CERN-TH.5426/89

The Z boson width: higher order effects and influence on the Z line shape

WOLFGANG HOLLIK *
CERN, Geneva, Switzerland

Abstract

The width of the Z boson has two faces: the one which determines, in the interplay with bremsstrahlung, the peak maximum of the Z cross section, and the other one which is a prediction of the Standard Model allowing a confrontation with experimental data as a test of the theory. We give an overview how the total and partial Z widths are calculated including the $O(\alpha^2)$ contributions. The dependence on the Higgs and top mass is discussed as well as the effect of these presently unknown parameters on the Z line shape. The relation of the peak maximum to the physical Z mass turns out to be independent of the Higgs and top mass within 2 MeV which allows to measure M_Z as an independent input parameter. Some observable effects in Z decays resulting from non-standard charged Higgs bosons are also presented.

*Talk given at the Ringberg Workshop on "Electroweak Radiative Corrections", April 3 - 7, 1989, Ringberg Castle, Germany

1 Introduction.

One of the basic measurements at the e^+e^- colliders LEP and SLC will be the determination of the shape of the Z resonance. This will provide us with two of the most interesting and important electroweak parameters: the mass and the width of the neutral vector boson. For precision tests of the Standard Model and for searches for signals of possible new physics it is indispensable to know the predictions of the Standard Model with high accuracy, including higher order corrections. The QED corrections [1], in particular real and virtual photonic corrections in the initial state, constitute the largest part of the radiative corrections and lead to a distortion of the shape of the resonance and to a shift in the peak location. In view of the high accuracy with which the mass and width will probably be measured (± 20 MeV [2]) we are forced to go beyond the $O(\alpha)$ contributions in these observables aiming at an accuracy of 10 MeV. The effect of $O(\geq \alpha^2)$ initial state radiation on the Z shape has been studied in [3]. It was found that the 2-loop QED corrections reduce the shift of the Z peak by about 90 MeV. The combination of the weak corrections in the non-radiative amplitude (with the s -dependence of the width and 2-loop contributions to the imaginary part of the Z self energy) and the initial state bremsstrahlung has also been performed [4] to give the most complete result for the prediction of the line shape.

The higher order corrections to the Z width are of twofold importance:

- They influence the shape of the resonance and have consequently to be considered for precision measurements of the Z mass. Both the reduction of the peak height and the shift of the peak maximum depend on the width:

$$\sigma \approx \sigma_0 \cdot \left(\frac{\Gamma_Z}{M_Z} \right)^\beta, \quad (1)$$

$$\sqrt{\sigma_{\max}} \approx M_Z + \frac{\pi}{8} \beta \Gamma_Z - \frac{\Gamma_Z^2}{2M_Z} \quad (2)$$

with

$$\beta = \frac{2\alpha}{\pi} \left(\log \frac{M_Z^2}{m_e^2} - 1 \right).$$

- Being a prediction of the Standard Model after the Z mass is known, the width serves as a first test of theory. The partial widths for $Z \rightarrow f\bar{f}$ will allow the investigation of the weak coupling constants of the various fermions at the level of quantum corrections.

In this talk we discuss in detail the radiative corrections to the fermionic partial widths $\Gamma(Z \rightarrow f\bar{f})$, $f = \nu, l, q (\neq t)$, and the total Z width which enters the Z line shape [5]. Other calculations have been performed for the leptonic widths [6] and also for $Z \rightarrow q\bar{q}$ [7,8]. In [8] the influence of the top quark on the $Z \rightarrow b\bar{b}$ decay width has been considered in a unitary gauge calculation.

The basis for our calculation is the on-shell scheme in the version specified in [9,10]. In contrast to [8] we perform our calculation in the renormalizable 't Hooft-Feynman gauge. Since we have to include virtual top quarks and unphysical Higgs bosons in the $Z \rightarrow b\bar{b}$ decay vertex corrections finite mass effects of the type m_t^2/M_W^2 have to be kept.

Electroweak corrections to open top final states in case of $m_t < M_Z/2$, a possibility which is experimentally not completely ruled out, have been considered in [11]. They are

less important in view of the uncertainties from the top mass in the phase space factors and from large QCD corrections near threshold [12]. Therefore we restrict ourselves to the case $m_t > M_Z/2$.

The presentation is organized as follows: Section 2 contains the lowest order discussion. A brief description of the underlying strategy for the next order calculation is given in section 3. In section 4 the one-loop electroweak corrections to the fermionic width are summarized, together with the QCD corrections and other decay channels of higher than $O(\alpha)$ in the coupling constants which enter the total width as well. This is followed by a discussion of the Z line shape (section 5) and of some non-standard effects in models with charged Higgs bosons.

2 The Z width in lowest order

In lowest order the Z propagator has the Breit-Wigner form

$$D_Z^0(s) = \frac{1}{s - M_Z^2 + iM_Z\Gamma_Z^0}. \quad (3)$$

The lowest order total width Γ_Z^0 is related to the one-loop self energy $\Sigma^Z(s)$ of the Z boson by

$$M_Z\Gamma_Z^0 = \text{Im} \Sigma^Z(s = M_Z^2). \quad (4)$$

It can be written as the sum of the partial fermionic decay widths $\Gamma_Z^0(f\bar{f})$ with $m_f < M_Z/2$:

$$\Gamma_Z^0 = \sum_f \Gamma_Z^0(f\bar{f}). \quad (5)$$

These partial widths can be expressed in terms of the vector and axial vector coupling constants of the fermion f to the Z

$$v_f = \frac{I_f^3 - 2Q_f s_W^2}{2s_W c_W} \quad a_f = \frac{I_f^3}{2s_W c_W} \quad (6)$$

with

$$s_W = \sin \theta_W, \quad c_W = \cos \theta_W$$

as follows:

$$\Gamma_Z^0(f\bar{f}) = N_C^f \frac{\alpha}{3} M_Z \sqrt{1 - 4\mu_f} \left[v_f^2 (1 + 2\mu_f) + a_f^2 (1 - 4\mu_f) \right] \quad (7)$$

with $N_C^f = 3$ for quarks, $N_C^f = 1$ for leptons, and

$$\mu_f = \frac{m_f^2}{M_Z^2}.$$

The mixing angle is used in the standard on-shell definition in terms of the boson masses:

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \quad (8)$$

Making use of the tree level relation between s_W^2 and M_Z by means of the muon decay constant

$$M_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu} \cdot \frac{1}{s_W^2 c_W^2} \quad (9)$$

we obtain another possible tree level representation of the partial decay width

$$\Gamma_Z^0(f\bar{f}) = N_C^f \frac{G_\mu M_Z^2}{2\pi\sqrt{2}} \sqrt{1 - 4\mu_f} \left[1 - A\mu_f + (2I_3^f - 4Q_f s_W^2)^2 (1 + 2\mu_f) \right] \quad (10)$$

leading to the Born total width in the G_μ representation:

$$\bar{\Gamma}_Z^0 = \sum_f \bar{\Gamma}_Z^0(f\bar{f}) \quad (11)$$

In both parametrizations no dependence on the unknown standard model parameters, Higgs and top mass (M_H , m_t), is present if the mixing angle is derived from the experimental boson mass ratio (8).

For actual calculations the dependence on M_W is usually eliminated in favor of the precisely measured Fermi constant G_μ by means of the radiatively corrected form [13] of relation (9)

$$M_W^2 (1 - M_W^2/M_Z^2) = \frac{\Lambda}{1 - \Delta r(\alpha, M_W, M_Z, M_H, m_t)} \quad (12)$$

with

$$\Lambda = \frac{\pi\alpha}{\sqrt{2}G_\mu} = (37.281 \text{ GeV})^2.$$

In this way a top and Higgs mass dependence is induced.

In table 1 we give the values for the total width obtained by this method for the two parametrizations specified above (for $M_Z = 92 \text{ GeV}$, $M_H = 100 \text{ GeV}$) as functions of the top mass. This shows the differences in the values as well as the different behaviour with m_t which are a clear indication for the need of including next order contributions.

Table 1: Lowest order total width

m_t (GeV)	Γ_Z^0 (GeV)	$\bar{\Gamma}_Z^0$ (GeV)
50	2.307	2.487
100	2.358	2.501
200	2.506	2.510

3 On-shell renormalization

The Standard Model has a certain number of free parameters which are not fixed by the theory. The definition of these parameters and their relation to measurable quantities is the task of a renormalization scheme, which completes the definition of the quantized theory.

The favoured renormalization scheme in QED is the on-shell scheme with the fermion masses m_f and the fine structure constant (the on-shell $e\bar{e}\gamma$ coupling) as input parameters. The most direct and natural extension to the electroweak theory leads to the on-shell (OS)

scheme of $SU(2) \times U(1)$, which has been widely used for practical applications (see e.g. Refs. [10, 11]). Differences in the treatment of field renormalization and in the unphysical sector disappear in the final relations between physical quantities. Here we follow the OS scheme as specified in [9, 10].

Starting point is the classical Lagrangian

$$L_d = L_C(\bar{\psi}, B, g_2, g_1) + L_H(\phi, \mu^2, \lambda) + L_{FC}(\psi_L, \psi_R, \bar{\psi}, B) + L_{FH}(\psi_L, \psi_R, \phi, g_f). \quad (13)$$

L_C is the gauge part with the $SU(2)$ and $U(1)$ fields $\bar{\psi}$ and B and the corresponding gauge couplings g_2, g_1 ; L_H is the Higgs part with the scalar doublet ϕ and the potential parameters μ^2, λ ; L_{FC} describes the fermion gauge field interaction with left- and right-handed fermion fields $\psi_{L,R}$, and L_{FH} is the Higgs-fermion Yukawa term that induces the fermion masses.

In the fields and parameters of (13) the $SU(2) \times U(1)$ symmetry of L_d is manifestly apparent. The physical content, however, becomes more transparent after switching to the “physical” fields and parameters

$$W^\pm, Z, \gamma, e, M_W, M_Z, m_f. \quad (14)$$

There is no room for $\sin^2 \theta_W$ as an additional independent quantity. The simplest choice in terms of (14) is to maintain relation (8) which will be used throughout the forthcoming discussion.

Since it is convenient to work in a renormalizable gauge (’t Hooft-Feynman gauge) the gauge fixing term L_{fix} and the corresponding Faddeev-Popov ghost term $L_{\phi\bar{\phi}}$ have to be added to L_d in order to obtain the Lagrangian for the quantized theory. Multiplicative field and parameter renormalization introduces renormalization constants $\sqrt{Z_i}$ for each field multiplet and Z_i for each free parameter in the original manifest symmetric version. These renormalization constants are then determined by the renormalization conditions.

The renormalization conditions give the parameters in (14) their correct physical meaning. The first subset consists of the OS conditions for the 2-point functions which make the particle content of the theory evident: (The bubbles mean the one-loop contributions together with the counter terms.)

$$\begin{aligned} \text{Re} \left[\text{bubble} \right]_{\bar{Z}} \Big|_{k^2 = M_Z^2} &= 0 \\ \text{Re} \left[\text{bubble} \right]_W \Big|_{k^2 = M_W^2} &= 0 \\ \text{Re} \left[\text{bubble} \right]_f \Big|_{k^2 = m_f^2} &= 0 \end{aligned}$$

The second subset defines the electric charge in the Thomson limit and allows to recover the ordinary QED as a simple substructure:

where $\hat{\Sigma}^{W(0)}$ can be expressed in terms of the unrenormalized self-energies as follows:

$$\frac{\hat{\Sigma}^{W(0)}}{M_W^2} = \frac{\Sigma^{W(0)} - \Sigma^{W(M_W^2)}}{M_W^2} + \Pi^W(0) + 2 \frac{\cos \theta_W}{\sin \theta_W} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} - \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) \quad (16)$$

with

$$\delta M_Z^2 = \text{Re } \Sigma^Z(M_Z^2), \quad \delta M_W^2 = \text{Re } \Sigma^{W(M_W^2)}.$$

4 Higher order contributions to the Z width

4.1 The dressed Z propagator

In lowest order, after diagonalization of the neutral boson mass matrix, the propagator matrix is diagonal. But mixing due to quantum corrections prohibits the photon and Z boson from propagating independently of each other in higher orders. Consequently, the propagator of the γZ system has to be considered as a 2×2 matrix. The radiative corrections to the propagator system can be obtained by inversion of the matrix (transverse parts only)

$$(D_{\mu\nu})^{-1} = i g_{\mu\nu} \begin{pmatrix} k^2 + \hat{\Sigma}^{\gamma}(k^2) & \hat{\Sigma}^{\gamma Z}(k^2) \\ \hat{\Sigma}^{\gamma Z}(k^2) & k^2 - M_Z^2 + \hat{\Sigma}^Z(k^2) \end{pmatrix} \quad (17)$$

with the 1-particle irreducible (1PI) renormalized self-energies specified in section 3 to one-loop order, yielding:

$$D_{\mu\nu} = -i g_{\mu\nu} \begin{pmatrix} D_\gamma & D_{\gamma Z} \\ D_{\gamma Z} & D_Z \end{pmatrix} \quad (18)$$

where ($s = k^2$)

$$D_\gamma(s) = \frac{1}{s + \hat{\Sigma}^\gamma(s) - \frac{\hat{\Sigma}^{\gamma Z}(s)^2}{s - M_Z^2 + \hat{\Sigma}^Z(s)}} \quad (19)$$

$$D_Z(s) = \frac{1}{s - M_Z^2 + \hat{\Sigma}^Z(s) - \frac{\hat{\Sigma}^{\gamma Z}(s)^2}{s + \hat{\Sigma}^\gamma(s)}} \quad (20)$$

$$D_{\gamma Z}(s) = - \frac{\hat{\Sigma}^{\gamma Z}(s)}{[s + \hat{\Sigma}^\gamma(s)][s - M_Z^2 + \hat{\Sigma}^Z(s)] - \hat{\Sigma}^{\gamma Z}(s)^2}. \quad (21)$$

Obviously the matrix (17) can be diagonalized only for one specific value of k^2 . This has been done by fixing the mixing counter term in such a way that (17) is diagonal for $k^2 = 0$.

In $O(\alpha)$, with the leading log terms resummed to all orders, the propagators are simplified to

$$\begin{aligned} D_\gamma &= \frac{1}{s + \hat{\Sigma}^\gamma(s)} \\ D_Z &= \frac{1}{s - M_Z^2 + \hat{\Sigma}^Z(s)} \\ D_{\gamma Z} &= - \frac{1}{s} \frac{\hat{\Sigma}^{\gamma Z}(s)}{\hat{\Sigma}^{\gamma Z}(s)} \frac{1}{s - M_Z^2 + i \text{Im } \hat{\Sigma}^Z(s)} \end{aligned} \quad (22)$$

The results can be summarized in terms of renormalized self-energies $\hat{\Sigma}^j$ (with $j = \gamma, Z, W, \gamma Z$) dressing the propagators as described in the next section, and the vector and axial vector form factors $F_{iA}^{jV}(q^2)$ for the Zff , γff and Wff vertices. A complete list can be found in [10].

Field renormalization ensures that we obtain finite Green functions. For physical S matrix elements the results are equivalent to those derived without field renormalization, as done in [13, 15]. Our field renormalization is performed according to the gauge symmetry by introducing the minimal number of field renormalization constants. The price for this, however, is that not all residues of the propagators can be normalized to one. As a consequence, any calculation with the renormalized Lagrangian will have to include finite multiplicative wave function renormalization factors for some of the external lines in S matrix elements.

It is of course possible to perform the renormalization in such a way that these finite corrections do not appear [16, 17, 18]. But then the Lagrangian will contain many constants which have to be calculated in terms of the few fundamental parameters.

The advantages of the OS scheme are obvious:

- The input parameters have a clear physical meaning and can be measured directly.
- Except the Higgs and top mass M_H , m_t all parameters are known.
- It has a natural separation into "QED corrections" (virtual and real bremsstrahlung) and infrared finite "weak corrections" for NC processes. This is of practical importance since the QED corrections in a realistic experiment are in general detector dependent.

The W mass M_W is not known as precisely as to make the uncertainty in the radiative corrections negligible ($\Delta M_W = 100$ MeV with LEP200). This drawback can easily be overcome by including the OS radiative correction to the μ lifetime: The non-QED correction $\Delta\tau$ in (12) can be written in terms of the renormalized W self-energy $\hat{\Sigma}^W$ (which depends on all particle masses of the model) and the sum of vertex, box, and wave function renormalization contributions:

$$\Delta\tau = \frac{\hat{\Sigma}^{W(0)}}{M_W^2} + \frac{\alpha}{4\pi \sin^2 \theta_W} \left[6 + \frac{7 - 4 \sin^2 \theta_W}{2 \sin^2 \theta_W} \log(\cos^2 \theta_W) \right] \quad (15)$$

The further approximation of the Z propagator in (20)

$$\text{Re } \hat{\Sigma}^Z \approx 0, \quad \text{Im } \hat{\Sigma}^Z(s) \approx \text{Im } \hat{\Sigma}^Z(M_Z^2)$$

leads to the Breit-Wigner form

$$D_Z^0(s) = \frac{1}{s - M_Z^2 + i M_Z \Gamma_Z^0}$$

which corresponds to our Born formula (3).

Off resonance, in the continuum region, the approximation (22) is adequate. Around the Z peak, however, (22) becomes insufficient:

1. The on-resonance value of the amplitude for $e^+e^- \rightarrow f\bar{f}$ in lowest order is of $O(1)$ and not of $O(\alpha)$ as in the continuum: The tree level width which is given by the imaginary part of the one-loop Z self-energy $\text{Im } \hat{\Sigma}^Z(M_Z^2)$ cancels the coupling constants in the numerator of the matrix element. For the next order corrections to the cross section around the Z peak the $O(\alpha^2)$ contributions to the Z width have to be included. One part of them is given by the imaginary part of the $(\hat{\Sigma}^{\gamma Z})^2$ term in (20).
2. The real part of $(\hat{\Sigma}^{\gamma Z})^2$ in (20) gives a $O(\alpha^2)$ correction to the resonance amplitude. In a systematic expansion up to $O(\alpha)$ it would therefore not appear. A numerical study shows that it is indeed negligible if the top quark is not too heavy (< 150 GeV). For a large mass splitting in the (t, b) doublet, however, the $O(\alpha^2)$ term matches the experimental accuracy aimed in LEP experiments.

4.2 The corrected Z width

For an appropriate discussion of the Z width we proceed as follows:

1st step:

The resummed form (20) is still insufficient for the imaginary part of the Z propagator: besides the reducible $O(\alpha^2)$ term $\text{Im } (\hat{\Sigma}^{\gamma Z})^2$ we need also the 2-loop irreducible part $\text{Im } \hat{\Sigma}_{(2)}^Z$ of the diagonal Z self-energy contributing to the $O(\alpha^2)$ width as well. Note that the s -dependence of the imaginary part is also significant: the replacement

$$M_Z \Gamma_Z(M_Z^2) \rightarrow \sqrt{s} \Gamma_Z(s) = \text{Im } \hat{\Sigma}^Z(s)$$

causes a shift of the resonance peak on the energy scale of about 35 MeV [4, 19] to lower values.

Altogether, we have to replace the imaginary part of the denominator in (20) by the proper expression

$$\text{Im} \left[\hat{\Sigma}^Z(s) - \frac{(\hat{\Sigma}^{\gamma Z}(s))^2}{s + \hat{\Sigma}^{\gamma Z}(s)} \right] + \frac{s}{M_Z^2} \cdot \text{Im } \hat{\Sigma}_{(2)}^Z(M_Z^2) \quad (23)$$

where $\hat{\Sigma}^Z$ still denotes the [P] one-loop part (a similar discussion has been given by Wetzel in [7]). For $s = M_Z^2$ we obtain the usual on-shell Z width.

2nd step:

The term $\text{Im } \hat{\Sigma}_{(2)}^Z(M_Z^2)$ is related by unitarity to the corrections to the Z width which are not of the reducible type:

$$\text{Im } \hat{\Sigma}_{(2)}^Z(M_Z^2) = M_Z \Delta \Gamma_Z \quad (24)$$

The term $\Delta \Gamma_Z$ summarizes all $O(\alpha^2)$ contributions which are missing in (20):

- weak vertex corrections to the decays $Z \rightarrow f\bar{f}$;
- QED corrections to the decays $Z \rightarrow f\bar{f}$, $f \neq \nu$;
- QCD corrections to the decays $Z \rightarrow q\bar{q}$;
- other decay channels of higher order in the coupling constants. In practice only the decay $Z \rightarrow \sum_f H f \bar{f}$ is of some importance for a light Higgs (≈ 5 MeV for $M_H = 10$ GeV); other decay channels can be neglected (see [20] and the references given there).

With the weak form factors $F_{V,A}^{Zf}$ we can write for $\Delta \Gamma_Z$

$$\begin{aligned} \Delta \Gamma_Z = & \sum_f N_{C,3}^f \alpha M_Z \left[v_f \text{Re} F_V^{Zf}(M_Z^2) + a_f \text{Re} F_A^{Zf}(M_Z^2) \right] \\ & + \sum_f N_{C,3}^f \frac{\alpha}{3} M_Z (v_f^2 + a_f^2) \cdot \delta_{QED} \\ & + \sum_{f=q} N_{C,3}^f \frac{\alpha}{3} M_Z (v_f^2 + a_f^2) \cdot \delta_{QCD} \\ & + \sum_f \Gamma(Z \rightarrow H f \bar{f}), \end{aligned} \quad (25)$$

The weak form factors (where the photon exchange has been removed) are listed in [10]. They correspond to the diagrams in Figure 1 (neutral Higgs boson exchange can be neglected because of small Yukawa couplings). For the light fermions ($\neq b, t$) also the charged “Higgs” diagrams are negligible. For $Z \rightarrow b\bar{b}$, however, due to the virtual top presence, they become important for large top masses exhibiting also a quadratic rise with m_t . The corresponding form factors have been calculated in [5] and [8].

The second and third terms in (25) are the QED and QCD corrections from virtual and real photon resp. gluon contributions, in the massless limit given by

$$\delta_{QED} = \frac{3\alpha}{4\pi} Q_f^2 \quad (26)$$

$$\delta_{QCD} = \frac{\alpha_s(M_Z^2)}{\pi} + 1.405 \left(\frac{\alpha_s(M_Z^2)}{\pi} \right)^2. \quad (27)$$

Taking into account the mass dependence of the QCD corrections in the partial width for $Z \rightarrow b\bar{b}$ increases Γ_Z by 2 MeV if $\alpha_s(M_Z^2) = 0.12 \pm 0.02$ [21] is used. The uncertainty in α_s induces an uncertainty in the total width of about 12 MeV.

The fourth term in (25), the decay width into Higgs and fermion pairs, is taken from [22] (for earlier work see [23]). Other 3- and 4-body decay channels, which contribute in principle also to the Z width in higher order, remain below 1 MeV. The ratios of the corresponding decay channels to the partial muonic width $\Gamma(Z \rightarrow \mu^+ \mu^-)$ are [20]:

$$\begin{aligned}
Z &\rightarrow H \gamma &< 10^{-4} \\
Z &\rightarrow gg, g\gamma &\sim 10^{-4} \\
Z &\rightarrow \sum_{f,l,b} W f_l f_b &\sim 10^{-6} \\
Z &\rightarrow \sum_{l,q} llqq &\sim 10^{-3}
\end{aligned}$$

3rd step:

Neglecting for the moment the real part of the $(\hat{\Sigma}^{ZZ})^2$ term in the Z propagator (20) and performing a Taylor expansion in $\text{Re } \hat{\Sigma}^{ZZ}$ around the Z mass

$$\text{Re } \hat{\Sigma}^{ZZ}(s) = (s - M_Z^2) \Pi_Z \quad (28)$$

with

$$\Pi_Z = \text{Re} \frac{\partial \hat{\Sigma}^{ZZ}}{\partial s}(M_Z^2) \quad (29)$$

we can write for the propagator after step 1 and 2:

$$D_Z(s) = \frac{1}{1 + \Pi_Z} \cdot \frac{1}{s - M_Z^2 + i \frac{\Gamma_Z}{M_Z}} \cdot \frac{1}{1 + \Pi_Z} \cdot \frac{\Gamma_Z^2 + \Delta\Gamma_Z^2}{1 + \Pi_Z} \quad (30)$$

Therein the expression

$$\frac{\Gamma_Z^2 + \Delta\Gamma_Z^2}{1 + \Pi_Z} \equiv \Gamma_Z \quad (31)$$

can now be identified with the physical Z width. The factor in front gives a correction to the overall normalization of the Z exchange amplitude; it has to be combined with the other corrections at the external vertices for the complete one-loop matrix element for $e^+e^- \rightarrow f\bar{f}$. The appearance of this factor in the imaginary part has to be understood as the correction to the Z width coming from the wave function renormalization of the Z line in the decay matrix elements. Writing it in the denominator

$$\frac{1}{1 + \Pi_Z} = 1 - \Pi_Z + \Pi_Z^2 \dots$$

takes into account the leading log summation for the light fermions in Π_Z which appear in terms of the photon vacuum polarization $\Pi_\gamma(M_Z^2)$.

The combination of the wave function renormalization with the parametrization (5) - (7) leads to the following common factor in the fermionic decay channels and hence in the total fermionic width:

$$\frac{\alpha}{3} M_Z \cdot \frac{1}{4s_W^2 c_W^2} \cdot \frac{1}{1 + \Pi_Z} = \frac{\sqrt{2} G_F M_Z^2}{12\pi} \cdot \frac{1 - \Delta r}{1 + \Pi_Z} \quad (32)$$

The structure of Δr and Π_Z in the leading terms from the light fermions and a potentially heavy top quark

$$\Pi_Z = \Pi_\gamma(M_Z^2) + \frac{c_W^2 - s_W^2}{s_W^2} \Delta\rho + \dots \quad (33)$$

$$\Delta r = -\Pi_\gamma(M_Z^2) - \frac{c_W^2}{s_W^2} \Delta\rho + \dots$$

with

$$\Delta\rho = \frac{\alpha}{4\pi} \cdot \frac{3}{4s_W^2 c_W^2} \left(\frac{m_t}{M_Z} \right)^2 \quad (34)$$

shows that the correction factor in (32) is very close to 1 unless the top is very heavy. In that case

$$\frac{1 - \Delta r}{1 + \Pi_Z} \approx 1 + \Delta\rho \quad (35)$$

yields an approximation of better than 0.5% (see Figure 2).

The parametrization of the Z width in terms of G_F , as given in (10) and (11) in lowest order, approximates the total fermionic width with an accuracy of better than 10 MeV if the top is not too heavy, as shown explicitly in table 2 for the fermionic decay width (10) and with weak corrections. For top masses above 200 GeV, however, the corrections are bigger than 20 MeV.

Table 2: Total width. Born, eq. (11), and with inclusion of weak corrections ($M_Z = 92$ GeV, $M_H = 100$ GeV)

m_t (GeV)	Born (with G_F)	with weak corrections
50	2.4869	2.4876
100	2.5010	2.4978
200	2.5401	2.5240

The variation with the Higgs mass is much weaker: keeping M_Z and m_t fixed, the variation of the total width is 6 MeV for M_H between 10 GeV and 1 TeV. This is too small to be experimentally visible.

4th step: Finally we have to discuss the effect of the $(\hat{\Sigma}^{ZZ})^2$ term in the Z propagator (20). The inclusion of this real part takes care of the fact that the Z mass gets a contribution in higher order originating from mixing with the photon. In our iterative approach, where $\hat{\Sigma}^{ZZ}$ is the renormalized mixing including the corresponding counter term with a piece proportional to $\Delta\rho$ in (34), the presence of $(\hat{\Sigma}^{ZZ})^2$ means a contribution of the type $\sim \alpha^2(m_t/M_Z)^4$ and can become of some influence in case of a very heavy top. The effect on the physical Z mass is absorbed in an additional contribution to the Z mass counter term δM_Z^2 . Since δM_Z^2 enters the quantity Δr in (15),(16) we have a modification of Δr according to:

$$\Delta r \rightarrow \Delta r + \frac{c_W^2}{s_W^2} \text{Re} \left(\frac{\hat{\Sigma}^{ZZ}(M_Z^2)}{M_Z^2[M_Z^2 + \hat{\Sigma}^{ZZ}(M_Z^2)]} \right). \quad (36)$$

The same additive contribution appears in Π_Z as well (with the opposite sign) which means that the correction factor in the Z width $(1 - \Delta r)/(1 + \Pi_Z)$ is insensitive to this term. The tiny effect observed in the total width comes from the slight change in the mixing angle when it is calculated from (12) with help of the modified Δr in (36). For more details we refer to [10,24]. Table 3 shows quantitatively the influence of the new term in the total width. The upper number is always derived without this second order term. The differences remain below 2.7 MeV.

Table 3: Total Z width ($M_Z = 92$ GeV, $M_H = 100$ GeV)

m_t (GeV)	50	100	150	200	230
Γ_Z (GeV)	2.5589	2.5699	2.5820	2.5982	2.6101
	2.5593	2.5694	2.5807	2.5961	2.6074

4.3 Partial widths

The total width shows an increase with increasing top mass which is due to the quadratic top mass term in (32) via $\Delta\rho$, eq. (34), and to the decrease of the mixing angle in the vector coupling constants. This behaviour is encountered also in each fermionic partial width with exception of the $Z \rightarrow b\bar{b}$ decay where the top mass dependence is much weaker. The reason for this is the additional top dependence of the vertex corrections in $Z \rightarrow b\bar{b}$ which cancels the top contributions from the gauge boson 2-point functions. This is shown in more detail in table 4 where the partial decay widths for $Z \rightarrow d\bar{d}$ and $Z \rightarrow b\bar{b}$ are compared. For the b quark channel, the partial width remains practically constant over the whole range of the top mass, whereas the d partial width increases by about 10 MeV. A global difference is present due to the finite b mass in the phase space factors.

Table 4: Partial widths, no QED and QCD corrections ($M_Z = 92$ GeV, $M_H = 100$ GeV)

m_t (GeV)	$\Gamma(Z \rightarrow d\bar{d})$ (MeV)	$\Gamma(Z \rightarrow b\bar{b})$ (MeV)
50	378.6	374.6
100	380.4	375.3
150	382.7	374.7
200	385.9	373.6
230	388.2	372.8

5 The Z line shape

Besides α and G_F we need the Z mass M_Z as an experimental quantity for completion of our input to fix the theory. M_Z will be measured from the shape of the resonance, in particular from the location of the maximum. The relation between $\sqrt{s_{\max}}$ and M_Z is sizably influenced by the initial state QED corrections and is to a good approximation described by eq. (2). For a final answer the QED corrections have to be combined with the non-QED weak corrections. This can be done by a convolution of the total non-radiative cross section σ_W (which contains the weak corrections) with the spectrum $\rho(k)$ of the energy carried away by the radiated photons:

$$\sigma(s) = \int_0^{s_{\max}} dk \rho(k) \sigma_W(s(1-k)). \quad (37)$$

The energy spectrum has been calculated up to $O(\alpha^2)$ in the hard photon part and to all orders in the leading soft photon contributions (see F.A. Berends, these proceedings). σ_W contains the dressed propagators and vertices. The form factor contributions to the coupling constants are practically energy independent over the resonance range; therefore they do not influence the location of the peak maximum. On the other hand, the width in the Z propagator has a

direct effect on the displacement of the resonance peak. Hence, in principle, the extraction of M_Z from the line shape depends on the values given to the unknown parameters.

The main results following from (37) are [1,19,25]:

- The position of the peak maximum is shifted to lower values by about 35 MeV. This shift comes from the s -dependence of the width and corresponds to the third term in eq. (2).
- The dependence of the maximum position on the unknown standard model parameters M_H , m_t is insignificant. This can also be understood in terms of eq. (2), where the variation of Γ_Z is too small to be of experimental importance. This is demonstrated explicitly in table 5 for the total cross section in $e^+e^- \rightarrow \mu^+\mu^-$.

Table 5: Peak maximum and position in $e^+e^- \rightarrow \mu^+\mu^-$ ($M_Z = 92$ GeV, $\alpha_s = 0.12$)

m_t (GeV)	M_H (GeV)	$\sigma_{\max}$ (nb)	$\sqrt{s_{\max}}$ (GeV)
40	100	1.369	92.094
60	100	1.452	92.094
90	10	1.446	92.094
90	100	1.453	92.094
90	1000	1.454	92.094
230	10	1.454	92.096
230	100	1.461	92.096
230	1000	1.461	92.095

The large difference in the peak cross section between the first two lines in table 5 is due to the open top production for $m_t = 40$ GeV which contributes to Γ_Z already at the tree level. The location of the maximum, however, is not influenced. It remains stable within 2 MeV.

6 Non-standard effects from a second Higgs doublet

As an example for new physics effects which can manifest themselves in Z decays we consider the minimal extension of the Standard Model which has two Higgs doublets in $SU(2) \times U(1)$ leaving the relation

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1$$

unchanged. The strongest motivation for extending the Higgs sector may come from supersymmetry. But also non-supersymmetric arguments advocate two Higgs doublets, such as the Peccei-Quinn mechanism to solve the strong CP problem [26], and the discussion of CP violation [27].

The vacuum expectation values v_1 , v_2 of the complex doublets ($j = 1, 2$)

$$\phi_j = \begin{pmatrix} \phi_j^+(x) \\ (v_j + \eta_j(x) + i\chi_j(x))/\sqrt{2} \end{pmatrix}$$

induce the masses of the vector bosons in the following way:

$$M_W = \frac{1}{2} g_2 \sqrt{v_1^2 + v_2^2}, \quad M_Z = \frac{1}{2} \sqrt{g_1^2 + g_2^2} \sqrt{v_1^2 + v_2^2}$$

3 of the eight degrees of freedom of the doublet fields are absorbed in forming the longitudinal polarization states of the $W^\pm$, Z , and γ remain as physical particles: a pair of charged Higgs bosons $H^\pm$, two neutral scalars H_0 , H_1 , and a single neutral pseudoscalar H_2 . These physical states are obtained by diagonalizing the mass matrix coming from the Higgs potential:

$$\begin{aligned} H^+ &= -\phi_1^+ \sin \beta + \phi_2^+ \cos \beta \\ H_2 &= -\chi_1 \sin \beta + \chi_2 \cos \beta. \end{aligned} \quad (38)$$

for the charged Higgs and the neutral pseudoscalar, and

$$\begin{aligned} H_0 &= \eta_1 \cos \alpha + \eta_2 \sin \alpha \\ H_1 &= -\eta_1 \sin \alpha + \eta_2 \cos \alpha \end{aligned}$$

for the 2 neutral scalars. The mixing angle β is determined by the ratio

$$\tan \beta = v_2/v_1 \quad (39)$$

whereas α depends on all parameters of the Higgs potential.

The structure of the Yukawa couplings that would arise from a supersymmetric model implies that Φ_1 gives masses to the down type quarks and Φ_2 to the up type quarks. A non-SUSY argument suggesting such a pattern is the absence of flavor changing neutral currents at the tree level.

The appearance of charged physical scalar states and the looser constraints on the Yukawa couplings may give rise to phenomenologically appealing consequences in the decay modes of the neutral vector boson. Two different scenarios yielding a set of enhanced Yukawa couplings are possible. The situation $v_1 > v_2$ which enhances the d couplings to $H^\pm$ by v_2/v_1 , and $v_2 > v_1$ enhancing the u couplings by v_1/v_2 , according to the Yukawa Lagrangian

$$\mathcal{L}_{Y,u} = \frac{g_2}{\sqrt{2}} \left(\frac{m_d}{M_W} \tan \beta \cdot \bar{d} \frac{1 + \gamma_5}{2} u + \frac{m_u}{M_W} \cot \beta \cdot \bar{u} \frac{1 - \gamma_5}{2} d \right) H^+ + h.c. \quad (40)$$

A situation of particular interest, also for the non-enhanced case $\tan \beta = 1$, is encountered in the $Z - bb$ vertex where the charged Higgs coupling to the (t, b) family involves the term

$$\frac{g_2}{\sqrt{2}} \cdot \frac{m_t}{M_W} \cdot \frac{v_1}{M_W} \cdot \frac{1 \pm \gamma_5}{2}$$

which becomes large if the top is heavy. The non-standard $H^\pm$ bosons enter the $Z - bb$ vertex in connection with virtual t quarks as follows:

They give a negative contribution to the partial decay width $\Gamma(Z \rightarrow b\bar{b})$ which is displayed in Figure 3 for the minimal model and for the presence of charged Higgs bosons (with $M_{H^\pm} = 100$ GeV). The standard model result is practically top independent, as discussed already in section 4.3, whereas the charged Higgs diminish the width if the top becomes heavy. The dashed line shows the $d\bar{d}$ partial width for illustration. Such an increase would also be expected from extra fermion generations with large doublet mass splitting. In a conventional extension of the minimal model they have only couplings to the gauge bosons and increase $\Delta\rho$ in (34) similarly as the top quark does.

Flavor changing Z decays

In the minimal model the decay rates for flavor changing decays of the Z boson like $Z \rightarrow b\bar{s}$ are so small to be experimentally detectable [28]. The reason can be found in a twofold suppression: the higher order in the coupling constants and the small non-diagonal Kobayashi-Maskawa matrix elements. The (virtual) presence of charged Higgs bosons with $v_1 > v_2$ may enhance these decay rates by several orders of magnitude. The following diagrams involving the top quark yield the dominant contribution:

The branching ratio

$$\frac{\Gamma(Z \rightarrow b\bar{s}) + \Gamma(Z \rightarrow \bar{b}s)}{\Gamma_Z} \sim |U_{ts}U_{tb}^*|^2 \left(\frac{m_t}{M_W} \cdot \frac{v_1}{v_2} \right)^4 \quad (41)$$

$$U_{cb} = 1, \quad U_{cs} \leq 0.05$$

contains the enhancement factor in the fourth power. Obeying the constraint from the experimental mass splitting between the neutral B mesons [30] the branching ratio becomes larger than 10^{-6} for top masses above 100 GeV [31].

7 Summary

In the Standard Model, the Z width is a prediction after the Z mass has been measured and the values of m_t , M_H have been specified. Together with the QED initial state bremsstrahlung corrections, the width is the essential ingredient in forming the Z resonance line shape.

The high precision in the experimental determination of the line shape at LEP and SLC requires a theoretical treatment of Γ_Z aiming an uncertainty of ± 10 MeV. To this end we have to include, together with non-fermionic decay channels, the next order corrections in the fermionic partial widths which in lowest order are the only contributions to the total width Γ_Z . These consist of the electroweak one-loop corrections (including the QED part) and the QCD corrections for the hadronic partial widths. The unknown parameters of the Standard Model, M_H and m_t , influence the prediction in a calculable way. Whereas the variation of Γ_Z with the top mass is sizeable (more than 20 MeV) the dependence on the Higgs mass is not very significant (≤ 6 MeV for M_H between 10 GeV and 1 TeV). The position of the peak maximum, from which M_Z will be determined experimentally, is stable within 2 MeV for all values of M_H and m_t in the considered range. This is important for M_Z being an independently measurable input parameter.

Among the other decay channels contributing to Γ_Z in higher order the decay into Higgs and fermion pairs is the only significant one yielding several MeV if the Higgs is light (< 10 GeV); others are below 1 MeV. The largest uncertainty in Γ_Z is induced by α_s in the hadronic decay modes yielding $\Delta\Gamma_Z = 12$ MeV. The recent re-evaluation [32] $\alpha_s = 0.11 \pm 0.01$ would reduce this error to $\Delta\Gamma_Z = 6$ MeV.

Among the partial widths the $Z \rightarrow b\bar{b}$ decay channel is of specific interest since the top quark in the vertex corrections cancels the increase for large m_t observed in the other partial widths. As a result, $\Gamma(Z \rightarrow b\bar{b})$ is constant within 2 MeV over the whole top mass range up to 250 GeV. For this reason it is an ideal probe for several kinds of new physics objects: a sequential fourth generation with mass splitting in the doublet would increase the b partial width, whereas charged Higgs bosons predicted by the extended Standard model with two scalar doublets diminish it, in particular in the case of enhanced Yukawa couplings. Such non-standard charged Higgs bosons with enhanced Yukawa couplings can also give rise to flavor changing Z decays with branching ratios $> 10^{-6}$ if the top mass is above 100 GeV.

References:

1. M. Greco, G. Paucher, Y. Srivastava, Nucl. Phys. B 171 (1980) 118; E. Nucl. Phys. B 197 (1982) 543;
F.A. Berends, R. Kleiss, S. Jadach, Nucl. Phys. B 202 (1982) 63;
M. Böhm, W. Hollik, Nucl. Phys. B 204 (1982) 45
2. Physics with LEP, CERN 86-02, eds. J. Ellis and R. Peccei
3. F.A. Berends, G. Burgers, W.L. van Neerven, Phys. Lett. 185 B (1987) 395;
G. Altarelli, G. Martinelli, in [2];
O. Nicrosini, L. Trentadue, Phys. Lett. 196 B (1987) 551; Z. Phys. C 39 (1988) 479;
E.A. Kuraev, V.S. Padin, Sov. J. Nucl. Phys. 41 (1985);
V.S. Padin, V.A. Khoze, Yad. Fiz. 47 (1988) 1693
4. F.A. Berends, G. Burgers, W. Hollik, W.L. van Neerven, Phys. Lett. 203 B (1988) 177
5. W. Beenakker, W. Hollik, Z. Phys. C 40 (1988) 141
6. P. Antonelli, M. Consoli, C. Corbo, Phys. Lett. 99 B (1981) 475;
M. Consoli, S. Lo Presti, L. Maiani, Nucl. Phys. B 223 (1983) 474;
F. Jegerlehner, Z. Phys. C 32 (1986) 425
7. W. Wetzel, in [2] and Nucl. Phys. B 227 (1983) 1
8. A.A. Akhundov, D.Yu. Bardin, T. Riemann, Nucl. Phys. B 276 (1986) 1
9. M. Böhm, W. Hollik, H. Spiesberger, Fortschr. Phys. 34 (1986) 687
10. W. Hollik, DESY 88-188 (1988), to appear in Fortschr. Phys.
11. W. Beenakker, W. Hollik, in: CERN 87-08, ECFA 87-08 (1987), eds. A. Böhm, W. Hoogland
12. J. Jersak, E. Laerman, P.M. Zerwas, Phys. Rev. D 25 (1980) 1218
13. A. Sirlin, Phys. Rev. D 22 (1980) 971
14. A. Barroso et al., in CERN/ECFA 87-08 (1987), eds. A. Böhm and W. Hoogland
15. D.C. Kennedy, B.W. Lynn, SLAC-PUB 4039 (1986, revised 1988)
16. D. Yu. Bardin, P. Ch. Christova, O.M. Fedorenko, Nucl. Phys. B 175 (1980) 435; Nucl. Phys. B 197 (1982) 1
17. J. Fleischer, F. Jegerlehner, Phys. Rev. D 23 (1982) 2001
18. K.I. Aoki, Z. Hlilki, R. Kawabe, M. Konuma, T. Muta, Suppl. Progr. Theor. Phys. 73 (1982) 1
19. D.Yu. Bardin, A. Leike, T. Riemann, M. Sachwitz, Phys. Lett. 206 (1988) 539;
D.Yu. Bardin, M.S. Hilenky, G.V. Mithselmakhov, T. Riemann, M. Sachwitz,
Zeuthen preprint PHE 89-05

20. E. Franco, in [2]
21. W. de Boer, SLAC-PUB-4428 (1988)
22. F.A. Berends, R. Kleiss, Nucl. Phys. B 260 (1985) 32
23. J.D. Bjorken, SLAC-198 (1977);
J. Finjord, Phys. Scripta 21 (1980) 143
24. M. Consoli, W. Hollik, F. Jegerlehner, CERN-TII.5395/89 (1989)
25. G. Burgers, in "Polarization at LEP", CERN 88-06, eds. G. Alexander et al.
26. R.D. Peccei, H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D 16 (1977) 1719
27. A.J. Buras, Proc. of the EPS Conference, Bari 1985
28. G. Mann, T. Riemann, Annalen der Physik 40 (1983) 334;
M. Glencens, C. Footman, A. Kronfeld, S. Navasimhan, D. Photiadis, Phys. Rev. D 27 (1983) 570;
V. Ganapathi, T. Weiler, E. Laerman, I. Schmitt, P.M. Zerwas, Phys. Rev. D 27 (1983) 579
29. Particle Data Group, Phys. Lett. 204 B (1988) 1
30. H. Albrecht et al. (ARGUS Coll.), DESY 87-029 (1987)
31. J.L. Hewett, S. Nandi, T. Rizzo, Ames preprint OSU 201, IS-J-2983 (1988);
M.J. Savage, CALTECH preprint CALT-68-1496 (1988);
C. Busch, DESY 88-148 (1988)
32. G. Allarelli, CERN-TII.5290/89 (1989)