HIGHER ORDER QED CORRECTIONS TO FORWARD-BACKWARD ASYMMETRY

G. Montagna and O. Nicrosini

I.N.F.N., Sezione di Pavia, and
Dipartimento di Fisica Nucleare e Teorica dell'Università
Pavia, Italy

and

L. Trentadue ^{*)}

CERN - Geneva

ABSTRACT

The problem of evaluating QED radiative corrections to forward-backward asymmetries at LEP/SLC energies is addressed. Using a formalism based on structure functions for initial and final states, a result is derived which includes exact $O(\alpha)$ -expression and box-interference corrections as well as resummation to all orders of the radiation. A comparison with the existing calculations is discussed.

*) Permanent address: Dipartimento di Fisica dell'Università, Parma, I.N.F.N., Gruppo Collegato di Parma, Sezione di Milano, Italy

1. Introduction

As is well known, precision tests of the Standard Model are among the main goals of LEP and SLC colliders. These accelerators will be able to measure with a high degree of accuracy several observables such as total cross sections, the Z^0 peak position, width and line shape and asymmetries. The forward-backward asymmetry, in particular, is a very sensitive and direct measurement of the parity violating couplings^[1]. The experimental uncertainty is expected to be of order 0.3 %, so that the problem of achieving a comparable theoretical accuracy is a very stringent one^[2].

The theoretical uncertainty is dominated by the electromagnetic radiative corrections^[2]. If, on the one hand, order α QED radiative corrections are known^[3], it is on the other hand also known that higher order QED corrections crucially affect any physical observable^[4]. These higher order corrections, related to infra-red and collinear singularities, have been computed by using the formalism of the structure functions^[5,6].

The aim of this note is to apply the structure function formalism to the calculation of all-orders forward-backward asymmetries. It is organized as follows: in sect. 2 the Born approximation formulae are collected for the process $e^+e^- \rightarrow \gamma, Z^0 \rightarrow \mu^+\mu^-$; sect. 3 is devoted to the description of the structure function formalism and its application to the asymmetry; in sect. 4 numerical results are presented and discussed; in sect. 5 a comparison with existing calculations is performed and some conclusions are sketched.

2. Forward-backward asymmetries in the Born approximation

We confine ourselves to the process $e^+e^- \rightarrow \gamma, Z^0 \rightarrow \mu^+\mu^-$. The zeroth order differential cross-section is given by^[3]

$$\frac{d\sigma_0}{d\Omega_\mu} = \frac{\alpha^2}{4s} [W_1(s)(1+c^2) + W_2(s)c], \quad (2.1)$$

where $c = \cos\vartheta$, ϑ being the angle between the momenta of e^- and μ^- , and the functions $W_1(s)$ and $W_2(s)$ are given by

$$\begin{aligned} W_1(s) &= 1 + \frac{2(s-M^2)sc_v^2}{|Z(s)|^2} + \frac{s^2(c_v^2 + c_a^2)^2}{|Z(s)|^2}, \\ W_2(s) &= \frac{4(s-M^2)sc_a^2}{|Z(s)|^2} + \frac{8s^2c_v^2c_a^2}{|Z(s)|^2}, \\ Z(s) &= s - M^2 + iM\Gamma. \end{aligned} \quad (2.2)$$

In eq. (2.2) M and Γ are the mass and width of the Z^0 respectively. c_a and c_v are given by

$$c_a = -\frac{1}{2\sin(2\vartheta_W)}, \quad c_v = -c_a(4\sin^2\vartheta_W - 1).$$

The forward-backward asymmetry is defined as

$$A_0(s) = \frac{\sigma_0^{(+)}(s) - \sigma_0^{(-)}(s)}{\sigma_0(s)}, \quad (2.3)$$

where $\sigma_0^{(+)}(s)$ and $\sigma_0^{(-)}(s)$ are the forward and backward hemisphere cross-sections respectively, and $\sigma_0(s)$ is the total cross-section. An explicit calculation gives

$$\begin{aligned} \sigma_0^{(+)}(s) &= \frac{\pi\alpha^2}{4s} \left[\frac{8}{3}W_1(s) + W_2(s) \right], \\ \sigma_0^{(-)}(s) &= \frac{\pi\alpha^2}{4s} \left[\frac{8}{3}W_1(s) - W_2(s) \right], \\ \sigma_0(s) &= \sigma_0^{(+)}(s) + \sigma_0^{(-)}(s) = \frac{4\pi\alpha^2}{3s}W_1(s). \end{aligned} \quad (2.4)$$

Inserting eq. (2.4) into (2.3) one obtains the expression for the Born approximation asymmetry

$$A_0(s) = \frac{3}{8} \frac{W_2(s)}{W_1(s)}, \quad (2.5)$$

where $W_1(s)$ and $W_2(s)$ are given in eq. (2.2).

3. QED radiative corrections

We now introduce the formalism of structure functions to handle the computation of the radiative corrections to the Born approximation expression of eqs. (2.3) and (2.5). The e^+e^- annihilation process can be regarded as the analogue of the QCD Drell-Yan process^[5,6], provided that the initial fermions are not considered as elementary particles but as particles "dressed" by a cloud of photons. From this point of view the calculation of the QED radiative corrections is reduced to the calculation of the structure function for initial and final states. In particular the corrected cross-section can be written as a convolution of a "bare" cross-section with a "radiator" H defined as follows:

$$\begin{aligned} \sigma(s) &= \int_0^\varepsilon dx H(x, s) \sigma_0((1-x)s), \\ H(x, s) &= \int_{1-x}^1 \frac{dz}{z} D(z, s) D\left(\frac{1-x}{z}, s\right). \end{aligned}$$

$D(x, s)$ are the structure functions for the electrons and positrons: they represent the probability of finding inside a parent electron at the scale s an electron with x fraction of longitudinal momentum; $H(x, s)$ is the probability of radiating a fraction x of the available energy; ε represents the maximum fraction of energy radiated away. The radiator H contains QED radiative

contributions to all orders of perturbation theory. In particular it allows a complete $O(\alpha^2)$ calculation and resummation of the leading and part of the next-to-leading logarithms in the soft approximation^[6].

The structure function approach can be generalized to include also final state radiative corrections together with box and interference contributions^[7]. The totally corrected cross-section can be written in the form

$$\sigma(s) = \int_0^\varepsilon dx H_e(x, s) F_\mu(\varepsilon - x, (1-x)s) \sigma_0((1-x)s) + \int d\Omega_\mu \frac{d\sigma_B}{d\Omega_\mu}, \quad (3.1)$$

where $\frac{d\sigma_B}{d\Omega_\mu}$ is the box and interference contribution (see for example ref. [3]). H_e and F_μ represent initial and final state radiation respectively. They are given by^[7]

$$\begin{aligned} H(\xi, s) &= \Delta \beta \xi^{\beta-1} - \frac{1}{2} \beta (2 - \xi) \\ &\quad + \frac{1}{8} \beta^2 \left[(2 - \xi) [3 \ln(1 - \xi) - 4 \ln \xi] - \frac{4}{\xi} \ln(1 - \xi) + \xi - 6 \right], \\ F(\xi, s) &= \Delta \xi^\beta - \beta \left(\xi - \frac{1}{4} \xi^2 \right) - \frac{1}{2} \beta^2 \left[\text{Li}_2(1 - \xi) \right. \\ &\quad \left. + \ln(1 - \xi) \ln \xi + \frac{3}{2} \ln(1 - \xi) \left[(1 - \xi) + \frac{1}{4} (\xi^2 - 1) \right] \right. \\ &\quad \left. + 2 \ln \xi \left(\xi - \frac{1}{4} \xi^2 \right) - \frac{1}{16} \xi^2 + \frac{5}{8} \xi - \zeta(2) \right]. \end{aligned} \quad (3.2)$$

In eq. (3.2) β and Δ are defined in terms of the collinear logarithm $L = \ln \frac{s}{m^2}$ as

$$\begin{aligned} \beta &= 2 \frac{\alpha}{\pi} (L - 1), \\ \Delta &= 1 + \frac{\alpha}{\pi} \left[\frac{3}{2} L + 2 (\zeta(2) - 1) \right] \\ &\quad + \left(\frac{\alpha}{\pi} \right)^2 \left[\left(\frac{9}{8} - 2\zeta(2) \right) L^2 + \left(3\zeta(3) + \frac{11}{2} \zeta(2) - \frac{45}{16} \right) L \right. \\ &\quad \left. - \frac{6}{5} (\zeta(2))^2 - \frac{9}{2} \zeta(3) - 6\zeta(2) \ln 2 + \frac{3}{8} \zeta(2) + \frac{57}{12} \right]. \end{aligned} \quad (3.3)$$

The indices e and μ in eq. (3.1) refer to the fact that the collinear logarithm in H_e and F_μ have to be evaluated at the electron and muon mass respectively.

Radiative corrections to the Born approximation asymmetry are implemented by simply applying eq. (3.1) to the forward and backward hemisphere Born approximation cross-sections $\sigma_0^{(+)}$ and $\sigma_0^{(-)}$ of eq. (2.4)

$$\begin{aligned} \sigma^{(+)}(s) &= \int_0^\varepsilon dx H_e(x, s) F_\mu(\varepsilon - x, (1-x)s) \sigma_0^{(+)}((1-x)s) + \int_0^1 d\Omega_\mu \frac{d\sigma_B}{d\Omega_\mu}, \\ \sigma^{(-)}(s) &= \int_0^\varepsilon dx H_e(x, s) F_\mu(\varepsilon - x, (1-x)s) \sigma_0^{(-)}((1-x)s) + \int_{-1}^0 d\Omega_\mu \frac{d\sigma_B}{d\Omega_\mu}, \end{aligned} \quad (3.4)$$

and then combining the results according to the definition

$$A(s) = \frac{\sigma^{(+)}(s) - \sigma^{(-)}(s)}{\sigma(s)}. \quad (3.5)$$

In eq. (3.5) therefore exact $O(\alpha^2)$ real and virtual photonic radiation from initial- and final-state, $O(\alpha)$ box and interference contributions and exponentiation of the leading and part of next-to-leading logarithms are included. Finite order results can be obtained by simply expanding the initial- and final-state radiators to the desired order. Pure initial-state results are obtained by putting $F_\mu = 1$ in eq. (3.4).

It has been recently argued^[8] that the correct definition of the forward-backward radiatively corrected asymmetry differs from the convolution expression^[9] given in eqs. (3.4) and (3.5) by terms of $O(\alpha^2)$. However in the region around the Z^0 mass an excellent agreement of the convolution radiator formula with the exact result has been found^[10]. This example shows again here, as in the line shape case, the dominant role played around the Z^0 peak by the soft and collinear radiation, thus supporting the factorization mechanism of the structure function method.

4. Numerical results

In order to quantify the effects of QED radiative corrections, we have numerically evaluated eqs. (3.4) and (3.5) with the following choice for the parameters: $M = 93.2$ GeV, $\Gamma = 2.6$ GeV, $\sin^2 \vartheta_W = 0.223$.

In fig. 1 we have plotted the forward-backward asymmetry as a function of the centre-of-mass energy $\sqrt{s}$. The Born approximation asymmetry eq. (2.5) is the dotted line. The continuous and dashed lines represent the resummed and $O(\alpha)$ results respectively, obtained by using eqs. (3.4) and (3.5) with the resummed radiators of eq. (3.2) or their $O(\alpha)$ expression. No cut on the photon energy ($\varepsilon = 1$) has been assumed. Since in this kinematical situation the effects due to box and interference terms are negligible^[7], we have not taken them into account. As is well known, the effect of radiative corrections is to shift the position of the zeroes and to change sensibly the shape of the asymmetry under and above the resonance energy.

In the same situation as before (no photon-energy cut and no box-interference contribution), in fig. 2 we have plotted the forward-backward asymmetry as a function of the centre-of-mass energy in a very strict region around the Z^0 mass. The specific position of the zeroes for various orders can be evaluated. In particular, the dashed line corresponds to the Born approximation; the contribution of the initial and final state $O(\alpha)$ radiation to the asymmetry is represented by the dash-dotted curve; the only initial state $O(\alpha)$ result is, on this scale, indistinguishable from the initial-final one and

for this reason does not appear in the figure; according to eq. (3.4), resummation is included in the solid line which lies between the results for Born and $O(\alpha)$ asymmetries; the correction due to resummation of the initial-state radiation only gives a curve which is, in the scale considered, coincident with the total-resummation solid line as in the $O(\alpha)$ case. $O(\alpha)$ corrections shift the zero of the asymmetry of about 300 MeV towards higher energies; resummation lowers the zero of about 100 MeV with respect to the $O(\alpha)$ result.

In fig. 3 the box and interference contributions are taken into account when the energy radiated by the photon is less than 1 GeV ($\epsilon = 2/\sqrt{s}$). It has already been pointed out, in fact, that initial-final interference diagrams are important if photon-energy cuts are considered^[11]. Our results support this conclusion. The sparsely dotted line is the $O(\alpha)$ asymmetry without box-interference terms while the closely dotted line corresponds to $O(\alpha)$ asymmetry with the inclusion of box-interference corrections. The effect on the shape is remarkable: more precisely, the inclusion of box and interference effects gives larger values for the asymmetry at the same centre-of-mass energy and the position of the zero is changed towards a lower energy of about 100 MeV. It is worth noting that this correction is of the same order of magnitude and sign as the one due to resummation only (dash-dotted line). The same kind of conclusion can be achieved by comparing the resummed result without box and interference terms (dash-dotted line) with the one obtained by including their contributions (solid line).

5. Conclusions

The structure function formalism gives a compact and powerful method for computing all-orders QED radiative corrections to forward-backward asymmetries around the Z^0 peak. Finite order results are obtained by expanding the radiators given in eq. (3.2) to the desired order. In particular, expanding our result to $O(\alpha)$ and using the proper parameters, the results given in ref. [3] can be reproduced. Our results also agree with the behaviour for the forward-backward asymmetry given in ref. [11], where our formalism, but with slightly different structure functions and parameters, has been adopted.

Our conclusion is that the effect of resummation is important in the calculation of the asymmetry, causing both a sizeable shift in the location of the zero and an impressive modification of the shape, especially above the Z^0 resonance. Moreover, initial state resummation is completely exhaustive, being the bulk of the effect.

As far as inclusive quantities are concerned, this approach is absolutely satisfactory. If instead one is interested in less inclusive observables, i.e. for example in imposing cuts, the effect of resummation becomes of the same order of magnitude of the correction due to $O(\alpha)$ box and interference contributions so that, to be consistent, one has to take both into account.

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Figure captions

- Fig. 1 Forward-backward asymmetry as a function of the centre-of-mass energy. Born approximation of eq. (2.5) (dotted line), all-order result of eqs. (3.4) and (3.5) (solid line) and $O(\alpha)$ result obtained by expanding the radiators to this order (dashed line).
- Fig. 2 The same as fig. 1 (blow-up): Born approximation (dashed line), $O(\alpha)$ (dash-dotted line) and resummed (solid line) results.
- Fig. 3 $O(\alpha)$ corrected asymmetry without (sparsely-dotted line) and with (closely-dotted line) box and interference contributions; all order corrected asymmetry without (dash-dotted line) and with (solid line) box and interference contributions. Cut on the photon energy: $E_\gamma < 1$ GeV.

Fig. 1

Fig. 2

Fig. 3