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Phenomenology of three generation superstring models with naturally suppressed proton decay

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Abstract

We study the phenomenology of a recently identified class of three generation superstring models in which there is a natural suppression mechanism of the most dangerous proton decay processes occurring through dimension five operators.

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1 Introduction

The Calabi-Yau (C-Y) approach [1] to the compactification of the $E_8 \times E_8$ heterotic superstring [2] is certainly among the most promising ones. It delivers four dimensional gauge models with $N=1$ supersymmetry and chiral fermions belonging to 27 or $\overline{27}$ representations of the gauge group E_6 . For multiply connected C-Y spaces the Wilson loop mechanism (flux breaking) offers the possibility of further breaking E_6 down to a proper subgroup H at the compactification scale M_c . The net number of light chiral fermion generations (defined as the difference between the number of 27's and $\overline{27}$'s) is a topological invariant of the C-Y manifold equal to one half the absolute value of its Euler characteristic. A phenomenologically important property of the C-Y approach is the appearance in the compactified theory of certain discrete symmetry groups directly related to the internal space.

In spite of the lack of a complete classification of all C-Y spaces, an extensive search among the large class of complete intersection C-Y manifolds modded out by projectively inherited freely acting discrete groups revealed the existence of only one topologically distinct such space with three generations of ordinary quarks and leptons [3]. This is the manifold originally constructed by Yau [4] and extensively studied by many authors [5] [6] [7] [8]. A common feature of all so far phenomenological studies is that the discrete groups considered (which in turn reflect the choice of the complex structure of the C-Y space) are subgroups of a maximal symmetry group with 72 elements [5] which does not contain any subgroup of R-symmetries (symmetries under which the covariantly constant spinor and the harmonic (3,0) form ω of the C-Y space transform non-trivially).

There are another two distinct choices of the complex structure of the three generation manifold which lead to maximal discrete groups with 108 elements [9].

It was recently realized that both 108 element discrete groups contain a Z_3 factor which is an R-symmetry [10] [7] [11]. This observation might have important phenomenological consequences since R-symmetries impose very powerful constraints on renormalizable as well as nonrenormalizable terms in the superpotential and are known to lead in some cases to exact flat directions which could break the E_6 symmetry down to $SO(10)$ or $SU(5)$ already at the compactification scale [12]. No detailed phenomenological study of models of this type exists so far.

In a recent note [13] we observed the existence of a class of three generation models (complex structures), with discrete groups contained in one of the 108 element maximal symmetry groups, in which there is a natural suppression mechanism of the ‘most dangerous’ proton decay processes occurring through dimension five operators. This mechanism is the result of the combined effect of a Z_3 matter parity and a Z_3 R-symmetry. Both of these symmetries are subgroups of the 108 element maximal discrete symmetry group under discussion. Our purpose here is to examine in more detail the phenomenology of this very interesting class of three generation models.

In section 2 we review the construction of the three generation manifold as a division of a simply connected space by a freely acting discrete group. We define the complex structures leading to the series of models possessing a mechanism of natural suppression of tree level proton decay through dimension five operators and identify the discrete symmetries compatible with these complex structures. We also discuss the constraints that R-symmetries impose on the four dimensional superpotential.

Section 3 is devoted to the detailed phenomenological study of the least symmetric model ($Z_3 \times Z_3$ symmetry) possessing the natural proton decay suppression mechanism.

Section 4 is devoted to the modifications of the discussion of section 3 as the discrete symmetry group is enlarged. Particular attention is paid to the maximally symmetric 108 element group, containing the minimal $Z_3 \times Z_3$ symmetry as a subgroup, and the next largest discrete group in the series possessing 54 elements. For their study use is made of the existing calculation [11] of the 27^3 Yukawa couplings in terms of the complex parameters defining the complex structure.

Finally section 5 contains our conclusions.

2 Three generation manifold, discrete symmetries and massless spectrum

The (first realization of the) three generation manifold [4] [5] is defined as the complete intersection of two cubic and one bilinear constraints

$$P_1 = \sum_{i=0}^3 x_i^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3 = 0,$$

$$P_2 = \sum_{i=0}^3 y_i^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3 = 0,$$

$$P_3 = \sum_{i=0}^3 c_i x_i y_i + c_4 x_2 y_3 + c_5 x_3 y_2 = 0,$$

in $CP^3 \times CP^3$ with homogeneous coordinates (x_0, x_1, x_2, x_3) and (y_0, y_1, y_2, y_3) , respectively, divided out by the freely acting Z_3 generated by $g = \text{diag}(1, \alpha^2, \alpha, \alpha, 1, \alpha, \alpha^2, \alpha^2)$ acting on $[x_i, y_i]$ ($i = 0, 1, 2, 3$), with $\alpha = \exp(2\pi i/3)$. The complex parameters $a_1, a_2, b_1, b_2, c_0, c_1, c_2, c_3, c_4, c_5$ define the complex structure and are only constrained by the transversality condition

$$P_1 \wedge P_2 \wedge P_3 \neq 0.$$

For the flux breaking we assume the embedding of the freely acting Z_3 into E_6 defined by the homomorphism $g \mapsto U_g \equiv (I_3, \alpha I_3, \alpha I_3) \in SU(3)_c \times SU(3)_L \times$

$SU(3)_R$. At energies below the compactification scale M_c the gauge group is $H \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$. Under H the left-handed lepton (λ), quark (Q) and antiquark (Q^c) fields in a 27 of E_6 transform as follows: $\lambda = (1, \bar{3}, 3) = \begin{pmatrix} H^1 & H^2 & l \\ e^c & \nu^c & N \end{pmatrix}$, $Q = (3, 3, 1) = \begin{pmatrix} q \\ g \end{pmatrix}$, $Q^c = (\bar{3}, 1, \bar{3}) = (u^c, d^c, g^c)$. There are 9 λ lepton fields, 7 Q (Q^c) quark (antiquark) fields, 6 $\bar{\lambda}$ mirror lepton fields and 4 $\bar{Q}$ ($\bar{Q}^c$) mirror quark (antiquark) fields. The group of honest discrete symmetries of the compactified theory depends on the coefficients of the constraint equations. For the flux breaking in question the classification of the models according to their honest discrete symmetries can be found in ref. [7].

We will be interested here in the complex structures with the coefficient $c_3 = 0$. The maximal discrete symmetry consistent with $c_3 = 0$ corresponds to the choice of the complex structure with non-vanishing parameters $c_0 = c_1 = 1$ and $c_2 \neq 0, 1$. This symmetry has 108 elements and the following generators:

$$B: x_2 \rightarrow \alpha x_2 \quad , \quad y_2 \rightarrow \alpha^2 y_2,$$

$$F: x_{0,1} \rightarrow \alpha^2 x_{1,0} \quad , \quad y_{0,1} \rightarrow \alpha y_{1,0},$$

$$G: x_3 \rightarrow \alpha x_3 \quad , \quad y_3 \rightarrow \alpha y_3,$$

$$S: x_i \rightarrow y_i \quad , \quad y_i \rightarrow x_i \quad (i = 0, 1, 2, 3).$$

In table 1 are given the transformation properties of the massless (compared to M_c) superfields of the theory under the above generators (and the important combination $F^2 B$). Notice that G generates an R-symmetry and under it the unique harmonic (3,0) form ω of the C-Y space transforms like α^2 .

One can easily show that the combination of the Z_3 symmetry generated by $F^2 B$ and the Z_3 symmetry generated by the element $diag(\alpha^2, \alpha, 1) \in SU(3)_R$ satisfies the conditions of a Z_3 matter parity [14]. This is the matter parity that

will be adopted in the following discussion. Under it the light $SU(2)_L$ quark and lepton doublets remain invariant, the light up antiquarks transform like α and the light down antiquarks and charged antileptons like α^2 .

In table 2 are listed the models in which the honest discrete symmetry group contains the Z_3 matter parity generated by F^2B and the Z_3 R-symmetry generated by G . There the reader can find the non-zero parameters defining the complex structure, the honest discrete symmetry group and its order. To simplify the notation the generator of a symmetry is given as a factor instead of the group itself. The symbol $\otimes$ denotes non-commutativity. [For instance $F^2B \times G$ means that the symmetry group is $Z_3 \times Z_3'$. Z_3 is generated by F^2B and Z_3' is generated by G .] These are the models with the natural proton decay suppression mechanism mentioned in the introduction.

Before we conclude this section we will explain the way in which an R-symmetry restricts couplings in the superpotential. Let $27 \rightarrow \beta 27$ and $\overline{27} \rightarrow \beta' \overline{27}$ be the ‘naive’ (i.e. obtained without realizing that the discrete symmetry is an R-symmetry) transformation property of (a component of) a 27 and a $\overline{27}$ respectively under the R-symmetry (table 1 gives the ‘naive’ transformations). Also assume that under the R-symmetry $\omega \rightarrow \gamma^{-1} \omega$. From the topological formulas for (renormalizable) superpotential couplings [15] it follows that a $27_1 27_2 27_3$ coupling is allowed by the R-symmetry if $\beta_1 \beta_2 \beta_3 = \gamma^2$ and a $\overline{27}_1 \overline{27}_2 \overline{27}_3$ coupling is allowed if $\beta'_1 \beta'_2 \beta'_3 = 1$. Therefore the superpotential, naively, does not seem to have a uniform behavior under the R-symmetry. This raises the question of what one should assume for nonrenormalizable terms for which there is no topological formula. The answer to this question was given in ref. [12]. There it was observed that the ‘naive’ transformations apply only to the components of a 27 of E_6 that transform as 10’s under the $SO(10)$ subgroup of E_6 . The E_6 generators which do not commute with

$SO(10)$ are constructed the way the supersymmetry generators are constructed and therefore pick up additional phases under an R-symmetry. It follows that the so defined R-symmetry does not commute with E_6 . One can, however, redefine the R-symmetry such that it commutes with E_6 . The ‘new’ transformations are $27 \rightarrow \gamma^{-1/3} \beta 27$ and $\overline{27} \rightarrow \gamma^{1/3} \beta' \overline{27}$. The allowed superpotential terms are the ones transforming like $\theta^2 \sim \gamma$, as expected for an R-symmetry. This reproduces the transformation properties expected from the topological formulas and also determines the allowed nonrenormalizable couplings for which there is no topological formula. For the R-symmetry G under discussion $\gamma = \alpha$.

3 Phenomenology of the least symmetric model

In this section we will discuss in detail the phenomenology of the least symmetric model in table 2, namely model (6) with a $Z_3 \times Z_3$ honest symmetry group generated by $F^2 B$ and G . This is the minimal symmetry necessary for the natural suppression of the most dangerous proton decay processes. No computation of Yukawa couplings of this model in terms of the complex structure is so far available. We will therefore treat them as free parameters.

The scenario of ref. [13], that we are implementing here, works better with a dominant vacuum expectation value (vev) in the N direction and a low vev ($\sim M_s \simeq 10^4$ GeV, the supersymmetry breaking scale in the visible sector) in the ν^c direction. This means that the $SU(3)_c \times SU(3)_L \times SU(3)_R$ gauge symmetry breaks down to only an $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry at an intermediate scale.

According to the Z_3 matter parity scenario that we follow here the candidate lepton fields for an intermediate vev in the N direction are N_1, N_3, N_5, N_6 and N_7

(matter parity 1). Under the discrete symmetries of the model the first four of these fields have identical quantum numbers and therefore, without loss of generality, we can assume that only N_1 and N_7 are candidates. For the $\bar{N}$ direction the candidates are $\bar{N}_1$ and $\bar{N}_2$ which are indistinguishable. We will assume an intermediate vev for both, subject to the conditions $|\langle N_1 \rangle| = |\langle \bar{N}_1 \rangle|$, $|\langle N_7 \rangle| = |\langle \bar{N}_2 \rangle|$.

Another important fact, which is an immediate consequence of the symmetries F^2B and G , is that the only candidate Higgs doublets coupling to the light quarks (and antiquarks) are the ones contained in λ_7 . To see this we observe that, according to the matter parity scenario chosen, the light $SU(2)_L$ doublet quarks are contained in Q_3 , Q_4 and Q_6 and the light $SU(2)_L$ singlet antiquarks are contained in Q_3^c , Q_4^c and Q_6^c . All these quarks and antiquarks remain invariant under the R-symmetry G . The only lepton field having couplings of the form λQQ^c involving these quarks and antiquarks is λ_7 since this is the only lepton field transforming under the R-symmetry G like α^2 . Therefore, if we want to give some tree level masses to our quarks, we should make sure that the $SU(2)_L$ doublets with the correct quantum numbers to act as Higgs fields do not acquire intermediate masses.

Before we embark on the evaluation of the physical spectrum we will quote some useful mathematical facts concerning mass matrices. The mass matrices M we deal with are, in general, non-hermitian ones which are diagonalizable through left and right multiplication with two different unitary matrices. The physical masses are not their eigenvalues but are equal, in absolute value, to the square root of the eigenvalues of the hermitian matrix $MM^\dagger$. In ref. [8] we proved that for an arbitrary square matrix M of order n , the sum of the products of the eigenvalues of $MM^\dagger$ in groups of $l \leq n$ is equal to the sum of the squares of the absolute values of all order l minor determinants of M . An immediate consequence of this proposition is that the order of magnitude of the product of the l largest

physical masses associated with a general mass matrix M is equal to the order of magnitude of the largest order l minor determinant of M .

Since the $SU(2)_R$ symmetry does not commute with the Z_3 matter parity we will work with the components of the $SU(2)_R$ doublets. We will also work with fermion mass matrices since we are not interested in fermion-boson mass splittings of order M_s or smaller. In all mass matrices given in the tables, a field in the N or $\bar{N}$ direction stands for its vev and an appropriate power of M_c^{-1} is understood in order to correctly restore the dimensions. The notation also suppresses all coupling constants.

The mass matrix of the matter parity α $SU(2)_L$ doublet leptons with $I^R_3 = -1/2$ ($H^2_1, H^2_3, H^2_5, H^2_6, H^2_7, \bar{H}^1_1, \bar{H}^1_2$) and the matter parity α^2 $SU(2)_L$ doublet leptons with $I^R_3 = 1/2$ ($H^1_1, H^1_3, H^1_5, H^1_6, H^1_7, \bar{H}^2_1, \bar{H}^2_2$) is given in table 3. These are the states (actually their supersymmetric partners) with the correct quantum numbers to play the role of the electroweak Higgs doublets. Matrix elements of order M_s are not shown explicitly. [Since the bosonic superpartners of $\bar{N}_1, \bar{N}_2$ have intermediate vevs their masses must be at most of order M_s . This implies that all mass terms of the form $m_{ij}\bar{\lambda}_i\lambda_j$ $i = 1, 2$ $j = 1, 3, 5, 6, 7$ satisfy the constraint $m_{ij} \leq M_s$.] As we already indicated earlier we would like H^1_7 and H^2_7 to acquire a mass not larger than $\sim M_s$. This immediately suggests that N_1 ($\bar{N}_1$) do not have an intermediate vev. However, as we will see later, an intermediate vev for N_1 ($\bar{N}_1$) is required in order that some ‘wrong’ matter parity colored fields acquire appreciable masses. We will now show that if $|\langle N_7 \rangle| \gg |\langle N_1 \rangle|$, some sort of a see-saw mechanism guarantees that the pair H^1_7, H^2_7 acquires a mass much smaller than the intermediate scales. The determinant of the 7×7 mass matrix of table 3 is easily estimated to be of the order of $(f_7\langle N_7 \rangle)^3(f_2\langle \bar{N}_2 \rangle)^2 \max\{f_7f'\langle N_7 \rangle^3\langle \bar{N}_2 \rangle M_c^{-2}, (f_1\langle N_1 \rangle)^2\}$,

where M_c is the compactification scale and f_7, f_2, f_1 and f' are typical values of the Yukawa couplings multiplying the terms proportional to $\langle N_7 \rangle, \langle \bar{N}_2 \rangle, \langle N_1 \rangle$ and $\langle N_7 \rangle^2 \langle \bar{N}_2 \rangle M_c^{-2}$ respectively (which are suppressed in the notation of table 3). The minor 6×6 determinant obtained from the original one by deleting the row and column labelled by the fields H^2_7 and H^1_7 is easily estimated to be $\sim (f_7 \langle N_7 \rangle)^4 (f_2 \langle \bar{N}_2 \rangle)^2$ which is (for $f_7 \sim f_2 \geq |\langle N_1 \rangle|/|\langle N_7 \rangle|$) the largest possible value (as an order of magnitude) of a 6×6 determinant in the problem under discussion. This means that the physical masses of the six states involved are all of order $\langle N_7 \rangle \sim \langle \bar{N}_2 \rangle$ while the seventh state, mainly consisting of the pair H^1_7 and H^2_7 , has a mass of the order of the ratio of the 7×7 determinant over the 6×6 minor, namely $(f_7 \langle N_7 \rangle)^{-1} \max\{f_7 f' \langle N_7 \rangle^2 \langle \bar{N}_2 \rangle M_c^{-2}, (f_1 \langle N_1 \rangle)^2\}$. With $|\langle N_7 \rangle| = |\langle \bar{N}_2 \rangle| \sim 10^{14} \text{ GeV}$, $|\langle N_1 \rangle| = |\langle \bar{N}_1 \rangle| \sim 10^{10} \text{ GeV}$, $M_c \sim 10^{18} \text{ GeV}$, and $f_7 \sim f_1 \sim f' \sim 10^{-2}$ one obtains a pair of light Higgs doublets with masses $\sim 10^1 \text{ GeV}$. Clearly this is a satisfactory solution of the gauge hierarchy problem.

The mass matrix of the matter parity $\alpha^2 SU(2)_L$ doublet leptons with $I^{R_3} = -1/2 (H^2_4, H^2_8, \bar{H}^1_4, \bar{H}^1_6)$ and the matter parity $\alpha SU(2)_L$ doublet leptons with $I^{R_3} = 1/2 (H^1_2, H^1_9, \bar{H}^2_3, \bar{H}^2_5)$ is given in table 4. [Contributions from nonrenormalizable terms are not important and are suppressed.] It is obvious that there are two states (involving mainly H^2_4, H^2_8 , and H^1_2, H^1_9) with masses $\sim \langle N_1 \rangle$ and two states (involving mainly $\bar{H}^1_4, \bar{H}^1_6$ and $\bar{H}^2_3, \bar{H}^2_5$) with masses $\sim \langle \bar{N}_2 \rangle$.

The mass matrix of the matter parity $1 SU(2)_L$ doublet leptons with $I^{R_3} = -1/2 (H^2_2, H^2_9, \bar{H}^1_3, \bar{H}^1_5)$ and the matter parity $1 SU(2)_L$ doublet leptons with $I^{R_3} = 1/2 (H^1_4, H^1_8, \bar{H}^2_1, \bar{H}^2_6)$ is given in table 5. [Contributions from nonrenormalizable terms are suppressed.] There are two states (involving mainly H^2_2, H^2_9 ,

and H^1_4, H^1_8) with masses $\sim \langle N_1 \rangle$ and two states (involving mainly $\bar{H}^1_3, \bar{H}^1_5$ and $\bar{H}^2_4, \bar{H}^2_6$) with masses $\sim \langle \bar{N}_2 \rangle$.

The mass matrix of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons of $U(1)_{B-L}$ charge +1 (−1) and matter parity α^2 (α) is a 2×2 matrix given in table 6 whose rows (columns) are labelled by the fields $\bar{l}_3, \bar{l}_5$ (l_4, l_8). There is one state of mass $\sim \langle N_7 \rangle < \bar{N}_2 > M_c^{-1}$ and one state of mass $\sim \langle N_7 \rangle^2 < \bar{N}_2 \rangle^2 M_c^{-3}$.

The mass matrix of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons of $U(1)_{B-L}$ charge +1 (−1) and matter parity α (α^2) is a 2×2 matrix given in table 7 whose rows (columns) are labelled by the fields $\bar{l}_1, \bar{l}_6$ (l_2, l_9). There is one state of mass $\sim \langle N_7 \rangle < \bar{N}_2 > M_c^{-1}$ and one state of mass $\sim \langle N_7 \rangle^2 < \bar{N}_2 \rangle^2 M_c^{-3}$.

The mass matrix of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons of matter parity 1 is a 2×5 matrix (table 8) whose rows and columns are labelled by the fields $\bar{l}_1, \bar{l}_2$ and l_1, l_3, l_5, l_6 and l_7 respectively. All matrix elements are constrained to be of order M_s due to the fact that $\bar{l}_1$ and $\bar{l}_2$ belong to the same gauge multiplets with $\bar{N}_1$ and $\bar{N}_2$ respectively whose bosonic components have intermediate vevs. Therefore there are two states of mass $\sim M_s$ involving $\bar{l}_1, \bar{l}_2$ and two combinations out of the set $\{l_1, l_3, l_5, l_6, l_7\}$. The three remaining combinations are the light $SU(2)_L$ ordinary lepton doublets.

The mass matrices of the $SU(2)_R$ doublet, $SU(2)_L$ singlet leptons (e^c, ν^c) are analogous in form and magnitude to the ones of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons with the same generation indices. Only the matter parity labels are subject to trivial changes. The matter parity of $e^c_i, \bar{e}^c_j, \nu^c_k, \bar{\nu}^c_m$ is obtainable from the matter parity of $l_i, \bar{l}_j, l_k, \bar{l}_m$ by a multiplication with $\alpha^2, \alpha, \alpha, \alpha^2$ respectively.

We now turn to the quark mass matrices. The mass matrix of the $SU(2)_L$ and $SU(2)_R$ singlet, electric charge $-1/3$ ($1/3$) quarks (antiquarks) of matter parity 1

(1) involving g_3, g_4 and g_6 (g^c_3, g^c_4 and g^c_6) is given in table 9. [Contributions of nonrenormalizable terms are suppressed.] There are three states of mass $\sim \langle N_7 \rangle$.

The mass matrix of the $SU(2)_L$ and $SU(2)_R$ singlet, electric charge $-1/3(1/3)$ quarks (antiquarks) of matter parity α^2 (α) involving $g_1, g_7, \bar{g}^c_1$ and $\bar{g}^c_3$ ($g^c_1, g^c_7, \bar{g}_1, \bar{g}_3$) is given in table 10. [Contributions of nonrenormalizable terms are suppressed.] There are two states involving g_1, g_7 and g^c_1, g^c_7 with mass $\sim \langle N_1 \rangle$ and two states involving $\bar{g}^c_1, \bar{g}^c_3$ and $\bar{g}_1, \bar{g}_3$ with mass $\sim \langle \bar{N}_2 \rangle$.

The mass matrix of the $SU(2)_L$ and $SU(2)_R$ singlet, electric charge $-1/3(1/3)$ quarks (antiquarks) of matter parity α (α^2) involving $g_2, g_5, \bar{g}^c_2$ and $\bar{g}^c_4$ ($g^c_2, g^c_5, \bar{g}_2, \bar{g}_4$) is given in table 11. [Contributions of nonrenormalizable terms are suppressed.] There are two states involving g_2, g_5 and g^c_2, g^c_5 with mass $\sim \langle N_1 \rangle$ and two states involving $\bar{g}^c_2, \bar{g}^c_4$ and $\bar{g}_2, \bar{g}_4$ with mass $\sim \langle \bar{N}_2 \rangle$.

The mass matrix of the $SU(2)_L$ singlet, electric charge $-1/3(1/3)$ quarks (antiquarks) with matter parity 1 (1) which are components of an $SU(2)_R$ doublet involving $\bar{d}^c_1$ and $\bar{d}^c_3$ (d^c_1 and d^c_7) is given in table 12. There is one state of mass $\sim \langle N_7 \rangle \langle \bar{N}_2 \rangle M_c^{-1}$ and one state of mass $\sim \langle N_7 \rangle^2 \langle \bar{N}_2 \rangle^2 M_c^{-3}$.

The mass matrix of the $SU(2)_L$ singlet, electric charge $-1/3(1/3)$ quarks (antiquarks) with matter parity α^2 (α) which are components of an $SU(2)_R$ doublet involving $\bar{d}^c_2$ and $\bar{d}^c_4$ (d^c_2 and d^c_5) is given in table 13. There is one state of mass $\sim \langle N_7 \rangle \langle \bar{N}_2 \rangle M_c^{-1}$ and one state of mass $\sim \langle N_7 \rangle^2 \langle \bar{N}_2 \rangle^2 M_c^{-3}$.

The three $SU(2)_L$ singlet, electric charge $1/3$ antiquarks transforming like $SU(2)_R$ doublets with matter parity α^2 , namely d^c_3, d^c_4 and d^c_6 have no mass partners and therefore remain massless before any further ($\sim M_s$) symmetry breaking. These are the ordinary down antiquarks.

The mass matrices of the $SU(2)_L$ singlet, electric charge $\pm 2/3$ quarks ($\bar{u}^c$) and antiquarks (u^c) are equal to the ones of the electric charge $\pm 1/3$ quarks ($\bar{d}^c$) and

antiquarks (d^c) which are their $SU(2)_R$ doublet partners. Only the matter parity assignments have to change. The matter parity of a u^c ($\bar{u}^c$) is obtained from the matter parity of the corresponding d^c ($\bar{d}^c$) through a multiplication by α^2 (α). In particular the three light ordinary up antiquarks are u^c_3 , u^c_4 and u^c_6 and have matter parity α .

The mass matrices of the $SU(2)_L$ doublet quarks (q) and antiquarks ($\bar{q}$) are analogous to the ones of the $SU(2)_L$ singlet antiquarks d^c and quarks ($\bar{d}^c$). The matter parity of the q ($\bar{q}$) is obtained from the matter parity of d^c ($\bar{d}^c$) through a multiplication by α (α^2). In particular the three light ordinary quarks q_3 , q_4 and q_6 and have matter parity 1.

When a vev in the ν^c - $\bar{\nu}^c$ directions (respecting the Z_3 matter parity) is turned on the distinction between H^2 's and U 's and between g^c 's and d^c 's disappears. As a result of this the mass matrices of some sectors merge into larger mass matrices. For a low ($\sim M_s$) vev in the ν^c - $\bar{\nu}^c$ directions the induced mixings are very small. In particular, one could choose a non-zero vev only in the $\bar{\nu}^c$ direction which makes the most relevant (for proton decay) mixings between states tiny. We will therefore not discuss in the following these effects any further.

The most dangerous proton decay processes in this type of theories (with a low $U(1)_{B-L}$ breaking scale) involve the exchange of a g^c fermion between vertices corresponding to superpotential couplings $u^c d^c g^c$ and $g^c q l$ or the exchange of a g fermion between vertices corresponding to superpotential couplings qqg and $gu^c e^c$ or a cross combination of the above vertices via a g - g^c mass term, where u^c , d^c , q , l , e^c are among the 'light' ones. These processes are under control only if g and g^c are heavier than $\sim 10^{14}$ GeV and simultaneously the coupling constants of the above mentioned vertices are smaller than $\sim 10^{-6}$.

In the Z_3 matter parity scenario employed here, the most dangerous baryon

number violating terms of the type mentioned above are $u^c(\alpha)d^c(\alpha^2)g^c(1)$ and $q(1)q(1)g(1)$ (in parentheses we give the Z_3 matter parity of the fields). These couplings are all contained in the terms $Q_i Q_j Q_k$ and $Q_i^c Q_j^c Q_k^c$ with i, j, k taking independently values from the set $\{3, 4, 6\}$. From table 1 is evident that all these terms are invariant under the R-symmetry G and therefore forbidden since the allowed 27^3 terms must transform under G like α^2 (using the naive transformations of table 1). The conclusion is that there is no tree level proton decay through dimension five operators (in the limit of zero $U(1)_{B-L}$ breaking and no nonrenormalizable terms). This is the most remarkable property of the models under discussion.

The next-most-dangerous baryon number violating couplings involve one only ‘light’ field, necessarily of the q , u^c or d^c type, and two ‘heavy’ fields out of which one must necessarily be of the g or g^c type. With the mixing between ‘heavy’ and ‘light’ fields suppressed, the dominant proton decay processes (only ‘light’ fields in external legs) involve a loop with four vertices out of which one at least is baryon number violating. Since one g or g^c is necessarily involved in the loop and g ’s and g^c ’s have masses at least $\sim \langle N_1 \rangle$, the dominant diagrams are suppressed by at least one power of $\sim \langle N_1 \rangle \sim 10^{10}$ GeV, one power of $M_s \sim 10^1$ GeV, one more loop factor $\sim 10^{-2}$ compared to the usual tree diagrams and the product of the four coupling constants associated with the vertices of the loop. We see that an average coupling constant $\sim 10^{-3}$ and a g - g^c mass $\sim 10^{10}$ GeV are adequate to sufficiently suppress the proton decay rate.

Finally we will make a few remarks concerning the neutral sector. Due to the fact that the $SU(2)_R$ symmetry breaks at a low scale the combination of e^c ’s (d^c ’s) that correspond to the light charged antileptons (down antiquarks) and of ν^c ’s (u^c ’s) that enter the Dirac mass terms of the neutrinos (up quarks) are $SU(2)_R$

partners and therefore have the same Yukawa coupling to the lepton λ_7 which contains both the Higgs bosons H^1_7 and H^2_7 . Since the mass of the τ lepton and of the b quark are not very different and since both get their mass from the vev of H^2_7 , it follows that the corresponding Yukawa couplings are not very different. Consequently, at least for the third generation, Dirac neutrino masses are not very different than up quark masses. Therefore one needs some sort of a see-saw mechanism.

In the case of Z_3 matter parities there is a general problem with the neutral sector because the mass matrices of some matter parity sectors are non-square ones and consequently there are some massless right-handed neutrinos. One solution is to break the Z_3 matter parity at a low scale which, however, does not solve the problem in the present case (right-handed neutrinos cannot become adequately heavy). Another way out would make use of the E_6 singlets. The number of these singlets, however, and their transformation properties are not known today for the realization of the three generation manifold under discussion. It is conceivable that use of these singlets will make neutrino masses compatible with at least the present laboratory limits.

4 Enlarging the symmetry

In this section we will discuss the modification of the results of the previous section as the symmetry group is enlarged from the 9 element $Z_3 \times Z_3$ group of model (6) to the maximal 108 element symmetry group of model (1) in table 2. We will mostly concentrate on models (1) and (1) for which there is a recent computation [11] of the 27^3 Yukawa couplings of the not properly normalized fields in terms of the complex parameters defining the complex structure of the C-Y space. The

computation of the normalization constants of the fields is not available for the realization of the three generation manifold considered here. In the following we will work assuming that the normalization constants are not very different than unity.

Tables 14, 15 and 16 (model (1)) contain the relative strength of the non-zero Yukawa couplings of the form $\lambda_i \lambda_j \lambda_k$, $Q_i Q_j Q_k$ ($Q_i^c Q_j^c Q_k^c$) and $\lambda_i Q_j Q_k^c$ in terms of one complex parameter $c = c_2 \neq 0, 1$. There is a common undetermined normalization A for all the couplings given in tables 14, 15 and 16 (model (1)).

As a consequence of the minimal $Z_3 \times Z_3$ symmetry generated by $F^2 B$ and G , the light quarks (antiquarks) are contained again in Q_i (Q_i^c), $i = 3, 4, 6$ and the Higgs doublets in λ_7 . The relevant Yukawa couplings giving rise to quark masses are $-Ac/2\lambda_7 Q_3 Q_3^c$, $-A/(1+c^3)\lambda_7 Q_4 Q_4^c$ and $-A/(1+c^3)\lambda_7 Q_6 Q_6^c$. We see that we have two (almost) degenerate light quarks (antiquarks), namely Q_4 (Q_4^c) and Q_6 (Q_6^c). The only possibility left is that the third generation quarks (antiquarks) are contained in Q_3 (Q_3^c) and that the tree level masses of the other two generations of quarks are suppressed such that only t and b have tree level masses. This uniquely singles out the choice $|c| \gg 1$. Also, since the third generation Yukawa is of order $10^{-1} - 1$, the overall normalization of the couplings of tables 13, 14 and 15 should be of the order of $(10^{-1} - 1)c^{-1}$. With the choice $|c| \gg 1$ and the overall normalization $A \sim (10^{-1} - 1)c^{-1}$ the only 27^3 coupling involving three leptons which is of order 1 is $[12c(2+c^3)/(1+c^3)]\lambda_5 \lambda_5 \lambda_7$. In this scenario of a low $SU(2)_R$ breaking scale, it is easily seen that l_5 will be among the light lepton doublets since under the permutation symmetry F^3 , λ_5 is the only odd field among the ones involved in the mass matrix of table 8. [The lepton fields with vevs in the $N-\bar{N}$ directions, in the matter parity scenario employed, are F^3 even.] It follows that the τ lepton is contained in λ_5 . Comparing the Yukawa coupling of

b and τ we conclude that $m_\tau = 4m_b$ (the vev of H^2_7 is a common factor in the two mass terms). Luckily there are the unknown normalization constants which could change this phenomenologically unacceptable relation. When these normalizations will be computed this theory will be immediately testable in detail.

Now, due to the enlarged symmetry, there are several inequivalent options for the intermediate vevs in the N - $\bar{N}$ directions. We will choose $|\langle N_7 \rangle| = |\langle \bar{N}_2 \rangle| \sim 10^{14}$ GeV and $|\langle N_1 \rangle| = |\langle N_3 \rangle| \simeq |\langle \bar{N}_1 \rangle| \sim 10^{11}$ GeV.

The states involved in the mass matrices of the different sectors are the same as in the previous section. The elements of the mass matrices are, of course, different since the allowed Yukawa couplings are different due to the enlarged symmetry. We now, of course, have more information since we roughly know the relative magnitude of many couplings.

In the Higgs sector, arguments similar to the ones used in the previous section show that the pair H^1_7 - H^2_7 has a mass $\sim A \langle N_1 \rangle \langle N_3 \rangle \langle N_7 \rangle^{-1} \sim A 10^8$ GeV. With $c \sim 10^3$, $A \sim 10^{-3} - 10^{-4}$ and the light Higgs mass is $\sim 10^4 - 10^5$ GeV.

Another important difference with respect to the previous section is that the masses of $g_1, g_2, g_5, g_7, g^c_1, g^c_2, g^c_5, g^c_7$ are of order $A \langle N_1 \rangle \sim A \langle N_3 \rangle \sim 10^8$ GeV, to be compared with the value 10^{10} GeV in the previous section. Also many states, which rely on nonrenormalizable terms to acquire a mass, have dangerously low masses because the appropriate terms occur in higher than previously orders.

The situation with respect to tree level proton decay through dimension five operators is not different than in the least symmetric case. However, now, one can make more definite statements concerning proton decay processes occurring through dimension five operators in loop diagrams. The superpotential terms with couplings $\sim 10^{-1} - 1$ are $\lambda_5 \lambda_5 \lambda_7, \lambda_2 Q_3 Q^c_7, \lambda_1 Q_3 Q^c_5, \lambda_2 Q_5 Q^c_3, \lambda_4 Q_7 Q^c_3, \lambda_7 Q_3 Q^c_3,$

$\lambda_5 Q_5 Q_5^c$, $\lambda_5 Q_7 Q_7^c$, $Q_3 Q_5 Q_7$, $Q_3^c Q_5^c Q_7^c$. Using one of the last two baryon number violating couplings and three of the $\lambda Q Q^c$ couplings one can write down simple baryon number violating box diagrams which do not have any large suppression due to small coupling constants. This, however, is compensated by the fact that the only light leptons, quarks and antiquarks in external legs are λ_5 , Q_3 and Q_3^c , all belonging to the third generation. Therefore there is a suppression by a product of mixing angles, some of which are unknown. If we use a baryon number violating coupling different than $Q_3 Q_5 Q_7$ ($Q_3^c Q_5^c Q_7^c$), there is a coupling constant suppression by at least a factor $\sim c^{-3}$. We therefore see that the proton decay suppression scenario works well even if we use the expressions of the Yukawa couplings in terms of only one complex parameter.

The situation clearly improves as the symmetry is reduced from the 108 element maximal symmetry of model (1) down to the 9 element minimal symmetry of model (6) since the Yukawa couplings are parametrized in terms of more than one ‘free’ complex number. The only other model in table 2 for which there is available today an analogous computation of the 27^3 Yukawa couplings in terms of the complex structure is model (4) with a 54 element discrete symmetry group and complex structure parametrized in terms of two ‘free’ complex parameters c_1 and c_2 . The Yukawa couplings are given in tables 14, 15 and 16 (model (4)). With arguments analogous to the ones given earlier we see that a favorable choice of the complex structure is $|c_1| \gg 1$, $|c_2| \ll 1$ and the overall normalization constant $A \sim (10^{-1} - 1)c_1^{-1}$. With this choice the third generation quarks, antiquarks and leptons are contained in Q_6 , Q_6^c and one combination out of λ_5 and λ_6 , respectively. We assume the same choice, as in the 108 element group case, of intermediate vevs in the $N-\bar{N}$ directions. The situation with respect to tree level proton decay remains unchanged. However, in loops, even in the case that the

external legs correspond to third generation quarks and leptons there is an extra suppression by one power of c_2 .

5 Conclusions

In the present work we discussed in some detail the phenomenology of a recently identified [13] series of three generation superstring models possessing R-symmetries, in which there is a natural suppression mechanism of the most dangerous baryon number violating processes leading to tree level proton decay through dimension five operators. This mechanism offers the option of a proton lifetime compatible with observations without tiny ($\sim 10^{-6}$) Yukawa couplings or super-heavy ($\sim 10^{15}$ GeV) g quarks. The gauge hierarchy problem is also solved without any contrived arrangements. The necessity of a low ($\sim 10^4$ GeV) $SU(2)_R$ breaking scale offers the best motivation for the existence of right-handed gauge bosons with masses much less than the typical intermediate scale.'

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	B	F	S	G	$F^2 B$		B	F	S	G	$F^2 B$
λ_1	α	α	λ_3	1	1	$\bar{\lambda}_1$	1	1	$\bar{\lambda}_1$	1	1
λ_2	1	α	λ_4	α	α^2	$\bar{\lambda}_2$	1	1	$-\bar{\lambda}_2$	1	1
λ_3	α^2	α^2	λ_1	1	1	$\bar{\lambda}_3$	α	$-\alpha^2$	$\bar{\lambda}_4$	α	α^2
λ_4	1	α^2	λ_2	α	α	$\bar{\lambda}_4$	α^2	$-\alpha$	$\bar{\lambda}_3$	α	α
λ_5	1	$-\alpha$	λ_5	1	1	$\bar{\lambda}_5$	α	$-\alpha^2$	$\bar{\lambda}_6$	α^2	α^2
λ_6	1	1	λ_6	1	1	$\bar{\lambda}_6$	α^2	$-\alpha$	$\bar{\lambda}_5$	α^2	α
λ_7	1	1	λ_7	α^2	1						
λ_8	α	1	λ_9	α	α						
λ_9	α^2	1	λ_8	α	α^2						
Q_1	α	$\alpha^2 Q_2^c$	Q_1^c	α	α^2	$\bar{Q}_1$	α	$\frac{Q_4^c}{Q_1}$	$\frac{Q_1^c}{Q_2}$	α	α
Q_2	α^2	αQ_1^c	Q_2^c	α	α	$\bar{Q}_2$	α^2	$\frac{Q_3^c}{Q_2}$	$\frac{Q_2^c}{Q_3}$	α^2	α^2
Q_3	1	Q_3^c	Q_3^c	1	1	$\bar{Q}_3$	α	$\frac{Q_2^c}{Q_3}$	$\frac{Q_3^c}{Q_4}$	α^2	α
Q_4	α^2	$\alpha^2 Q_6^c$	Q_4^c	1	1	$\bar{Q}_4$	α^2	$\frac{Q_1^c}{Q_4}$	$\frac{Q_4^c}{Q_1}$	α	α^2
Q_5	1	$\alpha^2 Q_7^c$	Q_5^c	α	α						
Q_6	α	αQ_4^c	Q_6^c	1	1						
Q_7	1	αQ_5^c	Q_7^c	α	α^2						
Q_1^c	α^2	αQ_2	Q_1	α	α	$\frac{Q_1^c}{Q_2}$	α^2	$\frac{Q_4}{Q_3}$	$\frac{Q_1}{Q_2}$	α	α^2
Q_2^c	α	$\alpha^2 Q_1$	Q_2	α	α^2	$\frac{Q_2^c}{Q_3}$	α	$\frac{Q_3}{Q_2}$	$\frac{Q_2}{Q_3}$	α^2	α
Q_3^c	1	Q_3	Q_3	1	1	$\frac{Q_3^c}{Q_4}$	α^2	$\frac{Q_2}{Q_1}$	$\frac{Q_3}{Q_4}$	α^2	α^2
Q_4^c	α	αQ_6	Q_4	1	1	$\frac{Q_4^c}{Q_1}$	α	$\frac{Q_1}{Q_4}$	$\frac{Q_4}{Q_1}$	α	α
Q_5^c	1	αQ_7	Q_5	α	α^2						
Q_6^c	α^2	$\alpha^2 Q_4$	Q_6	1	1						
Q_7^c	1	$\alpha^2 Q_5$	Q_7	α	α						

Table 1: Transformation properties of the ‘massless’ superfields under the symmetries generated by B , F , S , G and $F^2 B$.

Model	Non-zero parameters	Symmetry	Order
(1)	$c_0 = c_1 = 1; c_2 \neq 0, 1$	$[(F \times B) \otimes S] \times G$	108
(2)	(1) and $a_1 = b_1 \neq 0$	$[F B^2 \otimes S] \times G$	36
(3)	(1) and $a_1 \neq 0; b_1 \neq 0, a_1$	$F B^2 \times G$	18
(4)	$c_0 = 1; c_1 \neq 0, 1; c_2 \neq 0, 1, c_1$	$[(F^2 \times B) \otimes S] \times G$	54
(5)	(4) and $a_1 = b_1 \neq 0$	$[F^2 B \otimes S] \times G$	18
(6)	(4) and $a_1, b_1 \neq 0; a_1 \neq b_1$	$F^2 B \times G$	9

Table 2: Models with honest discrete symmetries containing the symmetries generated by $F^2 B$ and G .

	H^1_1	H^1_3	H^1_5	H^1_6	H^1_7	$\overline{H}^2_1$	$\overline{H}^2_2$
H^2_1	N_7	N_7	N_7	N_7	N_1		
H^2_3	N_7	N_7	N_7	N_7	N_1		
H^2_5	N_7	N_7	N_7	N_7	N_1		
H^2_6	N_7	N_7	N_7	N_7	N_1		
H^2_7	N_1	N_1	N_1	N_1	$N_7^2 \overline{N}_2$		
$\overline{H}^1_1$						$\overline{N}_2$	$\overline{N}_2$
$\overline{H}^1_2$						$\overline{N}_2$	$\overline{N}_2$

Table 3: Mass matrix of the matter parity α (α^2) $SU(2)_L$ doublet leptons with $I^R_3 = -1/2(1/2)$.

	H^1_2	H^1_9	$\overline{H}^2_3$	$\overline{H}^2_5$
H^2_4	N_1	N_1		
H^2_8	N_1	N_1		
$\overline{H}^1_4$				$\overline{N}_2$
$\overline{H}^1_6$			$\overline{N}_2$	

Table 4 : Mass matrix of the matter parity α^2 (α) $SU(2)_L$ doublet leptons with $I^R_3 = -1/2(1/2)$.

	H^1_4	H^1_8	$\overline{H}^2_4$	$\overline{H}^2_6$
H^2_2	N_1	N_1		
H^2_9	N_1	N_1		
$\overline{H}^1_3$				$\overline{N}_2$
$\overline{H}^1_5$			$\overline{N}_2$	

Table 5 : Mass matrix of the matter parity 1 $SU(2)_L$ doublet leptons with $I^R_3 = \pm 1/2$.

	l_4	l_8
$\bar{l}_3$	$N_7 \bar{N}_2$	$N_7 \bar{N}_2$
$\bar{l}_5$	$N_7^2 \bar{N}_2^2$	$N_7^2 \bar{N}_2^2$

Table 6 : Mass matrix of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons of $U(1)_{B-L}$ charge $+1(-1)$ and matter parity α^2 (α).

	l_2	l_9
$\bar{l}_4$	$N_7 \bar{N}_2$	$N_7 \bar{N}_2$
$\bar{l}_6$	$N_7^2 \bar{N}_2^2$	$N_7^2 \bar{N}_2^2$

Table 7 : Mass matrix of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons of $U(1)_{B-L}$ charge $+1(-1)$ and matter parity α (α^2).

	l_1	l_3	l_5	l_6	l_7
$\bar{l}_1$					
$\bar{l}_2$					

Table 8 : Mass matrix of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons of matter parity 1.

	g^c_3	g^c_4	g^c_6
g_3	N_7		
g_4		N_7	
g_6			N_7

Table 9 : Mass matrix of the $SU(2)_L$ and $SU(2)_R$ singlet, electric charge $-1/3(1/3)$ quarks (antiquarks) of matter parity 1.

	g^c_1	g^c_7	$\bar{g}_1$	$\bar{g}_3$
g_1	N_1	N_1		
g_7	N_1	N_1		
$\bar{g}^c_1$				N_2
$\bar{g}^c_3$			N_2	

Table 10 : Mass matrix of the $SU(2)_L$ and $SU(2)_R$ singlet, electric charge $-1/3(1/3)$ quarks (antiquarks) of matter parity α^2 (α).

	g^c_2	g^c_5	$\bar{g}_2$	$\bar{g}_4$
g_2	N_1	N_1		
g_5	N_1	N_1		
$\bar{g}^c_2$				N_2
$\bar{g}^c_4$			N_2	

Table 11 : Mass matrix of the $SU(2)_L$ and $SU(2)_R$ singlet, electric charge $-1/3(1/3)$ quarks (antiquarks) of matter parity α (α^2).

	d^c_1	d^c_7
$\bar{d}^c_1$	$N_7 \bar{N}_2$	$N_7 \bar{N}_2$
$\bar{d}^c_3$	$N_7{}^2 \bar{N}_2{}^2$	$N_7{}^2 \bar{N}_2{}^2$

Table 12.: Mass matrix of the $SU(2)_L$ singlet, $SU(2)_R$ doublet, electric charge $-1/3(1/3)$ quarks (antiquarks) of matter parity 1.

	d^c_2	d^c_5
$\bar{d}^c_2$	$N_7{}^2 \bar{N}_2{}^2$	$N_7{}^2 \bar{N}_2{}^2$
$\bar{d}^c_4$	$N_7 \bar{N}_2$	$N_7 \bar{N}_2$

Table 13 : Mass matrix of the $SU(2)_L$ singlet, $SU(2)_R$ doublet, electric charge $-1/3(1/3)$ quarks (antiquarks) of matter parity α^2 (α).

(i, j, k)	Model (1)	Model (4)
$(1, 3, 7)$	1	1
$(1, 4, 9)$	1	1
$(2, 3, 8)$	1	1
$(2, 4, 6)$	1	1
$(5, 5, 7)$	$2c(2 + c^3)(1 + c^3)^{-1}$	$(c_1 c_2 + 2c_2 + c_2 c_1^{-1})(1 + c_1^3)^{-1} +$ $+ c_1 c_2 (1 + c_2^3)^{-1} + c_2 c_1^{-1} (c_1^3 + c_2^3)^{-1}$
$(6, 6, 7)$	$2c^{-1}(1 + c^3)^{-1}$	$c_1 c_2^{-1} [(1 + c_2^3)^{-1} + (c_1^3 + c_2^3)^{-1}]$
$(6, 8, 9)$	$2c^{-1}(1 + c^3)^{-1}$	$c_1 c_2^{-1} [(1 + c_2^3)^{-1} + (c_1^3 + c_2^3)^{-1}]$
$(2, 4, 5)$	0	$c_2(c_1^2 - 1)(1 + c_1^3)^{-1}$
$(5, 6, 7)$	0	$-c_1(1 + c_2^3)^{-1} + (c_1^3 + c_2^3)^{-1}$
$(5, 8, 9)$	0	$-c_1(1 + c_2^3)^{-1} + (c_1^3 + c_2^3)^{-1}$

Table 14 : Non vanishing 27^3 couplings of the form $\lambda_i \lambda_j \lambda_k$ for models (1) and (4) in terms of the complex structure.

(i, j, k)	Model (1)	Model (4)
$(1, 2, 3)$	1	1
$(1, 4, 5)$	$(1 + c^3)^{-1}$	$c_1(c_1^3 + c_2^3)^{-1}$
$(2, 6, 7)$	$(1 + c^3)^{-1}$	$c_1(1 + c_2^3)^{-1}$
$(3, 5, 7)$	$-c/2$	$-c_2(1 + c_1^3)^{-1}$

Table 15 : Non vanishing 27^3 couplings of the form $Q_i Q_j Q_k$ ($Q_i^c Q_j^c Q_k^c$) for models (1) and (4) in terms of the complex structure.

(i, j, k)	Model (1)	Model (4)	(i, j, k)	Model (1)	Model (4)
(1, 2, 5)	1	1	(2, 2, 4)	1	1
(3, 1, 7)	1	1	(4, 1, 6)	1	1
(1, 7, 1)	1	1	(2, 6, 1)	1	1
(3, 5, 2)	1	1	(4, 4, 2)	1	1
(2, 3, 7)	$c/2$	$c_2(1 + c_1^3)^{-1}$	(4, 3, 5)	$c/2$	$c_1 c_2(1 + c_1^3)^{-1}$
(2, 5, 3)	$c/2$	$c_1 c_2(1 + c_1^3)^{-1}$	(4, 7, 3)	$c/2$	$c_2(1 + c_1^3)^{-1}$
(7, 3, 3)	$-c/2$	$-c_2(1 + c_1^3)^{-1}$	(6, 5, 5)	$-(1 + c^3)^{-1}$	$-(c_1^3 + c_2^3)^{-1}$
(7, 4, 4)	$-(1 + c^3)^{-1}$	$-(c_1^3 + c_2^3)^{-1}$	(8, 4, 5)	$-(1 + c^3)^{-1}$	$-(c_1^3 + c_2^3)^{-1}$
(9, 5, 4)	$-(1 + c^3)^{-1}$	$-(c_1^3 + c_2^3)^{-1}$	(6, 7, 7)	$-(1 + c^3)^{-1}$	$-c_1(1 + c_2^3)^{-1}$
(7, 6, 6)	$-(1 + c^3)^{-1}$	$-c_1(1 + c_2^3)^{-1}$	(8, 7, 6)	$-(1 + c^3)^{-1}$	$-c_1(1 + c_2^3)^{-1}$
(9, 6, 7)	$-(1 + c^3)^{-1}$	$-c_1(1 + c_2^3)^{-1}$	(6, 1, 1)	$-c^2(1 + c^3)^{-1}$	$-c_2^2(c_1^3 + c_2^3)^{-1}$
(6, 2, 2)	$-c^2(1 + c^3)^{-1}$	$-c_1 c_2^2(1 + c_2^3)^{-1}$	(8, 1, 4)	$c(1 + c^3)^{-1}$	$c_2(c_1^3 + c_2^3)^{-1}$
(8, 6, 2)	$c(1 + c^3)^{-1}$	$c_1 c_2(1 + c_2^3)^{-1}$	(9, 2, 6)	$c(1 + c^3)^{-1}$	$c_1 c_2(1 + c_2^3)^{-1}$
(9, 4, 1)	$c(1 + c^3)^{-1}$	$c_2(c_1^3 + c_2^3)^{-1}$			

(i, j, k)	Model (1)	Model (4)
(5, 1, 1)	$(2 + c^3)(1 + c^3)^{-1}$	$1 + c_1^2(c_1^3 + c_2^3)^{-1}$
(5, 2, 2)	$-(2 + c^3)(1 + c^3)^{-1}$	$-1 - c_1(1 + c_2^3)^{-1}$
(5, 5, 5)	$-c(2 + c^3)(1 + c^3)^{-1}$	$-c_2 c_1^{-1}[(c_1 + 1)(1 + c_1^3)^{-1} + (c_1^3 + c_2^3)^{-1}]$
(5, 7, 7)	$c(2 + c^3)(1 + c^3)^{-1}$	$c_2(1 + c_1^3)^{-1} + c_1 c_2[(1 + c_1^3)^{-1} + (1 + c_2^3)^{-1}]$

Table 16: Non vanishing 27^3 couplings of the form $\lambda_i Q_j Q_k^c$ for models (1) and (4) in terms of the complex structure.