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Instantons and Chiral Dynamics*

M.A. Nowak¹

Physics Department, SUNY
Stony Brook, New York, 11794, USA

J.J.M. Verbaarschot

CERN Theory Division
CH1211 Geneva 23, Switzerland

I. Zahed

Physics Department, SUNY
Stony Brook, New York, 11794, USA

ABSTRACT

We show that the local effective interactions between light quarks in an instanton liquid can be described by 't Hooft vertices. In the long-wavelength approximation, the effective action describing the interaction between light quarks and instantons satisfies both the chiral and scale anomaly. The latter determines the compressibility of the instanton liquid. In the large N_c limit the instanton liquid becomes incompressible.

¹On leave of absence from The Institute of Physics, Jagellonian University, Cracow, Poland.

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An intriguing, yet unsettled issue in QCD relates to the mechanism behind the spontaneous breakdown of chiral symmetry. Instantons constitute an important class of gauge configurations that can account for chiral symmetry breaking. This was first noted by 't Hooft [1,2] and discussed later by Callan, Dashen and Gross [3,4,5]. However, a quantitative understanding of the problem was missing since calculations could only be envisioned in the dilute gas approximation. In this limit instantons and anti-instantons in the vacuum tend to inflate leading to severe infrared divergences. Recently, Dyakonov and Petrov [6,7] have shown that by departing from the dilute gas approximation, the average interaction between instantons is repulsive, thereby providing a natural infrared cutoff and curing the infrared problem.

The relevance of instantons in QCD has also been substantiated by elaborate lattice QCD calculations [8,9,10,11,12]. Numerical simulations indicate that the density of instantons on the lattice is not very different from phenomenological estimates (see *e.g.* refs. [13,14]). Moreover, recent (infrared-finite) calculations using the instanton liquid model of the QCD vacuum yield very reasonable bulk parameters, and call upon more quantitative calculations. In particular, we want to mention the recent Monte Carlo simulation based on the instanton liquid picture by Shuryak [15] and ourselves [16,17].

It was first pointed out by 't Hooft that at scales larger than the size of instantons, the effects of a single instanton on fermions can be described in terms of nonrenormalizable point-like effective interactions [2,1]. For massless fermions these interactions reduce to $N_F \times N_F$ interactions in the color singlet variables that break explicitly the $U_A(1)$ symmetry, leading to a natural resolution of the the long-standing $U_A(1)$ problem. The relevance of these effective interactions was also pointed out by Shifman, Vainshtein and Zakharov [18] in the context of the QCD sum-rules. Recently, we were able to obtain this effective action (apart from smearing effects) [17] in the framework of the instanton liquid model [19]. Our arguments were based on the assumption that the fermion generating function for a

dilute instanton liquid can be derived from the one instanton generating function.

At low-energy and in the large N_c limit flavor dynamics is generic (see for example refs. [20,21]). To leading order, the light pseudoscalar mesons decouple from the heavier (scalar and pseudoscalar) glueballs. The dynamics can be described in the context of nonlinear-sigma models suitably coupled to heavy scalar and pseudoscalar glueball fields. The generic form of flavor dynamics in the long-wavelength approximation is reminiscent to the Landau-Ginzburg construction.

It is the purpose of this work to show that a long-wavelength description of the instanton liquid model leads to an effective action that agrees with the one discussed in a previous work [17,22].

To leading order in N_c , the resulting partition function is equivalent to the partition function of constituent quarks interacting via 't Hooft vertices [1]. After a suitable coarse graining, we show that our effective action is consistent with the ones discussed in the literature and based on nonlinear sigma-models. In our effective action the role of the glueballs is played by the instanton and anti-instanton densities. It will be shown that our effective action satisfies both the chiral anomaly and the scale anomaly. The latter is directly related to the compressibility of the instanton liquid.

The fermion propagator in the field A_μ of N_+ instantons and N_- anti-instantons is given by [19,23]

$$S = (-i\gamma_\mu\partial_\mu - \gamma_\mu A_\mu - im)^{-1}, \quad (1)$$

where the field A_μ is the sum of one-instanton field configurations,

$$A_\mu = \sum_{I=1}^{N_+} A_\mu^I + \sum_{I=1}^{N_-} A_\mu^I. \quad (2)$$

In the following (unless specified otherwise), we will restrict the discussion to the instanton liquid model [7,19] for which the color orientations and the positions of the instantons are distributed according to their invariant measures. Correlations induced by the fermion determinant and by the interaction between the pseu-

doparticles are not taken into account. The sizes of the instantons are distributed according to the weight function originally derived by 't Hooft.

The fermion propagator in the many-body system can be rewritten in terms of the propagators S_I in the field of one instanton I ,

$$S = S_0 + \sum_I (S_I - S_0) + \sum_{I \neq J} (S_I - S_0) S_0^{-1} (S_J - S_0) + \dots, \quad (3)$$

where S_0 is the free propagator,

$$S_0 = (-i\gamma_\mu \partial_\mu - im)^{-1}. \quad (4)$$

Since the instanton fields are short ranged, S_I can be approximated by the sum of a free propagator and a nonperturbative contribution. At low energies the latter contribution is dominated by dilatational zero modes ϕ_I , suggesting the following approximation for S_I [23,19]

$$S_I \sim S_0 + \frac{\phi_I \phi_I^\dagger}{-im}. \quad (5)$$

All results to be presented in this letter are based on this approximation. It should be noted that explicit calculations based on (5) lead to reasonable results for bulk parameters (as for example the chiral condensate). The second term in eq. (5) explicitly breaks chiral $U_L(3) \times U_R(3)$ invariance and is singular in the chiral limit. This singularity is harmless since the instanton distribution in the vacuum vanishes for $m \rightarrow 0$. In this limit¹ (explicit) chiral symmetry is restored. By using (5), the full propagator (3) can be rewritten as [19]

$$S = S_0 + \phi_I (T + imK - im)^{-1}_{IJ} \phi_J^\dagger, \quad (6)$$

where

$$K_{IJ} = \int d^4x \phi_I^\dagger(x) \phi_J(x) - \delta_{IJ} \quad \text{and} \quad T_{IJ} = \int d^4x \phi_I^\dagger(x) i\gamma_\mu \partial_\mu \phi_J(x). \quad (7)$$

¹The chiral limit has to be taken with care. First, one has to take the mass m in the inverse propagator (eq. (4)) to zero, and then the mass in (5) together with the mass in the size distribution of the instantons (see eq. (8)). Otherwise, the latter mass and the mass in the inverse propagator may generate spurious contributions for $m \rightarrow 0$ (depending on how the limit is taken).

Both K and T are Hermitean. The sum $T + imK$ is Hermitean up to a constant linear transformation. The eigenvalues of this matrix occur in pairs $(\lambda, -\lambda^*)$.

In this letter we will investigate the many-body determinant $\langle \det S^{-1} \rangle$ in the instanton liquid state where $\langle \dots \rangle$ denotes the average over the collective coordinates,

$$\langle \dots \rangle = \int \prod_{I=1}^{N_+ + N_-} dz_I dU_I d\rho_I w(\rho) \prod_f (m_f \rho_I) \exp(-S_{int}) \dots \quad (8)$$

Here z_I , U_I and ρ_I are respectively, the position, the color orientation and the size of the instanton (anti-instanton) I . S_{int} is the two-body (Euclidean) interaction between an instanton and an anti-instanton, and $w(\rho)$ the instanton size-distribution. Dyakonov and Petrov [7] have shown that the interaction between instantons provides a Gaussian cutoff for the ρ -integration, leading to a strongly peaked distribution over the instanton size in (8). From their results it can be concluded that the variance of this distribution is inversely proportional to N_c . Thus, we will assume in this paper that the size integration can be performed by a lowest order cumulant expansion where the average size is determined by the variational calculations of Dyakonov and Petrov [7]. We will not calculate the grand-partition function explicitly. Rather, we will evaluate the many-body fermion propagator for the density of instantons that minimizes the free energy of the instanton liquid. The fluctuations in the number of the pseudoparticles will be taken into account via a simple effective Lagrangian in the density of instantons.

The expectation value of the inverse of the fermion determinant in the presence of a source term λ can be written in the following form

$$Z(\lambda) \equiv \langle \det -S^{-1} \rangle = \det -S_0^{-1} \det \left(\frac{T + imK - im - \lambda}{-2im} \right) \quad (9)$$

As a side remark we want to note that to leading order in a cumulant expansion the fermion determinant can be expressed in terms of the resolvent $G(\lambda) =$

$\partial_\lambda \log Z(\lambda)$ of the matrix $T + imK - im$ by

$$\langle \det(T + imK - im) \rangle = \exp \oint \frac{1}{2\pi i} d\lambda G(\lambda) \log(\lambda). \quad (10)$$

The contour encloses all singularities of the resolvent and avoids the cut of the logarithm. This expansion is of relevance for obtaining a mean-field estimate of the fermion determinant [24]. Since we will not use the λ -dependence of $Z(\lambda)$ in the remainder of this paper, we will evaluate $Z(\lambda)$ for $\lambda = 0$.

To display the dynamical properties of the partition function we use a trick due to Ilgenfritz [25] to rewrite the determinants in eq. (9) in terms of integrals over Grassmann variables (the sum over pseudoparticles, Dirac, flavor and color indices is understood in the exponent)

$$\begin{aligned} \det -S_0^{-1} &= \int d\psi d\psi^\dagger \exp \int \psi^\dagger (i\gamma_\mu \partial_\mu + im) \psi, \\ \det(T + imK - im) &= \int d\chi d\chi^\dagger \exp \chi^\dagger \left(\int \phi^\dagger (i\gamma_\mu \partial_\mu + im) \phi - 2im \right) \chi. \end{aligned} \quad (11)$$

For fixed values of χ and $\chi^\dagger$ a shift in the Grassmann variables ψ and $\psi^\dagger$ by $\psi \rightarrow \psi + i\phi\chi$, and $\psi^\dagger \rightarrow \psi^\dagger + i\chi^\dagger\phi^\dagger$ yields

$$\begin{aligned} \langle \det -S^{-1} \rangle &= \langle \int d\psi d\psi^\dagger \exp(-\int \psi^\dagger S_0^{-1} \psi) d\chi d\chi^\dagger (-2im)^{-NN_f} \\ &\quad \times \exp[i\chi^\dagger \int \phi^\dagger S_0^{-1} \psi + \int i\psi^\dagger S_0^{-1} \phi \chi - 2im\chi^\dagger \chi] \rangle. \end{aligned} \quad (12)$$

Using the chiral properties of the zero modes one can show that in the limit $m \rightarrow 0$, $\det S^{-1}$ is invariant under $U_L(N_f) \times U_R(N_f)$ transformations of the fermion field ψ . One should note that the quantity $2m$ in eq. (12) is the sum of the mass in S_0^{-1} and the mass in the denominator of the zero mode contribution to eq. (5).

First, we show that (12) reduces to the partition function considered in Refs. [17] and [22]. For that, we rewrite the sum over the instanton and anti-instanton indices I and flavor indices f in the exponent of (12) as a product over I and

f . Due to the Grassmannian character of the variables only the zeroth and the first order terms of the Taylor expansion of the exponent are nonzero. Thus, the $\chi_{If}^\dagger, \chi_{I,f}$ -integrals are Gaussian and can be evaluated in a straightforward way. The result is,

$$\begin{aligned} \langle \det -S^{-1} \rangle &= \int d\psi d\psi^\dagger \exp\left(-\int \psi^\dagger S_0^{-1} \psi\right) \\ &\times \langle \prod_{If} \left(1 - \frac{1}{2im} \int \psi^\dagger S_0^{-1} \phi_I \int \phi_I^\dagger S_0^{-1} \psi\right) \rangle. \end{aligned} \quad (13)$$

Note that in order to obtain the last term of this equation, we reversed the order of ψ and $\psi^\dagger$. For the case where the mass matrix in S_0 is set to zero (13) coincides with the partition function used as a starting point in refs. [17,22]. By using the Grassmannian properties of the fermion fields, in this case eq. (13) can be rewritten in terms of a determinant in flavor space as

$$\begin{aligned} \langle \det -S^{-1} \rangle &= \int d\psi d\psi^\dagger \exp\left(-\int \psi^\dagger S_0^{-1} \psi\right) \\ &\times \prod_I \frac{1}{N_f!} \langle \det_{fg} \left(\frac{1}{-im} \int \psi^{\dagger f} S_0^{-1} \phi_I \int \phi_I^\dagger S_0^{-1} \psi^g \right) \rangle. \end{aligned} \quad (14)$$

Since the zero modes have definite chirality, the determinantal factor in (14) breaks explicitly the $U_A(1)$ symmetry and induces nonlocal interactions between fermions on the scale of the size of the instantons. At the mean-field level they also induce the spontaneous breakdown of chiral symmetry. In Ref. [17] we calculated the average of this determinant for one, two and three flavors. Apart from smearing effects, the resulting effective action coincides exactly with the effective action obtained by Shifman, Vainshtein and Zakharov [18] using the operator product expansion.

Second, we show that the partition function (13) is analogous to the partition function composed of 't Hooft determinants. To prove this, we evaluate the average over the color orientations and the size distribution using a cumulant expansion,

$$\langle \exp(i\chi^\dagger \int \phi^\dagger S_0^{-1} \psi + i \int \psi^\dagger S_0^{-1} \phi \chi) \rangle_{U_\rho} = \exp \langle -\chi^\dagger \int \phi^\dagger S_0^{-1} \int \psi \psi^\dagger S_0^{-1} \phi \chi \rangle_{U_\rho}, \quad (15)$$

where the brackets $\langle \dots \rangle_{U_\rho}$ denote averaging over the gauge group and the normalized size distribution. To leading order in N_c the higher order cumulants

do not contribute. In the integrand the $\chi^\dagger, \chi$ -variables occur as the exponent of a quadratic form, diagonal in the pseudoparticles and nondiagonal in the flavor indices. Performing this integral yields

$$\begin{aligned} \langle \det -S^{-1} \rangle &= \int d\psi d\psi^\dagger \exp\left(-\int \psi^\dagger S_0^{-1} \psi\right) \\ &\times \prod_I \int dz_I \bar{w}(\rho_I) d\rho_I \det\left(m\rho - \frac{\rho}{2i} \left\langle \int \psi^\dagger S_0^{-1} \phi_I \int \phi_I^\dagger S_0^{-1} \psi \right\rangle_{U_\rho}\right). \end{aligned} \quad (16)$$

Here, $\bar{w}(\rho)$ is the effective one-body size distribution that includes the average repulsive interaction between the pseudoparticles. The factor $\prod_f m_f \rho$ has been absorbed into the determinant. The determinant in this equation is over the flavor indices and can be identified as a smeared 't Hooft determinant. This result was first derived by Dyakonov and Petrov [22] using different arguments. For an *equal* number of instantons and anti-instantons the partition function (16) *is invariant* under $U_L(N_f) \times U_R(N_f)$ transformations of the fermion field ψ . If the number of instanton *differs* from the number of anti-instantons, the partition function *is not invariant* under $U_A(1)$ transformations. Hence, in general it is important to account for fluctuations in the number of instantons and anti-instantons in order to account for the $U_A(1)$ anomaly in the instanton liquid model of the QCD vacuum.

Notice that the effective interactions induced by (16) in the vacuum, yield zero off-diagonal matrix elements for the chiral condensates $\sigma^{f_1 f_2}$ (which can be obtained by introducing suitably chosen source terms in eq. (9)). The diagonal matrix elements factorize

$$\langle \sigma_{f_1 f_1} \sigma_{f_2 f_2} \rangle_Z = \langle \sigma_{f_1 f_2} \rangle_Z \langle \sigma_{f_2 f_2} \rangle_Z, \quad (17)$$

where $\langle \cdot \rangle_Z$ denotes the expectation value with respect to the partition function given in eq. (16). This result is in agreement with the QCD result that there is no flavor mixing to leading order in $1/N_c$.

Finally, we show that the effective action implied by the partition function given in eq. (16) is reminiscent to an effective action proposed some time ago by

Schechter *et al.* [21]. For this we choose to rewrite the integral over the positions of the instantons as an integral over the density of the instantons by allowing for the number of instanton and anti-instanton to *change*. To achieve this we partition the total Euclidean volume V in blocks ΔV_i containing $n^+(z_i)$ instantons and $n^-(z_i)$ anti-instantons, where z_i denotes the position of the block ΔV_i . Assuming a sufficient amount of coarse graining we have

$$\int \prod_I dz_I \bar{w}(\rho_I) d\rho_I F(z_I) = \int \prod_{\Delta V_i} dn^+(z_i) dn^-(z_i) F_+^{n^+(z_i)}(z_i) F_-^{n^-(z_i)}(z_i). \quad (18)$$

In the limit $\Delta V \rightarrow 0$ this integral becomes a functional integral,

$$\int \prod_z dn^+(z) dn^-(z) F_+^{n^+(z)}(z) F_-^{n^-(z)}(z). \quad (19)$$

So far we have only discussed the leading term in the grand-partition function: the instanton density is fixed. In what follows, we will also include fluctuations about this density. (In general, in the grand-partition function these fluctuations are induced through correlations in the collective variables.) The attitude we are taking here is that the mean-field approximation for the liquid phase provides a good description for the bulk parameters of the QCD vacuum. Therefore we expect that correlations in the density of instantons can be described by few low order moments.

With this in mind, we take the pseudoparticle densities at different space-time points to be uncorrelated. For the pseudoparticles densities at the same space-time points we have

$$\begin{aligned} \langle\langle n^+(z) \rangle\rangle &= \langle\langle n^-(z) \rangle\rangle = \frac{1}{2} \frac{N}{V}, \\ \langle\langle n^+(z) n^-(z) \rangle\rangle &= \langle\langle n^+(z) \rangle\rangle \langle\langle n^-(z) \rangle\rangle. \end{aligned} \quad (20)$$

The functional average over the distribution of the pseudoparticles has been denoted by $\langle\langle \dots \rangle\rangle$. Since different pseudoparticles are uncorrelated we have

$$\int d^4z \langle\langle (n^+(z) - n^-(z))^2 \rangle\rangle = \frac{N}{V}. \quad (21)$$

Thus the variance in the number difference is of order N_c . The variance of the total number of instantons and anti-instantons in the vacuum

$$\sigma^2 = \int d^4z \langle \left(n^+(z) + n^-(z) - \frac{N}{V} \right)^2 \rangle \quad (22)$$

is related to the compressibility of the instanton liquid. Both the variational calculations of Dyakonov and Petrov and low energy theorems indicate that $\sigma^2 \sim N_c^0$. In our case it will be fixed by the requirement that the scale anomaly is correctly reproduced at tree level. This requirement is consistent with the low energy theorems. Therefore, we expect $\sigma^2 / \frac{N}{V} \sim N_c^{-1}$ to vanish in the large N_c limit, indicating a sharply peaked distribution in the total instanton number density.

Using the coarse grained version of (16) and (20-22), we obtain the following effective action for the instanton liquid state

$$\begin{aligned} S = & \frac{1}{\sigma^2} \frac{N}{V} \int d^4z (n^+(z) + n^-(z)) \left(\log \frac{n^+(z) + n^-(z)}{\frac{N}{V}} - 1 \right) \\ & + \frac{V}{2N} \int d^4z (n^+(z) - n^-(z))^2 \\ & + \int d^4z \left(\psi^\dagger S_0^{-1} \psi - n^+(z) \log \det_+(z) - n^-(z) \log \det_-(z) \right) \end{aligned} \quad (23)$$

where we have used the short-hand notation for the smeared 't Hooft determinants (the z dependence relates to the global position of an instanton or anti-instanton)

$$\det_\pm(z) = \frac{1}{N_f!} \langle \det_{fg} \left(m\rho - \frac{\rho}{2i} \int \psi^\dagger f S_0^{-1} \phi_\pm \int \phi_\pm^\dagger S_0^{-1} \psi g \right) \rangle_{U\rho} \quad (24)$$

The fluctuations in the number densities in (23) stand for the "gluonic" contribution to the effective action. The distribution of the difference $n^+(z) - n^-(z)$ is Gaussian with a width given by (21). The distribution of the sum $n^+(z) + n^-(z)$ is logarithmic and consistent with (20-22) in the Gaussian approximation (to be justified below)². The latter is exact in the large N_c limit.

²The logarithmic choice is also motivated by the Stirling form of the pre-exponential factorials in the grand-partition function.

The chiral limit of eq. (23) is obtained by first taking the mass originating from S_0^{-1} to zero, and then taking the mass of the zero-mode terms in $\det_{\pm}$ (see eq. (5)) to zero. In doing this, the factor $\frac{1}{2}$ in the determinant in eq. (24) is replaced by 1. In this limit we will discuss the properties of S under chiral and scale transformations.

Under an infinitesimal $U_A(1)$ transformation $\psi(z) \rightarrow (1 + i\alpha(z)\gamma_5)\psi(z)$ where α varies slowly on the scale of the size of the instantons, we have

$$\partial_{\mu} j_{5\mu} = -2N_f(n^+(z) - n^-(z)). \quad (25)$$

The axial singlet current vanishes in a CP symmetric vacuum. Since the variance of the number difference (21) (topological susceptibility) does not vanish, we expect (25) to generate nonvanishing matrix elements between the vacuum (topologically neutral state) and pseudoscalar singlet excitations (*e.g.* η_0). This result is consistent with QCD.

To exhibit the scale anomaly, let $\rho \rightarrow \lambda^{-1}\rho$, $\psi(z) \rightarrow \lambda^{\frac{3}{2}}\psi(\lambda z)$, and $n^{\pm}(z) \rightarrow \lambda^4 n^{\pm}(\lambda z)$ in (23). Under this transformation the first term in the effective action (23) transforms as

$$\frac{\delta}{\delta\lambda} \left(\frac{1}{\sigma^2} \int d^4z (n^+(z) + n^-(z)) \left(\log \frac{n^+(z) + n^-(z)}{\frac{N}{V}} - 1 \right) \right) = + \frac{4}{\sigma^2} (n^+(z) + n^-(z)). \quad (26)$$

It follows that the divergence of the dilatational current does not vanish at tree level (*i.e.* only classical instanton and antiinstanton configurations). The result is

$$\partial_{\mu} j_{\mu}^{dil} = \left(\frac{4}{\sigma^2} \left(\frac{N}{V} \right) + \mathcal{O}(1/\log^2(\rho\Lambda)) \right) (n^+(z) + n^-(z)). \quad (27)$$

The first term is the gluonic contribution to the trace anomaly, while the second term is the fermionic contribution. The latter contribution is obtained by using the one-loop beta function in the size distribution $\bar{w}(\rho)$. Since it is subleading in $1/N_c$, we will not take it into account any further.

If we were to identify the instanton density N/V in the liquid phase with the value of the gluon condensate $\langle 0|F_{\mu\nu}^2/32\pi^2|0\rangle$, then a comparison of the vacuum expectation value of (27) with the usual QCD trace anomaly yields the following equation for the variance in the large N_c limit

$$\sigma^2 = \frac{4}{b} \frac{N}{V} \quad (28)$$

where $b = \frac{11}{3}N_c$. This result was first obtained by Dyakonov and Petrov [7] in the instanton liquid using a detailed variational calculation. Notice that in the large N_c limit the instanton liquid becomes incompressible. For $N_c = 3$, the ratio $\sigma^2/N/V = 4/11$ indicates a peaked distribution in the total number of instantons. *A posteriori*, this result justifies the use of a Gaussian approximation for the distribution of the total density of the pseudoparticles.

By introducing the coarse grained fields $H(z) = \frac{1}{2}(n^+(z) + n^-(z))$ (scalar glueball) and $Q(z) = \frac{1}{2}(n^+(z) - n^-(z))$ (pseudoscalar glueball), the grand-canonical partition function Ξ corresponding to the effective action (24) can be rewritten as

$$\begin{aligned} \Xi = & \int d\psi d\psi^\dagger \exp\left(-\int \psi^\dagger S_0^{-1} \psi\right) \int \prod_z 2dH(z)dQ(z) W(H, Q) (\det_+(z) \det_-(z))^{H(z)} \\ & \times \left(\frac{\det_+(z)}{\det_-(z)}\right)^{Q(z)}. \end{aligned} \quad (29)$$

The instanton and anti-instanton distribution functional $W(H, Q)$ can be read immediately from the effective action (23). It is logarithmic in H and Gaussian in Q . Notice that the combination $(\det_+ \det_-)$ is $U_A(1)$ invariant, while the combination $(\det_+ / \det_-)$ breaks $U_A(1)$. As mentioned above, the fluctuations in the number difference Q are at the origin of the $U_A(1)$ anomaly. In general the distribution functional $W(H, Q)$ is more complicated than indicated above. However, for a system of instantons and anti-instantons in the liquid phase, we expect the above approximations to be satisfactory. These approximations should be contrasted with the dilute gas approximation where the collective variables decouple. In this case n^+ and n^- are uncorrelated, and we find that the compressibility for the total number of pseudoparticles is equal to one. However, this case does not have a well

behaved infrared limit. Therefore it is clear that the 't Hooft effective actions that have been derived with the dilute gas approximation can be at best qualitative.

To conclude, we have shown that the 't Hooft effective interactions between quarks of different flavors can be obtained in a uncorrelated instanton liquid model for the QCD vacuum. In the long wavelength limit, chiral dynamics in the liquid state can be mapped onto an effective action that has the usual chiral and scale anomalies. In particular, the compressibility of the instanton liquid is determined by the scale anomaly and vanishes in the large N_c limit. Our results agree with detailed estimates based on the Feynman variational principle. In future work, we will investigate the character of the long-wavelength excitations as implied by the effective action (23), and the relationship between the topological susceptibility (fluctuations in the number difference of instantons) and the mass of the pseudoscalar singlet excitations.

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