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"n-flavour Thirring models from compact Chern-Simons CP^1 theories and high- T_c Superconductivity"

By

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NOTE ADDED

To avoid confusion, perhaps it would be useful to make the following comment regarding the fermionisation of the Chern-Simons term in eqs. (12) and (16). In the CP^1 -CS theories under consideration, the fermion fields ψ^i are "isodoublets" as mentioned above eq. (12), which explains the extra factor of 2 in the coefficient of the CS term in comparison with the results of ref. [14]. Such CP^1 factors have been suppressed in the notation of the Maxwell terms, whose precise magnitude is irrelevant for our qualitative considerations. Note therefore the difference of the present situation from that of a CS QED where the coefficient of the CS term is quantised to $n/8\pi$ (n integer) instead, due to the parity anomaly of the gauge field coupled to single fermions. It worths noticing that our quantisation condition (8), pertaining to the CP^1 σ -model case in the non-zero flux sector, is consistent with the fermionisation discussed in the present work.

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n - FLAVOUR THIRRING MODELS FROM COMPACT CHERN - SIMONS CP^1 THEORIES AND HIGH - T_c SUPERCONDUCTIVITY.

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Abstract

We examine the low-energy properties of a system involving a CP^1 σ -model and an Abelian Chern-Simons term whose coefficient is fixed to $n/4\pi$, n =integer, by gauge invariance requirements on a three-dimensional manifold of the form $S^2 \times S^1$; we find that the system is equivalent at low momenta to a system of n neutral fermions with strong Thirring-type interactions between currents corresponding to different flavours, which cannot be fine-tuned to zero. Given that such systems may be considered as long wavelength limits of appropriate statistical systems describing high T_c superconductors, we relate these qualitative considerations to Laughlin's ideas on pairing of spinons and/or holons in order to explain the experimentally observed charge $-2e$ order parameter.

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The field theoretic approach to high T_c superconductivity, initiated by the work of Soviet scientists [1], revived interest in $2+1$ -dimensional Abelian gauge theories with (parity (P) and time-reversal (T) violating) Chern-Simons (CS) terms. The latter are known to be responsible for spin-statistics transmutations of the solitonic solutions of the modified non-linear σ models, describing the long wavelength physics of microscopic models relevant to high T_c superconductivity [2].

Although this result was known for some time [3], only after the work of the Soviet physicists was it realised that there are two phases [2b,4], a charge phase and a flux phase, related to each other by a duality transformation. Specifically, if $n(x)$ denotes the effective spin in the modified (with CS terms) non-linear σ model in $2+1$ dimensions, then one can pass into the so-called CP^1 representation by defining complex point-like excitations $z(x)$, which are, in some sense, the square root of n , $n(x) = z^\dagger(x)\sigma z(x)$, with σ the Pauli matrices. The two phases are obtained by identifying the solitons as corresponding to flux tubes of flux Φ_{sol} and spin S_{sol} , and the z -quanta as being particle-like excitations of charge Q and spin S_z . Following elementary steps of classical electrodynamics, one can obtain the following generic relations between the spin S , the flux Φ and the charge Q of the excitations of a theory with only Chern-Simons terms [5] (Maxwell-like kinetic terms are ignored in the low momenta approximation),

$$\begin{aligned}\frac{1}{4\pi}Q &= k\Phi \\ \frac{1}{4\pi}S &= \frac{k}{2}\Phi^2\end{aligned}\tag{1}$$

where k is the coefficient of the CS action

$$S_{CS} = k \int_{\mathcal{M}} d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho\tag{2}$$

In the flux phase the elementary excitations create units of flux $\Phi = 1$, whilst in the particle (z -quanta) phase they create units of charge $Q = 1$. Thus, the induced spin change of the solitons is given by the familiar expression [3]

$$\Delta S_{sol} = 2\pi k \quad (3)$$

and the soliton statistics is of order $2S_{sol}$ (the latter is found by a Bohm–Aharonov type of experiment involving the circulation of the gauge field along a curve surrounding two non – intersecting flux tubes [5]).

The corresponding spin change of the z – quanta is given by

$$\Delta S_z = \frac{1}{8\pi k} \quad (4)$$

and the statistics is of order $1/4k\pi$. These results can also be found by an explicit calculation [4] extending Polyakov’s approach [1c] for $\theta = \pi$ to theories with arbitrary coefficient of the CS term.

Comparing (2) with (3) one obtains the duality relation [4]

$$\Delta S_{sol} = \frac{1}{4\Delta S_z} \quad (5)$$

From an effective Lagrangian point of view [6,7] this duality relation can be understood by considering a theory with Chern – Simons terms coupled to an identically conserved current J_μ , written in terms of a new gauge field a_μ as

$$J^\mu = \frac{1}{2\pi} \epsilon^{\mu\nu\rho} \partial_\nu a_\rho \quad (6)$$

Integrating out the field A , we obtain a new theory which at low momenta contains an induced Chern – Simons term for the a – field of the form

$$\frac{1}{16\pi^2 k} \int_{\mathcal{M}} d^3x \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \quad (7)$$

Eq. (5) follows from computing the spin of the relevant a – field Wilson lines [4]. This equation also shows that in the case of spin $1/2$ both excitations are fermionic, thereby suggesting the identification

of the solitons with the neutral fermions of the RVB theory [8]. There are problems, however, with such an identification: it leads to a flux quantization (charge e) which has not been observed experimentally. We shall explain below how this problem can be overcome.

For the time being we note that, as explained in refs. [2a,4], the relevant models for a high T_c analysis are formulated not on a compactified three-sphere, but on a product $S^2 \times S^1$ as a result of a high temperature expansion (for all practical purposes, e.g. handling possible infra-red infinities, we compactified the 2D space). Following a standard differential geometric analysis [9] one is then led to a topological quantization rule for the coefficient of the CS term in the Abelian case¹ (unlike the S^3 case),

$$4\pi k = n, \quad n \in \mathbb{Z} \quad (8)$$

thus obtaining a fractional spin-statistics in the "particle phase", for $n > 1$, but an ordinary integer or half-integer spin and integral statistics for the flux phase [4].

In connection with these results one can now try to apply Laughlin's ideas about the relation of fractional statistics excitations and high T_c superconductivity [10]. In particular, in ref. [10] it has been suggested that a gas of particles obeying ν statistics exhibits an attractive gauge force between pairs of particles and is equivalent to a system of freely moving fermions in a magnetic field with strength $(1-\nu)e^{-1}hc\rho$, where ρ is the particle density. Now, an energy gap exists in the fermionic spectrum whenever the quantity $(1-\nu)^{-1}$ is an integer, which implies that in such a case the system exhibits superfluidity with an order parameter of charge $(1-\nu)^{-1}$. For the case at hand, gauge invariance restricts ν to $1/n$, n =integer, and the excitations are "anyons", in standard terminology [11]. The (phenomenologically relevant) conditions for superfluidity are satisfied for $n=2$ only, in which case one obtains the experimentally observed charge $-2e$ order parameter. From our point of view, it is the "particle phase" in which statistics $1/2$ is observed. In the soliton flux phase the statistics is integral. The above considerations obviate the pressing need to understand Laughlin's results in a more field-theoretic (qualitative) approach.

¹ The flux through any two-dimensional subsurface of $S^2 \times S^1$ is assumed to be a member of the integer Čech cohomology class $H^2_C(M, \mathbb{Z})$, which is the Dirac monopole quantization condition; in the case of the CP^1 theories under consideration this may be verified by an explicit calculation [9c].

It is the purpose of this letter to attempt this problem; in particular we shall show that under the quantization condition (8) – necessary for gauge $U(1)$ invariance on $S^2 \times S^1$ – a sort of fermionisation of the CS theory at low energies takes place, in the following sense. The partition function – ignoring the kinetic terms in the action, of higher order in derivatives – is equivalent, after integration over the gauge field, to a system of n fermionic species (flavours), neutral w.r.t the $U(1)$ gauge group. These fermions interact via a Thirring interaction appropriately extended to the many-flavor case. In ref. [12] this result was obtained for $n=1$ where, however, the charge $-2e$ flux quantization could not be understood; it must be stressed that this type of fermionisation pertains to anyons and is, therefore, different from the one discussed in ref. [13], where the particle excitations that induced fermionisation were not anyons, as corresponding to CS coefficients $1/n$. Of course the formalism of ref. [13] was developed on S^3 and, hence, there was no problem with the gauge invariance, unlike our case. It must be noticed that the results of ref. [13] reproduced, in canonical quantization, the conclusions about the induced statistics and spin of refs. [1c,4], in the path integral approach.

To start with, we consider the CP^1 – formulation of a non-linear σ model with a CS term; we remind the reader that such a system may, in principle, arise in the long wavelength limit of a doped Hubbard model, for small doping concentrations [2] (doped Mott insulators, which exhibit high T_c superconductivity can be described by such a system, in which a commensurate Wigner crystal of holes is formed at a critical but small doping concentration).

For simplicity we shall restrict ourselves to the case where the space-time manifold is flat, but we shall fix the coefficient of the topological CS term to the one dictated by gauge invariance on curved manifolds of the form $S^2 \times S^1$. It is understood that a complete analysis would require fermionisation on these manifolds, but the above simplification is sufficient for the qualitative considerations of this work. We use Minkowskian formalism throughout.

The action to be studied takes the form

$$S_0 = \frac{1}{\gamma_0} \int |(\partial_\mu - iA_\mu)z|^2 + \frac{n}{4\pi} \int \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \quad (9)$$

where $\gamma_0 = g_0^2/2$, with g_0 the (bare) gauge coupling constant, and $z^T = (z_1, z_2)$ a doublet of complex fields satisfying the constraint $z^\dagger z = 1$. Integrating out the magnons z in the path integral and expanding the result in powers of momenta we obtain Maxwell kinetic terms for the gauge field $F_{\mu\nu}^2(\Lambda)$ and terms of higher order in derivatives. The result is

$$\exp\{iW_{\text{eff}}[A]\} \simeq \exp\left\{\pm i\frac{n}{4\pi} \int \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho\right\} \left(1 - \frac{i}{24\sqrt{\sigma_0}} \int F_{\mu\nu}^2(A) + \dots\right) \quad (10)$$

where $(\sigma_0)^{1/2} = M - 4\pi/g^2$, with g^2 the renormalized gauge coupling constant and M a transmutation mass. They are related via,

$$\frac{1}{g^2} = \frac{1}{g_0^2} - \frac{\Lambda}{2\pi^2} + \frac{\sqrt{M^2}}{4\pi} \quad (11)$$

with Λ an ultra-violet (UV) cut-off.

In ref. [14] it has been shown that for $n=1$, the CS term can be fermionised at low momenta, i.e. it can be traded off for a minimally coupled (massive) two-component spinor

$$\begin{aligned} & \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp\left\{i \int_{\mathcal{M}} \bar{\psi}(i\gamma^\mu(\partial_\mu + iA_\mu) \pm m)\psi\right\} \\ & \simeq \exp\left\{i \int_{\mathcal{M}} (\pm \frac{1}{4\pi}) \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho\right\} \end{aligned} \quad (12)$$

where the coefficient of the (P and T violating) topological CS term depends only on the sign and not the magnitude of the fermion mass, which, nevertheless, must be heavier than any momentum scale in the system.

In a recent study [15], the fermionisation procedure outlined above has been extended to include the Maxwell term². We quote the results for future use:

² In ref. [15] it was claimed that a Maxwell term with the wrong sign emerged. According to G. Semenoff this is not correct; the correct result yields a kinetic term for the gauge field with the correct sign, and does not disturb unitarity of the effective theory. We use the corrected term here, although its signature is, actually, irrelevant for our qualitative considerations.

$$\begin{aligned} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS_\psi} &= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp \left\{ i \int_{\mathcal{M}} \bar{\psi} (i\gamma^\mu (\partial_\mu + iA_\mu) \pm m) \psi + \frac{c}{m} (J_\psi^\mu)^2 \right\} \\ &\simeq \exp \left\{ \pm \frac{i}{4\pi} \int_{\mathcal{M}} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - i \left[\frac{1}{24\pi m} + \frac{c}{16\pi^2 m} \right] \int_{\mathcal{M}} F_{\mu\nu}^2(A) \right\} \end{aligned} \quad (13)$$

where $J_\psi^\mu = \bar{\psi}\gamma^\mu\psi$. The interaction term on the l.h.s. of (13) is a Thirring interaction. Demanding equivalence of (13) with (10), for $n=1$, fixes the value of c

$$c = \frac{2\pi}{3} \left(\frac{m}{\sqrt{\sigma_0}} - 1 \right), \quad \sqrt{\sigma_0} = M - \frac{4\pi}{g^2} > 0 \quad (14)$$

(at low momenta we are working in the region $(\sigma_0)^{1/2} > 0$; the region $(\sigma_0)^{1/2} < 0$ is probably reached by a phase transition, but in our low momentum approximation we are far from the critical point).

We tackle now the Lagrangian (9). Since we have fixed the coefficient of the CS term to be $n/4\pi$, n =integer, we can split the effective action for the A -field in a product of n factors, and fermionise – at low momenta – each one separately, thereby introducing an n -flavour multiplet of fermionic fields $\psi^i, i=1,2,\dots,n$. Hence,

$$\begin{aligned} \mathbf{I} &= \int \mathcal{D}\bar{z}\mathcal{D}z\mathcal{D}A \exp \left\{ \pm i \frac{n}{4\pi} \int_{\mathcal{M}} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \frac{i}{\gamma_0} \int_{\mathcal{M}} |(\partial_\mu - iA_\mu)z|^2 \right\} \\ &\Rightarrow \int \mathcal{D}\bar{z}\mathcal{D}z\mathcal{D}A \prod_{j=1}^n \left[\exp \left\{ \pm \frac{i}{4\pi} \int_{\mathcal{M}} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \right\} \right] \exp \left\{ i \int_{\mathcal{M}} \mathcal{L}_z(A) \right\} \end{aligned} \quad (15)$$

For simplicity we use a common (heavy) fermionic mass (since its magnitude is irrelevant). We also use a common cut-off and a common transmutation mass, since the A -theory contains one single gauge field and only one z -doublet. We now extend the analysis of ref. [12] for the $n=1$ case to the multiflavour case of eq. (9), which is physically more interesting (as explained above, the case $n=2$ is relevant for high T_c phenomena). First we trade off each one of the CS terms in each factor for a minimally coupled fermion ψ^i (at low momenta):

$$\mathbf{I} = \int \mathcal{D}\bar{z}\mathcal{D}z\mathcal{D}A \mathcal{D}\bar{\psi}^i \mathcal{D}\psi^i \exp \left\{ i \int_{\mathcal{M}} \sum_{j=1}^n \bar{\psi}^j (i\gamma^\mu (\partial_\mu + iA_\mu) \pm m) \psi^j + i \int_{\mathcal{M}} \mathcal{L}_z(A) \right\} \quad (16)$$

Integrating out the gauge field in (16), in the Landau gauge for simplicity, one is left with kinetic terms for the z and ψ^i fields and a current squared term of the form

$$-\frac{1}{4\gamma_0} \left(J_z^\mu - \gamma_0 J_\psi^\mu \right)^2; \quad J_z^\mu = 2i \bar{z} \partial^\mu z \quad (17)$$

The J_z^2 term exactly cancels the z -kinetic terms (this can be checked very easily using the CP^1 matrix formulation [12], in which case eq. (17) becomes $(\frac{-1}{8\gamma_0}) \text{Tr} |J_z - \gamma_0 \sigma_3 J_\psi|^2$, where the current J_z is now a matrix valued quantity given by an expression similar to (17), but with z a 2×2 unitary matrix of the form :

$$\begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}$$

where the constraint $\det z = 1$ is taken care of).

Defining new fermion fields,

$$\chi^i = z \psi^i, \quad i = 1, \dots, n \quad (18)$$

we can absorb the cross-term $J_z J_\psi$ and the ψ kinetic terms into the χ kinetic terms. We thus write the partition function as:

$$\mathbf{I}_{z\psi^i} = \mathbf{I}_{\chi^i} = \int \mathcal{D}\bar{\chi}^i \mathcal{D}\chi^i \exp \left\{ i \int_{\mathcal{M}} \sum_{j=1}^n \bar{\chi}^j (i\gamma^\mu \partial_\mu \pm m) \chi^j - \frac{\gamma_0}{4} \left(\sum_{l=1}^n \bar{\chi}^l \gamma^\mu \chi^l \right)^2 \right\} \quad (19)$$

The Jacobian of the χ redefinition yields terms proportional to $z^\dagger z = 1$, so the z -integration contributes only to an overall normalization factor.

Expression (19) is the main result of this article, extending the conclusion of ref. [12] to the multi-flavour case. Note that the charge of the z and ψ^i fields are equal in magnitude and opposite in sign, which implies the neutrality of the χ -fermion fields with respect to the $U(1)$ gauge symmetry. The low-energy version of (9) yields an n -flavour Thirring model with current-current interactions among different flavours in general. It should be stressed that the cancellation of the z -kinetic terms, after the fermion field redefinition and the integration over the gauge field, is exact, to all orders in momenta. The potential among the fermionic (physical) degrees of freedom χ^i , on the other hand, is a

non-local expression, which to lowest non-trivial order in the long-wavelength limit yields the Thirring interaction shown above.

We now make use of the results in ref. [15] to establish that, at the next order in momenta, a Thirring interaction DIAGONAL IN FLAVOUR (i.e. $\delta_{ij}(\bar{\psi}^i \gamma_\mu \psi^i)^2 (\bar{\psi}^j \gamma^\mu \psi^j)$) can be added to the Dirac term, resulting in the following equivalence:

$$\begin{aligned} \int \mathcal{D}\bar{\psi}^i \mathcal{D}\psi^i \exp \left\{ i \sum_{i=1}^n \left[\int_{\mathcal{M}} \bar{\psi}^i (i\gamma^\mu (\partial_\mu + iA_\mu) \pm m) \psi^i + \frac{c}{m} \int_{\mathcal{M}} (\bar{\psi}^i \gamma^\mu \psi^i)^2 \right] \right\} \\ \simeq \exp \left\{ \pm i \frac{n}{4\pi} \int_{\mathcal{M}} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - in \left[\frac{1}{24\pi m} + \frac{c}{16\pi^2 m} \right] \int_{\mathcal{M}} F_{\mu\nu}^2(A) \right\} \end{aligned} \quad (20)$$

This implies that one can view the coefficient of the Thirring self-interactions between currents corresponding to the SAME flavour as a coefficient to be tuned. From the physical point of view this result is relevant for pairing mechanisms which could be related to Laughlin's ideas [10]; indeed a renormalization of γ_0 implies that the strength of the Thirring interaction grows as the momentum decreases [15]. Because of (20), with an appropriate choice of c , one can probably fine-tune the self-interactions of a given flavour to zero, leaving only strongly coupled (off-diagonal) attractive current-current Thirring terms of the form:

$$-\frac{\gamma_R}{2} \sum_{i < j} \left(\bar{\chi}^i \gamma^\mu \chi^i \right) \left(\bar{\chi}^j \gamma_\mu \chi^j \right) \quad (21)$$

with $\gamma_R = g^2/2$.

This last result may be helpful in establishing a qualitative understanding of Laughlin's ideas [10]; his model requires $n=2$, and, therefore, a two-flavour Thirring model whose interactions between different flavours grow with decreasing momenta, thereby dominating against pairings of the fermions χ^i (NEUTRAL w.r.t to the original U(1) symmetry [12]) with holes (or electrons); of course a Lagrangian formalism is not capable by itself of describing the mechanisms for spinon-spinon-spinon-

holon and holon-holon bound states, but we think that the qualitative analysis presented above is sufficient to explain why in the $n=2$ case a charge e (spinon-hole) pairing is ruled out³, in agreement with experimental data.

On the other hand, to see the formation of holon-holon states, one has to consider the motion of a fermion ψ_h (e.g. hole) minimally coupled to a gauge field in a CS environment with coefficient $n/4\pi$. This is equivalent to considering one more flavour in our Thirring model, whose self-interactions, however, cannot be fine-tuned to zero. In this way holon-holon and spinon-holon interactions can be generated in accordance with Laughlin's suggestions [10].

Given the superfluidity analogy in the $n=2$ case, it would be interesting to examine whether the dressing of fermions and bosons could be studied by introducing a supersymmetric extension of the CS theories [16], in analogy with the two-dimensional superfluid helium case where 2D supersymmetry is physically realized [17]. The planar high T_c superconducting oxides might constitute physical realizations of 3D supersymmetry between holes and magnons; the simplest model has a single supersymmetry. It can be most easily studied using superfield formalism, where the incorporation of constraints consistent with $N=1$ supersymmetry, is straightforward. Restricting ourselves to the quantization condition (8), trading off the bosonic part of CS terms for n -minimally coupled fermions, then integrating out gauge fields and gauginos, performing fermion redefinitions similar to the ones above and, finally, integrating out magnons, we are left with kinetic terms for "holes" (obtained from a redefinition of the magnino with its bosonic z -partner) and neutral fermions plus interactions among these fermionic fields. The physical meaning of the resulting potential terms has not yet been studied to a satisfactory level, neither is it clear whether such models are physically relevant (i.e. can be obtained from, say, supersymmetric Hubbard models describing high T_c phenomena). Such important issues fall beyond the scope of this article and are left for future work.

Some comments concerning the parity-violating mass terms $m\bar{\psi}^i\psi^i$ in (20) are in order. Notice that these (heavy) mass terms come with the same sign for all flavours and, therefore, necessarily violate parity and time-reversal symmetry. The situation is, thus, different from multiflavour massless

³ In the $n=1$ case, such pairing might take place, provided that the coefficient of the Thirring interaction can be fine-tuned to zero (even after radiative corrections are included). Of course, it is understood that explaining the physical reasons for ruling out the $n=1$ case (and selecting the $n=2$ one) requires more than a simple field theoretic analysis.

QED, for which an interesting dynamical (Schwinger – Dyson) generation of a parity – conserving mass takes place for an even number of flavours smaller than a critical value (which in two – component spinor notation turns out to be $64/\pi^2$), in the lowest order in $1/N$ expansion (with N the number of flavours of two component spinors)⁴ [18]. In such a case, $N/2$ spinors acquire masses m , $N/2$ acquire masses $-m$, so there is no overall breaking of parity. Such parity – conserving mass components can always be generated, therefore, in the case $N=2$ we are interested in; they obviously survive the limit of taking the CS coefficient to zero. This might be interesting from the experimental point of view were such models to describe the long – wavelength physics of high T_c superconducting oxides.

To recapitulate, in this letter we have examined the low – energy properties of a field – theoretic model involving an extended CP^1 non – linear σ – model with a CS term, whose coefficient is quantized by gauge invariance requirements on $S^2 \times S^1$. The model arises as the continuum limit in a high temperature expansion of microscopic models pertaining to high T_c superconductors. Gauge invariance fixes the CS coefficient to $n/4\pi$, $n = \text{integer}$, which leads to representing the model at low energies as an n – flavour Thirring model with interactions that grow with decreasing momenta. The latter observation is relevant in discussing pairings among the elementary excitations which lead to realistic high T_c models. The question of selecting the coefficient n is dynamical in nature, and cannot be answered by Lagrangian considerations only. Laughlin’s choice ($n=2$) is a physically appealing one, which however awaits a better understanding in terms of the underlying microscopic theory. In our approach we demonstrated the field theoretic description of Laughlin’s ideas, which can be viewed as the (approximate) fermionisation of the anyon case. The duality considerations in the beginning of the article allow one to characterize Laughlin’s approach involving fractional statistics excitations as corresponding to the “particle” phase, the flux phase being characterized by integer (1) spin and statistics.

⁴ Recently there have been some numerical studies [19] indicating parity – violating dynamical mass generation in a theory with a single flavour of two – component spinors.

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References

- [1]a. I. Dzyaloshinskii, A.M. Polyakov and P.B. Wiegmann, Phys. Lett. A127, 112 (1988).
b. P.B. Wiegmann, Phys. Rev. Lett. 60, 821 (1988); Landau Inst. preprint 1988.
c. A.M. Polyakov, Mod. Phys. Lett. A3, 325 (1988).
- [2]a. I.J.R. Aitchison and N.E. Mavromatos, Phys. Rev. B39, 6544 (1989).
b. P.B. Wiegmann, Phys. Scr. T27, 160 (1989).
- [3]F. Wilczek and A. Zee, Phys. Rev. Lett. 51, 2250 (1983);
Y.S. Wu and A. Zee, Phys. Lett. 147B, 325 (1984);
R. Jackiw, in "Current Algebra and Anomalies", ed. Treiman et. al. (World Scient. 1987).
- [4]I.J.R. Aitchison and N.E. Mavromatos, Mod. Phys. Lett. A4, 521 (1989).
- [5]c.f. for example A.P. Polychronakos, Phys. Lett. B192, 385 (1987);
see also, I.J. R. Aitchison and N. Mavromatos, contribution to the ESPRIT meeting 3041 – MESH, at
Heraklion – Crete, Greece, 8 – 9 June 1989, to be published.
- [6]X.G. Wen and A. Zee, Santa Barbara preprint NSF – ITP – 145, 1989.
- [7]I.J.R. Aitchison and N.E. Mavromatos, Oxford preprint OUTP – 89 – 12P.
- [8]P.W. Anderson, Science 235 (1987);
G. Baskaran, Z. Zou and P.W. Anderson, Solid State Comm. 63, 973 (1987);
Z. Zou and P.W. Anderson, Phys. Rev. B37, 627 (1988).
- [9]a. O. Alvarez, Comm. Math. Phys. 100, 279 (1985);
b. A.P. Polychronakos, Nucl. Phys. B281, 241 (1987).
c. I.J.R. Aitchison, Act. Phys. Polonica B18, 207 (1987).
- [10]R.B. Laughlin, Phys. Rev. Lett. 60, 2677 (1988); Science 242, 525 (1988).
- [11]R. Mackenzie and F. Wilczek, Int. J. Mod. Phys. A3, 2877 (1988).
- [12]L. Huerta and M. Ruiz – Altaba, Phys. Lett. B216, 371 (1989).
- [13]M. Luscher, DESY preprint 89 – 050, April 1989.
T. Matsuyama, Kyoto preprint RIPP – 803, 1989.
- [14]S. Deser, R. Jackiw and S. Templeton, Ann. Phys. NY 140, 372 (1982).
I. Affleck, J. Harvey and E. Witten, Nucl. Phys. B206, 413 (1982).

A.N. Redlich, Phys. Rev. D29, 2366 (1984).

[15]S. Deser and A.N. Redlich, Phys. Rev. Lett. 61, 1541 (1988).

[16]J. Schonfeld, Nucl. Phys. B185, 157 (1981).

[17]Z. Qiu, EFI - preprint 1986 and references therein;

A.M. Polyakov, "Gauge Fields and Strings" (Harwood, New York 1987).

[18]T. Appelquist, M. Bowick, D. Karabali and L.C. Wijewardhana, Phys. Rev. D33, 3774 (1986);

T. Appelquist, D. Nash and L.C. Wijewardhana, Phys. Rev. Lett. 60, 2575 (1988);

E. Dagotto, J. Kogut and A. Kocic, preprint ILL - TH - 88 - 30, 1988;

[19]Y. Hoshino and T. Matsuyama, Kyoto preprint RIFP - 791, 1989.