expected bounds mentioned above ($\sqrt{s} < 4\pi v$ and the first resonance mass) we have of course the unitarity bound Λ_U . In fact, the precise value of Λ_U depends on the specific values of the parameters M_R and N_R but for "reasonable" values of these parameters such as the ones corresponding to the two examples considered here, we find that this dependence is weak and we will give a unique value of Λ_U . There are several ways of visualizing unitarity breaking in the η^2 , δ and $|a|$ curves. First of all, η^2 has to be less than or equal to one. By requiring unitarity breaking not to be larger than a 20% effect, in η^2 units, we can set Λ_U in our model around 1.5 TeV from the η^2 plots. Note that in the case of the LET predictions the breaking of unitarity, as defined above, occurs (from the η_{00}^2 plot) at 800 GeV. This is much lower than the usually advocated of 1.7 TeV which is based on the weaker condition $|a_{00}| \leq 1$. From this point of view the $|a_{00}|$ plot shows, in contrast, that all the models considered here behave like the LET predictions. Another independent constraint dictated by unitarity (in fact by causality) is that δ has to increase with energy. From the δ_{00} plot we again get the bound $\sqrt{s} < 1.5 \text{ TeV}$. The conclusion is that for typical values of the parameters the model breaks down from unitarity considerations around 1.5 TeV. Therefore we could safely set the range of applicability of the model as $m_W \ll \sqrt{s} < 1.5 \text{ TeV}$ which covers roughly one order of magnitude. In fact this should be taken as a conservative estimate for the case of higher spin channels.

Another important point to be considered concerns to the sensitivity of the model for LWB scattering amplitudes on simple but phenomenologically relevant properties of the SBS dynamics such as an eventual scalar or vector dominance. Again we will consider the minimal standard model and the QCD rescaled case as representative of these two situations. Looking at Fig.2 it is possible to compare the values of δ , η^2 and $|a|$ obtained for these two examples in the scalar and vector channels ($I = J = 0$ and $I = J = 1$). As would be expected, $|a_{00}|$ is larger in the scalar dominated theory than in the vector dominated one (for instance $|a_{00}^{MSM}| \sim 1.4 |a_{00}^{QCD}|$ at $\sqrt{s} \sim 1250 \text{ GeV}$ (MSM stands for

Minimal Standard Model)) and the converse occurs in the $|a_{11}|$ case ($|a_{11}^{QCD}| \sim 1.7 |a_{11}^{MSM}|$ at $\sqrt{s} \sim 1250 \text{ GeV}$). Similar patterns are found in the δ and η^2 plots. Therefore, having theories as different as a vector dominated one or a scalar dominated one is manifested in the LWB scattering as an effect of roughly 30 or 40% in the parameters describing the process.

In practice, the task of measuring the LWB scattering amplitudes is hard for several reasons. First of all one should be able to separate longitudinal from transverse gauge bosons by reconstructing events with leptons and/or jets of hadrons in the final state. Besides, the energy and angular distributions of the LWB's, which are the relevant physical quantities to be reconstructed, have to be convoluted with some assumed structure functions depending on the type of experiment to be considered (e^+e^- , $p\bar{p}$, pp or e^-p). So far, the proposed experiments with some chances for studying the SBS of the Standard Model in the strong interacting case are CLIC, UNK, LHC and SSC. As usual pp and $p\bar{p}$ machines are able to cover higher energies than the e^+e^- ones, but have more problems with the background. At this point we think that it is of great importance to research in more detail which one of these machines, if any, will be the more appropriate for the study of the LWB scattering and eventually to get some insight on how the underlying SBS dynamics would show up. These topics are under study.

Acknowledgements

We would like to thank A. Pich for reading the manuscript and for useful discussions on the subject of Chiral Perturbation Theory. One of the authors (M. J. H) is grateful to D. Diakonov for his interesting comments on Chiral Lagrangians. The authors also thank the " Instituto de Estudios de Postgrado", CSIC (Spain) and the " Ministerio de Educacion y Ciencia" (Spain) for financial support.

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Figure Captions

- Fig. 1: Representation of the position in the parameter space of the two models discussed in the text. The renormalized parameters are defined at $\nu = 1 \text{ TeV}$. The solid line is the effective Higgs-like model as a function of the Higgs mass parameter. The vertical crosses on the line correspond to $m_H = 1, 2, 3, 4, 5 \text{ TeV}$ going from bottom to top respectively. The diagonal crosses on the bottom of the figure represent the QCD-scaled model. The crosses correspond to values of N_{TC} from 3 to 10, going from top to bottom respectively in the plot.
- Fig. 2: The behaviour of the parameters defining the lowest partial wave amplitudes as a function of the energy. Figures a),c) and e) are for $I = J = 0$. Figures b),d) and f) are for $I = J = 1$. δ_{IJ} is plotted in $\pi/2$ units. Solid and dotted lines correspond to the Higgs-like model for $m_H = 1 \text{ TeV}$ and $m_H = 1.3 \text{ TeV}$ respectively. Dot-dashed lines correspond to the QCD-scaled model for $N_{TC} = 3$. The LET predictions (dashed curves) are also included for comparison. If curves are not visible they are coinciding with the solid line.

Fig.1

Fig.2