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THE CONSTRUCTION OF "REALISTIC" FOUR-DIMENSIONAL STRINGS THROUGH ORBIFOLDS

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Abstract

We discuss the construction of "realistic" lower rank 4-dimensional strings, through symmetric orbifolds with background fields. We present Z_3 three-generation $SU(3) \times SU(2) \times U(1)$ models as well as models incorporating a left-right $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry in which proton stability is automatically guaranteed. Conformal field theory selection rules are used to find the flat directions to all orders which lead to these low-rank models and to study the relevant Yukawa couplings. A hierarchical structure of quark-lepton masses appears naturally in some models. We also present a detailed study of the structure of the $Z_3 \times Z_3$ orbifold including the generalized GSO projection, the effect of discrete torsion and the conformal field theory Yukawa coupling selection rules. All these points are illustrated with a three-generation $Z_3 \times Z_3$ model. We have made an effort for a self-contained presentation in order to make this material available to non-string experts interested in the phenomenological aspects of this theory.

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I. INTRODUCTION

It is well known that historical developments in Theoretical Physics do not usually follow a logical order. Most of the time, we do not need to have solved all the formal mysteries of a given theory in order to try to extract some physical consequences from it. The development of gauge theories constitutes an appropriate example to illustrate the former claims. After the formal work describing spontaneously broken gauge theories was achieved, Weinberg and Salam applied it to construct an explicit model (out of an infinite number) which happens to be the basic block of the Standard Model of particle physics. Notice, however, that this was done several years before the renormalizability of the theory was established.

String theories [1] face a number of unsolved problems, which will have to be elucidated before strings can be claimed to be the consistent candidates for a fundamental theory of nature. On the other hand, the structure of classical vacua is sufficiently understood, i.e. they are given by 2-D (super) conformal field theories (CFT) [2]. Even though these theories are not completely classified, large classes of models can be constructed [3-5]. In fact, it is remarkable how, starting from 2-D CFT techniques, it is possible to obtain information about 4-D properties of string models such as the spectrum, symmetries and couplings between the physical particles. Actually there are a few models with properties resembling very much the Standard Model [6-12]. Nevertheless, it is fair to say that so far none of them can be claimed to be realistic. This, we believe, is a point which has been underemphasized in the string literature. The experimental results available are rich enough as to provide nontrivial tests for the models and to discard all those constructed so far in a purely stringy manner, i.e. with no additional inputs from outside the theory itself.

The field of string model building is still open. There are many avenues yet to be explored in detail. We are in a similar situation to that of gauge theories before 1967. The progress made during the past three years has been very encouraging. It has led us to pass from the so-called superstring inspired models, which were merely 4-D gauge models based on E_6 , to honest-to-God string models. Presently we can construct explicit 4-D string models which are exact solutions of the string equations and satisfy general consistency requirements, such as multiloop modular invariance that puts them at the same level as the flat 10-D models. We know string mechanisms that break

the original $(E_8 \times E_8)$ gauge symmetry and moreover, using CFT techniques, we can discover flat directions to all orders on each model and obtain e.g. three-family models with $SU_3 \times SU_2 \times U(1)$ gauge symmetry. Also the structure of Yukawa couplings can be studied without invoking any field theory argument other than the general structure implied by $N = 1$ supersymmetry. Therefore, we can go a long way to actually study low-energy effects of string theories using only stringy techniques and it is worth investigating the possibilities offered by these constructions.

After the big success of the Standard Model, many attempts were made to go beyond it, especially in the context of Grand Unified Theories. There was then a complete freedom to choose not only the gauge group but also the spectrum of particles in any representation of the group as well as the discrete and global symmetries in order to get realistic models. Similar things can be said for supersymmetric model building. For superstrings however, the situation is far more restrictive; all of the symmetries and spectrum as well as the couplings, are completely determined by the properties of 2-D conformal field theories. So, for instance, representations other than the fundamental one are very rare and global continuous symmetries are usually absent [13]. Therefore, problems such as neutrino masses and proton stability are more challenging in string models.

The ideal output of a string model building program would be to find an existence proof of the fact that strings are rich enough to be a fundamental theory of nature. Notice that it is already known that, even though type II superstrings are as consistent as heterotic strings to be good quantum gravity theories, they seem to be of no use for physics since none of their vacua have simultaneously the right gauge group and matter content to contain the Standard Model [14]. If a realistic heterotic string model is found, it will probably have extra properties beyond the Standard Model which may give indications of possible predictions of string theory. Notice however that a definite prediction is much more than that. Assuming that all 2-D CFT's are stable vacua, we must find all the models with realistic perspectives and extract common properties of those models which are beyond the Standard Model and which may be testable. Then this may be called a prediction. There is still the logical possibility that the real (quantum) vacua are not given by CFT's at all and then most of the work in CFT (including this) would be less relevant. We will not consider this possibility further, but

we want to make clear how difficult is to extract a definite string prediction. On the other hand if the true vacua are (subsets of) CFT's, finding a realistic model will give hints on where to look for testable predictions (e.g. extra gauge groups and/or matter fields, etc.).

The aim of the present paper is to give a detailed description of how to construct explicit string models with properties very similar to those of the Standard Model. This is done in the context of orbifold [3] constructions for which the space of vacua and the explicit computation of effective couplings in 4-D, are well understood [15] Several models are presented in order to illustrate how close they can be to realistic and discuss also the main problems that we have encountered so far. Low energy interactions such as Yukawa couplings are analyzed in detail in order to explore the pattern of quark-lepton masses, proton stability, neutrino masses etc. We have made an effort to be as self-contained as possible in order to reach also non-string physicists who are interested in the phenomenological aspects of this theory. The plan of the paper is as follows.

In the next section, we briefly review the general way to construct 4-D Abelian-orbifold models. The techniques needed to find flat directions to all orders in those models are discussed in section III. In section IV, a detailed discussion of model building based on the Z_3 orbifold is presented, and some particular examples are discussed. The structure of Yukawa couplings is found for these models, including the quark and lepton mass matrices, neutrino masses and proton decay. An interesting left-right symmetric model is presented where proton stability is automatic and some hierarchy of quark and lepton masses can be observed. Models based on the $Z_3 \times Z_3$ orbifold are discussed in section V and the general techniques are illustrated in a particular example with 3 families and $SU(3) \times SU(2) \times U(1)$ symmetry. These models present new features not observed in Z_3 , such as the existence of a new degree of freedom given by discrete torsion and the possibility of having the complete three generations only in the twisted sectors, offering different alternatives to the hierarchy of quark-lepton masses than the ones existing in Z_3 models. Finally we have three Appendices. In Appendix A the partition functions for Abelian orbifolds are reviewed for completeness and in particular the explicit form of the generalized GSO projector is presented for both Z_N and $Z_N \times Z_M$ orbifolds. Appendix B is a short review of how to obtain the vertex operators needed to compute the couplings in orbifold models and Appendix C is a short discussion of the

weak mixing angle in these models.

II. CONSTRUCTION OF ABELIAN ORBIFOLD MODELS

In this section we will explain the construction of orbifold compactifications of the heterotic string. This discussion is aimed at introducing the basic tools and notation needed in the following chapters.

We start by describing the world-sheet degrees of freedom of the heterotic string. In the light-cone these include eight right-moving bosons $x_R^a(\sigma + t)$, $a = 1 \dots 8$ and Neveu-Schwarz-Ramond (NSR) fermions $\lambda_R^a(\sigma + t)$. Left-movers include eight bosons $x_L^a(\sigma - t)$ and another sixteen bosons $F_L^I(\sigma - t)$, $I = 1, \dots, 16$ which are equivalent to thirty-two fermions $\psi_L^K(\sigma - t)$, $K = 1, \dots, 32$.

A four-dimensional string can be constructed in general by compactifying the 6 right-moving and 22 left-moving extra bosonic coordinates on a torus $T = T_R^6 \otimes T_L^{22}$ and then identifying points related by an isometry of T . This identification, called modding, defines an orbifold as the quotient of T by the action of the isometry. Compactification on just T leads to a 4-dimensional theory with $N = 4$ supersymmetry which is thus non-chiral. The modding procedure reduces the number of supersymmetries. To obtain a chiral theory we need $N = 0$ or $N = 1$ supersymmetry. The latter case is more interesting as it provides a solution to the hierarchy problem. Furthermore, in this case the 4-dimensional spectrum is always free of tachyons.

In what follows we will only deal with symmetric orbifolds. In this case six of the left-moving coordinates are compactified on the same torus as the six right-moving ones whereas the remaining 16 left coordinates are compactified on the $E_8 \times E_8$ torus (i.e. a torus whose defining lattice is the $E_8 \times E_8$ root lattice). The orbifolds we will consider can then be represented as

$$O = \frac{T_{R+L}^6}{P} \otimes \frac{T^{E_8 \times E_8}}{G} \quad (2.1)$$

P is called the orbifold point group and G the gauge twisting group. Both P and G must be discrete groups. T^6 is defined by means of a 6-dimensional lattice denoted Λ . In fact, the 6-dimensional orbifold can be thought of as the quotient R^6/S where now the space group S includes elements $(\theta, \vec{\ell})$ where $\theta \in P$ and $\vec{\ell} \in \Lambda$. This means that in the orbifold the points $\vec{x}$ and $\theta\vec{x} + \vec{\ell}$ are identified. Under this identification there are points, such as the origin, left invariant up to a lattice vector. These invariant points are called

fixed points.

The physical 4-D spectrum of a string propagating on an orbifold is relatively simple to find. This is because an orbifold is basically flat except at the fixed points. The “internal” degrees of freedom x_R^i , λ_R^i , x_L^i , $i = 1, \dots, 6$ satisfy free field equations of motion as in a flat background but with boundary conditions dictated by the orbifold geometry. In an orbifold the spectrum is divided into untwisted and twisted sectors. In the untwisted sector boundary conditions are of the form

$$\vec{x}(\sigma = 2\pi, t) = \vec{x}(\sigma = 0, t) + \vec{\ell} \quad (2.2)$$

where (σ, t) are the world-sheet coordinates. In the twisted sectors boundary conditions take the form

$$\vec{x}(\sigma = 2\pi, t) = \theta \vec{x}(\sigma = 0, t) + \vec{\ell} \quad (2.3)$$

Twisted sectors correspond to strings that are closed up to the action of the space group. One can see that in this case the position of the string center of mass is given by a fixed point of θ . Then, roughly speaking, twisted sectors are labelled by the element θ and the corresponding fixed point. The correct statement is that twisted sectors are in 1–1 correspondence with the conjugacy classes of the space group [15,16].

Different choices of lattice Λ , point group P and gauge twisting group G lead to different 4-D models. These choices are limited by various constraints listed below.

(1) Elements of P act on the internal bosonic coordinates (and on the right-moving internal fermions) as $SO(6)$ rotations denoted θ . The twists θ must satisfy the following conditions.

i) θ must be an automorphism of the torus lattice, i.e. $\theta \vec{\ell} \in \Lambda$ for $\vec{\ell} \in \Lambda$. Notice that this condition is necessary so that the identification of points is well defined. In particular this implies that there is a basis in which the θ entries are integers. Thus, in any basis $\det \theta$ and $\text{Tr} \theta$ must be integers. Also, the number of points left invariant by θ up to a lattice vector is given by $\det(1 - \theta)$ according to the Lefschetz fixed point theorem. Therefore $\det(1 - \theta)$ must also be an integer.

ii) In order to have $N = 1$ supersymmetry in the 4-dimensional theory θ must belong to $SU(3)$, i.e. the point group P must be a discrete subgroup of $SU(3)$. If we further restrict P to be Abelian we are left with only two choices for P , either Z_N or

$Z_N \times Z_M$. Products of more Z_N 's are not contained in $SU(3)$. A Z_N group is generated by a twist θ with $\theta^N = 1$, i.e. in this case $P = \{\theta^n, n = 0 \dots N-1\}$. The twist θ can be written as

$$\theta = \exp 2\pi i [v_1 J_{12} + v_2 J_{34} + v_3 J_{45}] \quad (2.4)$$

where $Nv_i \in \mathbb{Z}$ and the J_{mn} are the $SO(6)$ Cartan generators. The condition of 4-D $N = 1$ supersymmetry then requires $\pm v_1 \pm v_2 \pm v_3 = 0$ for some choice of signs [16]. If we define complex coordinates $X^1 = \frac{1}{\sqrt{2}}(x^1 + ix^2)$, etc., the θ action on these coordinates corresponds to multiplication by a phase $e^{2\pi i v_i}$. The internal right-handed fermions are treated in the same way.

Z_N twists satisfying conditions i) and ii) were classified in ref [16]. There are only six possible values of N , $N = 3, 4, 6, 7, 8, 12$, leading to exactly one 4-D supersymmetry. For $N = 6, 8, 12$ there are in fact two different choices for θ . For $N = 2, 3, 4, 6$ there are also θ 's leading to two space-time supersymmetries. A $Z_N \times Z_M$ group is generated by two twists θ and ω with $\theta^N = 1$, $\omega^M = 1$ and M a multiple of N . $Z_N \times Z_M$ point groups with elements satisfying conditions i) and ii) were classified in [17].

In every case it is possible to find a lattice on which P elements act as automorphisms. Different lattices may be chosen with the same P . For example, one can have Z_3 acting on an $SU(3)^3$ or E_6 root lattice. Many properties of orbifolds do not depend on the specific lattice but rather on the point group. However, there are some properties (such as those related to the Wilson lines discussed later) which depend on the whole space group.

(2) The action of the space group must be supplemented with some action on the remaining left "gauge coordinates". In the bosonic representation of these coordinates the gauge twisting group G can be realized by a shift $V = (V_1, \dots, V_8) \times (V'_1, \dots, V'_8)$ on top of the $E_8 \times E_8$ lattice $\Lambda_{E_8 \times E_8}$. For a Z_N action $NV \in \Lambda_{E_8 \times E_8}$ is required. In the fermionic (complex) representation of the gauge degrees of freedom this G action corresponds to multiplication by a phase $e^{2\pi i V_I}$. Modular invariance of the orbifold partition function gives the restriction

$$N(V^2 - v^2) = 0 \text{ mod } 2 \quad (2.5)$$

The standard embedding consists in setting $V = (v_1, v_2, v_3, 0, \dots, 0) \times (0, \dots, 0)$. This choice gives a model in which the $N = 2$ world-sheet supersymmetry of the right-movers

is extended to the left-movers. Such models are called (2,2). A generic choice of V leads to (0,2) models, i.e. only right-moving world-sheet supersymmetry.

Without loss of generality, the shifts V can be chosen to have length less than one on each E_8 factor. Using this fact and eq. (2.5) for a given v all possible embeddings V for each Z_N orbifold can be found. This has been done explicitly for Z_3 [16], and the other Z_N orbifolds [18].

For $Z_N \times Z_M$ orbifolds eq. (2.5) is generalized as follows. We introduce shift vectors a and b corresponding to the θ and ω generators. The embedding in the gauge degrees of freedom is realized through shifts A and B with $NA, MB \in \Lambda_{E_8 \times E_8}$. The modular invariance condition can then be written as [17,19]

$$N'[(kA + \ell B)^2 - (ka + \ell b)^2] = 0 \text{ mod } 2 \quad (2.6)$$

$$k = 0, \dots, N-1 \quad ; \quad \ell = 0, \dots, M-1$$

where N' is the order of the $(ka + \ell b)$ shift. In $Z_N \times Z_M$ orbifolds there is a freedom to include phases $\epsilon, \epsilon^N = 1$, in the partition function as will be discussed in Appendix A. Then, for a choice of shifts satisfying (2.6) there are different models labelled by ϵ .

The gauge twisting group can also be realized through automorphisms Θ of the $E_8 \times E_8$ lattice. In the absence of Wilson lines this procedure is equivalent to the embedding by shifts, i.e. any inner automorphism Θ is equivalent to a shift in the lattice [16].

The gauge twisting group can be enlarged by introducing Wilson lines. Usually, Wilson lines $A_m^I, I = 1, \dots, 16; m = 1, \dots, 6$ can be viewed as extra phases $\int A_m^I dx^m$ acquired by the (left) fermions after being transported around a non-contractible loop of the 6-D space. Analogously, they can be introduced by embedding the whole space group $S = \{(\theta, \vec{\ell})\}$ in the gauge degrees of freedom. That is, Wilson lines provide an extra shift $m_i a_i^I$ in the $E_8 \times E_8$ lattice. Here $m_i = 0, 1, \dots, N-1$ for a Z_N point group and $a_i^I = A_m^I e_i^m$ where e_i^m are the 6-D lattice vectors which are in 1-1 correspondence with the torus non-contractible loops. In this case the a 's are also quantized, $Na_1^I \in \Lambda_{E_8 \times E_8}$ and the modular invariance constraint becomes [8]:

$$N[(nV^I + m_i a_i^I)^2 - n^2 v^2] = 0 \text{ mod } 2 \quad (2.7)$$

$$n = 0, \dots, N-1$$

Eq. (2.7) can be interpreted as having different (non-standard) embeddings on each fixed point (or equivalently on each twisted sector). Not all the six Wilson lines a_i^I are independent since the corresponding non-contractible loops are related by $\theta e_i \rightarrow \theta_{ij} e_j, \theta_{ij} \in \mathbb{Z}$. Therefore

$$a_i^I = \theta_{ij} a_j^I + L_i^I, \quad L_i^I \in \Lambda_{E_8 \times E_8} \quad (2.8)$$

Including Wilson lines increases enormously the number of 4-D string models obtained in this approach. For instance, for the Z_3 orbifold there are four different models (given by the four independent solutions of eq. (2.5)) without Wilson lines and an estimated $10^6 - 10^9$ models with Wilson lines. We will see later that Wilson lines are responsible for breaking the gauge group to smaller and more realistic groups such as $SU(3) \times SU(2) \times U(1)^5 \times G$ where G is a rank 8 subgroup of the second E_8 . Furthermore, the number of families can be controlled and it is relatively easy to obtain models with only three families.

When the gauge twisting group is realized through automorphisms it is possible to have Wilson lines that are not quantized. They can then be varied continuously and the gauge group can be broken to a lower rank one. One can see that this situation corresponds to a symmetry breaking process due to the untwisted chiral fields getting vacuum expectation values (vev's) along flat directions [20]. We will not consider this construction in the rest of this paper.

In addition to the different choices for modding groups described above we can also use the freedom to choose very specific values for backgrounds of g_{ij} and B_{ij} fields. In this way the rank of the gauge group can be increased from 16 up to 22 (usually up to 6 extra $U(1)$ factors for orbifolds). However, we prefer to work at a generic point in field space for which these extra symmetries are absent. For phenomenological reasons it is desirable to have smaller rather than bigger gauge symmetries.

To construct a model explicitly we need to find the massless spectrum. Massive states have $M^2 \sim 1/\alpha' \sim M_P^2$ and are thus less relevant for low-energy phenomenology. In the untwisted sector, the mass formulae for the right and left movers are given by

$$\begin{aligned} \frac{m_R^2}{8} &= N_R + \frac{1}{2}r^2 - \frac{1}{2} \\ \frac{m_L^2}{8} &= N_L + \frac{1}{2}P^2 - 1 \end{aligned} \quad (2.9)$$

where we have set $\alpha' = 1$. Here r is a vector in the $SO(8)$ weight lattice and it must satisfy the GSO projection $\sum_{i=0}^3 r_i = \text{odd}$. For instance, in the NS sector we can have vectorial weights $(\pm 1, 0, 0, 0)$ and permutations. In the Ramond sector we have spinorial weights such as $r = (\pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2})$. Notice that the vectorial and spinorial weights just mentioned give the vacuum (massless) state. P is a vector in the $E_8 \times E_8$ lattice. Vectors in the E_8 lattice have the form

$$(n_1, \dots, n_8) \\ (n_1 + \frac{1}{2}, \dots, n_8 + \frac{1}{2}) \quad (2.10)$$

with $\sum_i n_i = \text{even}$. N_R and N_L are oscillator numbers. The numerical factors in (2.9), $-\frac{1}{2}$ and -1 , arise from normal ordering.

For Z_N orbifolds mass formulae in a sector twisted by $\theta^n, n = 1 \dots N-1$ are given by

$$\frac{m_R^2}{8} = N_R + \frac{1}{2}(r + nv)^2 + E_0 - \frac{1}{2} \\ \frac{m_L^2}{8} = N_L + \frac{1}{2}(P + nV)^2 + E_0 - 1 \quad (2.11)$$

Here $v = (0, v_1, v_2, v_3)$ with v_i the twist vector introduced before. V is the shift vector that realizes the gauge twisting group. E_0 appears from normal ordering and is given by

$$E_0 = \sum_{i=1}^3 \frac{1}{2} |v_i| (1 - |v_i|) \quad (2.12)$$

N_R and N_L are again oscillator numbers which in this case can be fractional, e.g. $N_L = k/N$, because the boundary conditions imply fractional moddings for (internal) oscillators. Notice that in the absence of Wilson lines θ^n twisted sectors corresponding to different fixed points are degenerate. In the presence of Wilson lines the effective $E_8 \times E_8$ shift is $nV + m_i a_i$ and the degeneracy is broken because the m_i depend on the fixed points. The twisted mass formulae eq. (2.11) are generalized to the $Z_N \times Z_M$ case in an obvious way.

States in the spectrum are tensor products of right and left modes satisfying the level-matching condition $m_R^2 = m_L^2$. It is also necessary to project out states that are not invariant under the full orbifold group (space group plus gauge embeddings). For example, 4-D gauge vectors appear in the untwisted sector. They have $r = (\pm 1, 0, 0, 0)$

and $P^2 = 2$. This gives $240 + 240$ possibilities for P corresponding to the charged generators of $E_8 \times E_8$. Now, invariance under the orbifold group imposes $P \cdot V = 0 \bmod$ integer and $P \cdot a_i = 0 \bmod$ integer. Thus, the $E_8 \times E_8$ gauge group is broken.

The orbifold projection in the untwisted sector is straightforward to find, whereas for twisted sectors a detailed analysis of the one-loop partition function is required. We will present the relevant information on these matters in Appendix A. Fortunately, in the Z_3 orbifold that we mostly consider all twisted states with zero mass survive the projection. The less trivial case of the $Z_3 \times Z_3$ orbifold will be also discussed in Appendix A.

III. FLAT DIRECTIONS

We know that each of the models considered in the previous section are in fact classes of continuously degenerated models. This is because we can vary continuously the angles and radii of the underlying torus, as well as (in the case of the embeddings through automorphisms) the Wilson lines, which in some cases turn out to be continuous parameters also.

A general analysis of continuously degenerated models can be made in terms of the 4-D effective theory. Note that from the 4-D point of view, this continuous degeneracy can be seen as corresponding to 4-D scalar fields with zero potential, i.e. flat directions. Notice that the 6-D metric modes as well as the Wilson lines correspond only to scalar 4-D fields in the untwisted sector. Flat directions for untwisted and twisted sector fields have been considered in references [21],[20]. The general procedure is to use 4-D supersymmetry to impose the conditions for flatness, which are

$$\langle D_\alpha \rangle = \langle g_\alpha \phi_i^* (T_\alpha)_j^i \phi^j \rangle = 0 \quad (3.1)$$

$$\langle W \rangle = \langle \partial W / \partial \phi_i \rangle = 0 \quad (3.2)$$

where ϕ_i are the scalar fields, W the superpotential and g_α and T_α the couplings and generators of the gauge group. In $N = 1$ supergravity, the solutions of equations (3.1) and (3.2) represent supersymmetric vacua with vanishing cosmological constant. The condition for vanishing D-terms is tractable from field theory methods. The only stringy effect we have to remember is that if there is an anomalous $U(1)$, as usual in (0,2) models, the D-term for that $U(1)_A$ includes a Fayet-Iliopoulos term. This has been explicitly

calculated for 4-D strings to be [22]

$$D_A = \sum_i q_A^i |\phi_i|^2 + (g/192\pi^2) \text{Tr} Q_A \quad (3.3)$$

Note that for the usual “vacuum” $\langle \phi_i \rangle = 0$, the extra term may induce a breaking of supersymmetry for $g \neq 0$ (the dilaton vev). However generally, the equation $D_A = 0$ has a solution forcing $\langle \phi_i \rangle \neq 0$ for some i ’s. We can say that the presence of the Fayet-Iliopoulos term forces some of the fields to have a non-vanishing vev and therefore to break the gauge symmetries rather than supersymmetry.

For the vanishing of the D-terms coming from the factors of the gauge group other than $U(1)_A$, a sufficient condition is the existence of a function $I(\phi_i)$ [23] which is gauge invariant. In order to satisfy also (3.3), I must have a charge under $U(1)_A$ having sign opposite to that of $\text{Tr} Q_A$. Equations (3.1) are automatically satisfied if

$$\frac{\partial I}{\partial \phi_i} = c \phi_i^* \quad (3.4)$$

where c is an arbitrary constant.

It is to satisfy equations (3.2), where stringy considerations are required. Notice that since supergravity is non-renormalizable, the superpotential is in fact an infinite series of monomials of the chiral superfields. To see if in a particular model we can satisfy equations (3.2), we will need more information about the complete superpotential. Conformal field theory techniques have been developed for orbifold vacua and some general results concerning arbitrary couplings of strings are available. In particular, a set of “selection rules” has been extracted which allows us to know directly whether a coupling vanishes or not. This is all we need to know in order to look for flat directions.

The general coupling that we are interested in, is a Yukawa-type coupling of the type FFB^m , where F and B represent fermionic and bosonic components of different chiral superfields. These terms can be probed by studying the string amplitude

$$\langle V_F V_F V_B \cdots V_B \rangle \quad (3.5)$$

where V_F and V_B are the vertex operators which create the states F and B respectively. A brief description of how to construct the vertex operators for any massless field in the spectrum is provided in Appendix B, following the formalism of ref. [15]. Here we will

restrict ourselves to writing down the corresponding selection rules, which are the most needed information for practically looking for solutions of (3.2). We will state these rules for a Z_N orbifold, for simplicity. Extension to any other orbifold is straightforward.

1. Space and Point Group Selection rules. Every field entering in the interaction can be labelled by an element (more precisely, a conjugacy class) of the space group $h = (\theta^m, e)$, where θ^m indicates to which twisted sector the field belongs and e specifies the fixed point (or combination thereof) at which the corresponding field sits. The space group selection rule states that the set of products of all the elements h corresponding to each field in a given interaction, must include the identity element $(1,0)$. Otherwise the coupling vanishes identically. The restriction of this rule to the rotation part of S is the point group selection rule, which for practical purposes is very simple to check. That is, the sum of all the powers of the exponents in h must vanish modulo the order of the orbifold. This implies that a general coupling

$$\prod_{i,j} U_i^{\ell_i} T_j^{\ell_j} \quad (3.6)$$

vanishes, unless $\sum j\ell_j = 0 \bmod N$. Here U_i 's are untwisted sector fields whereas T_j 's are twisted fields in the θ^j sectors. The translation part of the space group rules is essentially a restriction on the location of the twisted sector fields. For instance, for the Z_3 orbifold, cubic couplings between twisted sector fields are allowed only if the fixed points where the fields sit are all equal or all different. Notice that the untwisted fields couplings are not restricted from these rules. It can be easily checked that from the 4-D field theory point of view, these selection rules are equivalent to a set of discrete symmetries [24].

2. Conservation of $\vec{H}$ momenta. As explained in Appendix B, the right-moving vertex operators have factors proportional to the NSR fermions which, when bosonized can be written as $e^{i\vec{\alpha}\cdot\vec{H}}$ where $\vec{\alpha}$ is an $SO(10)$ weight. The boson fields $\vec{H}$ then enter into the amplitudes in the same way as the coordinate field $\vec{x}$ in the momenta terms $e^{i\vec{k}\cdot\vec{x}}$ and, similar to the conservation of the momenta $\vec{k}$, a conservation of momenta $\vec{\alpha}$ for any interaction can be proved. In the next section we will see how this rule is implemented in Z_3 , illustrating also how it forbids many couplings for both untwisted and twisted fields. A similar analysis can be done for the left-moving vertex operators. Here, conservation

of the left momenta in the $E_8 \times E_8$ lattice is just equivalent to gauge invariance of the corresponding amplitude.

3. Invariance under independent phase rotations of the internal 6-D coordinates. If the six dimensional lattice is a product of several 2-D sublattices, independent orbifold rotations of each of the (complex) coordinates can be proved to be also a symmetry of the interactions. This invariance restricts the number of derivatives entering into a given amplitude.

Rules 2 and 3 together have been found to be equivalent to discrete R symmetries in the 4-D theory [24].

In addition to these well-understood rules, there are some other couplings that are known to vanish. We have

$$U^{n+3} = 0 \quad (3.7)$$

$$M_{ij}^m = 0 \quad n, m \in \mathbb{Z}_+ \quad (3.8)$$

where U represents any untwisted sector field and M_{ij} the untwisted moduli (angles and radii of the underlying 6-D torus). Equation (3.8) implies that the potential for the moduli is flat. Also, for twisted sector fields, when they all sit at the same fixed point, there are extra Z_2 -types of symmetries of the amplitude implying that the couplings vanish. For example, for the Z_3 orbifold, the following couplings including only twisted sector fields are allowed by the selection rules above

$$T^{18n+12} \quad n \in \mathbb{Z}_+$$

where T represents any twisted sector field (non-oscillator). However if all the T fields sit at the same fixed point, these couplings will also vanish identically. (This “rule” was stated incorrectly in references [7], [20]). These vanishing couplings do not seem to correspond to known 4-D symmetries in field theory [24].

IV. MODEL BUILDING: Z_3 MODELS

Let us turn now to the construction of “realistic” four dimensional strings. We will be interested in three-generation models since supersymmetric models with more generations have problems with the evolution of the gauge coupling constants (they get out of the perturbative regime before unification). Since no (2,2) orbifold model

with 3 generations is known we will concentrate on (0,2) compactifications. Only (0,2) Abelian orbifolds have been constructed up to now. They correspond to the Z_N and $Z_N \times Z_M$ orbifolds considered in some detail in refs. [16] and [17]. Some non-standard embeddings of these orbifolds contain semi-realistic gauge groups like E_6 , $SO(10)$, $SU(5)$ etc. Within them some will have three generations (e.g. the Z_7 orbifold) but only a few are expected to have this property. Furthermore, in order to break these large gauge groups down to the standard model, Wilson line backgrounds are in general required. They in general not only break the gauge symmetry but alter the chiral matter content, thus also the number of generations may be changed. That is for example what happens with the three-generation Z_7 model of refs. [8],[25]. It contains 3 E_6 generations in the untwisted sector but they are partially killed if we add a Wilson line to break the E_6 symmetry. Thus it is quite tricky to get three generations in a model with gauge group $SU(3) \times SU(2) \times U(1)$ or some simple extension of it in a generic Z_N or $Z_N \times Z_M$ model.

In the case of the Z_3 orbifold the situation is better. This is because, due to the Z_3 structure, particles in the untwisted sector come always in three copies (one per complex dimension). Furthermore, particles in the twisted sectors come in three copies if there are two Wilson lines ($3 = 27/(3 \times 3)$) [26]. The first of these properties is a unique feature of the Z_3 orbifold. The second has analogues in some other orbifolds, particularly in those employing an underlying $SU(3)^3$ torus. The simplest example of this is the $Z_3 \times Z_3$ orbifold in which some twisted sectors have 9 fixed tori which split into 3 sets of three tori in the presence of a Wilson line. Thus one may easily obtain three-generation models using Z_3 and $Z_3 \times Z_3$ orbifolds. In what follows we describe how one can construct semi-realistic models based on these orbifolds and give some explicit examples. We start in this chapter with models based on the Z_3 orbifold.

i) Z_3 models. General properties.

All models which can be obtained from the Z_3 ($SU(3)^3$) orbifold by taking arbitrary (modular invariant) gauge embeddings and one single Wilson line were classified in ref. [8]. In particular one can obtain three-generation models with gauge groups $SU(5) \times SU(2)^2 \times U(1)^2$, $SO(10) \times SU(2) \times U(1)^2$, $SU(7) \times U(1)^2$, $E_6 \times U(1)^2$ etc. To break the symmetry further one has to add a second Wilson line. In this way one can obtain a large class of $SU(3) \times SU(2) \times U(1)^n$ models with three generations [8]. Although we

have not made a complete classification up to now, we have found some regularities in the type of models obtained. There are models (e.g. example 1 in ref [7]) in which there are no extra colour-triplet chiral particles (i.e. just the 6 usual quarks) and models (e.g. examples 2 and 3 in ref. [7]) in which there are extra $(\underline{3} + \overline{\underline{3}})$ pairs. The former have no oscillator twisted modes (relatives of the “blowing up” modes) whereas the second type of models do have such modes. It turns out that the existence of these twisted oscillator modes (“Y-fields”) facilitates the existence of flat directions lowering the rank of the gauge group as we will see in the examples below. Even though models with no extra colour triplets seem at first sight attractive to avoid fast proton decay, it turns out that typically these models are phenomenologically problematic. An example of these is model 1 in ref. [7]. One can easily find a linear combination of the eleven unbroken $U(1)$ ’s of that model which may be identified with the hypercharge. However there are very few $SU(3) \times SU(2) \times U(1)$ singlets and no flat directions along those singlets seem to exist lowering the rank of the model. On the other hand models of the second type (e.g. model 3 in [7]) do have plenty of $SU(3) \times SU(2) \times U(1)$ singlet flat directions which not only break the unwanted $U(1)$ ’s but at the same time give large masses to the extra colour triplets through Yukawa couplings. All these models have an anomalous $U(1)$ and hence an associated Fayet-Iliopoulos term. To cancel this term in the scalar potential generically some singlets in the model are dynamically forced to acquire a vev and thus produce further symmetry breaking. Thus very often the existence of an anomalous $U(1)$ triggers further symmetry breaking. We will see examples of this mechanism below.

Before discussing specific examples let us describe a few properties of the selection rules described in previous sections for the case of the Z_3 orbifold. Point group invariance only allows couplings of the form $U^\ell T^{3m}$. $\vec{H}$ -conservation does also restrict couplings. Trilinear U^3 couplings are constrained by this symmetry to be of the form

$$\eta \epsilon_{ijk} U^i U^j U^k \quad (4.1)$$

Therefore, there are no self-couplings among untwisted fields. If one of the U^i fields is a Weinberg-Salam Higgs, this implies that the induced mass matrix will have one zero and two degenerate eigenvalues. The coefficient η is some numerical factor of order 1. $\vec{H}$ -conservation also constraints twisted couplings with more than three twisted vertices. Correlation functions of this type (of order $3(1+m)$) always involve a factor

$(\partial X^1 \partial X^2 \partial X^3)^m$ as one can easily check. Selection rule 3 further constrains the couplings by requiring invariance under the automorphisms of the lattice. In particular, if a coupling is proportional to $(\partial X^i)^m (\partial \bar{X}^i)^n (\partial \bar{X}^i)^p (\partial X^i)^q$ it will vanish unless $m + q - n - p = 0 \bmod 3$. Many couplings are allowed by point group, gauge invariance and $\vec{H}$ conservation but not by selection rule 3. Typical examples are the couplings of the form

$$S_a S_b Y^{\bar{j}} \quad (4.2)$$

which always vanish because the corresponding correlation is proportional to $\partial \bar{X}^j$, which is not invariant under phase rotations of the j -th coordinate. Many couplings forbidden by rule 3 are allowed if one adds moduli to the given coupling. Vev's for moduli correspond to deformations of the shape of the orbifold. For the Z_3 orbifold a general analysis can be carried out, indicating that only couplings proportional to

$$(\partial X^1)^a (\partial X^2)^b (\partial X^3)^c, \quad a + b + c = 0 \bmod 3 \quad (4.3)$$

can be made allowed by adding enough moduli $M^{i\bar{j}}$. For example, the coupling $(S_a S_b Y^{\bar{1}})^2 (S_c S_d Y^{\bar{2}})$ does not satisfy rule 3, but

$$(S_a S_b Y^{\bar{1}})^2 (S_c S_d Y^{\bar{2}}) M^{3\bar{2}} \quad (4.4)$$

is an allowed coupling. On the other hand a coupling of the form

$$U^i (TTT) \quad (4.5)$$

will be proportional to ∂X^i and hence will be forbidden for any shape of the orbifold (no moduli insertions can make it an allowed coupling). Sometimes it is convenient to assume the most symmetric shape for the underlying torus (i.e. $\langle M^{i\bar{j}} \rangle = 0$ for all $i \neq \bar{j}$) since in this case some couplings which could ruin flatness of the potential are absent. We will see examples of this type below.

ii) A three generation $SU(3) \times SU(2)_L \times U(1)$ model

To illustrate the methods described above we will now consider an example based on one of the $SU(3) \times SU(2) \times U(1)^n$ models built in ref. [7]. We have already discussed some aspects of this model in ref. [10] and it has also been studied by other

authors [11]. The model consists of a Z_3 orbifold with non-standard gauge shift $V = \frac{1}{3}(1, 1, 1, 1, 2, 0, 0, 0) \times (2, 0, \dots, 0)'$ in the $E_8 \times E'_8$ lattice. There are two discrete Wilson lines (For a pedagogical introduction to the construction of (0,2) orbifolds in the presence of Wilson lines see ref. [27].) with $a_1 = a_2 = \frac{1}{3}(0, 0, \dots, 2) \times (0, 1, 1, \dots, 0)'$ and $a_3 = a_4 = \frac{1}{3}(1, 1, 1, 2, 1, 0, 1, 1) \times (1, 1, \dots, 0)'$. The 27 twisted sectors split into nine different sectors (with multiplicity = 3) labelled by (n_1, n_3) , $n_1, n_3 = 0, 1, -1$, referring to the position of the fixed points in the first two $SU(3)$ sublattices of the torus. The associated shifted gauge momenta for these 9 twisted sectors is

$$(P + V + n_1 a_1 + n_3 a_3)^I \quad I = 1, \dots, 16 \quad (4.6)$$

The gauge group of the model is $SU(3) \times SU(2) \times U(1)^8 \times SO(10)'$ and the complete massless spectrum is shown in table 1. It consists of three quark-lepton generations plus a number of extra $SU(3)$ triplets, $SU(2)$ doublets and singlets

$$\begin{aligned} & 3[(3, 2) + 2(\bar{3}, 1) + (1, 2) + (1, 1)] + \\ & + 3[4(3, 1) + 4(\bar{3}, 1)] + 3[12(1, 2) + 56(1, 1)] + 3(16'). \end{aligned} \quad (4.7)$$

As one can see, there are three copies of each of the states in the table. For the twisted matter this is because for each of the nine sectors one can vary the fixed points in the 3^{rd} sub-lattice, $n_5 = 0, 1, -1$. For the untwisted states the factor 3 is due to the existence of three complex compactified dimensions. The states Y_1, Y_2, Y_3 have an extra multiplicity of three because they have oscillators $\bar{\alpha}_{-1/3}^i, i = 1, 2, 3$ acting on the vacuum. Notice that only the fields $16'$ carry $SO(10)'$ quantum numbers while the other fields are neutral.

The $U(1)$ generators in this model are given by $Q^a = iq^a \cdot \bar{\partial} F$ where the corresponding q^a vectors are given in table 2. We give a brief discussion of the normalization of general $U(1)$ charges and the computation of the Weinberg angle in this class of models in appendix C. One can readily check that $Tr Q^a = 0$ for $a = 1, \dots, 7$ and all the mixed anomaly terms cancel for them [28]. The generator X in table 2 is anomalous and this anomaly is cancelled by the known mechanism involving a $B^{\mu\nu} F_{\mu\nu}^X$ coupling.

The physical hypercharge generator will be a linear combination of the Q^a $a = 1, \dots, 7$. There is some ambiguity in choosing this linear combination but it has to be such that it gives the correct hypercharge to the candidate quarks and leptons. There

is still an ambiguity in assigning the $SU(2)_L$ doublets in table 1 to leptons and Higgs doublets. The left-handed quarks are unambiguously assigned to the $(3,2)$ fields in the untwisted sector and the same happens with the right-handed U -quarks. There is also little ambiguity concerning the right-handed D -quarks since, as we will see soon, most of the colour-triplets in table 1 become massive once the Fayet-Iliopoulos D-term is made to cancel. The simplest hypercharge generator satisfying these constraints is

$$6Y = \frac{1}{3}Q_1 - \frac{1}{2}Q_2 + Q_4 \quad (4.8)$$

The values of the hypercharge of all fields in the model are shown in table 1. Of the 56×3 singlets in eq. (4.7), 20×3 are singlets under this hypercharge. Another admissible hypercharge may be obtained by adding $\pm Q_3$ to eq. (4.8). In this case the number of hypercharge singlets would be 34×3 but the general properties of the model (in particular the flat directions) remain unchanged so that we will adopt eq. (4.8) which is the simplest one.

Since the generator X is anomalous, the corresponding gauge contribution to the scalar potential has the form in eq. (3.3). Thus some of the fields $S_a, Y_b (a = 1, \dots, 11 ; b = 1, \dots, 3)$ will be forced to acquire a vev to cancel the Fayet-Iliopoulos term. As we described in section III, in order to find a direction in field space with vanishing D-terms we must construct a function I of the $SU(3) \times SU(2) \times U(1)_Y$ singlets which is invariant under all the $Q^a, a = 1, \dots, 7$ but has non-vanishing Q_X charge with sign opposite to that of $\text{tr} X$. In the present case, one can easily find that one must have $Q_X < 0$ (since $\text{tr} X > 0$). One can easily make a computer search for flat directions and the simplest I functions appear at order 6 and correspond to

$$\begin{aligned} \text{a)} & (S_1 S_2 S_3 S_6 S_8 Y_1) \\ \text{b)} & (S_1 S_2 S_3 S_8^2 S_{10}) \\ \text{c)} & (S_1 S_2 S_3 S_6^2 S_{11}) \\ \text{d)} & (S_1^2 S_2 S_3 S_5 S_8) \\ \text{e)} & (S_1^2 S_2 S_3 S_6 S_9) \end{aligned} \quad (4.9)$$

where we have also checked that there are no allowed superpotential couplings between the fields in each flat direction. The first point to remark is that necessarily the fields

S_1 , S_2 and S_3 have to acquire a vev in order to cancel the Fayet-Iliopoulos term. In fact this is a general property of the model which was proved in general in ref. [10]. This fact is remarkable because as we will see shortly these are precisely the fields whose vev's give masses to all unwanted color triplets and many other extra particles. Something similar happens for other choices of the hypercharge generator. One can search for flat directions involving vev's for any $SU(3) \times SU(2)$ singlet in the model and one finds 5 further directions to be added to those in eq. (4.9). They can be obtained from the latter by making the replacements $S_1 \rightarrow A_7$, $S_2 \rightarrow \bar{A}_7$. In this case the unbroken hypercharge would be $Y + Q_3/6$ but the fields $A_7, \bar{A}_7$ and S_3 are forced again to acquire vev's in order to cancel the Fayet-Iliopoulos term.

The flat directions in eq. (4.9) break a large number of $U(1)$'s. The unbroken gauge groups for each case are:

$$\begin{aligned} a) &\rightarrow SU(3) \times SU(2)_L \times U(1)_Y \\ b), d) &\rightarrow SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_Z \\ c), e) &\rightarrow SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_{Z'} \end{aligned} \quad (4.10)$$

as one can easily check using table 1. To this one has to add a “hidden” gauge symmetry $U(1)_3 \times SO(10)'$ where $U(1)_3$ is the invariance associated to the Q_3 generator. The extra Z, Z' generators in eq. (10) correspond to the charges

$$Z = (6Y) + \frac{11}{30}(Q_2 - 4Q_4 - 2Q_5 + 2Q_6 - 2Q_7) \quad (4.11a)$$

$$Z' = (6Y) + \frac{11}{30}(Q_2 - 4Q_4 + 2Q_5 + 2Q_6) \quad (4.11b)$$

We thus see that the existence of an anomalous $U(1)$ forces most of the $U(1)$'s to be broken and only the standard hypercharge and, at most, another extra $U(1)$ are allowed to survive. In fact one can see that both Z and Z' are linear combinations of Y and a charge generator which can be interpreted as a gauged lepton number.

In cases b) – e) the extra $U(1)_{Z, Z'}$ symmetries may also be broken by giving vev's to further fields along flat directions with vanishing anomalous charge. In particular the present model has “invariant blocks”

$$Y_1 S_3 S_4 \qquad Y_1 S_2 S_7 \qquad Y_2 S_1 S_9 \qquad Y_2 S_6 S_{11}$$

$$Y_3 S_1 S_5 \quad Y_3 S_8 S_{10} \quad Y_1 Y_3 S_6 \quad Y_1 Y_2 S_8 \quad (4.12)$$

All these couplings are forbidden by selection rule 3 and thus in principle one can give equal vev's to the fields in each block and obtain in this way a flat direction without dangerous F-terms. One has to be careful though. Although the couplings in eq. (4.12) are forbidden, products of powers of them are allowed and care should be taken not to ruin flatness. One way to solve this is to allow at most one of the fields in each invariant block to have a vev. However this has the shortcoming that some non-renormalizable Yukawa couplings will not be generated, which, as we shall see, may be phenomenologically problematic. We shall discuss this problem in a specific flat direction below.

Let us discuss now the simplest flat direction (4.9-a) which breaks directly down to the Standard Model (+ hidden sector). One assumes $\langle S_1 \rangle = \langle S_2 \rangle = \langle S_3 \rangle = \langle S_6 \rangle = \langle S_8 \rangle = \langle Y_1 \rangle \sim M_P$. As it stands this direction is flat to all orders since all the invariant blocks in eq. (4.12) have one field with no vev. Thus allowed F-terms (like e.g. $(Y_1 S_3 S_4 \cdot Y_1 Y_3 S_6)$, allowed by all selection rules) will involve more than one vanishing field and hence are harmless.

The renormalizable couplings of the model (see table 3) give masses $\sim M_P$ to most of the particles that are non-chiral under $SU(3) \times SU(2) \times U(1)_Y$. Thus,

- i) All extra colour triplets $u_2, D_{1,2,3,4}, \bar{D}_{1,2}$ and the linear combination $(d_1 + d_2)$ get large masses due to the vev's given to $S_{1,2,3}$.
- ii) All extra pairs of $SU(2)$ -singlet leptons also get large masses except for the fields $\ell_5, (\ell_1 - \ell_3 + \ell_2 - \ell_4)$ (with $Y = 1$) and $(\bar{\ell}_1 - \bar{\ell}_3 + \bar{\ell}_2 - \bar{\ell}_4)$ (with $Y = -1$). All of these come in three copies.
- iii) All exotic $Q = \pm 1/2$ states ($\bar{A}_b, A_b$ fields) disappear from the low energy spectrum
- iv) The only $SU(2)_L$ doublets surviving at low energies are $G_5, (G_1 - G_4)$ and $(G_2 - G_3)$ ($Y = -1/2$) and $\bar{G}_1, \bar{G}_2$ ($Y = 1/2$). All come in three copies and hence at this level we have 3 doublets and 6 doublet-antidoublet pairs. They will give rise to leptons and Higgs multiplets.

It is remarkable in this model that, already from the outset, it admits no extra massless colour triplets but necessarily has prospective Higgs doublets. In fact this is

independent of the flat direction chosen since it is a consequence (see table 3) of the fact that S_1, S_2 and S_3 get a vev, which is necessarily true in any flat direction, as we mentioned above. Somehow the model knows that light doublets are needed at low energies.

The massless particle content may also be affected by non-renormalizable terms. Some of them are non-vanishing even in the absence of invariant blocks. That is the case of the couplings of the form ($UTTY$). The only allowed relevant couplings involving $SU(2)_L$ doublets are

$$(\overline{G}_1^i G_2 S_3 Y_3^{\bar{i}}), (\overline{G}_1^i G_3 S_2 Y_3^{\bar{i}}), (\overline{G}_1^i G_1 S_2 Y_2^{\bar{i}}), (\overline{G}_1^i G_4 S_3 Y_2^{\bar{i}}) \quad (4.13)$$

where i -labels the three complex internal coordinates.

These couplings may be used to get rid of further $SU(2)_L$ doublets by giving a vev e.g. to $Y_3^{\bar{i}}$. But we do not want to get rid of all $\overline{G}_1^i$ doublets since, as we shall see below, they will provide us with the Higgs field giving masses to the U -quarks. Thus let us assume

$$\begin{aligned} \langle Y_3^{\bar{1}, \bar{2}} \rangle &\neq 0 \\ \langle Y_1^{\bar{1}, \bar{2}} \rangle &\neq 0 \\ \langle Y_3^{\bar{3}} \rangle &= 0 \\ \langle Y_1^{\bar{3}} \rangle &= 0 \end{aligned} \quad (4.14)$$

Under these circumstances $\overline{G}_1^2, \overline{G}_1^1$ and two of the $(G_2 - G_3)$ doublets will get a large mass. We have now then 3 doublets (G_5) and 4 doublet-antidoublet pairs ($3(G_1 - G_4), 3\overline{G}_2, 1(G_2 - G_3)$ and $1\overline{G}_1$).

Since a vev for $Y_3^{\bar{1}, \bar{2}}$ was not included in the original flat direction we have to be careful again not to spoil flatness. It is easy to get a vanishing D-term by giving simultaneously vev's to Y_1 and S_6 (notice $Y_1 Y_3 S_6$ form an invariant block) so that the modified flat direction reads

$$(S_1 S_2 S_3 S_6 S_8 Y_1)(Y_1 Y_3 S_6)^a. \quad (4.15)$$

The F -terms do also vanish. The coupling $(Y_1 Y_3 S_6)$ vanishes because of rule 3. Higher couplings vanish due to the combination of rule 3, $\vec{H}$ -momentum conservation and the

fact that $\langle Y_{3,1}^3 \rangle = 0$. Thus, e.g., the dangerous coupling

$$(Y_1 Y_3 S_6)(Y_1 S_3 S_4) \sim (\partial X_1 \partial X_2 \partial X_3)(\bar{\partial} \bar{X}_1 \bar{\partial} \bar{X}_2 \bar{\partial} \bar{X}_3) \sim 0 \quad (4.16)$$

because we are assuming there is no vev for either $Y_3^{\bar{3}}$ nor $Y_1^{\bar{3}}$, which are the only fields with a $\bar{\partial} \bar{X}_3$ term in their vertices. Notice that here we are also taking the most symmetric form of the $SU(3)^3$ lattice, i.e. we take $\langle M_{i\bar{j}} \rangle = 0$ for $i \neq \bar{j}$. Otherwise $Y_3^{\bar{2}} M_{\bar{2}3}$ could make the job of $Y_3^{\bar{3}}$. Notice that there is a nice connection between flatness and the survival of at least one $\bar{G}_1^i (i = 1, 2, 3)$ Higgs doublet field. Flatness requires at least one of the $Y_3^{\bar{i}} (i = 1, 2, 3)$ fields to vanish. Then the couplings in eq. (13) can give a large mass at most to two of the $\bar{G}_1^i$ (and $(G_2 - G_3)$) fields. Thus necessarily a massless Higgs doublet set must survive in order to guarantee flatness.

There is another coupling of the (*UTTY*) type which is non-vanishing and affects the massless spectrum. It is

$$(u_1^i D_2 S_1 Y_1^{\bar{i}}). \quad (4.17)$$

Once S_1 and $Y_1^{\bar{1},\bar{2}}$ get a vev, $u_1^{1,2}$ (candidate *U*-right quarks) will combine with the D_2 fields. However, since already a renormalizable coupling $(u_2 D_2 S_1)$ exists giving a large mass to D_2 , the final outcome will be that the three physical massless right-handed *U*-quarks will be u_1^3 and two other states, each one of which will be a linear combination of u_1^2, u_2 and u_1^1, u_2 . We will loosely denote these combinations as $u_1^2 \sim u_2$ and $u_1^1 \sim u_2$ (notice that u_2 comes in three copies). This mixing turns out to be relevant for the resulting *U*-quark mass spectrum.

The non-renormalizable couplings discussed above (eqs. (4.13) and (4.17)) are all non-vanishing without the addition of any invariant block factor. It turns out that (in the absence of $M_{i\bar{j}}, i \neq \bar{j}$ vev's) one cannot find other non-vanishing non-renormalizable couplings giving further mass contributions to otherwise massless $SU(2)$ doublets. Thus the massless spectrum of the model consists of (table 4) the standard particle content plus three sets of vector-like $|Y| = 1/2$ ($L', \bar{L}'$) and $|Y| = 1$ ($E'_R, \bar{E}'_R$) extra leptons. To these one has to add a number of singlets N . Notice that the assignments of $SU(2)$ doublet fields to leptons, Higgses or extra leptons is tentative and one has to see if the corresponding Yukawa couplings respect these assignments. Finally, there are three 16 fields couplings to the hidden sector gauge group $SO(10) \times U(1)_3$.

Let us now consider the Yukawa couplings among the quark, lepton and Higgs fields which appear at low energies.

U-quark Yukawas. The untwisted fields $Q_L^i, u_1^i, \bar{G}_1^i$ have couplings at the renormalizable level (of the type in eq. (4.1))

$$(Q_L^i u_1^j \bar{G}_1^k), \quad i \neq j \neq k \neq i \quad (4.18)$$

However since the physical Higgs $\bar{H} = \bar{G}_1^3$, only $Q_L^{1,2}, u_1^{1,2}$ can couple to $\bar{H}$. This suggests to identify Q_L^3, u_1^3 with a massless (at this level) up quark. The charm and top quarks (corresponding to $Q_L^{1,2}$ and the two linear combinations $(u_1^1 \sim u_2), (u_1^2 \sim u_2)$) will have couplings:

$$\begin{aligned} \bar{H} Q_L^1 (u_1^2 \sim u_2) \\ \bar{H} Q_L^2 (u_1^1 \sim u_2) \end{aligned} \quad (4.19)$$

and depending on the amount of mixing between $u_1^{1,2}$ and the u_2 fields the corresponding Yukawa coupling will be large or small. If, e.g., the field u_1^2 does not significantly mix with a u_2 ($| \langle Y_1^2 \rangle | \ll | \langle S_1 \rangle |$), the first coupling in eq. (4.19) would be of order g (the gauge coupling constant), which would suggest taking u_1^2, Q_L^1 as the top-quark fields. In this case the top-quark would have a relatively large mass in the range $m_t \sim 160\text{--}180$ GeV. At this level one cannot make a definite prediction since the size of the Yukawa coupling heavily depends on the amount of mixing due to eq. (4.17). Furthermore one would also need to know the normalization of the kinetic terms of each field. It is however attractive to see how a hierarchy of U -quark masses appears naturally in the context of this model.

D-quark Yukawas. Point group invariance forbids renormalizable couplings of this type because Q_L is in the untwisted sector and the D_L^c quarks and H -Higgs belong to the twisted sector. Thus D -quark masses are absent in flat directions without invariant blocks. In the present model, if an invariant block $(Y_1 Y_3 S_6)$ as in eq. (15) is used there are couplings allowed

$$Q_L^i d_1 G_2 < S_8 (Y_1 Y_3 S_6)^2 > \quad (4.20a)$$

$$Q_L^i d_2 G_3 < S_8 (Y_1 Y_3 S_6)^2 > \quad (4.20b)$$

which give rise to $(Q_L D_L^c H)$ Yukawa terms. It is easy to check that rule 3 and $\vec{H}$ conservation force these couplings to vanish except for Q_L^3 . This means only one of the D -quarks is massive at this level. Let us however remark that the precise form of the D -quark mass matrix depends also on the kinetic terms normalization which may also be non-diagonal.

Lepton Yukawas. In this case Yukawa couplings exist at the renormalizable level. It is easy to see from table 1 that the following couplings exist:

$$H L E^c \quad , \quad H L' E'_R$$

The actual size of these couplings depends on the position of each fixed point associated to each L, E^c, L', E'_R field. Recall that H is a linear combination of fields generically sitting at different fixed points. Taking into account these facts, a hierarchy of lepton masses along the lines discussed in ref. [29] occurs naturally. The same happens with the extra leptons L', E'_R , although in this case one expects that, once supersymmetry is broken, vectorlike masses for the pairs $(L', \bar{L}')$ and $(E'_R, \bar{E}'_R)$ will be generated. Thus only the usual leptons would be observable at presently available energies. Another allowed renormalizable coupling is $(L' L E^c)$.

Neutrino Yukawas. One can easily check that no renormalizable coupling of the form $L \bar{H} N$ is allowed by the $U(1)$ symmetries of the model. The term

$$\bar{G}_1^3 G_5 Y_2 < Y_3 > \quad (4.21)$$

is allowed by point group, gauge invariance and $\vec{H}$ momentum conservation but it is forbidden by rule 3, and cannot be resurrected by multiplying by the available invariant blocks. Thus no Dirac neutrino mass exists for the standard leptons. On the other hand the couplings

$$\bar{G}_1^3 G_4 Y_2 < S_3 >, \quad \bar{G}_1^3 G_1 Y_2 < S_2 > \quad (4.22)$$

are allowed (see eq. (4.13)) and lead to neutrino mass couplings of the type $(\bar{H} L' Y_2)$ but for the extra (unobserved) leptons.

" μ -term" couplings. These are couplings of the type $H \bar{H}$ or $\bar{H} \bar{H} N$ (once N gets a vev). One can easily check the presence of allowed non-renormalizable couplings

$$\bar{G}_1^3 G_2 Y_3^3 < S_3 > \quad , \quad \bar{G}_1^3 G_3 Y_3^3 < S_2 > \quad (4.23)$$

which lead to terms of the form $(\overline{H}HN)$. These type of terms are useful in order to obtain appropriate $SU(2)_L \times U(1)_Y$ breaking through radiative corrections [30]. Typically radiative corrections generate a negative (mass)² for $\overline{H}$ and thus a $\langle \overline{H} \rangle \neq 0$ is generated. A term of the form $(\overline{H}HN)$ in the superpotential transmits this vev to H . Notice that since flatness requires $\langle Y_3^3 \rangle = 0$ (or $\langle Y_3^3 \rangle \sim M_W$ once SUSY is broken) a μ -term is necessarily small in this model.

Baryon and lepton violating Yukawas. The coupling $(u_1^i d_1 d_2 S_1)$ is allowed by space-group, $\vec{H}$ -momentum and gauge symmetries but it is forbidden by rule 3. However the coupling

$$u_1^i d_1 d_2 \langle S_1 (Y_1 Y_3 S_6)^2 \rangle \quad (4.24)$$

is allowed. This leads to $\Delta B = 1$ effective interactions of the form $(u_R s_R b_R)$ where s_R and b_R are the right-handed s and b quarks. In spite of this, there is no fast proton decay because there are no quark-antiquark-lepton couplings allowed. There are couplings of the type $(Q_L D_L^c L')$ originating from non-renormalizable terms similar to those in eq. (4.20). However one can easily see that the dangerous $(Q_L D_L^c L)$ couplings are proportional to the vev's of S_4 and S_7 ($= 0$ in the flat direction considered).

Let us make some comments on the phenomenological properties of this model along the flat direction chosen. In this model a hierarchy of quark and lepton masses may occur naturally depending on the specific values of the vev's of the symmetry breaking singlets. Only the heaviest quarks (t, b, c) but all leptons have masses at this level. One would expect the lightest quarks (u, d, s) to get masses through radiative corrections, as often happens in supersymmetric models [31]. Furthermore in the present analysis we have not considered possible effects due to non-diagonal quark-lepton kinetic terms which could modify the shape of the mass matrices. These effects could be generated in loops even if they were not present at the tree level.

One of the U -quarks (presumably the top) may have a Yukawa coupling as big as g (the gauge coupling constant). After renormalization group effects are taken into account this would imply $m_t \sim 170$ GeV. We remind the reader, however, that the uncertainties in the normalization of the fields as well as the amount of $u_1^i \sim u_2$ mixing does not allow to make a definite prediction.

R -parity is explicitly broken along the chosen flat direction. The lightest supersymmetric particles are thus not stable and decay into quarks and leptons (e.g. $\tilde{\gamma} \rightarrow uds$, $\tilde{\nu}_e \rightarrow \nu_e uds$). This would make milder the existent experimental limits on s -quark and gluino masses. The $\Delta B = 1$ interaction ($u_R d_R s_R$) has been considered as a possible source of the baryon asymmetry of the universe [32]. It may also give rise to neutron-antineutron oscillations at rates consistent with experimental limits.

One point which is a bit deceptive in the model is the origin of proton stability. As we remarked above, there are Yukawa couplings violating baryon number and the proton is stable just because along the chosen flat direction ($\langle S_4 \rangle = 0, \langle S_7 \rangle = 0$) no quark-antiquark-lepton couplings exist. However one may worry that once $SUSY$ is broken there could be some $L - L'$ mixing so that the stability of the proton would be jeopardized. (Another way to get rid of the proton unstability danger is to dispose of the invariant block in eq. (4.15) and just use the original flat direction (4.9.a). Then the $\Delta B = 1$ coupling in eq. (4.24) vanishes and the proton is automatically stable. Unfortunately in this case the D -quark masses eq. (4.20) also vanish). One would like to have a more elegant explanation for proton stability. Another unattractive point is the existence of the extra leptons $L', \bar{L}', E_R, E'_R$. In particular, the presence of six extra chiral $SU(2)_L$ doublets spoils the renormalization group prediction for the unification scale (the coupling constants tend to join much below the Planck mass) and the weak angle $\sin^2 \theta_W$ (see appendix C). These problems might be solved by resorting to an intermediate stage of symmetry breaking (using approximate flat directions and not true flat directions as the ones we have used here) in which one would give large masses to the extra $L', \bar{L}', E_R, E'_R$. This may be done by adding invariant blocks and moduli to the coupling ($\bar{G}_2 G_5 Y_1$) which is allowed by all but rule 3. We did not use this possibility above because the direction chosen was only flat to all orders if $\langle M_{i\bar{j}} \rangle = 0$ ($i \neq \bar{j}$). If we admit approximate flat directions to generate an intermediate scale, $\langle M_{i\bar{j}} \rangle \neq 0$ would be admissible and all extra leptons would gain large masses. This approach would be similar to the one used for the three-generation Calabi-Yau model studied in refs. [6,33,34].

Variations of this model can also be constructed along the other flat directions in eq. (4.9). Flat direction b) was described in ref. [10]. In this case there is an unbroken extra $U(1)$ (eq. 4.11.a)) which is in fact a linear combination of the hypercharge Y and

a generator:

$$Q_{LN} = -Q_1 + Q_2 - Q_4 + Q_5 - Q_6 + Q_7. \quad (4.25)$$

One can check that $Q_{LN}(Q_L, D_L^c, U_L^c, H, \overline{H}) = 0$, whereas $Q_{LN}(L, L') = 6$, $Q_{LN}(E^c, E_R') = -6$, $Q_{LN}(\overline{L}') = -12$ and $Q_{LN}(\overline{E}_R') = 12$, so that Q_{LN} behaves as some sort of lepton number. (The identification of standard model particles with orbifold massless states is slightly different in this flat direction, see ref. [10]). By giving a vev to Y_3 , e.g. along the flat direction $(Y_3 S_8 S_{10})$ one can break this Z gauge symmetry at an arbitrary scale and give masses to two pairs of doublets through the couplings (4.13) as in flat direction a). The model thus left is very similar to the one we considered in the previous pages. There are however some differences. Since in this second flat direction also $\langle S_6 \rangle = 0$ one can check that not only the lepton violating coupling $(Q_L D_L^c L)$ but also $(Q_L D_L^c L')$ is forbidden. This makes this version of the model more stable with respect to proton stability once SUSY is broken. In order to get masses for D -quarks one may introduce an invariant block $(Y_1 Y_2 S_8)$ with vev's as we did with the case of flat direction a). The models through the other flat directions 4.9.c), 4.9.d) and 4.9.e) are really very similar to the 4.9.b) case and lead to similar physics. Notice that in all these models the possibility exists of having an extra Z^0 gauge boson at low energies which may be available to experiments. The couplings of this gauge boson to quarks and leptons are given by eq's (4.11) and table 1. Notice that this $U(1)$ is not of the E_6 -model type studied by many authors in the context of the so called "superstring inspired" models. Notice also that this orbifold model does not allow for more than one extra Z^0 .

iii) A three generation left-right symmetric model

We would like now to build a four-dimensional string model with an unbroken low energy symmetry of the type [35]

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times G \quad (4.26)$$

where $U(1)_{B-L}$ corresponds to a generator coupling to baryon-lepton number of quarks and leptons. G contains the hidden sector gauge interactions and (possibly) some further extra $U(1)$. There are a number of motivations to consider such a type of model. First, it is well known that in a supersymmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, gauge

invariance automatically forbids possible couplings $q-q-q$ and $q-\bar{q}-\ell$ which violate R -parity and baryon and lepton number. This is not the case in the supersymmetric standard model where such couplings are in principle allowed. In a generic 4-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ string, those terms will in general appear although, as we have shown in the previous section, along certain field directions they may be absent. Left-right symmetric models as in eq. (4.26) automatically guarantee the absence of these potentially dangerous couplings which may lead to fast proton decay.

There is another (practical) reason to consider $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models. In a standard model string like the one we considered in the previous section, there is to start with, a large number of $SU(2)_L$ doublets and it is difficult to distinguish which ones correspond to Higgs doublets and which to leptons. In a left-right symmetric model the gauge couplings distinguish between them

$$(B-L)(\text{Higgs}) = 0 \quad ; \quad (B-L)(\text{Lepton}) = -1 \quad (4.27)$$

so that it is easy to make an a priori distinction. Something similar happens with the right-handed quarks which are contained in $(\bar{3}, 1, 2, -\frac{1}{3})$ reps. of $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and thus are easy to distinguish from other $SU(3)$ antitriplets usually present in this type of orbifold models.

Finally, there is a third motivation to consider this type of models within the context of this particular type of orbifold constructions. In the model of the previous section the Yukawa couplings for U -quarks and D -quarks had different origins, renormalizable for the first, non-renormalizable for the second. In such a situation the experimentally observed structure of doublets $(U, D)_L$ with a small off-diagonal mixing in flavour space (Cabbibo suppression) is not to be expected. Is a situation possible in which both U -quark and D -quark Yukawas have a similar origin? In the context of the $(0, 2)$ Z_3 orbifolds we are studying this is equivalent to the question whether one can have both U -quarks and D -quarks sitting in the untwisted sector.* This question has an interesting answer: "Within the context of $(0, 2)Z_3$ orbifolds with Wilson lines, one can have both left-handed and right-handed quarks† in the untwisted sector as long as the gauge symmetry

* In the context of $(0, 2)$ Z_3 orbifold models, the left-handed quarks can never be in the twisted sectors [8].

† If one insists in having simultaneously quarks and leptons in the untwisted sector the

is promoted to $SU(2)_L \times SU(2)_R$ and the right-handed quarks form an $SU(2)_R$ doublet.” This can be shown by explicit construction of the generic untwisted particles and gauge bosons in these models. One can check that if one wants to break the $SU(2)_R$ symmetry with a Wilson line, one automatically eliminates from the untwisted spectrum one of the two right-handed triplets (D_L^c -quarks). Notice that the opposite implication is not necessary: the existence of an $SU(2)_R$ gauge symmetry does not imply that all quarks belong to the untwisted sector.

To construct a model as in eq. (4.26) we will make a slight modification to the Wilson lines used in the model of previous section but we will keep the same gauge shift V^I :

$$\begin{aligned} V &= \frac{1}{3}(1, 1, 1, 1, 2, 0, 0, 0) \times (2, 0, \dots, 0) \\ a_1 &= \frac{1}{3}(0, 0, \dots, 0, 2) \times (0, 0, 1, 1, 0, \dots, 0) \\ a_3 &= \frac{1}{3}(1, 1, 1, 2, 1, 1, 1, 0) \times (1, 1, 0, \dots, 0). \end{aligned} \quad (4.28)$$

At this stage the gauge group of the model is

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)^7 \times [SO(8)' \times SU(2)'] \quad (4.29)$$

and the unbroken charged generators in the first E_8 correspond to the roots $\pm(\underline{1}, -1, 0, \dots, 0)$ ($SU(3)$), $\pm(0, 0, 0, 1, 1, 0, 0, 0)$ ($SU(2)_R$) and $\pm(0, 0, 0, 0, 0, 1, -1, 0)$ ($SU(2)_L$), where the underlining implies permutations. The untwisted matter in the first E_8 includes left-handed quarks Q_L : $(\underline{1}, 0, 0, 0, 0, 0, -1, 0)$, right-handed quarks Q_R : $(-1, 0, 0, -1, 0, 0, 0, 0)$ $(-1, 0, 0, 0, 1, 0, 0, 0)$ and the Higgs doublets $H, \bar{H}$: $(0, 0, 0, 0, -1, \underline{1}, 0, 0)$, $(0, 0, 0, 1, 0, \underline{1}, 0, 0)$. In the second E_8 sector the untwisted sector includes a $(\underline{8}, 2)'$ of $SO(8)' \times SU(2)'$ and a singlet u . The complete massless charged matter sector of the model is shown in table 5. Altogether one gets a representation content under $SU(3) \times SU(2)_L \times SU(2)_R \times SO(8) \times SU(2)'$:

$$\text{U - sector : } \quad 3[(3, 2, 1) + (\bar{3}, 1, 2) + (1, 2, 2)] + 3[(8, 2)' + 1']$$

$$\text{T - sector : } \quad 9[(3, 1, 1) + (\bar{3}, 1, 1)] + 15[(1, 2, 1) + (1, 1, 2)] + 3(1, 2, 2) + \quad (4.30)$$

gauge symmetry would have to be promoted to $SO(10)$. When breaking this group some quarks and/or leptons will disappear from the untwisted spectrum.

$$+3[(1, 2, 1)(2, 1)' + (1, 1, 2)(2, 1)'] + 24(2, 1)' + 72(1, 1, 1).$$

The identification of quarks and Higgses in the model is obvious: $Q_L = (3, 2, 1)$, $Q_R = (\bar{3}, 1, 2)$ and Higgs = $(1, 2, 2)$. To identify candidate leptons one has to study what the massless spectrum is after the extra gauge symmetry breaking takes place, but in general we will have $L = (1, 2, 1)$, $R = (1, 1, 2)$. The $U(1)$ generators in the model are given by $Q^a = iq^a \cdot \bar{\partial}F$ where the corresponding q^a vectors are given in table 6. All $U(1)$'s are traceless and anomaly-free except for Q_5 and Q_6 . The combination $Q_5 + Q_6$ is also free of anomalies and $X = Q_5 - Q_6$ is anomalous with $Tr(X) = +864$. This anomaly can be shown to be cancelled by the usual mechanism.

As usual, the existence of this anomalous $U(1)$ generates a one-loop modification of the D -term scalar potential as in eq. (3.3) and some scalars in the model will get a vev in order to cancel the Fayet-Iliopoulos term. In the example of the previous section a minimum of 5 different scalar fields had to acquire a vev so that the minimum of the potential remained at zero. Interestingly enough, in the present model one field is enough! The untwisted singlet “ u ” is only charged with respect to the anomalous $U(1)$ and the sign of its charge is the appropriate one in order to cancel the F-I term. Thus in the present model, unlike the one in the previous section, the existence of an anomalous $U(1)$ does not force further symmetry breaking, only the anomalous X generator gets broken. The singlet u is forced to slide and cancel the F-I term. This makes the analysis of flat directions simpler.

A vev for “ u ” only gives mass to two of the hidden sector $(\underline{8}, \underline{2})'$ fields and one would like to get rid of much of the matter in eq. (4.30). One can achieve this by giving vev's to some $3 \times 2 \times 2$ singlet fields (S_i and T_i 's in table 5). To have an idea of which fields become massive when some fields get vev's we show in table 7 the allowed renormalizable couplings involving colour triplets and $SU(2)_L$ and $SU(2)_R$ doublets. Notice that a vev for S_1 gives large masses to all extra colour triplets and the Higgs-like fields H' . In fact one can check that all singlets $S_i (i = 2 - 17)$, $N_i (i = 1 - 4)$ except for u and Y get masses if S_1 gets a vev. Instead of S_1 one can give a vev to Y since the combination (uY) has the same charges and space group properties as S_1 . Furthermore, whenever a renormalizable coupling with S_1 exists a similar allowed non-renormalizable coupling

with (Yu) does exist; e.g.

$$(S_1)D_a\overline{D}_a \rightarrow (u^i Y^i)D_a\overline{D}_a \quad (4.31)$$

etc. Notice that the second coupling is allowed by all selection rules.

To get rid of some of the extra lepton-like objects L_i, R_i ($i = 1 - 5$) one needs to give vev's to some $SU(2)'$ doublets T_i . We need to do that anyhow since we have to break this $SU(2)'$ in order that the “observable” and “hidden” worlds do not mix. One can see that the following invariant blocks exist

$$(YT_3T_6), \quad (YT_1T_7), \quad (YT_5T_4) \quad (4.32)$$

One can e.g. give vev's to T_3, T_4, T_5 and T_6 . One cannot give vev's to all six T_i 's because there are trilinear couplings of type $(T_iT_jS_k)$ which will spoil flatness if some pairs of T_i fields simultaneously have vev's. The subset of dangerous couplings is also given in table 7. In fact, in order to give vev's to T_3, T_4, T_5 and T_6 without spoiling flatness (recall the dangerous couplings $(T_3T_5S_{11})$ and $(T_6T_4S_{16})$) one must choose the $SU(2)'$ doublet components with vev as follows:

$$\begin{aligned} \langle T_3 \rangle &= \langle T_5 \rangle = \begin{pmatrix} \nu \\ 0 \end{pmatrix} \\ \langle T_4 \rangle &= \langle T_6 \rangle = \begin{pmatrix} 0 \\ \nu \end{pmatrix} \end{aligned} \quad (4.33)$$

Thus altogether our flat direction is given by the I-function:

$$(u)(YT_3T_6)(YT_4T_5) \quad (4.34)$$

Alternatively one can also give a vev to S_1 and consider the (to all purposes equivalent) flat direction $(u)(YT_3T_6)(S_1T_4T_5)$. In order for this field direction to be flat to all orders in (vev's) insertions we impose

$$\langle Y^{\overline{3}} \rangle = 0, \quad \langle M_{i\overline{j}} \rangle = 0 \quad i \neq \overline{j} \quad (4.35)$$

as we did with the previous model. Then, any powers of the invariant blocks will vanish and are not allowed as Yukawas. We remark at this point that in all models discussed in this paper a computer scan of possible renormalizable and non-renormalizable Yukawa couplings has been made in order to check that the field directions are really flat.

Instead of $T_{3,4,5,6}$ a different set of T_i fields could have been chosen to break the $SU(2)'$ symmetry. The results would have been completely analogous.

Let's consider now the effect of non-renormalizable couplings. A systematic search shows that (apart from the ones obtained from renormalizable couplings through the replacement $S_1 \rightarrow uY$) the only interesting couplings are

$$(H^i H^j S_1 Y^{\bar{i}} Y^{\bar{j}}). \quad (4.36)$$

These couplings are allowed by all selection rules and once $S_1, Y^{\bar{i}}$ get vev's they give masses to the Higgs doublets H^i . However since $\langle Y^{\bar{3}} \rangle = 0$, one Higgs (1,2,2), H^3 remains light. This is similar to what happened in the previous model.

All in all, the massless particle content of the model given by the flat direction (4.34) is shown in table 8 (we do not display singlet fields). In this case the vev's for the scalar fields are not able to break a couple of extra $U(1)$'s which we denote by Y_1 and Y_2 . One can check that the most general unbroken $U(1)$ generator in the flat direction considered has the form

$$Q = \alpha(Q_1 + 3Q_5 + 3Q_6) + \beta(Q_4 - Q_7) + \delta(Q_2 - Q_3 + 2Q_5 + 2Q_6) \quad (4.37)$$

For the $B-L$ generator one must have $\alpha + \delta = 1/6$, $\beta + \delta = 1/2$ in order to get the correct charges for quarks, leptons and Higgses. In order that the extra leptons $L', \bar{L}', R', \bar{R}'$ have non-exotic charge assignments one gets $\delta = 0$ so that a reasonable $B-L$ generator is given by

$$3Q_{B-L} = \frac{1}{6}Q_1 + \frac{1}{2}(Q_4 + Q_5 + Q_6 - Q_7) \quad (4.38)$$

Two orthogonal generators are

$$Y_1 = \frac{1}{4}(Q_1 - Q_2 + Q_3 - Q_4 + Q_5 + Q_6 + Q_7) \quad (4.39a)$$

$$Y_2 = \frac{1}{6}(Q_1 + 3Q_5 + 3Q_6) + \frac{1}{2}(Q_4 - Q_7) - \frac{2}{3}(Q_2 - Q_3 + 2Q_5 + 2Q_6) \quad (4.39b)$$

We give the Y_1 and Y_2 charges of the massless fields in table 8. Notice that the standard model matter does couple to both of the extra $U(1)$'s but one can check that a linear combination of them exists which decouples from ordinary matter.

The Yukawa couplings in the model go as follows. The quarks have renormalizable Yukawa couplings of the type:

$$Q_L^i Q_R^j H^3 \quad i \neq j, \quad i, j \neq 3. \quad (4.40)$$

Once H^3 gets a vev, two generations of quarks get mass. Q_L^3, Q_R^3 remain massless at this level and would contain the up and down quarks. The massive quarks will have $m_c = m_t$ and $m_b = m_s$ at this level and no Cabbibo-Kobayashi-Maskawa mixing since the mass matrices for U and D quarks are proportional. However, as we already mentioned, the form of the kinetic term matrix may alter this situation (at tree level or in loops). For example, once $SU(2)_R$ is broken, the wave function renormalization of Q_R is different from that of Q_L and the mass matrix will no longer be antisymmetric. Thus one would have $m_t \neq m_c, m_b \neq m_s$ and quark mixing.

The leptons do not have renormalizable couplings but do have non-renormalizable ones allowed by the CFT selection rules. Thus one has

$$(H^3 LR) < u^3 Y Y T_3 T_6 > \quad (4.41)$$

which couples the standard leptons to the physical Higgs H^3 . The extra leptons do also have similar couplings

$$(H^3 L' \bar{R}') < u^3 Y Y T_3 T_6 > \quad (4.42a)$$

$$(H^3 \bar{L}' R') < u^3 Y Y T_4 T_5 > \quad (4.42b)$$

All leptons get masses through these couplings and a hierarchical structure of masses as in ref. [29] is again possible.

Eventually the symmetry breaking process

$$SU(2)_R \times U(1)_{B-L} \times U(1)_1 \times U(1)_2 \rightarrow U(1)_Y \quad (4.43)$$

has to take place. Within this model this breaking cannot occur at the Planck mass since, as we have shown, flat (to all orders) field directions only allow the breaking down to $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)^3 \times SO(8)'$. The above breaking can take place at an intermediate scale if we relax the condition of absolute flatness and admit approximate flat directions. In this case masses could be generated for the right-handed neutrinos through couplings of the type

$$(RN_2) < R' u^i Y^{\bar{i}} T_3 T_6 > < M_{3\bar{1}} M_{3\bar{2}} >$$

$$(RS_{10}) < \bar{R}' u^i Y^{\bar{i}} T_3 T_6 > < M_{3\bar{1}} M_{3\bar{2}} > \quad (4.44)$$

etc. Once fields like R' or $\bar{R}'$ get a large vev the right-handed neutrinos would combine with singlets like N_2, S_{10} to acquire a mass. The moduli insertions in eq. (4.44) are required since otherwise the couplings would be forbidden by the selection rules 2 and 3. Notice that absolute flatness forbids moduli vev's (deformation of torus) and hence terms like those in eq. (4.44) are not present if the most symmetric shape of the tori is assumed.

The massless particle content of this model is very similar to the one of the previous section: usual standard particles plus 3 sets of left- and right-handed leptons (and singlets). As happened with the $3 \times 2 \times 1$ model, the presence of these leptons spoils the prediction for the unification scale (the coupling constants tend to join much below the Planck mass). Again, the consideration of an intermediate symmetry breaking scale could alleviate this problem.

Let us close this section on Z_3 models with a few comments. The particular models studied have the property that extra colour triplets easily get large masses whereas there is a tendency to have extra light $SU(2)_L$ doublets. This is good and bad; good because this shows that models of this type have Higgs-doublet candidates to break $SU(2)_L \times U(1)_Y$ at low energies, and bad because usually there are too many doublets (lepton-like) and the standard predictions for the unification mass are jeopardized. Another interesting point is the relationship between the existence of at least one physical Higgs doublet (H^3 in the examples studied) and the flatness of the scalar potential. Also, no μ -terms are generated at the tree level but couplings singlet- H - H exist. This may be important for the radiative generation of electroweak breaking. Concerning Yukawa couplings, usually only a few quarks and leptons have renormalizable couplings with the physical Higgs. This is probably a property of most superstring models. Some others get couplings through non-renormalizable terms. Finally, some do not get tree-level masses at all. The latter should probably be identified with the first quark-lepton generation and it is reasonable to assume that they may get their tiny masses from loop corrections, as often happens in supersymmetric models. The overall structure of quark and lepton Yukawas is encouraging since a natural hierarchy of masses seems possible. A study of the effects of supersymmetry breaking in this type of models is certainly interesting

since it would tell us if complete consistency with experimental constraints is possible. The existence of possible heavy leptons or extra gauge couplings of the type discussed in the text could also be checked experimentally. It could well turn out that a large class of string models lead to new sets of heavy leptons.

V. $Z_3 \times Z_3$ MODELS

Up to now the only “realistic” orbifolds built are based on the Z_3 orbifold. It is interesting to investigate to what extent the features found in those models are general or depend on the details of the Z_3 orbifold structure. The $Z_3 \times Z_3$ orbifold is specially interesting in this context since, as we will see, examples of models with three generations are easy to obtain.

It turns out that the generalization from Z_3 to $Z_3 \times Z_3$ is not completely straightforward and a number of new features are found. As is well known, the standard embedding $Z_3 \times Z_3$ model has 84 $\underline{27}$'s of E_6 and no antigenerations. There are a good number of non-standard embeddings consistent with modular invariance leading to a variety of models with different gauge groups. On top of that one can add up to three Wilson lines to modify further the spectrum and break the symmetry. An important difference with respect to Z_3 is that the GSO-like projection in the twisted sectors is non-trivial and many massless states are projected out. Thus one has to build the projectors carefully to get the correct massless spectrum. Furthermore (also unlike Z_3) the partition function admits possible relative (quantized) phases between different twisted sectors of the $Z_3 \times Z_3$ orbifold (“discrete torsion”). Thus any given model will have several versions, depending on the value of the discrete torsion. The twisted massless spectra of two models with different discrete torsion are completely different, although both are modular invariant. Other different features concern the Yukawa couplings. Unlike the Z_3 case, renormalizable couplings of the type (UTT) exist in $Z_3 \times Z_3$. Notice that the fact that no such couplings exist in Z_3 leads to difficulties in trying to give masses to some quarks in the Z_3 models we discussed above. This is a potentially interesting feature of $Z_3 \times Z_3$. Another interesting point is that in fact all quarks and leptons may be contained in the twisted sector (including the Q_L quarks which never live in the twisted sector of $(0,2)Z_3$ orbifolds). This is again potentially interesting in order to allow all the fermions to get mass. The structure of the CFT selection rules for Yukawa couplings

are slightly more delicate in the $Z_3 \times Z_3$ orbifold. This is particularly the case for the $\vec{H}$ -momentum conservation rule.

In spite of all the above potential advantages of the $Z_3 \times Z_3$ orbifold, the large number of massless (to start with) fields in these models makes their analysis a bit complicated. We will present below a three generation $Z_3 \times Z_3$ model to illustrate some of the points discussed above.

Let us first discuss the general properties of the $Z_3 \times Z_3$ orbifold. To our knowledge no discussion of this model is available in the literature. We think this is interesting in itself and not only for realistic model building. The $Z_3 \times Z_3$ orbifold can be equivalently constructed as nine copies of ($k = 1$) $N = 2$ superconformal minimal models [5] and it provides the simplest example of this type of constructions. Understanding the properties of the $Z_3 \times Z_3$ orbifold may be useful to generalize well-known orbifold techniques (e.g. (0,2) models) to the $N = 2$ model tensoring method [36].

The $Z_3 \times Z_3$ point group is generated by twists θ and ω with twist vectors $a = \frac{1}{3}(1, 0, -1)$ and $b = \frac{1}{3}(0, 1, -1)$ respectively. We take the 6-dimensional lattice to be given by the product of three $SU(3)$ root lattices. The gauge embedding shifts are 16-dimensional vectors A and B satisfying $3A, 3B \in \Lambda_{E_8 \times E_8}$ as well as the modular invariance condition eq. (2.6). It is useful to define the auxiliary shifts $C = A + 2B \equiv A - B$ and $D = A + B$. Besides the untwisted sector there are eight twisted sectors, $\theta, \omega, \theta\omega^2, \theta\omega$ together with the inverses $\theta^2, \omega^2, \theta^2\omega, \theta^2\omega^2$. We will alternatively denote these sectors by their associated shifts, e.g. A -sector = θ -sector.

Having introduced some basic notation we now turn to a description of $Z_3 \times Z_3$ orbifold properties relevant for our subsequent model building. We will often compare with the Z_3 orbifold. For ease of presentation these properties are listed below.

- 1) In the Z_3 orbifold untwisted matter fields are degenerate, i.e. they appear in three copies. This is because the three right-moving modes that give rise to these fields transform in the same manner under the point group (they are all rotated by $e^{2\pi i/3}$). In the $Z_3 \times Z_3$ orbifold this degeneracy is broken because now the point group differentiates the three right-moving modes.
- 2) Since the twists $\theta, \omega, \theta\omega^2$ and their inverses leave one lattice unrotated these twisted sectors have nine fixed tori. This gives a degeneracy factor of 9 in the absence of Wilson lines. The sector $\theta\omega$ has 27 fixed points as in the Z_3 orbifold.

3) In the Z_3 orbifold matter supermultiplets of, say, positive chirality appear in the twisted sector whereas the corresponding antiparticles appear in the inversely twisted sector. In the $Z_3 \times Z_3$ orbifold the situation is different as supermultiplets of + chirality can also appear in the inversely twisted sectors. For instance, in the θ -twisted sector there are supermultiplets of + chirality together with supermultiplets of - chirality that are the antiparticles of + chirality supermultiplets appearing in the θ^2 -sector, and likewise for the ω and $\theta\omega^2$ sectors. The $\theta\omega$ sector behaves like the Z_3 twisted sector, thus it gives rise to + chirality multiplets with the corresponding antiparticles arising from the inverse $\theta^2\omega^2$ -sector. We will generically denote multiplets of + chirality in each sector as shown in table 9. The table also shows the $\vec{H}$ -momentum of the corresponding vertex operators in the -1 and $-\frac{1}{2}$ pictures.

4) In the Z_3 orbifold the only renormalizable Yukawa couplings in the superpotential allowed by the point group selection rule are U^3 and T^3 . UT^2 is forbidden and $UT\bar{T}$ is not a Yukawa coupling because $\bar{T}$ has chirality opposite to that of T . Because of property 3 discussed above in the $Z_3 \times Z_3$ orbifold we can have Yukawa couplings such as $UA\bar{A}$ allowed by point group invariance.

5) $\vec{H}$ -conservation also constrains the possible cubic couplings. For instance, $U_2A\bar{A}$ is allowed whereas $U_1A\bar{A}$ and $U_3A\bar{A}$ are forbidden. Table 10 shows all cubic and quartic couplings allowed by both point group and $\vec{H}$ -conservation selection rules. There are also many higher order couplings allowed by these selection rules. However, unlike the Z_3 case, it is not possible to give a simple general expression for them. A case-by-case analysis is better suited to the purpose of checking whether a coupling is allowed or not. In checking $\vec{H}$ -invariance of non-renormalizable couplings care must be taken to consider all possible field assignments to V_0 vertices in the correlator. For instance, the coupling $\bar{A}BCU_i$ respects $\vec{H}$ -conservation when we choose $U_i, \bar{A}$ or C to be in the 0-picture. However, it violates $\vec{H}$ -conservation if we instead choose B in the 0-picture. With this last choice it is then clear that the coupling $\bar{A}BCU_i$ vanishes.

6) Yukawa couplings are further constrained by selection rule 3. Thus, cubic couplings involving only one oscillator mode always vanish, just as in the Z_3 case. Quartic couplings are always proportional to a derivative ∂X^i arising from the V_0 vertex in the correlator. For example, $A^2\bar{B}D$ turns out to be proportional to ∂X^3 . This coupling then vanishes unless it involves oscillator modes that pick up a compensating phase

under a Z_3 rotation of the third sub-lattice. This type of analysis must be carried out to determine whether a coupling is allowed. For completeness, table 10 also shows to which derivatives the quartic couplings are proportional.

Armed with the above properties we are ready to discuss particular $Z_3 \times Z_3$ models. The $Z_3 \times Z_3$ orbifold projection needed to determine the massless spectrum is described in Appendix A.

A three-generation $Z_3 \times Z_3$ example

An easy method to get a three-generation model is to start from a non-standard embedding (shifts A and B) such that it contains nine generations in one of the singly twisted sectors (A , B or C ; $\overline{A}$, $\overline{B}$ or $\overline{C}$), each one of which contains 9 fixed tori. Then adding a Wilson line in one of the 3 lattices, the 9 twisted states will split (as happened with Z_3) into $3 + 3 + 3$ different states. Three of them will inherit the families. As an example consider the shifts

$$\begin{aligned} A &= \frac{1}{3}(2, 1, 1, 0, \dots, 0) \times (1, 1, 0, \dots, 0) \\ B &= \frac{1}{3}(0, 2, 0, \dots, 0) \times (0, -1, 1, 1, 1, 0, 0, 0) \end{aligned} \quad (5.1)$$

which fulfil the modular invariance constraints. This leads to an unbroken gauge group

$$SO(10) \times [SU(7)' \times SU(2)' \times U(1)^4] \quad (5.2)$$

This model has nine $SO(10)$ generations (16's) in the A -twisted sector. We can further break the $SO(10)$ symmetry and reduce the number of generations to three by adding the Wilson line

$$a_1 = \frac{1}{3}(0, \dots, 0, 1, 1, -2)(0, \dots, 0, 1, 1, -2) \quad (5.3)$$

in the first of the three $SU(3)$ tori. The shifted gauge momenta are now:

$$(P \pm A + na_1) \quad ; \quad (P \pm C + na_1) \quad ; \quad (P \pm D + na_1) \quad (5.4)$$

with $n = 0, \pm 1$. The twisted fields in the $\pm B$ sectors do not feel the presence of the Wilson line since the B -twist does not rotate the first $SU(3)$ torus in which the Wilson line resides. The gauge symmetry is broken down to

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)^6 \times [SU(4)' \times SU(3)' \times SU(2)'] \quad (5.5)$$

Let us assume for the moment the version of the model with no discrete torsion. One can compute the massless spectrum and after carefully projecting out a good number of states in the manner described in Appendix C, one finds the massless fields in table 11. There are three $SU(3)_C \times SU(2)_L \times SU(2)_R$ generations in the A sector and a large number of other fields. The six $U(1)$ generators are shown in table 12. In the same table the five traceless (anomaly-free) linear combinations U_i are shown along with the anomalous generator

$$X = (-5Q_1 + 9Q_2 + Q_3) - (Q_5 - Q_6) \quad (5.6)$$

One can easily make a (computer) search for flat directions in which the Fayet-Iliopoulos term corresponding to the above anomalous $U(1)$ is cancelled. The simplest $I(\phi)$ functions occur at order six and there are only two (out of a total of 7) which preserve $SU(2)_L$:

$$(5 \times 11 \times 12^2 \times 75^2) \quad (5.7a)$$

$$(6 \times 12^3 \times 75^2) \quad (5.7b)$$

where the numbers denote the fields in table 11. In fact there are untwisted renormalizable couplings $(5 \times 11 \times 2)$ and $(6 \times 12 \times 2)$ which endanger the above flat directions. In the case of the field direction (5.7.a) this can be mended by adding the invariant block $(25 \times 43 \times 73)$ which contains two oscillator fields and is forbidden as a Yukawa coupling by rule 3. Then the following field direction is flat:

$$\begin{aligned} 5 = 11 = \begin{pmatrix} \nu \\ 0 \end{pmatrix}_R \quad ; \quad 12 = \begin{pmatrix} 0 \\ \nu \end{pmatrix}' \quad ; \quad 75 = \begin{pmatrix} 0 \\ \nu \end{pmatrix}_R \\ 43 = 25 = \begin{pmatrix} \nu \\ 0 \end{pmatrix}' \quad ; \quad 73 = \nu \end{aligned} \quad (5.8)$$

The coupling $(5 \times 11 \times 2)$ does not spoil flatness because both 4 and 11 have the vev in the same $SU(2)_R$ entry. Notice that $SU(2)_R$ is necessarily broken (as well as $SU(2)'$) and thus the anomaly structure (F-I term) does not allow us to maintain a left-right symmetry in this model. The unbroken gauge group is thus of the form $SU(3) \times SU(2)_L \times U(1) \times G'$.

Once the vev's indicated in eqs. (5.8) appear, a substantial number of fields, as usual, get large masses. We give here some examples of allowed renormalizable Yukawa

couplings involving the fields with flat directions so that the reader can check the $Z_3 \times Z_3$ orbifold selection rules:

$$\begin{array}{lll}
(2 \times 6 \times 12) & (16 \times 22 \times 5) & (16 \times 58 \times 75) \\
(57 \times 64 \times 12) & (19 \times 24 \times 5) & (33 \times 67 \times 75) \\
& (33 \times 35 \times 5) & (37 \times 44 \times 75) \\
& (34 \times 36 \times 5) & (38 \times 71 \times 75)
\end{array} \tag{5.9}$$

etc. Notice that there are couplings of types $(U)^3$, (UTT) and (TTT) . There are also a good number of allowed non-renormalizable couplings of order 4 like e.g.

$$(6 \times 43 \times 26 \times 66) \quad (UTTY \text{ type}) \tag{5.10a}$$

$$(22 \times 23 \times 51 \times 73) \quad (TTTT \text{ type}) \tag{5.10b}$$

etc. We invite the reader to check that all selection rules (particularly $\vec{H}$ -momentum conservation) are fulfilled.

This particular model, however, does not look phenomenologically very promising. Although the existing renormalizable and non-renormalizable Yukawa couplings give large masses to a substantial part of the massless spectrum in table (11), there still remain too many massless fields, particularly $SU(2)_L$ doublets. We hope however that this example illustrates some of the particular properties of three generations $Z_3 \times Z_3$ orbifold models. We believe that the existence of many new types of Yukawa couplings ($UTT, TTTT$, etc.) may facilitate the obtaining of interesting quark-lepton spectra. A systematic search of phenomenologically interesting $Z_3 \times Z_3$ models, as has already been done for Z_3 , is still to be made.

Let us finally comment that one may obtain a new three-generation $Z_3 \times Z_3$ model starting from the above example and adding discrete torsion (a phase $\epsilon = \exp(i2\pi/3)$ in the projector). We have in fact computed the spectrum of this “torsion” model and although the twisted massless spectrum changes completely, the overall structure of the model is quite similar to the parent model. It is however amusing to check how all the spectrum gets reshuffled and still the final theory is anomaly free (as it should be since the procedure preserves modular invariance).

VI. CONCLUSIONS

We have tried to present in a systematic way the essential tools and formalism required for the construction of “realistic” 4-dimensional strings in terms of orbifold compactifications. The formalism is quite general and applies to the large class of models based on $(0,2)$ Abelian Z_N and $Z_N \times Z_M$ orbifolds including Wilson line backgrounds. Special use is made of selection rules derived from the conformal field theory of orbifolds to determine the renormalizable and non-renormalizable Yukawa couplings. Another general feature is the presence of an anomalous $U(1)$ whose anomaly is cancelled by the 4-dimensional version of the Green-Schwarz mechanism. This $U(1)$ has an associated Fayet-Iliopoulos term which very often triggers further gauge symmetry breaking which lowers the rank of the group, a very welcome property. Several models are discussed in the text including a three-generation $SU(3) \times SU(2) \times U(1)$ model and a $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)^3$ left-right symmetric model, both based on the Z_3 orbifold. The Yukawa couplings of these models are also analyzed. They present interesting features including the possibility of a hierarchy of quark-lepton masses. In addition a detailed study of the $Z_3 \times Z_3$ orbifold including a new 3-generation $SU(3) \times SU(2) \times U(1)$ model is presented.

A complete phenomenological analysis of the above models including the effects of supersymmetry breaking and (possibly) an intermediate scale of symmetry breaking is still to be done and certainly worthwhile. However we have decided to stop at the present level in order to make it clear how far we can go using only string theory techniques rather than field theory arguments. At present, one of the unsolved mysteries in strings is the understanding of supersymmetry breaking, which is supposed to occur non-perturbatively [37]. Therefore any discussion of 4-D models including supersymmetry breaking effects has to be done at the field theory level.* Another missing ingredient at the moment is the exact expression for the kinetic terms, which is certainly necessary to know the physical Yukawa couplings for instance. In principle the determination of those terms, given by the Kähler potential, can be done using supergravity and CFT techniques, although in practice it is not straightforward to find them. This is certainly a direction where more progress will be made in the near future. New models, built

* For an interesting discussion on these matters the reader can consult reference [34].

using the tools described in the text which could eventually lead to experimental tests of superstring theory, are still to be studied. We hope we have encouraged the patient reader to participate in this challenge.

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APPENDIX A

The general properties of the spectrum of a string propagating on an orbifold follow from the one-loop partition function. This partition function can be written as

$$Z = \int \frac{d^2\tau}{\tau_2^2} \mathcal{I}(\tau) \quad (\text{A.1})$$

where the integration is over the modular parameter $\tau = \tau_1 + i\tau_2$. $\mathcal{I}(\tau)$ is given by the path-integral of e^{-S} on a world sheet torus with coordinates (σ, t) and metric characterized by τ . $\mathcal{I}(\tau)$ must be invariant under modular transformations $\tau \rightarrow a\tau + b/(c\tau + d)$, where $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$.

In an orbifold $\mathcal{I}$ includes contributions from untwisted and twisted sectors, this is

$$\mathcal{I} = \frac{1}{|P|} \sum_{[h,g]=0} \epsilon(h,g) I(h,g) \quad (\text{A.2})$$

where $|P|$ is the order of the point group P . $I(h,g)$ is the path-integral evaluated with boundary conditions twisted by $h(g)$ in the $\sigma(t)$ direction and with τ fixed (the τ dependence is dropped for simplicity). The coefficients $\epsilon(h,g)$ are phases fixed by requiring modular invariance of $\mathcal{I}$. $\epsilon(h,g)$ is called discrete torsion. Modular transformations change the boundary conditions. For example, under $\tau \rightarrow \tau + 1$, $(h, 1) \rightarrow (h, h)$. We then require $I(h, 1) \rightarrow I(h, h)$ and $\epsilon(h, 1) \rightarrow \epsilon(h, h)$ under this transformation. In general $(h, g) \rightarrow (g^c h^d, g^a h^b)$ under $\tau \rightarrow a\tau + b/(c\tau + d)$. Thus, the various contributions to $\mathcal{I}$ are related by modular transformations. Keeping h fixed in (A.2) and summing over g implements the generalized GSO-projection as we will see shortly.

At one-loop there might be sectors not mixed by modular transformations thus leaving some $\epsilon(h,g)$ arbitrary. However, higher loop modular invariance and factorization imply restrictions on $\epsilon(h,g)$ [19]. For Z_N orbifolds with P elements $\theta^m, m = 0, \dots, N-1$ we find $\epsilon(\theta^m, \theta^n) = 1, \forall m, n$. For $Z_N \times Z_M$ with P elements $\theta^k \omega^\ell (\theta^N = \omega^M = 1, M$ multiple of $N)$ we find $\epsilon(\theta^k \omega^\ell, \theta^+ \omega^s) = \epsilon^{ks - \ell t}$ where ϵ is an N^{th} -root of unity. Thus, in $Z_N \times Z_M$ orbifolds there is a freedom in choosing ϵ .

The explicit form of $I(h,g)$ can be derived either in the path-integral or in the operator formalism. Here we skip the details since for the purpose of obtaining the degeneracy factor of states it is enough to deal with a simplified form of $I(h,g)$. For Z_N

orbifolds we find

$$\begin{aligned}
I(\theta^m, \theta^n) = & e^{i\pi mn(V^2 - v^2)} \tilde{\chi}(\theta^m, \theta^n) \times \\
& \left\{ \sum_{\Sigma r_i = \text{odd}} e^{2\pi i(r+mv)nv} [q^{\frac{1}{2}(r+mv)^2} - q^{\frac{1}{2}(r+mv+\frac{1}{2})^2}] q^{E_0 - \frac{1}{2}} (1 + \dots) \times \right. \\
& \left. \left\{ \sum_{P \in \Lambda_{E_8 \times E_8}} e^{-2\pi i(p+mV)nV} \bar{q}^{\frac{1}{2}(P+mV)^2} \bar{q}^{E_0 - 1} (1 + \dots) \right\} \right\} \quad (A.3)
\end{aligned}$$

where $q = e^{2\pi i\tau}$. Let us now explain the various terms in $I(\theta^m, \theta^n)$.

$\tilde{\chi}(\theta^m, \theta^n)$ is a numerical factor that takes into account the fixed point degeneracy. It is given by

$$\begin{aligned}
\tilde{\chi}(1, \theta^n) &= 1 \\
\tilde{\chi}(\theta^m, \theta^n) &= \chi(\theta^m, \theta^n) \quad \text{if } \chi(\theta^m) \neq 0 \\
\tilde{\chi}(\theta^m, \theta^n) &= \hat{\chi}(\theta^m, \theta^n) \quad \text{if } \chi(\theta^m) = 0
\end{aligned} \quad (A.4)$$

where $\chi(\theta^m, \theta^n)$ is the number of simultaneous fixed points of θ^m and θ^n . If θ^m leaves fixed tori, i.e. $\chi(\theta^m) = 0$, we must use $\hat{\chi}(\theta^m, \theta^n)$ which is the number of simultaneous fixed points in the subspace actually rotated by θ^m .

The first and second brackets in (A.3) are the contributions of the right and left movers respectively. r is a vector (r_0, r_1, r_2, r_3) with integer entries (the first entry refers to the uncompactified fermions). v is the twist vector, $v = (0, v_1, v_2, v_3)$. Notice the appearance of NS and R sectors together with the GSO projection $\sum r_i = \text{odd}$. V is the gauge embedding shift and P belongs to the $E_8 \times E_8$ lattice as mentioned in chapter II. E_0 is given in (2.12).

Powers of $q(\bar{q})$ in (A.3) give the right (left) mass levels. Higher order terms in the q and $\bar{q}$ expansions give oscillator states. The leading term gives massless non-oscillator states with $\frac{1}{2}(r+mv)^2 + E_0 = \frac{1}{2}$ and $\frac{1}{2}(P+mV)^2 + E_0 = 1$. Keeping m fixed and summing over n gives the degeneracy factor $D(\theta^m)$ of these states. We readily obtain [8]

$$\begin{aligned}
D(\theta^m) &= \frac{1}{N} \sum_{n=0}^{N-1} \tilde{\chi}(\theta^m, \theta^n) \Delta(m, n) \\
\Delta(m, n) &= \exp\{2\pi i[(r+mv)nv - (P+mV)nV + \frac{1}{2}mn(V^2 - v^2)]\} \quad (A.5)
\end{aligned}$$

The form of E_0 and the GSO-projection imply that there are no massless states with right oscillators. However, there can be massless states with left oscillators. In this case, the exponent in $\Delta(m, n)$ should include a term $2\pi i n \rho$ where $e^{2\pi i \rho}$ is the eigenvalue of the oscillator (or product thereof) under θ . There is also an extra multiplicity (given in the $\bar{q}$ expansion) that indicates the number of oscillators at the required energy level. (A.5) can be regarded as a generalized GSO-projection. States satisfying the massless condition but having zero degeneracy are effectively projected out of the massless spectrum.

We have then seen that states can be labelled by r , P and possible oscillator numbers. Notice that the first entry in r gives space-time properties of the state. For example, in the NS sector $(\pm 1, 0, 0, 0)$ corresponds to vectors whereas $(0, \pm \underline{1}, 0, 0)$ corresponds to scalars (underlining means permutations). In the left sector there might be space-time (uncompactified) oscillators, thus providing further Lorentz indices for the state. This is the case of the supergravity multiplet. The Yang-Mills and supergravity multiplets always appear in the untwisted sector. Massless matter (chiral) multiplets appear in the untwisted and twisted sectors. Gauge properties of states come from the left part. The possible $(P + mV)$ can be arranged into weights of irreducible representations of the gauge group. In particular, the weights of the adjoint representation (i.e. the roots) are those P 's satisfying $P^2 = 2$ and $P \cdot V = 0$ (also $P \cdot a_\alpha = 0$ if there are Wilson lines), so that $\Delta = 0$ and the degeneracy is 1. In this way we identify the gauge group.

For the singly twisted sector ($m = 1$) (A.5) yields

$$D(\theta) = \frac{\chi(\theta)}{N} [1 + \Delta + \dots + \Delta^{N-1}]$$

$$\Delta = \exp\{2\pi i[(r + v)v - (P + V)V + \frac{1}{2}(V^2 - v^2) + \rho]\} \quad (\text{A.6})$$

where $\rho = 0$ for non-oscillator states. (A.6) follows from $\chi(\theta, \theta^n) = \chi(\theta)$ and in turn this equality holds because every fixed point of θ is a fixed point of θ^n (if θ leaves fixed tori we should replace $\chi(\theta)$ by $\hat{\chi}(\theta)$). Furthermore, it is straightforward to show that $\Delta = 1$ for all massless states in this θ -sector. Therefore, $D(\theta) = \chi(\theta)$ for all massless θ -twisted states. Clearly this is also true for all massless states in the $\theta^{-1} = \theta^{N-1}$ sector. For $m \neq \pm 1$ a simple expression such as (A.6) is no longer possible because, in general, every fixed point of θ^m is not a fixed point of θ^n . This is particularly true for non-prime order orbifolds. For the Z_3 orbifold we only have $\theta^{\pm 1}$ sectors with a degeneracy factor

$D(\theta) = \chi(\theta) = 27$. For the other prime order orbifold, namely the Z_7 orbifold, we can show that $\chi(\theta^m, \theta^n) = 7 \forall m, n$.

A degeneracy factor similar to (A.5) is also obtained in the presence of Wilson lines for the twisted sectors. In this situation we have $V \rightarrow V + m_\alpha a_\alpha$ and the fixed point degeneracy $\tilde{\chi}$ must be appropriately changed. For example, consider the Z_3 orbifold with $[SU(3)]^3$ lattice and one Wilson line a_1 on the first sub-lattice. The θ -twisted sector now splits into three, one for each fixed point of θ in the first sub-lattice. These sectors have shifts $V + n_1 a_1, n_1 = 0, \pm 1$ and a fixed point degeneracy of 9 corresponding to the fixed points of θ in the sub-lattices not touched by a_1 . With two Wilson lines there are nine twisted sectors with shifts $V + n_1 a_1 + n_3 a_3, n_1, n_3 = 0, \pm 1$ and fixed point degeneracy of 3. With three Wilson lines the degeneracy is completely broken.

For $Z_N \times Z_M$ orbifolds we can follow the same steps as in the derivation of (A.5) to obtain an expression for the degeneracy factor in a $\theta^k \omega^\ell$ twisted sector. We recall that in this case there is an extra phase due to $\epsilon(h, g)$. The final result is [17]

$$D(\theta^k \omega^\ell) = \frac{1}{MN} \sum_{t=0}^{N-1} \sum_{s=0}^{M-1} \epsilon^{(ks-\ell t)} \tilde{\chi}(\theta^k \omega^\ell, \theta^s \omega^t) \Delta(k, \ell; t, s)$$

$$\Delta(k, \ell; t, s) = \exp\{2\pi i[r + ka + \ell b](ta + sb) - (p + kA + \ell B)(tA + \ell B) + \frac{1}{2}((kA + \ell B)(tA + sB) - (ka + \ell b)(ta + sb)) + t\rho_a + s\rho_b]\}$$
(A.7)

where $\rho_{a,b}$ appear in the case of oscillator states and indicate how the oscillator (or product thereof) is rotated under θ and ω respectively. $\tilde{\chi}$ is computed according to the generalization of (A.7).

In the presence of Wilson lines (A.7) must be appropriately modified. For example let us consider the $Z_3 \times Z_3$ orbifold with twist vectors $a = \frac{1}{3}(1, 0, -1), b = \frac{1}{3}(0, 1, -1), [SU(3)]^3$ lattice and one Wilson line a_1 in the first sub-lattice. Then, the twisted sectors split as follows. The sectors $\theta, \theta\omega^{-1}$ (and their inverses) each split into three sectors with gauge shifts $A + n_1 a_1, A + n_1 a_1 - B, n_1 = 0, \pm 1$, respectively (and similarly for the inverses). For these sectors we then make the replacement $A \rightarrow A + n_1 a_1$ in (A.7). The fixed point degeneracy is reduced from 9 to 3. Likewise, the twisted sector $\theta\omega$ (and its inverse) splits into three sectors with shifts $A + n_1 a_1 + B$ and fixed point multiplicity reduced from 27 to 9. The sector $\omega(\omega^{-1})$ does not feel the presence of the

Wilson line because ω does not touch the first sub-lattice. $D(\omega)$ is slightly more complicated to compute. The terms $\epsilon(\omega, \omega^s) \tilde{\chi}(\omega, \omega^s) \Delta(B; sB)$, where $\Delta(B; sB) = \Delta(0, 1; 0, s)$, are unchanged. However, a term $\epsilon(\omega, \theta) \tilde{\chi}(\omega, \theta) \Delta(B; A)$, for instance, splits into

$$\frac{\epsilon(\omega, \theta) \tilde{\chi}(\omega, \theta)}{3} \{ \Delta(B; A) + \Delta(B; A + a_1) + \Delta(B; A - a_1) \} \quad (A.8)$$

Here $\Delta(B; A + n_1 a_1)$ is given by

$$\begin{aligned} \Delta(B; A + n_1 a_1) = & \exp\{2\pi i[(r + b)a - (p + B)(A + n_1 a_1) \\ & + \frac{1}{2}((A + n_1 a_1)B - ab) + \rho_a + \rho_b]\} \end{aligned} \quad (A.9)$$

Other terms involving θ are split in a similar fashion. Using $\tilde{\chi}(\omega, \theta^t \omega^s) = 9$ for $Z_3 \times Z_3$ we readily find

$$\begin{aligned} D(B) = & \frac{1}{3} \{1 + \Delta(B; B) + \Delta^*(B; B)\} \times \\ & \{3 + [\epsilon^*(\Delta(B; A) + \Delta(B; A + a_1) + \Delta(B; A - a_1)) + c \cdot c]\} \end{aligned} \quad (A.10)$$

We can show that $\Delta(B; B) = 1$ for all massless states in the B -sector. Thus (A.9) can be further simplified.

APPENDIX B

Vertex Operators

In this appendix we will provide the vertex operators which create the massless fields in the spectrum. For this it is usually convenient to work in the covariant, Euclidean formulation of reference [38]. The world sheet parameters σ, t are expressed in terms of the complex variable z and its complex conjugate $\bar{z}$. The right moving coordinates are split as $x^\mu(z)$ $\mu = 1, \dots, 4$, and the complexified internal ones $X^i(z), \bar{X}^i(z)$, $i = 1, \dots, 3$, whereas the NSR fermions are represented by five free boson fields $H^m(z)$. The left-moving fields are then $x^\mu(\bar{z}), X^i(\bar{z}), \bar{X}^i(\bar{z})$ and sixteen bosons $F^I(\bar{z})$ with momenta in the $E_8 \times E_8$ lattice. Working with a covariant approach implies that conformal and superconformal ghosts have to be considered. In particular a free scalar $\phi(z)$, related to the superconformal ghost system usually appears in the form $e^{q\phi}$ where q is called the ghost charge.

Every particle in the spectrum is created by the vertex operators $V(z, \bar{z})$ which are conformal fields with dimension (1,1) and are usually expressed in terms of products $V_R(z) \otimes V_L(\bar{z})$ (complemented by an orbifold projection that relates the left and right parts). The right-moving vertex operators $V_R(z)$ can be written in different but equivalent “pictures”. This is due to the fact that the right moving sector is supersymmetric and each picture is labelled by a value of the ghost charge. Physical amplitudes are independent of the ghost picture used; however, due to the fact that the superconformal vacuum carries ghost charge, the total charge of the vertex operators entering into a given amplitude must add up to -2 for tree level interactions. The simplest pictures for boson and fermion fields are the ones with ghost charge -1 and $-\frac{1}{2}$ respectively. Vertex operators in any other picture can be obtained from V_{-1} and $V_{-\frac{1}{2}}$ by the picture changing operation, for instance

$$V_0(z) = \lim_{\omega \rightarrow z} e^{\phi(\omega)} T_F(\omega) V_{-1}(z) \quad (B.1)$$

where $T_F(\omega)$ is the world sheet supercurrent (supersymmetric partner of the stress energy tensor) which is represented as

$$T_F(\omega) = \partial X^\mu \lambda^\mu + \partial X^i \bar{\lambda}^i + \partial \bar{X}^i \lambda^i \quad (B.2)$$

The NSR fermions $\lambda^\mu, \lambda^i, \bar{\lambda}^i$ are expressed in terms of the H^m field mentioned above, i.e. $\lambda \sim e^{i\alpha_v \cdot H}$ where the α_v ’s are the $SO(10)$ weights $(\pm 1, 0, 0, 0, 0)$ and permutations. Cycle factors, needed to satisfy fermionic anti-commutation properties, are consistently neglected here.

Therefore we will only need to concentrate on the simplest pictures and, when needed, use (B.1) to find the representation in other pictures. For the untwisted sector we can immediately write:

$$V_{-1} = e^{-\phi} e^{i\alpha_v H} e^{ikx} \quad (B.3)$$

$$V_{-\frac{1}{2}} = e^{-\frac{\phi}{2}} e^{i\alpha_s H} e^{ikx} \quad (B.4)$$

where k^μ is the 4-D momentum, α_v was described above and α_s is a spinorial weight of $SO(10)$, $\alpha_s = \frac{1}{2}(\pm 1, \pm 1, \dots, \pm 1)$ with an even number of $+$ signs. Notice that the conformal dimension of $e^{q\phi}$ is $-\frac{1}{2}q(q+2)$ and therefore it is easy to check that the vertex

operators above indeed have conformal dimension 1. This is equivalent to the massless condition in equation (2.9) of the text.

We have seen that the construction of untwisted vertex operators is straightforward; they are just the same as for flat space with the only condition that the total vertex operator $V_R \otimes V_L$ has to be invariant under the orbifold rotations which is implemented by the generalized GSO projection of the physical states. The construction of twisted-sector vertex operators requires a new feature, however. This is the so-called twist field. We can define a twist field $\Lambda(z)$ as the field which creates the twisted vacuum out of the untwisted one, that is

$$\Lambda(0)|0\rangle = |\Lambda\rangle \quad (B.5)$$

There is then a different twist field for every sector in the Hilbert space and so the twist fields are in one to one correspondence with the space group conjugacy classes. Geometrically, we can think that in the vicinity of a twist field the coordinates X undergo an order N phase rotation. Twist fields for world-sheet fermions are just a generalization of the spin field $e^{i\alpha_s H}$, that is for a Z_N orbifold a fermionic twist field can be represented by e^{ivH} where $v = (0, 0, v_1, v_2, v_3)$ and v_i are defined in equation (2.4) of the text. Notice that this v has one more component than the one we have used in chapter II and Appendix A. This is because we are now working in the covariant formalism in which $SO(8)$ is replaced by $SO(10)$. The bosonic twist fields Λ however, do not have a simple representation in terms of free fields. The only information we have available is their OPE (operator product expansion) with the other conformal fields in the theory. In particular the OPE with the stress energy tensor give us their conformal dimension to be $\frac{1}{2} \sum_{i=1}^3 |v_i|(1 - |v_i|)$.

With this information at hand, we can now write down the vertex operators for the emission of twisted sector states in the right moving sector.

$$V_{-1} = e^{-\phi} e^{i(\alpha_v + v)H} e^{ikx} \Lambda \quad (B.6)$$

$$V_{-\frac{1}{2}} = e^{-\frac{\phi}{2}} e^{i(\alpha_v + v)H} e^{ikx} \Lambda \quad (B.7)$$

We can check again that the mass conditions of equations (2.11) are obtained by requiring the conformal dimension of the vertex operators above to be 1. Strictly speaking we have only described the vertex operators for the fields in a sector twisted by θ . Actually for

a sector twisted by θ^m we will have to change v by mv and consider the corresponding twist field; in particular, for $m = 0$ and $\Lambda = 1$ we obtain the untwisted vertex operators. Even though the twist fields have to be labelled by the space group conjugacy classes, we consistently omit those labels.

Before considering the left-moving parts of the vertex operators, we notice that the amplitudes we are considering are of the form

$$\langle V_{-\frac{1}{2}}(z_1)V_{-\frac{1}{2}}(z_2)V_{-1}(z_3)V_0(z_4)\cdots V_0(z_{n+3}) \rangle \quad (B.8)$$

This is because, as mentioned above, the total ghost charge must add up to -2 . Therefore, to consider non-renormalizable couplings, we need to know the expressions for V_0 both for twisted and untwisted sector fields. Plugging (B.3) and (B.5) on (B.1) we find the expression for V_0 in both sectors which we will write only at zero momentum, which is the relevant case for our purpose. For the untwisted sector we have

$$V_0^i(z) = \partial X^i \quad (B.9)$$

and for the twisted sector

$$V_0(z) = \lim_{w \rightarrow z} \sum_i e^{i(\alpha_v + v - \alpha^i)H(z)} (w - z)^{1-v_i} \partial_z X^i \Lambda(\omega) \quad (B.10)$$

where $\alpha^1 = (0, 0, 1, 0, 0)$, $\alpha^2 = (0, 0, 0, 1, 0)$, $\alpha^3 = (0, 0, 0, 0, 1)$.

We will now consider the left-moving part of the vertex operators. This is simpler since it is not supersymmetric and then there are no superghosts nor pictures. We only have to construct operators of dimension 1 out of combinations of $\bar{\partial}X^\mu$, $\bar{\partial}X^i$, $\bar{\partial}\bar{X}^i$ and $e^{iP \cdot F}$ with twist fields $\Lambda(\bar{z})$. For the untwisted sector we have the operators

$$e^{ikx} \bar{\partial}X^j, e^{ikx} \bar{\partial}\bar{X}^j, e^{ikx} e^{iP \cdot F} \quad (B.11)$$

where P belongs to the $E_8 \times E_8$ lattice and $P^2 = 2$ gives us the massless states. The first two, when combined with (B.3) (or (B.9)), create the untwisted moduli M_{ij} and $M_{i\bar{j}}$. The last term in (B.11) gives rise to untwisted charged matter fields. Notice that all these combinations of left and right states are implemented after projecting out the combinations which are not invariant under the orbifold transformation, i.e. the generalized GSO projection discussed in Appendix A.

Finally, the twisted left-moving states are created by

$$e^{ikx} e^{i(P+V)F} \Lambda \quad (B.12)$$

where V is the $E_8 \times E_8$ embedding introduced in section II. If Wilson lines are present, we simply make the replacement $(P + V) \rightarrow (P + V + n_\alpha a^\alpha)$. The states created by (B.12) are non-oscillator states. Twisted oscillator states, like the blowing-up modes, come from the same kind of vertices but Λ is substituted by an “excited” twist field Λ' . Effectively this introduces an extra factor of $\overline{\partial X}$ (or $\overline{\partial X}$ or products) in the vertex operator.

From all the information, we can see that the Yukawa-type amplitude

$$\langle V_{-\frac{1}{2}}(z_1, \bar{z}_1) V_{-\frac{1}{2}}(z_2, \bar{z}_2) V_{-1}(z_3, \bar{z}_3) V_0(z_4, \bar{z}_4) \cdots V_0(a_{n+3}, \bar{z}_{n+3}) \rangle \quad (B.13)$$

will take the general form

$$\langle \partial X \cdots \overline{\partial X} \cdots \overline{\partial X} \cdots \Lambda \cdots \Lambda \rangle \quad (B.14)$$

There are systematic procedures to compute such correlation functions and some general results can be extracted. In particular the formal proof of the selection rules and the vanishing of other couplings mentioned in section III can be obtained from this analysis. For our purposes, given an interaction, we only need to know the number of derivatives and twist fields and depending how they are arranged we can use the selection rules to see if the correlation vanishes identically. Examples of how to use this are provided in the text.

APPENDIX C

In the class of three-generation orbifold models with gauge group $SU(3) \times SU(2) \times U(1)^n \times G'$ the hypercharge Y that gives the correct assignments to candidate quarks and leptons is generally a combination of the various $U(1)$'s. We now want to explain briefly how to determine the normalization of Y and the $SU(N)$ generators, as these numbers are needed to fix the tree-level values of $\sin^2 \theta_W$ and other running quantities.

The vertex operator for gauge fields can be written as (in the 0-picture)

$$V^{\mu a} \sim e^{ikx} (\partial x^\mu + ik \cdot \psi \psi^\mu) J^a(\bar{z}) \quad (C.1)$$

where the currents $J^a(\bar{z})$ satisfy the operator product expansion (OPE)

$$J^a(\bar{z})J^b(\bar{w}) \sim \frac{\tilde{k}\delta^{ab}}{(\bar{z}-\bar{w})^2} + \frac{if^{abc}J^c(\bar{w})}{(\bar{z}-\bar{w})} + \dots \quad (C.2)$$

For the heterotic superstring, f^{abc} are the structure constants of $E_8 \times E'_8$ and $\tilde{k} = \frac{1}{2}$ so that the Virasoro algebra associated to the currents J^a has central charge $c = 16$, as it should. These currents satisfy a Kac-Moody algebra [39].

The currents $J^a(\bar{z})$ have a representation in terms of the left-handed internal bosons, $F_I(\bar{z})$, $I = 1, \dots, 16$, of the heterotic string. In the Cartan basis we have

$$E_\alpha = e^{iP_\alpha \cdot F} \quad , \quad P_\alpha^2 = 2 \quad (C.3)$$

$$H_i = ih_i \cdot \bar{\partial}F \quad (C.4)$$

where P_α belongs to the $E_8 \times E'_8$ root lattice and (C.3) should also include a cocycle factor. All the generators in the Cartan sub-algebra are invariant under the orbifold action which in the left sector is realized through a shift V and Wilson lines a_α . For the step operators, only those with $P_\alpha \cdot V = 0$ and $P_\alpha \cdot a_\alpha = 0$ are invariant.

A $U(1)$ generator, Q , of the unbroken gauge group has the form (C.4), i.e. $Q = iq \cdot \bar{\partial}F$ and satisfies the OPE

$$Q(\bar{z})Q(\bar{w}) \sim \frac{q^2}{(\bar{z}-\bar{w})^2} + \dots \quad (C.5)$$

Comparing with (C.2) shows that q must be normalized as $q^2 = \tilde{k} = \frac{1}{2}$ and similarly for the Cartan generators of the non-Abelian factors. This is equivalent to the standard normalization $\text{Tr} T^a T^b = \frac{1}{2} \delta^{ab}$ for $SU(N)$ generators in the fundamental representation.

The double pole term in (C.2) fixes the coupling of two gauge bosons to one graviton. In ref. [40], it is shown that the coupling g of the unbroken gauge group is related to the gravitational coupling κ by $\kappa/g \sim \sqrt{\tilde{k}}$. The hypercharge generator can also be written as $Y = iy \cdot \bar{\partial}F$ and it fulfils an OPE similar to (C.5). Thus, we see that the Y coupling g_Y satisfies $\kappa/g_Y \sim \sqrt{y^2}$. Since $\tilde{k} = \frac{1}{2}$ we readily obtain the tree-level relation

$$\left(\frac{g_Y}{g}\right)^2 = \frac{1}{2y^2} \quad (C.6)$$

y is determined by the requirement that candidate quarks and leptons in the spectrum have the correct Y -assignments. In general, y is not uniquely determined.

In the $SU(3) \times SU(2) \times U(1)$ model in section IV, the hypercharge in eq. (4.8) has $y^2 = 11/6$ and hence $g_Y^2/g^2 = 3/11$ leading to a Weinberg angle $\sin^2 \theta_W = g_Y^2/(g_Y^2 + g^2) = 3/15 \simeq 0.214$. This is to be compared with the standard $SU(5)$ prediction $\sin^2 \theta_W = 3/8$. This is just the tree-level value valid at the Planck mass and to compare it with the value measured at the weak scale one has to use the standard renormalization group equations. An analysis of the effect of renormalization group corrections on the computation of the Weinberg angle and unification scale can be found in ref. [41].

We wish to finish this discussion with a brief explanation of how $U(1)$ charges are computed. The gauge part of the vertex operator for a given particle has the form $e^{iP \cdot F}$ where P belongs to the $E_8 \times E'_8$ root lattice for the untwisted matter and to the shifted lattice for twisted matter. For a $U(1)$ generator $Q = iq \cdot \bar{\partial} F$ we have the OPE

$$Q(\bar{z})e^{iP \cdot F(\bar{w})} \sim \frac{q \cdot P}{(\bar{z} - \bar{w})} e^{iP \cdot F(\bar{w})} + \dots \quad (C.7)$$

Thus, the particle's charge is $q \cdot P$. Finally, from (C.7) and the generalization of (C.5) to two different $U(1)$'s we deduce that the $U(1)$ generators must be orthogonal to each other and to the Cartan and step operators of the non-Abelian factors. This is an expected result and provides a prescription for finding the various $U(1)$'s.

TABLE 1

(n_1, n_2)	Field	Rep.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	X	Y	U^1
(0,0)	Q_1	(3,2)	6	6	0	0	0	0	0	0	0	1
	u_1	(3,1)	6	0	0	6	0	0	0	0	0	4
	$\bar{G}_1$	(1,2)	0	6	0	6	0	0	0	0	0	3
	$16'$	1	0	0	0	0	0	0	0	9	0	0
	D_1	(3,1)	0	4	0	0	0	0	4	0	4	-2
	$\bar{G}_2$	(1,2)	6	-2	0	0	0	0	4	0	4	3
(1,0)	A_1	1	-3	-2	-3	-3	-3	4	0	4	4	-3
	A_2	1	-3	-2	3	-3	3	4	0	4	4	-3
	A_3	1	-3	-2	-3	3	3	4	0	4	4	3
	A_4	1	-3	-2	3	3	1	-6	-2	4	4	3
	A_5	1	-3	-2	-3	-3	1	0	-2	-8	-3	-3
	A_6	1	-3	-2	3	3	1	6	4	4	4	3
(-1,0)	S_4	1	6	4	0	0	-2	6	4	4	4	0
	S_5	1	6	4	0	0	-2	-6	-2	4	4	0
	S_6	1	6	4	0	0	-2	0	-2	-8	0	0
	A_7	1	-3	-2	-3	-3	1	6	4	4	4	-3
	A_8	1	-3	-2	-3	-3	1	-6	-2	4	4	-3
	A_9	1	-3	-2	-3	-3	1	0	-2	-8	-3	-3
(0,1)	d_1	(3,1)	0	0	0	2	2	-4	-4	4	4	2
	F_1	(1,2)	3	0	-3	-1	-1	-4	-4	4	4	0
	A_9	1	3	6	3	-1	-1	-4	-4	4	4	-3
	A_{10}	1	3	-6	3	-1	-1	-4	-4	4	4	3
	l_1	1	-6	0	0	-4	2	-4	-4	4	4	-6
	S_{10}	1	-6	0	0	2	-4	-4	-4	4	4	0
(1,1)	D_2	(3,1)	6	0	0	2	0	-2	0	4	4	4
	u_2	(3,1)	0	0	0	-4	0	-2	0	4	4	-4
	F_2	(1,2)	3	0	3	-1	-3	-2	0	4	4	0
	$\bar{F}_2$	(1,2)	3	0	-3	-1	3	-2	0	4	4	0
	S_1	1	-6	0	0	2	0	4	0	-8	0	0
	Y_1	1	-6	0	0	2	0	-2	0	4	4	0
(-1,1)	A_{10}	1	3	6	-3	-1	-3	-2	0	4	4	-3
	A_{11}	1	3	6	3	-1	3	-2	0	4	4	3
	A_{12}	1	3	-6	3	-1	3	-2	0	4	4	3
	A_{13}	1	3	-6	-3	-1	-3	-2	0	4	4	3
	d_2	(3,1)	0	0	0	2	-2	0	4	4	4	2
	F_3	(1,2)	3	0	3	-1	1	0	4	4	4	0
(0,-1)	D_3	(3,1)	3	2	3	1	1	0	-2	4	4	-1
	$\bar{G}_3$	(1,2)	0	2	0	4	-2	0	-2	4	4	3
	G_1	(1,2)	0	2	0	-2	4	0	-2	4	4	-3
	S_2	1	0	-4	0	-2	-2	0	4	-8	0	0
	Y_2	1	0	-4	0	-2	-2	0	-2	4	4	0
	l_2	1	0	-4	0	4	4	0	-2	4	4	6
(1,-1)	A_{13}	1	0	8	0	-2	-2	0	-2	4	4	-6
	A_{14}	1	-9	2	3	1	1	0	-2	4	4	-3
	A_{15}	1	9	2	-3	1	1	0	-2	4	4	3
	D_4	(3,1)	3	2	-3	1	-1	2	2	4	4	-1
	$\bar{G}_4$	(1,2)	0	2	0	4	2	2	2	4	4	3
	G_2	(1,2)	0	2	0	-2	-4	2	2	4	4	-3
(-1,-1)	S_3	1	0	-4	0	-2	2	-4	-4	-8	0	0
	Y_3	1	0	-4	0	-2	2	2	2	4	4	0
	l_3	1	0	-4	0	4	-4	2	2	4	4	6
	l_4	1	0	8	0	-2	2	2	2	4	4	-6
	A_{16}	1	-9	2	-3	1	-1	2	2	4	4	-3
	A_{17}	1	9	2	3	1	-1	2	2	4	4	3
(0,-1)	G_3	(1,2)	0	2	0	-2	0	-2	-6	4	4	-3
	G_4	(1,2)	0	2	0	-2	0	4	6	4	4	-3
	G_5	(1,2)	0	2	0	-2	0	-2	0	-8	-3	-3
	l_5	1	0	-4	0	4	0	4	6	4	4	6
	l_6	1	0	-4	0	4	0	-2	-6	4	4	6
	l_7	1	0	-4	0	4	0	-2	0	-8	6	6

Table 1: Orbifolds massless spectrum for the model of section (IV.ii).

TABLE 2

$Q_1 = 6(1, 1, 1, 0, \dots, 0) \times (0, \dots, 0)'$
$Q_2 = 6(0, 0, 0, 1, -1, 0, 0, 0) \times (0, \dots, 0)'$
$Q_3 = 6(0, \dots, 0, 1, 0, 0) \times (0, \dots, 0)'$
$Q_4 = 6(0, \dots, 0, 0, 1, 0) \times (0, \dots, 0)'$
$Q_5 = 6(0, \dots, 0, 0, 0, 1) \times (0, \dots, 0)'$
$Q_6 = 6(0, \dots, 0) \times (1, 1, 0, 0, \dots, 0)'$
$Q_7 = 6(0, \dots, 0) \times (0, 1, 1, 0, \dots, 0)'$
$X = 6(0, \dots, 0) \times (1, -1, 1, 0, \dots, 0)'$

Table 2: $U(1)$ charge generators for the model of section (IV.ii).

TABLE 3

$d_1 \ D_1 \ S_2$	$\bar{D}_1 \ D_3 \ S_2$	$u_2 \ D_2 \ S_1$	
$d_2 \ D_1 \ S_3$	$\bar{D}_2 \ D_4 \ S_3$		
<hr/>			
$\bar{G}_3 \ G_1 \ S_2$	$\bar{G}_4 \ G_2 \ S_3$	$F_1 \ F_4 \ S_1$	$*\bar{G}_2 \ G_5 \ Y_1$
$\bar{G}_3 \ G_4 \ S_3$	$\bar{G}_4 \ G_3 \ S_2$	$F_2 \ F_3 \ S_1$	
$*\bar{G}_3 \ G_5 \ Y_3$	$*\bar{G}_4 \ G_5 \ Y_2$		
<hr/>			
$\bar{l}_1 l_2 \ S_8$	$\bar{l}_2 l_1 \ S_6$	$\bar{l}_3 l_1 \ S_2$	$\bar{l}_4 l_2 \ S_3$
$\bar{l}_1 l_3 \ S_6$	$\bar{l}_2 l_4 \ S_8$	$\bar{l}_3 l_3 \ S_3$	$\bar{l}_4 l_4 \ S_2$
$\bar{l}_1 l_5 \ S_4$	$\bar{l}_2 l_5 \ S_7$	$*\bar{l}_3 l_5 \ Y_3$	$*\bar{l}_4 l_5 \ Y_2$

Table 3: Renormalizable couplings involving at most one hypercharge singlet in the model of section (IV.ii). Couplings marked with an asterisk are forbidden by selection rule 3.

TABLE 4

SM field	string state	#	Y
Q_L	Q_L^i	3	1/6
U_L^c	$u_1^3 ; (u_1^1 \sim u_2); (u_1^2 \sim u_2)$	3	-2/3
D_L^c	$(d_1 - d_2)$	3	1/3
L	G_5	3	-1/2
E^c	$(l_1 + l_2) - (l_3 + l_4)$	3	1
H	$(G_2 - G_3)$	1	-1/2
$\bar{H}$	$\bar{G}_1^3$	1	1/2
L'	$(G_1 - G_4)$	3	-1/2
$\bar{L}'$	$\bar{G}_2$	3	1/2
E_R'	l_5	3	1
$\bar{E}_R'$	$(\bar{l}_1 + \bar{l}_2) - (\bar{l}_3 + \bar{l}_4)$	3	-1
N	$S_i, Y_i, \dots$		0

Table 4: Massless fields after symmetry breaking in the model of section (IV.ii)

TABLE 5

	n_1	n_2	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_{B-L}	Y_1	Y_2	Rep
1			-6	-6	0	0	0	0	0	-1	0	3	Q_R
2			6	0	-6	0	0	0	0	1	0	-3	Q_L
3			0	6	6	0	0	0	0	0	0	0	H
4			0	0	0	0	3	-3	0	0	0	0	S_2
5			0	0	0	0	-6	6	0	0	0	0	u
6	0	0	0	4	0	0	4	0	0	2	0	-6	D_1
7	0	0	-3	-2	0	3	4	0	0	3	0	-1	L_1
8	0	0	6	-2	0	0	4	0	0	3	1	-1	R_1
9	0	0	-3	-2	-6	-3	4	0	0	0	0	-8	S_3
10	0	0	-3	-2	6	-3	4	0	0	0	1	0	S_3
11	1	0	-3	-2	6	1	-2	0	-2	0	0	8	T_1
12	1	0	-3	-2	-6	1	-2	0	-2	0	-1	0	T_2
13	1	0	6	4	0	-2	-2	0	-2	0	0	0	T_3
14	1	0	-3	-2	6	1	4	0	4	0	1	0	S_4
15	1	0	-3	-2	-6	1	4	0	4	0	0	-8	S_5
16	1	0	6	4	0	-2	4	0	4	0	1	-8	S_6
17	-1	0	-3	-2	0	-1	-2	0	2	-3	0	1	L_T
18	-1	0	-3	-2	0	-1	4	0	-4	3	0	-1	L_2
19	-1	0	6	4	0	2	-2	0	2	0	0	0	T_4
20	-1	0	6	4	0	2	4	0	-4	6	0	-2	S_7
21	0	1	0	0	4	0	0	-4	0	-2	0	6	D_1
22	0	1	-6	0	-2	0	0	-4	0	-3	-1	1	L_3
23	0	1	3	0	-2	-3	0	-4	0	-3	0	1	R_2
24	0	1	3	6	-2	3	0	-4	0	0	-1	0	S_8
25	0	1	3	-6	-2	3	0	-4	0	0	0	8	S_9
26	0	-1	3	2	2	3	2	-2	0	2	0	2	D_2
27	0	-1	-3	2	2	-3	2	-2	0	-2	0	-2	D_2
28	0	-1	0	2	2	0	2	-2	0	0	0	0	H'
29	0	-1	0	8	-4	0	2	-2	0	0	-1	-8	N_1
30	0	-1	0	-4	8	0	2	-2	0	0	1	8	N_2
31	0	-1	0	-4	-4	0	-4	4	0	0	0	0	S_1
32	0	-1	-9	2	2	3	2	-2	0	0	-1	0	N_3
33	0	-1	9	2	2	-3	2	-2	0	0	1	0	N_4
34	0	-1	0	-4	-4	0	2	-2	0	0	0	0	Y
35	1	1	3	0	-2	1	0	2	-2	3	0	-1	R_T
36	1	1	-6	0	4	-2	0	2	-2	0	0	0	T_5
37	1	1	3	0	-2	1	0	-4	4	-3	0	1	R_3
38	1	1	-6	0	4	-2	0	-4	4	-6	0	2	S_{10}
39	1	-1	-3	2	2	1	2	-2	4	-2	0	-2	D_3
40	1	-1	0	2	-4	-2	2	-2	4	-3	0	-7	R_4
41	1	-1	0	-4	2	-2	2	-2	4	-3	1	1	L_4
42	1	-1	0	-4	-4	4	2	-2	4	0	0	0	S_{11}
43	1	-1	9	2	2	1	2	-2	4	0	1	0	S_{12}
44	-1	1	3	6	-2	-1	0	-4	-4	0	-1	0	S_{13}
45	-1	1	3	-6	-2	-1	0	-4	-4	0	0	8	S_{14}
46	-1	1	-6	0	4	2	0	-4	-4	0	-1	8	S_{15}
47	-1	1	-6	0	4	2	0	2	2	0	0	0	T_6
48	-1	1	3	6	-2	-1	0	2	2	0	0	-8	T_7
49	-1	1	3	-6	-2	-1	0	2	2	0	1	0	T_8
50	-1	-1	3	2	2	-1	2	-2	-4	2	0	2	D_3
51	-1	-1	0	-4	2	2	2	-2	-4	3	0	7	L_5
52	-1	-1	0	2	-4	2	2	-2	-4	3	-1	-1	R_5
53	-1	-1	0	-4	-4	-4	2	-2	-4	0	0	0	S_{16}
54	-1	-1	-9	2	2	-1	2	-2	-4	0	-1	0	S_{17}

Table 5: Orbifold massless spectrum for the model of section (IV.ii)

TABLE 6

$Q_1 = 6(1, 1, 1, 0, \dots, 0) \times (0, \dots, 0)'$
$Q_2 = 6(0, 0, 0, 1, -1, 0, 0, 0) \times (0, \dots, 0)'$
$Q_3 = 6(0, \dots, 0, 1, 1, 0) \times (0, \dots, 0)'$
$Q_4 = 6(0, \dots, 0, 0, 0, 1) \times (0, \dots, 0)'$
$Q_5 = 6(0, \dots, 0) \times (1, 0, 0, 0, \dots, 0)'$
$Q_6 = 6(0, \dots, 0) \times (0, 1, 0, 0, \dots, 0)'$
$Q_7 = 6(0, \dots, 0) \times (0, 0, 1, 1, 0, \dots, 0)'$

Table 6: $U(1)$ charge generators for the model of section (IV.iii).

TABLE 7

$D_1 \bar{D}_1 S_1$	$L_1 T_3 L_T$	$R_2 T_6 R_T$
$D_2 \bar{D}_2 S_1$	$L_1 T_4 L_T$	$R_3 T_5 R_T$
$D_3 \bar{D}_3 S_1$	$L_T L_T S_7$	$R_T R_T S_{10}$
$Q_L Q_R H$	$L_5 L_7 L_T$	$R_4 T_1 R_T$
$T_3 T_5 S_{11}$	$T_6 T_4 S_{16}$	$H' H' S_1$
$T_3 T_1 S_5$	$T_1 T_4 S_2$	$H' L_T R_T$
$T_6 T_7 S_{14}$	$T_7 T_5 S_9$	

Table 7: Renormalizable couplings involving color triplets and/or $SU(2)_L$, $SU(2)_R$ and $SU(2)'$ doublets in the model of section (IV.iii).

TABLE 8

field	string state	#	$3Y_{B-L}$	Y_1	Y_2	$3 \otimes 2 \otimes 2$
Q_L	Q_L^i	3	1	0	-3	$(3, 2, 1)$
Q_R	Q_R^i	3	-1	0	3	$(\bar{3}, 1, 2)$
L	L_4	3	-3	1	1	$(1, 2, 1)$
R	R_5	3	3	-1	-1	$(1, 1, 2)$
H	H^3	1	0	0	0	$(1, 2, 2)$
L'	L_3	3	-3	-1	1	$(1, 2, 1)$
$\bar{L}'$	L_5	3	3	0	7	$(1, 2, 1)$
R'	R_4	3	-3	0	-7	$(1, 1, 2)$
$\bar{R}'$	R_1	3	3	1	-1	$(1, 1, 2)$

Table 8: Massless fields after symmetry breaking in the model of section (IV.iii)

TABLE 9

Sector	Field Notation	$\vec{H}_{-1}$	$\vec{H}_{-1/2}$
Untwisted	U_1	$(0,0,1,0,0)$	$\frac{1}{2}(\pm 1, \mp 1, 1, -1, -1)$
	U_2	$(0,0,0,1,0)$	$\frac{1}{2}(\pm 1, \mp 1, -1, 1, -1)$
	U_3	$(0,0,0,0,1)$	$\frac{1}{2}(\pm 1, \mp 1, -1, -1, 1)$
θ	A	$\frac{1}{3}(0,0,1,0,2)$	$\frac{1}{6}(\pm 3, \mp 3, -1, -3, 1)$
θ^2	$\overline{A}$	$\frac{1}{3}(0,0,2,0,1)$	$\frac{1}{6}(\pm 3, \mp 3, 1, -3, -1)$
ω	B	$\frac{1}{3}(0,0,0,1,2)$	$\frac{1}{6}(\pm 3, \mp 3, -3, -1, 1)$
ω^2	$\overline{B}$	$\frac{1}{3}(0,0,0,2,1)$	$\frac{1}{6}(\pm 3, \mp 3, -3, 1, -1)$
$\theta\omega^2$	C	$\frac{1}{3}(0,0,1,2,0)$	$\frac{1}{6}(\pm 3, \mp 3, -1, 1, -3)$
$\theta^2\omega$	$\overline{C}$	$\frac{1}{3}(0,0,2,1,0)$	$\frac{1}{6}(\pm 3, \mp 3, 1, -1, -3)$
$\theta\omega$	D	$\frac{1}{3}(0,0,1,1,1)$	$\frac{1}{6}(\pm 3, \mp 3, -1, -1, -1)$

Table 9: Notation and $\vec{H}$ -momentum for fields in the $Z_3 \times Z_3$ orbifold.

TABLE 10

$U_1 U_2 U_3$	$U_3 C \bar{C}$	$\bar{A} B C$	$\bar{A} \bar{B} D$
$U_1 B \bar{B}$	DDD	$A \bar{B} \bar{C}$	
$U_2 A \bar{A}$	ACD	$B \bar{C} D$	
$U_1 ACD (\partial X^1)$	$AA \bar{B} D (\partial X^3)$	$\bar{A} B B D (\partial X^3)$	
$U_1 DDD (\partial X^1)$	$A \bar{A} B \bar{B} (\partial X^3)$	$\bar{A} \bar{C} D D (\partial X^1)$	
$U_2 B \bar{C} D (\partial X^2)$	$A \bar{A} C \bar{C} (\partial X^1)$	$B \bar{B} C \bar{C} (\partial X^2)$	
$U_2 DDD (\partial X^2)$	$ABDD (\partial X^3)$	$BCCD (\partial X^2)$	
$U_3 \bar{A} \bar{B} D (\partial X^3)$	$A \bar{C} \bar{C} D (\partial X^1)$	$\bar{B} \bar{B} \bar{C} D (\partial X^2)$	
$U_3 DDD (\partial X^3)$	$\bar{A} \bar{A} C D (\partial X^1)$	$\bar{B} C D D (\partial X^2)$	

Table 10: Cubic and Quartic couplings in the $Z_3 \times Z_3$ orbifold. The derivatives to which the quartic couplings are proportional are shown in parentheses.

TABLE 11

	Sector	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Rep	Mult
1	U_1	-6	0	0	0	0	0	$(1, 2, 2_L, 2_R)$ 1	1
2		12	0	0	0	0	0		
3		0	0	0	6	6	-18		
4	U_2	-6	3	9	0	0	0	$(1, 1, 2_L, 1)$ $(1, 1, 1, 2_R)$ $(1, 2, 1, 1)$ $(4, 1)$ $(-4, 1)$ 1	1
5		-6	3	-9	0	0	0		
6		-6	-6	0	0	0	0		
7		0	0	0	0	-12	0		
8		0	0	0	0	6	18		
9		0	0	0	0	6	-18		
10	U_3	-6	-3	-9	0	0	0	$(1, 1, 2_L, 1)$ $(1, 1, 1, 2_R)$ $(1, 2, 1, 1)$ $(4, 1)$ 1 $(-4, 1)$	1
11		-6	-3	9	0	0	0		
12		-6	6	0	0	0	0		
13		0	0	0	-6	6	18		
14		0	0	0	12	6	-18		
15		0	0	0	-6	-12	0		
16	A	2	-1	3	4	2	-6	$(-3, 1, 1, 2_R)$ $(3, 1, 2_L, 1)$ $(1, 1, 1, 2_R)$ $(1, 1, 2_L, 1)$ 1 1	3
17		2	-1	-3	4	2	-6		
18		2	-1	-9	4	2	-6		
19		2	-1	9	4	2	-6		
20		-4	-4	0	-8	-4	12		
21		-4	-4	0	4	2	-6		
22	$-A$	4	-2	6	-4	-2	6	$(3, 1, 1, 1)$ $(-3, 1, 1, 1)$ $(1, 1, 2_L, 2_R)$ $(1, 2, 1, 1)$	3
23		4	-2	-6	-4	-2	6		
24		4	-2	0	-4	-2	6		
25		-2	-2	0	-4	-2	6		
26	$A + w$	-4	2	6	-2	8	0	$(1, 1, 2_L, 1)(4, 1)$ $(1, 1, 2_L, 1)(1, -3)$ 1	3
27		2	-1	-3	-2	-4	0		
28		2	-1	-3	4	2	-6		
29	$-A - w$	4	-2	-6	2	4	0	$(-4, 1)$ $(1, 3)$ $(1, 1, 2_L, 1)$	3
30		4	-2	-6	-4	-2	6		
31		-2	1	3	2	-2	-18		
32	$A - w$	-4	2	-6	-2	-10	6	$(1, 1, 1, 2_R)(1, 3)$ $(1, 1, 1, 2_R)$ 1	3
33		2	-1	3	4	2	-6		
34		2	-1	3	-2	-4	-12		
35	$-A + w$	4	-2	6	-4	-2	6	$(1, -3)$ 1 $(1, 1, 1, 2_R)(4, 1)$	3
36		4	-2	6	2	4	12		
37		-2	1	-3	2	-2	-6		
38	B	0	-2	0	-2	6	6	$(1, 1, 2_L, 2_R)$ $(3, 1, 1, 1)$ $(-3, 1, 1, 1)$ $(1, 3)$ $(1, -3)$ 1	9
39		0	-2	6	-2	6	6		
40		0	-2	-6	-2	6	6		
41		0	4	0	-2	-6	-6		
42		0	4	0	4	0	0		
43		6	2	0	2	-6	-6		
44	$-B$	0	-4	0	2	6	-6	$(1, 2, 1, 1)$ $(-4, 1)$ $(-3, 1, 1, 1)$ $(3, 1, 1, 1)$ $(1, -3)$ $(1, 3)$	9
45		0	2	-6	2	-6	-6		
46		0	2	6	2	-6	-6		
47		0	-4	0	2	6	6		
48		0	-4	0	-4	0	0		
49		0	-4	0	2	-6	-6		
50	$A - B$	2	-2	0	0	2	-6	$(1, 2, 1, 1)(1, 3)$ 1	3
51		-4	4	0	6	-4	-12		
52	$-A + B$	2	-2	0	0	2	6	$(1, 2, 1, 1)(4, 1)$ $(1, -3)$ $(1, 2, 1, 1)$	3
53		-2	2	0	0	-2	-6		
54		-2	2	0	6	10	-6		
55	$A - B + w$	4	-4	0	0	-2	-6	$(1, 1, 1, 2_R)$ 1 $(3, 1, 1, 1)$ $(-3, 2, 1, 1)$ 1 $(1, 1, 1, 2_R)$	3
56		2	1	-3	0	-4	12		
57		8	-2	6	0	2	-6		
58		-4	-2	-6	0	2	-6		
59		2	-2	0	0	2	-6		
60		-4	-2	6	0	2	-6		
61	$-A + B - w$	2	1	-3	0	2	-6	$(1, 1, 1, 2_R)$ 1 $(1, 1, 2_L, 1)$ $(1, 2, 1, 1)$ $(3, 1, 1, 1)$ 1	3
62		4	2	-6	0	4	-12		
63		-2	5	3	0	-2	6		
64		-2	-4	-6	0	-2	6		
65		4	-4	0	0	-2	6		
66	$A - B - w$	4	2	-6	0	-2	6	$(1, -3)$ 1 $(1, 1, 2_L, 1)(-4, 1)$	3
67		-4	-2	-6	0	2	-6		
68		-4	-2	-6	6	8	0		
69	$-A + B + w$	2	1	3	0	-4	0	$(4, 1)$ $(1, 1, 2_L, 1)$	3
70		-2	-1	-3	6	-2	-18		
71	$A + B$	8	0	0	2	8	0	1	9
72		-4	0	0	2	8	0		
73	$A + B + w$	2	-3	-3	2	-4	-12	$(1, 1, 1, 2_R)$	9
74		2	3	3	-4	-4	12		
75	$A + B - w$	2	3	3	-4	-4	12	$(1, 1, 1, 2_R)$	9

Table 11: Orbifold massless spectrum for the model of section V.

TABLE 12

$Q_1 = 6(1, 0, -1, 0, \dots, 0) \times (0, \dots, 0)'$
$Q_2 = 6(0, 1, 0, 0, \dots, 0) \times (0, \dots, 0)'$
$Q_3 = 6(0, \dots, 0, 1, 1, 1) \times (0, \dots, 0)'$
$Q_4 = 6(0, \dots, 0) \times (1, 1, 0, \dots, 0)'$
$Q_5 = 6(0, \dots, 0) \times (1, 0, 1, 1, 1, 0, 0, 0)'$
$Q_6 = 6(0, \dots, 0) \times (0, -3, 0, 0, 0, 1, 1, 1)'$

$U_1 = Q_2 - 3Q_3$
$U_2 = 42Q_1 + 45Q_2 + 5Q_3$
$U_3 = Q_4 - Q_5$
$U_4 = 3Q_4 + Q_6$
$U_5 = 8(-5Q_1 + 9Q_2 + Q_3) + 67(Q_5 - Q_6)$
$X = (-5Q_1 + 9Q_2 + Q_3) - (Q_5 - Q_6)$

Table 12: $U(1)$ charge generators for the model of section (V). Non-anomalous combinations are denoted by U_i and the anomalous combination by X

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