DENSITY-DEPENDENT COUPLINGS AND ASTROPHYSICAL BOUNDS ON LIGHT SCALAR PARTICLES

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ABSTRACT

The couplings of a light scalar to ordinary matter induce non-vanishing fields ϕ inside and around stars. In turn, the masses and couplings of ordinary particles depend on ϕ in general. Thus, fundamental coupling constants can be altered by dense matter nearby. We estimate the resulting effect on the cooling of neutron stars, the neutrino burst from supernova 1987a and the period of the pulsar recently discovered in its remnant, and obtain bounds on the mass and couplings of scalar particles. We also comment on cosmological and other constraints on such scalar particles, and discuss in particular bounds on the mass of the dilaton in string theories.

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Weakly-coupled scalar particles with a very small mass μ appear in a variety of theoretical contexts, notably spontaneously broken anomalous dilatation symmetry [1] and in string theories [2]. If the scalar field ϕ is a singlet of the low-energy gauge group $SU(3) \times SU(2) \times U(1)$, simple arguments based on renormalizability and chirality indicate that its couplings to ordinary particles (gauge bosons, quarks, ...) must be suppressed by the inverse of some mass scale f from beyond the Standard Model. If f is sufficiently large, such scalars cannot be produced in conventional particle physics experiments. On the other hand, forces radiated by the exchange of light scalar particles may compete with gravity if μ is sufficiently small. It has been proposed [1] that weakly-coupled scalars with $\mu \sim 10^{-9}$ eV ($\simeq (200 \text{ m})^{-1}$) could be responsible for the deviations from Newtonian gravity reported in some 'fifth force' experiments [3], although these experimental reports still require clarification. For a range $\lambda = \mu^{-1}$ of the scalar interaction between a few km and the extension of our solar system, a large body of astronomical and satellite observations impose stringent bounds [4] on the scalar coupling strength which depend on μ . Similar, albeit somewhat weaker, bounds are derived from Cavendish-type experiments in the centimetre range. These bounds follow directly from the observed validity of the $1/r$ dependence of the gravitational potential, since a scalar, unlike a pseudoscalar, mediates an attractive interaction that leads to a correction to the Newtonian potential of the form

$$\Delta V = -G_N \frac{Mm}{r} \alpha \exp(-\mu r) \quad (1)$$

Here α measures the strength of the scalar coupling to ordinary atoms, which we assume to be proportional to mass in leading order.

Another possible aspect of a scalar interaction is an induced density-dependence of fundamental couplings. Scalar couplings to nucleons and photons of the form

$$\mathcal{L}_\phi = C_N \frac{\phi}{f} m_N \bar{N}N + C_F \frac{\phi}{f} F_{\mu\nu} F^{\mu\nu} \quad (2)$$

generate not only a modification to the Newtonian potential of the form (1) with α given by

$$\alpha = \frac{C_N^2}{4\pi} \frac{m_p^2}{f^2} \quad (3)$$

but also imply that in a non-zero background field the nucleon mass and the electromagnetic coupling [5] differ from their vacuum values when $\phi \neq 0$:

$$\frac{\delta m_N}{m_N} = -C_N \frac{\phi}{f}, \quad \frac{\delta e}{e} = 2C_F \frac{\phi}{f} \quad (4)$$

Similar ϕ -dependences can arise for quark and electron masses and the strong gauge coupling. A ϕ -dependence of the Higgs mass and/or its self-interaction can induce a ϕ -dependence of the Fermi constant, via a ϕ -dependence of the vacuum expectation value (v.e.v.) of the Standard Model Higgs doublet*):

$$\delta G_F/G_F = -C_H \phi/f \quad (5)$$

*) We do not commit ourselves in this paper to any specific model for generating the electroweak Higgs v.e.v.. In the minimal Standard Model with $V(H) = -\nu^2 |H|^2 + \lambda(|H|^2)^2$, $\delta G_F/G_F = \delta\lambda/\lambda - 2\delta\nu/\nu$.

The local value of the scalar field ϕ in turn depends on the surrounding material according to the field equation

$$\square\phi + \mu^2\phi = \left\langle \frac{\partial \mathcal{L}_I}{\partial \phi} \right\rangle \simeq \frac{C_N}{F} \rho_N \quad (6)$$

A non-vanishing source $\rho_N \neq 0$ implies that $\phi \neq 0$ and hence there is an effective density-dependence of couplings. This effect may have cosmological implications [6], or could be studied by improved Pound-Rebka-type experiments or atomic clocks [7].

Let us consider a large body of material consisting mainly of nucleons, such as a star or the Earth. In the limit where ρ_N is homogeneous on a length scale large compared with $\lambda \equiv \mu^{-1}$, we can neglect the derivative term in (6) and obtain

$$\left(\frac{\delta m_N}{m_N} \right)_0 = -0.9 \times 10^{-18} \propto \left(\frac{\lambda}{300m} \right)^2 \rho [\text{g cm}^{-3}] \quad (7)$$

The variations in other parameters are scaled by the ratios of coefficients C_i/C_N , e.g.

$$\frac{\delta C_F}{C_F} = \frac{C_G}{C_N} \left(\frac{\delta m_N}{m_N} \right) \quad (8)$$

Upper limits on the material dependence of the scalar interaction constrain these ratios. The scalar would couple exactly to mass if the ratios C_i/C_N reflected the mass dimensions^{*)} of the corresponding quantities, e.g.

$$\frac{C_F}{C_N} = 0, \quad \frac{C_G}{C_N} = -2 \Rightarrow \frac{\delta C_F}{C_F} = -2 \left(\frac{\delta m_N}{m_N} \right), \text{ etc.} \quad (9)$$

and a material dependence of the scalar interaction would arise if the C_i/C_N deviated from these values.

When λ is large compared with the scale of inhomogeneity in the material medium, the derivative term in (6) must be taken into account. For ϕ at a distance R from the centre of a sphere with uniform density ρ and radius $\hat{R}$ one obtains

$$\phi = \begin{cases} \frac{C_N \rho}{F \mu^3 R} \exp(-\mu R) [\mu \hat{R} \cosh(\mu \hat{R}) - \sinh(\mu \hat{R})] : R > \hat{R} \end{cases} \quad (10a)$$

$$\phi = \begin{cases} \frac{C_N \rho}{F \mu^3 R} [\mu R - (\mu \hat{R} + 1) \exp(-\mu \hat{R}) \sinh \mu \hat{R}] : R < \hat{R} \end{cases} \quad (10b)$$

*) The naïve mass dimensions of operators should be corrected for anomalous dimensions, inducing small corrections in relations like (9).

and hence corresponding expressions for $(\delta m_N/m_N)$ etc. On the Earth, where $\rho \sim 5 \text{ gcm}^{-3}$, the density-dependences of masses and couplings are too small to be observed now, if one takes values of $\alpha (\approx 10^{-2})$ and $\lambda (\approx 300 \text{ m})$ suggested by 'fifth force' experiments [3].

In this letter, we investigate whether the large densities in the core of supernova 1987a and neutron stars could lead to observable changes in fundamental 'constants'. We keep α and λ as free parameters and derive bounds on their allowed values from observations [8] of the neutrino burst from supernova 1987a, from the period of its recently-discovered pulsar remnant [9], and from the cooling of neutron stars [10]. In the case of the supernova, a substantial change in G_F would have modified the energy and duration of the neutrino burst, whilst changes in G_F or m_N would change neutrino emissivities and hence the cooling rates of neutron stars. Changes in m_N would alter the central density of the SN 1987a pulsar remnant, and hence its period. As an example, we apply our bounds on α and λ to the case of a light dilaton with couplings suggested by string theories. We also comment on cosmological bounds on weakly-coupled light scalars.

We now consider neutrino emission from a supernova. We first discuss the effects due to a change in the Fermi constant G_F , and then more indirect effects due to variations in other fundamental quantities. The neutrinos detected [8] are emitted from a neutrinosphere located approximately at a radius R which is equal to the neutrino mean free path λ_ν , i.e. when $\lambda_\nu \approx R \approx (15 \text{ to } 20) \text{ km}$. The core profile is essentially independent of G_F [11]. However, the neutrino mean free path is given by $\lambda_\nu \approx 1/\sigma n$, where $\sigma \propto G_F^2 T^2$ is the low-energy neutrino-nucleon elastic scattering cross-section and $n \propto T^3$ is the nucleon density, so that $\lambda_\nu^{-1} \propto G_F^2 T^5$. The neutrino energy emission rate Q_ν is given by the black-body formula at $R \approx \lambda_\nu$, so that $Q_\nu \propto R^2 T^4 \propto G_F^{-4} T^{-6}$. The observed duration [8] of the supernova 1987a neutrino pulse agree with calculations [11] based on the Standard Model, and estimates [12] of the total energy $E_\nu = \int dt Q_\nu$ emitted in the form of neutrinos are in the range $E_\nu = (2 \text{ to } 4) \times 10^{53} \text{ egs}$ and consistent with the calculated [11] total binding energy of a newly-born neutron star. Variations in Q_ν by more than a factor of 2 would have led to significant observable changes in the neutrino pulse. We therefore constrain a possible variation in G_F by requiring that

$$\frac{1}{2} < \frac{Q_\nu}{Q_{\nu_0}} = \frac{R^2}{R_0^2} \cdot \frac{T^4}{T_0^4} < 2 \quad (11)$$

where quantities with (without) suffices are those calculated in the Standard Model (with a modified value of G_F). Since the star density varies very rapidly as a function of the radius in the range of interest, we expect that

$$R/R_0 \approx 1 \quad (12a)$$

and hence $\lambda/\lambda_0 \approx 1$, i.e.

$$G_F^2 T^5 / G_F^2 T_0^5 \approx 1 \quad (12b)$$

Substituting the approximations (12) into (11) we obtain the bounds

$$2^{-5/8} < G_F/G_{F_0} < 2^{5/8} \quad (13)$$

Using the parametrization (5) of variations in G_F , equation (13) can be used to bound ϕ/f and hence α and μ .

Since we are interested in G_F at $R \approx \lambda$, which is where the matter density $\rho \sim 10^{11} \text{ gcm}^{-3}$, we need to compute the field ϕ outside the core of the supernova where the matter has nuclear density. We approximate the core by a uniform sphere of radius $R_c = 10 \text{ km}$ and density $\rho_c = 10^{15} \text{ gcm}^{-3}$. If the range $\lambda = \mu^{-1}$ of the scalar interaction is larger than a few km we should use Eq. (10a) with $\hat{R} = R_c$ and $\rho = \rho_c$, obtaining the bound

$$-0.14 \lesssim \left(\frac{R_0}{20 \text{ km}}\right)^3 \alpha \frac{G_F}{G_N} (\mu R_0)^3 \exp(-\mu R_0) [\mu R_c \cosh(\mu R_c) - \sinh(\mu R_c)] \lesssim 0.093 \quad (14)$$

If $\lambda = \mu^{-1} \ll 1 \text{ km}$, one can neglect the derivative term in (6) and obtain an expression for $\delta G_F/G_F$ analogous to Eq. (7) for $\delta m_N/m_N$. Taking in this case $\rho \approx 10^{11} \text{ gcm}^{-3}$, one obtains

$$-6.3 \times 10^6 \lesssim \alpha \frac{G_F}{G_N} \left(\frac{\lambda}{300 \text{ m}}\right)^2 \lesssim 4.1 \times 10^6 \quad (15)$$

Bounds for $10 \text{ km} \gtrsim \lambda \gtrsim 1 \text{ km}$ can also be obtained, but they depend more sensitively on the details of the neutron star model.

The above bounds on $\delta G_F/G_F$ arise under the assumption that all couplings and masses except G_F are held fixed. However, the neutrino energy emission also depends implicitly on other parameters. For example, the radius of the core would be sensitive to a variation in the strong interaction scale, which would change the nucleon mass, nuclear density, and hence the binding energy of the neutron star. One can bound a possible variation in the nucleon mass by requiring the total binding energy to be within a factor 2 of the estimated energy of the neutrino pulse. The binding energy $E \sim G_N M^2/R_c \sim G_N N^2 m_N^2/R_c$, where N is the total number of nucleons in the core. From the balancing of gravitational and Fermi energies we have $N^{-2/3} \sim G_N m_N^2$, and the radius R_c of the core is related to N and the baryon number density n by $R_c \sim (N/n)^{1/3}$. The number density n is close to the nuclear matter density, so that $n \sim \Lambda_{\text{QCD}}^3 \sim m_N^3$, where the constant of proportionality depends on details of the equation of state. We allow an uncertainty of a factor of 10 in this constant, and hence of $10^{1/3}$ in the proportionality constants in

$$R_c \sim G_N^{-1/2} / m_N^2, \quad E \sim \frac{1}{G_N^{3/2} m_N^2} \quad (16)$$

Comparing the expected binding energy with that observed in neutrinos, we can constrain the density variation of m_N , assuming G_N to be fixed:

$$0.48 \approx 2^{-1/2} 10^{-1/6} \lesssim m_N/m_{N_0} \lesssim 2^{1/2} 10^{1/6} \approx 2.1 \quad (17)$$

For an estimate of ϕ/f inside the core we use Eq. (10b) with $\rho \approx 10^{15} \text{ gcm}^{-3}$ to obtain

$$-1.3 \times 10^3 \lesssim \left(\frac{\lambda}{300 \text{ m}}\right)^2 \alpha \left(1 - \frac{\mu R_c + 1}{\mu R} \exp(-\mu R) \sinh \mu R\right) \lesssim 6.1 \times 10^2 \quad (17_N)$$

It is worth noting that time, energy and temperature scales also become density-dependent inside the neutron star. The conversion to the vacuum time and temperature scales relevant at low densities on and beyond the surface introduces density-dependent factors. In the special case of a scalar field coupling exactly to mass the situation simplifies considerably. There is a common ϕ -dependent physics scale, since all masses are fixed in units of the neutron mass, and E , t^{-1} and T are all proportional to m_N . Since only the ratio m_N/m_P is observable, where m_P is the Planck mass, we can use an appropriate Weyl rescaling of the metric to make a reformulation where m_P is ϕ -dependent whilst all other physics masses and couplings are ϕ -independent. Then we have to examine the effects of a density-dependence of Newton's constant whereas m_N is kept fixed.

Next we consider the cooling of mature neutron stars. These are generally believed to have no internal source of heat. Starting with a high initial temperature $T_i \sim 10^{11}$ K provided by the collapse of a supernova, they cool at a rate that is expected to be dominated by neutrino emission until the temperature T falls to $\sim 10^8$ K, with [13, 10]

$$-\frac{dT}{dt} = \frac{1}{c_n} R_{\text{Urca}} \quad (18)$$

where c_n is the specific heat of normal neutrons:

$$c_n \simeq 2 \times 10^{20} T_9 \text{ erg cm}^{-3} \text{ K}^{-1} \quad (19)$$

For $\rho = 10^{15} \text{ g cm}^{-3}$, and $T_9 \equiv T/10^9 \text{ K}$, the heat emission rate for the modified Urca process $n + n \rightarrow n + p + e^- + \bar{\nu}_e$ is [13]

$$R_{\text{Urca}} \simeq 1.7 \times 10^{21} T_9^8 \text{ erg cm}^{-2} \text{ s}^{-1} \quad (20)$$

Density-induced changes in couplings would modify R_{Urca} (20) and/or c_n , and thus modify the cooling rate (18) and the temperature of the neutron star observed now. Defining $Q \equiv (R_{\text{Urca}}/c_n)/(R_{\text{Urca0}}/c_{n0})$, we can rewrite Eq. (18) as

$$-\frac{dT_9}{dt} = 0.9 \times 10^{-8} T_9^7 Q \text{ s}^{-1} \quad (21)$$

whose solution is [13, 10] (with t in seconds)

$$\log_{10} T_9 = \frac{1}{6} (7.3 - \log_{10} t - \log_{10} Q) \quad (22)$$

) We take the effective neutron mass $m_n^ = 0.8 m_n$.

The ratio R_{Urca}/c_n depends on several fundamental parameters:

$$R_{\text{Urca}}/c_n \propto G_F^2 m_N^2 n^{1/3} T^7 / m_\pi^4 \quad (23)$$

As a guide to estimating the sensitivity to density dependence, we assume that all particle physics mass parameters scale in the same way. Since $(R_{\text{Urca}}/c_n)/T^7$ has the dimension of $(\text{mass})^{-5}$, we can write $Q = \hat{Q}(m_{N0}/m_N)^5$ and derive from Eq. (22) the relation

$$\ln(m_N/m_{N0}) = \frac{1}{5} \ln(\hat{Q}/\hat{Q}_0) + \frac{6}{5} \ln(T/T_0) \quad (24)$$

where $\hat{Q}/\hat{Q}_0$ allows for a possible uncertainty in the calculation of R_{Urca}/c_n even if the fundamental physics is specified, and T/T_0 represents a deviation of the observed temperature (T) from that expected (T_0).

Observations of X-ray emission from the Crab pulsar [14] and RCW103 [15] are consistent with surface temperatures $T_s = (1.5 \text{ to } 2.5) \times 10^6$ K. The conversion from the surface temperature to the internal temperature T_I depends on the equation of state assumed. For example, $T_s = 1.5 \times 10^6$ K corresponds to $T_I \approx 5 \times 10^7$ K for a magnetized star with a soft equation of state, and to $T_I \approx 2 \times 10^8$ K for an unmagnetized star with a stiff equation of state [16]. A recent analysis [17] of the behaviour of the Crab pulsar following glitches in 1969 and 1975 indicates that $T_I = (4 \pm 2) \times 10^8$ K. Equation (22) would give $T_{I0} \approx 3 \times 10^8$ K if $Q = 1$, so we conclude that $(T_I/T_{I0}) = 2^{0 \pm 1}$. The prediction for RCW103 is comparably accurate.

Using $(T_I/T_{I0}) = 2^{0 \pm 1}$ and allowing $(\hat{Q}/\hat{Q}_0) = 10^{0 \pm 1}$, we obtain

$$0.27 \lesssim m_N/m_{N0} \lesssim 3.6 \quad (25a)$$

and hence

$$-2.6 \lesssim C_N \frac{\phi}{f} \lesssim 0.73 \quad (25b)$$

To obtain a constraint on the scalar mass and coupling strength, we use the previous solution (10b) for ϕ inside the neutron star, which we take to have uniform density and radius $\hat{R} = 10$ km. Averaging (10b) over the neutron star's interior, we find

$$\langle \frac{\phi}{f} \rangle = \frac{C_N \rho}{f^2 \mu^2} \left[1 - \frac{3(\mu \hat{R} + 1)}{(\mu \hat{R})^3} e^{-\mu \hat{R}} (\mu \hat{R} \cosh \mu \hat{R} - \sinh \mu \hat{R}) \right] \quad (26)$$

Imposing the upper limit (25b) on this expression, we obtain the constraint^{*)}.

*) Here we introduce C_π defined by $\delta m_\pi/m_\pi = -C_\pi \phi/f$.

$$-3.0 \times 10^{-3} \lesssim \alpha \left(\frac{\lambda}{300\text{m}} \right)^2 \left[1 - \frac{3(\mu\hat{R}+1)}{(\mu\hat{R})^3} e^{-\mu\hat{R}} (\mu\hat{R} \cosh \mu\hat{R} - \sinh \mu\hat{R}) \right] \lesssim 850 \quad (27)$$

The quantity in square brackets varies between 1 ($\mu \rightarrow \infty$) and 0 ($\mu \rightarrow 0$).

The recent observation [9] of a pulsar with short period $P \sim 5 \times 10^{-4}$ s in the remnant of supernova 1987a allows us to set a slightly more stringent constraint on m_N/m_{N_0} and hence on $\delta\phi/f$ and μ . This infant neutron star is believed to spin close to break-up, with a surface velocity close to the speed of light [18]. In this case, one estimates that $P \simeq 2\pi(G_N \rho)^{-1/2}$, where ρ is the core density [18], and the observed period of SN 1987a is compatible with a central density $\rho \sim 10^{15} \text{ g cm}^{-3}$ as expected. Assuming that all particle masses scale in the same way so that $\rho \propto m_N^4$, we can write $P^{-2} = d \cdot G_N \cdot m_N^4$ where the dimensionless coefficient d may be known to within a factor of 10: $d/d_0 = 10^{0 \pm 1}$ reflecting uncertainties in the equation of state, etc.. Comparing the observed period (P) with that expected (P_0), we therefore have

$$\ln(m_N/m_{N_0}) = -\frac{1}{2} \ln(P/P_0) - \frac{1}{4} \ln(d/d_0) \quad (24_P)$$

and taking $P/P_0 = 2^{0 \pm 1}$ we finally obtain

$$0.4 \lesssim m_N/m_{N_0} \lesssim 2.5 \quad (25a_P)$$

and hence

$$-1.5 \lesssim \frac{C_N \phi}{f} \lesssim 0.6 \quad (25b_P)$$

and finally

$$-1.7 \times 10^3 \lesssim \alpha \left(\frac{\lambda}{300\text{m}} \right)^2 \left[1 - \frac{3(\mu\hat{R}+1)}{(\mu\hat{R})^3} e^{-\mu\hat{R}} (\mu\hat{R} \cosh \mu\hat{R} - \sinh \mu\hat{R}) \right] \lesssim 700 \quad (27_P)$$

Our bounds are displayed in the figure, together with various previous bounds [4,19]. A few comments are in order:

- 1) The solid lines in the figure are calculated assuming a material-independent scalar interaction: $C_G = -2C_n$, etc. Bounds for other model couplings are easy to derive, and we will give an example at the end of this paper.
- 2) As compared to bounds from Cavendish-type experiments, astronomical and satellite observations etc., our bounds are competitive when $\lambda \lesssim 10^{-3}$ m.
- 3) The changes in masses and couplings corresponding to our bounds are not small. Therefore, the linear approximation to the scalar-matter couplings assumed above may not be valid. However, we expect that realistic non-linear couplings would yield qualitatively similar bounds.
- 4) The dotted lines in the figure are cosmological bounds [19]. These depend on some specific assumptions that we now discuss.

To treat cosmology, we replace the right-hand-side of Eq. (6) by a term $(C_U/f)q_U$ which parametrizes the coupling of ϕ to the dominant component of the cosmic energy density. We expect C_U to be negligible during the radiation-dominated epoch, since we can always rescale the fields so that kinetic terms are independent of ϕ from which it is clear that the scalar does not couple to an ideal gas in thermal equilibrium. A non-vanishing value of C_U could only arise from modifications of the equation of state due to ϕ -dependent interactions. In contrast, couplings of ϕ to visible or dark non-relativistic matter could induce a substantial value of C_U during the matter-dominated epoch. The earlier arguments strongly constrain ϕ couplings to matter, but couplings to dark matter could be much larger.

For an equation of state $p_n = (n/3 - 1)q_n$ with $n = 4$ (3) for the radiation (matter) dominated epoch, and assuming the Hubble constant H obeys the condition $H^2/\mu^2 \ll 1/12$ (2/9), the generic homogeneous and isotropic solution of the $k = 0$ cosmological equations [6]

$$\left. \begin{aligned} \ddot{\phi} + 3H\dot{\phi} + \mu^2\phi &= \frac{\beta}{M}\rho \\ \dot{\rho} + nH\rho + \frac{\beta}{M}\rho\dot{\phi} &= 0 \\ 6M^2H^2 &= \rho + V + \frac{1}{2}\dot{\phi}^2 \end{aligned} \right\} \quad (28)$$

(where $M^2 \equiv m_\phi^2/16\pi$, $\beta \equiv \frac{1}{2}\sqrt{\alpha} C_U/C_N$) is a damped oscillation of ϕ around a time-dependent minimum given by

$$\phi_0(t) = \frac{\beta}{\mu^2 M} \rho(t) \quad (29)$$

The oscillating field behaves like non-relativistic matter and should not dominate the energy density of the Universe too early. This gives bounds [20] on the initial amplitude $\hat{\phi} \equiv \phi - \phi_0$. Whether these bounds are satisfied or not depends on the detailed dynamics of the QCD phase transition or, in the case of small μ^2 , the dynamics of the onset of ϕ oscillations once H^2 decreases below μ^2 . The dotted line in the figure corresponds to maximal oscillations $|\hat{\phi}| = \alpha^{-1/2}M$ ($\simeq f$ when $C_N \simeq 1$) at $T = \min(100 \text{ MeV}, (\mu M)^{1/2})$, as would have been natural for the similar case of axion oscillations. For $\alpha < 1$ we take $|\hat{\phi}| = M$ as a maximal amplitude.

When $\mu^2 \ll H^2$ the mass term in (30) becomes negligible and the evolution of ϕ is entirely driven by its coupling to matter^{*)}. For a linear coupling the asymptotic solution of (30) is

$$\phi = c_0 + aM \ln t/t_0, \quad H = \eta t^{-1}, \quad \rho = \rho_0 t_0^2 t^{-2} \quad (30)$$

where

$$\eta = \frac{2}{(n + \frac{12\beta^2}{6-n})}, \quad a = \frac{24\beta}{(n(6-n) + 12\beta^2)}, \quad \rho_0 t_0^2 = (6\eta^2 - \frac{1}{2}a^2)M^2 \quad (31)$$

^{*)} This invalidates the bound given in ref. [21], since during nucleosynthesis $\ddot{\phi} + 3H\dot{\phi} \gg \mu^2\phi$ for the value of μ quoted.

Nucleosynthesis and the microwave background radiation place limits on η [22] and hence on β during the radiation-dominated epoch:

$$\beta \lesssim 0.28 \text{ (0.20)} \quad (32)$$

for a Helium abundance $Y_p \leq 0.26$ (0.25). However, as discussed above, we expect β to be very small during this epoch. When μ^{-1} is bigger than our present horizon size, β is bounded by the age of the Universe ηH_0^{-1} . It should be noted, though, that a linear approximation to the ϕ -matter coupling is not well-motivated when $\mu^2 \ll H^2$. If we postulate instead a scalar evolution equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\gamma}{M^2}(\phi - \bar{\phi})\rho = 0 : \beta = \frac{\gamma\bar{\phi}}{M} \quad (33)$$

we obtain standard cosmology with ϕ approaching ϕ_0 as a negative power of t :

$$\phi - \bar{\phi} = A t^{-m} : m = \frac{3\gamma-1}{2} - \sqrt{\frac{(3\gamma-1)^2}{4} - 6\gamma^2} : \gamma = \frac{2}{n} \quad (34)$$

When a small mass term is included, ϕ approaches a time-dependent effective minimum at

$$\phi_0(t) = \phi_0 \left(1 + \frac{\mu^2 M^2}{\gamma \rho(t)} \right)^{-1} \quad (35)$$

The evolution of ϕ before the onset of oscillations ($\mu^2 \ll H^2$) is relevant for the magnitude of the initial amplitude $\hat{\phi}$, since we expect β and γ to be non-negligible during the QCD phase transition (but also the dynamics to be more complex than in our simple discussion). This illustrates the difficulty of obtaining cosmological bounds in our case. In particular, a scalar with range $\lambda \simeq 300$ m and $\alpha \simeq 10^{-2}$ cannot be completely excluded, given our present ignorance of the cosmological evolution of such a scalar field.

Finally, we specialize the above analysis to one example of matter-dependent scalar couplings, namely a light ‘dilaton’ from string theory. In this case, the dominant effect comes from the ϕ -dependence of the strong coupling due to the term [2]

$$\mathcal{L} = -\frac{1}{4} \exp(-\phi/M) F_{\mu\nu} F^{\mu\nu} \quad (36)$$

which implies

$$\frac{\delta g_s}{g_s} = \frac{1}{2} \frac{\phi}{M} \quad (37)$$

This leads [6] to a variation in the strong coupling scale:

$$\frac{\delta\Lambda}{\Lambda} \approx \left(\frac{1}{2b_3 g_s^2(M_x)} + M \frac{d}{d\phi} \ln \frac{M_x}{M} \right) \frac{\phi}{M} \quad (38)$$

where

$$\Lambda \approx M_x \exp \left(- \frac{1}{2b_3 g_s^2(M_x)} \right) \quad (39)$$

with M_x the scale at which SU(3) is unified with other interactions in some higher symmetry and b_3 is the coefficient of g_s^3 in the renormalization group equation for g_s . The first term in Eq. (38) dominates, since $g_s^2(M_x)$ must be small. The nucleon mass is proportional to Λ at zeroth order in the small u, d and s quark masses, so that $\delta m_N / m_N \approx \delta \Lambda / \Lambda$ and one obtains

$$\alpha \approx \frac{1}{b_3^2 g_s^4(M_x)} \quad (40)$$

Numerical values of α (40) in the minimal SU(5) GUT where $b_3 = (33 - 2N_q)/48\pi^2$ and $\alpha_s(M_x) \equiv g_s^2(M_x)/4\pi \approx 1/42$, and the minimal supersymmetric SU(5) GUT where $b_3 = (27 - 3N_q)/48\pi^2$ and $\alpha_s(M_x) \approx 1/24$, for $N_q = 6$ are

$$\alpha \approx \begin{cases} 250 & \text{without} \\ 190 & \text{with} \end{cases} \text{supersymmetry} \quad (41)$$

To first order in the light u, d and s quark masses there is an additional contribution to the possible variation in the nucleon mass:

$$\left(\frac{\delta m_N}{m_N} \right)_q = \sum_{q=u,d,s} \left(\frac{\delta m_q}{m_q} \right) \frac{m_q \langle N | \bar{q}q | N \rangle}{m_N} \quad (42)$$

If we assume, for definiteness, that $\delta m_q / m_q$ is universal for the u, d and s quarks, and use the estimate [23] $\sum_{q=u,d,s} m_q \langle N | \bar{q}q | N \rangle \approx \frac{1}{3} m_N$, we find that

$$\left(\frac{\delta m_N}{m_N} \right)_q = \frac{1}{3} \frac{\delta m_q}{m_q} \quad (43)$$

Since quark masses $m_q = \lambda_q \cdot v$, where the λ_q are Yukawa couplings and v is a Higgs v.e.v., we have

$$\frac{\delta m_q}{m_q} = \frac{\delta \lambda_q}{\lambda_q} + \frac{\delta v}{v} \quad (44)$$

In the type of string models discussed here, $\delta\lambda_q/\lambda_q = \delta g/g$ as a first approximation and is hence given by Eq. (39). The value of $\delta v/v$ depends on the detailed mechanism of spontaneous electroweak symmetry breaking. There are contributions to $\delta v/v$ from the ϕ -dependence of the Higgs scalar mass ν and the quartic coupling λ mentioned above. One would expect that $\lambda \propto g^2$ and hence that $\delta\lambda/\lambda \approx 2\delta g/g$, but the ϕ -dependence of ν and/or λ would be more complicated if symmetry breaking is induced by radiative corrections. We assume conservatively that

$$\frac{\delta m_q}{m_q} = O(1) \frac{\phi}{M} \quad (45)$$

in which case the dominant source of dilaton couplings to matter is $\delta\Lambda/\Lambda$, resulting in a relatively large material-independent coupling α (40, 41) and smaller material-dependent couplings suppressed by $O(1/\alpha)$.

A scalar with such a large coupling strength (40, 41) and material-dependent couplings 2 or 3 orders of magnitude smaller would have been seen in Eotvos-type experiments or satellite observations, unless its range were very small. We therefore conclude that the dilaton mass must be bounded below by:

$$\mu \gtrsim 2 \times 10^{-4} \text{ eV} \quad (46)$$

The dilaton mass must therefore be generated by some physics beyond the QCD scale, since otherwise a typical order of magnitude would be $\mu \sim \Lambda^2/M$ as for an axion. We should note, however, the possibility that the spectrum of light string scalars is richer. A contribution of these with a small dilaton component would have a correspondingly reduced value of α , and could have a mass in the range detectable by experiment.

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Figure caption

Fig. 1 Compilation of constraints on the dilaton-matter coupling strength α as functions of the range $\lambda = 1/\mu$. The solid line is taken from Ref. 4, and is a compilation of bounds from Cavendish-type experiments, astronomical and satellite observations, etc. The hatched lines are the astrophysical and cosmological bounds discussed in this paper. The line marked SN is derived from the agreement [12] of neutrino observations [8] of SN 1987a with theoretical predictions [11]. The line marked N.S. is derived from the agreement between neutron star cooling calculations [13, 10] and observations [14, 15], and from the observed [9] period of the SN 1987a pulsar. The dotted line is the porous cosmological constraint [19] on coherent dilaton field oscillations. These bounds are shown for material-independent couplings. In addition, we display (dashed line) bounds from Eötvös-type experiments for a material-dependent interaction, assuming a scalar coupling proportional to baryon number.