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**MODEL INDEPENDENT ANALYSIS OF THE Z LINE SHAPE
IN e^+e^- ANNIHILATION^(*)**

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Abstract

A model-independent analysis of the shape and the size of the Z resonant cross section measured in e^+e^- collisions is performed. We focus on the process $e^+e^- \rightarrow \mu^+\mu^-$, but give results also for the inclusive hadronic cross-section. Explicit calculations in the case of the Standard Theory and a comparison with existing calculations are presented.

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1. Introduction and presentation of the results

We study in this paper the line shape at the Z-resonance as measured in the processes:

$$e^+e^- \rightarrow \mu^+\mu^- + \dots \quad (1.1)$$

$$e^+e^- \rightarrow q \bar{q} \rightarrow \text{hadrons} + \dots \quad (1.2)$$

where dots indicate that the final state is electromagnetically inclusive. Our aim is a model-independent expression of the cross-section, to an accuracy of about 1.10^{-3} , such as to allow detection of the one-loop electroweak corrections to the physical parameters of the Z-boson. This is required to match the accuracy of the LEP and SLC experiments now in preparation. We shall focus our discussion on the leptonic process, eq.(1.1), and then extend the results to the hadronic case, eq.(1.2).

The electroweak corrections to (1.1) predicted by the Standard Theory^[1] have been widely studied in the literature^[2-9]. In this paper, we take a point of view rather different from, and complementary to, previous studies, which can be illustrated as follows.

Usually, one parametrizes the cross section in terms of the basic parameters of the theory, namely the accurately determined fine structure and Fermi constants, α and G , the less accurately known parameter $\sin^2 \theta_w$, and the unknown t-quark and Higgs boson mass. The result of any experimental measurement would then be a determination of, or some restriction on, the above parameters. The disadvantage is that if one wants to change theory, e.g. enlarging the particle content at high energy, one has to redo the analysis of the data from the start, fitting them to the new predictions. A more economical procedure is to find a phenomenological expression of the cross-section, consistent with the general requirements of an underlying renormalizable quantum field theory, in terms of a number of model-independent parameters such as the vector boson mass, the partial and total widths etc. The result of a measurement is then a set of values for these parameters, which any theorist could compare with the predictions of his own favorite model (including the Standard Theory with m_t and m_H as unknown parameters) without going back to the raw data.

As a starting point we shall assume that the physical spectrum of the states below the Z mass is known. Furthermore, to simplify the discussion, we assume that the lowest order, tree level, amplitude arises only from photon and Z exchange. Equivalently, we are assuming that new physics enter only through loop diagrams (we comment at the end on

the effects of new, heavy vector bosons which can be exchanged at the tree level). Under these conditions our program can be explained as follows.

i) At resonance, the dominating Z-exchange amplitude of (1.1) is essentially imaginary. Its imaginary part is determined by the physical Z boson mass, M , and by the ratio^(*):

$$B = \left(\frac{\Gamma_e \Gamma_\mu}{\Gamma^2} \right)^{1/2} = \frac{\Gamma_e}{\Gamma} \quad (1.3)$$

The interference of the dominant imaginary part with the non-resonating amplitude involves only the absorptive parts of the latter, which are determined by known physics, i.e. the production amplitudes of light particles.

ii) At resonance, the Z-exchange amplitude has also a small real part which arises from the absorptive parts of the $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^-$ amplitudes. This real part is itself a higher order effect so that it interferes only with tree-level, non-resonating, amplitudes, i.e. one-photon exchange.

iii) To describe the line shape, we have to move in energy around the resonance by a small amount. Defining:

$$\xi = \frac{s - M^2}{M^2}$$

we need to know the variation for values of ξ such that^(**):

$$|\xi| \leq \frac{\Gamma}{M} \simeq O(\alpha)$$

We can develop in ξ the various complicated functions of s arising from the electroweak corrections to vertices, propagators etc., with the result that, to a given order in α , i.e. in ξ , only a finite number of derivatives are needed, which we can use as additional phenomenological parameters.

The final result can be illustrated as follows.

(*) Sometimes we shall distinguish, for the sake of clarity, the electronic and muonic widths but otherwise assume electron-muon universality, $\Gamma_e = \Gamma_\mu$.

(**) We shall not distinguish between powers of α and α_w , although numerically α_w is considerably larger than α .

Let us define:

$$\sigma_0(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{|D(s)|^2} \left[\frac{s}{M^2} + R \frac{s - M^2}{M^2} + \frac{\Gamma}{M} \mathcal{J} + \dots \right] \quad (1.4)$$

$\Gamma_{e,\mu}$, are the electromagnetically inclusive widths into electron and muon pairs, M and Γ the Z -mass and total decay rate and:

$$D(s) = s - M^2 + i M \Gamma \left[\frac{s}{M^2} + \epsilon \frac{s - M^2}{M^2} \right] + \dots \quad (1.5)$$

Dots denote higher order terms in $(s - M^2)/M^2$.

The parameter R in eq.(1.4) starts with a term of zeroth-order in α , which arises from the interference of the resonant amplitude with the one-photon exchange in lowest order. The remainder represents the interference of the real part of Z -exchange with the virtual corrections to the non-resonating amplitude, as well as the expansion of the resonating cross-section around $s = M^2$. In addition, R includes contributions arising from real soft-photon emission, required to cancel the infra-red divergences in γ - γ and γ - Z box diagrams. Already at this level $\sigma_0(s)$ is ultra-violet and infra-red finite.

The parameter $\mathcal{J}$ summarizes the effect of the absorptive parts in the amplitude at resonance described in (i) and (ii) above. As such, it is not a free parameter but it is completely determined from the composition of the real states at $s = M^2$, and it has to be computed and inserted in eq.(1.4), so as to be able to determine the value of B , eq.(1.3), from the peak cross-section.

Finally, ϵ has the very simple interpretation of representing scale-invariance violations in the total Z -width:

$$\epsilon = \frac{1}{\Gamma} \left[\left(s \frac{\partial}{\partial s} - 1 \right) \Gamma(s) \right]_{s=M^2} \quad (1.6)$$

where $\Gamma(s)$ is the decay-width at a generic value of s . To zeroth order in α , ϵ is directly related to the branching fraction for the decay of Z into massive particles. For example, the decay into $b\bar{b}$ pairs contributes a term:

$$\epsilon_{b\bar{b}} \simeq \frac{6 m_b^2}{M^2} \frac{a_b^2}{v_b^2 + a_b^2} B(Z \rightarrow b\bar{b}) \quad (1.7)$$

where a_b and v_b are the axial and vector couplings of b to Z and $B(Z \rightarrow b\bar{b})$ the corresponding branching ratio. Similarly to $\mathcal{J}$, ϵ depends on the real particles to the Z -energy scale, and it has to be computed and inserted in eq.(1.4).

In conclusion, the independent phenomenological parameters in σ_0 , are M , Γ , B and R .

Radiation from the final state is included in Γ_μ , for a fully inclusive measurement, and the interference of initial and final state radiation is included in R , to order α . On the other hand, the bremsstrahlung from the initial state has to be resummed to all orders, to obtain a meaningful observable cross-section around a narrow resonance^[3]. This is achieved by a convolution of σ_0 with the initial state radiation spectrum as described in refs.(10-14). In addition, we have to take into account the one-photon exchange cross-section, which is the only non-resonating term to be included to the level of precision considered here.

The observable cross-section is thus defined according to:

$$\sigma_{\text{obs}}(s) = \sigma_Z(s) + \sigma_\gamma \quad (1.8)$$

The first contribution is obtained from σ_0 according to:

$$\sigma_Z(s) = \int_0^{1-s_0/s} dx \sigma_0[s(1-x)] x \beta \left\{ \beta \frac{1+(1-x)^2}{2x} (1+\delta_{v+\text{soft}}) + \tilde{\Delta}_h(x) \right\} \quad (1.9)$$

where s_0 is the experimental threshold for the outgoing muon pair, and:

$$\delta_{v+\text{soft}} = \frac{3}{4} \beta + \frac{\alpha}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right) - \frac{3}{4} \frac{\alpha}{\pi} + \frac{\beta^2}{4} \left(\frac{9}{8} - \frac{\pi^2}{3} \right) + O(\beta^3, \frac{\beta\alpha}{\pi}) \quad (1.10)$$

$$\tilde{\Delta}_h(x) = \frac{\beta^2}{4} \left\{ \left[3 \frac{(2-x)}{2} - \frac{2}{x} \right] \ln(1-x) - x \right\} + O(\beta^3, \frac{\beta\alpha}{\pi}) \quad (1.11)$$

$$\beta = \frac{2\alpha}{\pi} \left(\ln \frac{s}{m_e^2} - 1 \right) \equiv 0.108 \text{ (for } \sqrt{s} = 92 \text{ GeV)} \quad (1.12)$$

The above expressions for $\delta_{v+\text{soft}}$ and $\tilde{\Delta}_h$ coincide with those given in the literature^[10-14] except for slight modifications:

i) We have placed the exponential factor x^β in front of all terms in the radiation spectrum. This takes into account all orders in $\beta \ln x$ and a certain number of terms of order β^2 .

ii) We have inserted the term $-3\alpha/4\pi$ in $\delta_{v+\text{soft}}$, to account for the use of the electromagnetically inclusive width, Γ_e , in σ_0 .

As for σ_γ , we keep only the leading term in the radiation spectrum of eq.(1.9), to obtain:

$$\sigma_\gamma = \frac{4}{3} \pi \frac{\alpha^2(M)}{M^2} \left[1 + \frac{\beta}{2} \ln\left(\frac{M^2}{s_0}\right) + O(\beta) \right] \quad (1.13)$$

Numerically, for $M=92$ GeV and $\alpha(M)^{-1} \simeq 128$:

$$\begin{aligned} \sigma_\gamma &= 1.12 \cdot 10^{-2} [1+0.49] \text{nb} = 1.67 \cdot 10^{-2} \text{nb} \quad (\sqrt{s_0} = 1 \text{GeV}) \\ \sigma_\gamma &= 1.12 \cdot 10^{-2} [1+0.66] \text{nb} = 1.86 \cdot 10^{-2} \text{nb} \quad (\sqrt{s_0} = 2m_\mu) \end{aligned} \quad (1.14)$$

We shall exclude, for simplicity, significant contributions to ε of yet undiscovered massive particles. Assuming $s_0/s \ll 1$ and approximating $\varepsilon \simeq 0$, we can integrate eq.(1.9) analytically. We obtain:

$$\begin{aligned} \sigma_Z(s) &= \frac{12\pi \Gamma_e \Gamma_\mu}{|D(s)|^2} \frac{1}{r^\beta} \frac{\beta\pi}{\sin(\beta\pi)} \cdot \\ &\left\{ \left[\frac{s}{M^2} + R \frac{s-M^2}{M^2} + \frac{\Gamma}{M} \right] \left[\cos(\beta\phi) + \frac{s-M^2+s\gamma^2}{M\Gamma} \sin(\beta\phi) \right] [1+\delta_{v+\text{soft}}] - \right. \\ &\quad \left. - (2+R) \gamma \frac{(s-M^2)^2 + M^2 \Gamma^2}{M^2 \Gamma^2} \sin(\beta\phi) + O(\beta^3, \beta^2 \frac{\Gamma}{M}, \frac{\beta\alpha}{\pi}) \right\} \quad (15.1) \end{aligned}$$

$$\gamma = \frac{\Gamma}{M}; \quad r = \frac{s\sqrt{1+\gamma^2}}{\sqrt{(s-M^2)^2 + s^2 \gamma^2}}$$

$$\phi = \frac{\pi}{2} + \text{tg}^{-1} \left[\frac{s-M^2+s\gamma^2}{M\Gamma} \right]$$

The result of the integration coincides, up to terms of order β^2 and $\frac{\alpha}{\pi}$, with the results reported in ref.(15).

The plan of the paper is the following.

We discuss the derivation of the above results in Sect.2, in the case of the leptonic process eq.(1.1). The numerical values of the parameters M , Γ , B , R and $\mathcal{J}$ predicted in the Standard Theory to one loop and the resulting values of the unfolded cross-section σ_0 for different values of m_t and m_H are presented in Sect.3. The numerical results for the observable cross-section of process (1.1) are given in Sects.4 and 5, with a comparison with previous calculations and a study of the sensitivity of the parameters to the experimental errors. The hadronic line shape is discussed in Sect.6. Unlike the leptonic case, we take into consideration only the leading electroweak corrections, in particular those corrections which increase quadratically with m_t . Finally, we comment about the effect of a new vector boson in Sect.7.

We give in the Appendix a guide to the formulae relevant for the calculation of R and $\mathcal{J}$ in the Standard Theory.

2. Diagram Classification.

In this Section we illustrate the derivation of σ_0 , eq.(1.4), and the calculation of the parameters R and $\mathcal{J}$ in the Standard Theory for process (1.1). Detailed formulae and the corresponding references are given in the Appendix.

2.1 The Vector Boson Propagator. We start from the vector boson propagator. One writes:

$$\Delta_{\mu\nu} = \frac{-i g_{\mu\nu}}{s - M_0^2 + \Pi(s)} + k_\mu k_\nu - \text{terms}$$

where M_0 is the bare mass and $\Pi(s)$ the sum of all self-energy diagrams, one-particle irreducible with respect to the Z line, see Fig.1. Two-loop contributions to $\Pi(s)$, both reducible and irreducible with respect to the photon line, are needed only in the imaginary part, to reconstruct the total width to order α^2 included^[14].

The physical Z -mass is defined by the equation:

$$M^2 - M_0^2 + \text{Re}[\Pi(M^2)] = 0 \tag{2.1}$$

Dropping longitudinal terms, which give contributions proportional to the external lepton masses, and expanding around $s=M^2$, we get:

$$\Delta_{\mu\nu} = -i g_{\mu\nu} [(s - M^2) (1 + \Pi') + \frac{1}{2} \frac{(s - M^2)^2}{M^2} \Pi'' + \dots + i \text{Im } \Pi(s)]^{-1}$$

where:

$$\Pi^{(n)} = \left[s^{n-1} \frac{\partial \text{Re } \Pi(s)}{\partial s^n} \right]_{s=M^2}$$

The term with Π'' is of order α^3 and will be dropped in the following. The physical total width, Γ , is defined by:

$$\text{Im}[\Pi(M^2)] = M\Gamma(1 + \Pi') \quad (2.2)$$

Therefore, introducing $\Gamma(s)$ through the relation (for $s \cong M^2$):

$$\text{Im}[\Pi(s)] = M\Gamma(s)(1 + \Pi') \quad (2.3)$$

we obtain:

$$\Delta_{\mu\nu} = \frac{-i g_{\mu\nu}}{1 + \Pi'} \frac{1}{D(s)} \quad (2.4)$$

with:

$$D(s) = s - M^2 + i M \Gamma(s) \quad (2.5)$$

A suitable expansion of $\Gamma(s)$ around $s=M^2$, which takes into account the contribution of massless and massive final states in Z-decay is:

$$\Gamma(s) = \Gamma \left[\frac{s}{M^2} + \varepsilon \frac{s-M^2}{M^2} + \dots \right] \quad (2.6)$$

where ε is given by eq.(1.6) and takes into account, to first order in $(s-M^2)/M^2$, the possible effect of massive states, i.e. of scale invariance violation, in Z-decay.

Following ref.(9), it is easy to see that all the linear dependence of $\Gamma(s)$, even in the case $\epsilon \neq 0$, can be mostly reabsorbed into a redefinition of M and Γ . In fact, we may rearrange the terms in $D(s)$ according to:

$$D(s) = (s - M^2) \left[1 + i(1+\epsilon) \frac{\Gamma}{M} \right] + i M \Gamma \cong \eta (s - M'^2 + i M' \Gamma')$$

with $(\gamma = \Gamma/M)$:

$$\eta = 1 + i(1+\epsilon)\gamma$$

$$M' \simeq M \left[1 - \frac{1+\epsilon}{2} \gamma^2 \right]$$

$$\Gamma' \simeq \Gamma \left[1 - \frac{1+3\epsilon}{2} \gamma^2 \right]$$

To lowest order, η is a phase factor. It gets lost when we compute $|D(s)|^2$, for the resonant cross-section and it can be set equal to unity in the interference of the resonant with the non-resonant amplitudes, since it would contribute higher order terms. Therefore, it is not possible to disentangle ϵ from M : if we want to extract from the data the value of M , defined according to eq.(2.1), we need to know ϵ a priori^(*).

The linear variation of Γ with s gives a shift to M and Γ quite at the limits of the precision implied by the present one-loop calculations, i.e. $\Delta M \simeq -35$ MeV, $\Delta \Gamma \simeq -1$ MeV, see ref.(9) and Sect.4.

2.2 The full resonating amplitude. The resonating amplitude is obtained by combining the Z -propagator with vertex corrections. We write:

$$A_{\text{res}} = M(Z \rightarrow e^+e^-)_s \frac{1}{D(s)} M(Z \rightarrow \mu^+\mu^-)_s \quad (2.7)$$

Expanding the numerator around $s=M^2$, we get:

$$A_{\text{res}} = \frac{N(M^2)}{D(s)} \left\{ 1 + \frac{s - M^2}{M^2} \left[s \frac{\partial}{\partial s} N(s) \right]_{s=M^2} + \dots \right\} \quad (2.8)$$

(*) One of us, L.M., would like to thank A.Martin for an enlightening discussion on this point.

where:

$$N(M^2) = [M(Z \rightarrow e^+e^-) M(Z \rightarrow \mu^+\mu^-)]_{s=M^2}$$

$N(M^2)$ is still infra-red divergent. The divergence is removed, in the square, by the corresponding divergence due to real photon emission or absorption from the initial or final state. For the final state, this amounts to replacing $|M(Z \rightarrow \mu^+\mu^-)|^2$ with the electromagnetically inclusive width, Γ_μ . For the initial state, we add to $|M(Z \rightarrow e^+e^-)|^2$ the appropriate terms to reconstruct Γ_e and subtract the same terms in the radiation spectrum with which σ_0 , eq. (1.4), will be folded, eq. (1.9). This procedure is the origin of the $-(3/4\pi)\alpha$ term in eq. (1.10). The interference between initial and final state radiation will be dealt with later on.

In conclusion, the resonant cross-section takes the form given in eq. (1.4), with a first contribution to R obtained from eq. (2.8), see Appendix:

$$R^{(1)} = 2 \left\{ \text{Re} \left[\frac{s}{N(s)} \frac{\partial N(s)}{\partial s} \right]_{s=M^2} \right\} - 1 \quad (2.9)$$

Non-vanishing contributions to $R^{(1)}$ arise from one-loop diagrams only, since the lowest order result yields $N(s)$ proportional to $\sqrt{s}$.

In computing the interference with the other amplitudes, we shall neglect the s -dependence of N , and simplify the form of A_{res} according to:

$$A_{\text{res}} \simeq \frac{N(M^2)}{D(s)} = \frac{N(M^2)}{|D(s)|^2} [s - M^2 - iM\Gamma(s)] \quad (2.10)$$

2.3 One-Photon Exchange. The interference of A_{res} with the one-photon exchange diagram, propagator and vertex corrections included, gives rise to contributions to both R and $\mathcal{I}$.

With reference to eq. (2.10), one sees that the real part of the one-photon-exchange amplitude contributes to R in conjunction with the real part of $N(M^2)$, while its absorptive part gives a contribution to $\mathcal{I}$ (these are the terms $R^{(2,3)}$ and $\mathcal{I}^{(1)}$ discussed in the Appendix). When we take the imaginary part of $N(M^2)$, we can restrict to the lowest order one-photon-exchange amplitude and we get a contribution to $\mathcal{I}$, $\mathcal{I}^{(2)}$.

One-photon exchange gives the largest contribution to R . An approximate expression for the sum of $R^{(2)}$ and $R^{(3)}$ is:

$$R^{(2)} + R^{(3)} = \frac{2Q_e Q_\mu v'_e v'_\mu}{(v'^2_e + a_e^2)(v'^2_\mu + a_\mu^2)} \frac{\alpha(M)}{\alpha(0)} \frac{(37.2805 \text{ GeV})^2}{(1 + \delta\rho)M^2} \quad (2.11)$$

where Q and a denote, respectively, the electric charge and the axial vector couplings and v' is the vector coupling expressed in terms of the effective $\sin^2\theta_W$ defined in ref.(16), useful in the large m_t regime ($s_W^2 = \sin^2\theta_W$):

$$v' = \frac{1}{2} T_3 - Q (s_W^2)' \quad (2.12)$$

$$a = \frac{1}{2} T_3 \quad (2.13)$$

$$(s_W^2)' = s_W^2 (1 + \delta\rho \frac{c_W^2}{s_W^2}) \quad (2.14)$$

$$\delta\rho = \frac{G}{\sqrt{2}} \frac{3m_t^2}{8\pi^2} \quad (2.15)$$

and $\sin^2\theta_W$ defined as usual from the W and Z mass, see eq.(4.1) below.

Eq.(2.11) reproduces quite well the full expression of $R^{(2)} + R^{(3)}$, in the range of m_t between 50 to 200 GeV. The complete expression for R is obtained by adding to eq.(2.11) a small constant contribution from $R^{(1)}$ and from the box diagrams discussed below.

For vanishing lepton masses, the electromagnetic virtual corrections to the Z and photon vertices are the same. Therefore the infra-red divergence compensation discussed for the resonating cross-section goes through unchanged in this case.

The square of the one-photon amplitude gives rise to the well-known cross-section:

$$\sigma_{0\gamma} = \frac{4}{3} \pi \frac{\alpha^2(M)}{s} \quad (2.16)$$

We shall use the running coupling at $s=M^2$, but otherwise ignore all remaining electroweak corrections to $\sigma_{0\gamma}$, at the level of precision we are interested.

In fact, with respect to the tree-level peak cross-section σ_0 , $\sigma_{0\gamma}$ is formally of order α^2 , considering that $(\Gamma_e \Gamma_\mu)/\Gamma^2$ is of order α^0 . In principle, we could neglect $\sigma_{0\gamma}$ altogether, since the computed one-loop corrections to σ_0 are of order α . However, the small leptonic branching ratio ($B \approx 3.3 \cdot 10^{-2}$) makes the effect of $\sigma_{0\gamma}$ non-negligible:

$$\sigma_{0\gamma} \equiv \frac{\alpha^2(M)}{9B^2} \quad \sigma_0(M^2) \equiv 6 \cdot 10^{-3} \sigma_0(M^2) \quad (2.17)$$

and we shall keep it. Folding $\sigma_{0\gamma}$ with the initial state radiation gives the result reported already in eq.(1.13).

2.4 Box Diagrams. Due to the vanishing photon mass, the γ -Z box diagram has a resonating part. The interference of the resonating part with Z and one-photon exchange has to be added to the interference of initial-final state radiation, to eliminate the residual infra-red divergences. The two amplitudes cancel exactly against each other.

The non-resonating γ -Z box amplitude interferes only with Z-exchange, to the order considered, and it gives further contributions to R and $\mathcal{I}$, both very small.

For the γ - γ box, we need to consider only interference with Z-exchange. The real part contributes to R. Initial-final state radiation interference has to be added to obtain an infrared finite result. The contribution to $\mathcal{I}$ due to the γ - γ absorptive part vanishes exactly, because a real two-photon state cannot have angular momentum $J=1$.

Weak box diagram contributions are purely real. The interference with A_{res} gives the last contributions to R (see the Appendix).

We conclude this Section with some considerations about the order of magnitude of the various corrections.

As stated already, the resonant amplitude at the peak is formally of order α^0 and the one-loop corrections to M and B that we want to detect are of order α . Therefore, we should keep the interference with the other diagrams only to order α . This means discarding all contributions except the lowest order interference with the one-photon exchange, $R^{(2)} + R^{(3)}$. However, the peak resonant amplitude is proportional to the leptonic branching ratio B, which gives another small parameter because of the many channels available (15 quark channels, for five flavours in three colors, and 6 lepton channels). Contributions such as the interference with the W-W box, formally of order α^2 , are in fact of order α^2/B , with respect to the peak cross-section, and could give sizeable corrections. This is why we computed all the interference terms (and kept the contribution of $\mathcal{I} \Gamma/M$ in eq.(1.4), which is also formally of order α^2). In practice, however, the effect of the $1/B$ factor turns out to be compensated by small numerical factors, in particular by the presence of the electron vector coupling:

$$v_e = \sin^2\theta_w - \frac{1}{4} \quad (2.18)$$

which is very small. Numerically, the addition of the potentially relevant α^2/B terms is not very significant.

3. Values of the Parameters in the Standard Theory and Numerical results for σ_0 .

In this Section, we give the values of the parameters appearing in σ_0 , as predicted by the Standard Theory. We assume a fixed value for the Z mass:

$$M = 92.00 \text{ GeV} \quad (3.1)$$

$\sin^2\theta_w$ is conventionally defined as:

$$\sin^2\theta_w = s_w^2 = 1 - \frac{M_w^2}{M^2} \quad (3.2)$$

Given the value of M, $\sin^2\theta_w$ is obtained from the relation ($c_w^2 = \cos^2\theta_w$):

$$M^2 = \frac{(37.2805 \text{ GeV})^2}{s_w^2 c_w^2} \left[\frac{\alpha(M)}{\alpha} + (\Delta r)_w \right] \quad (3.3)$$

$(\Delta r)_w$ is the electroweak correction^[16,17,18] which remains after taking into account the running of the electric charge up to the vector boson scale. The value of $\sin^2\theta_w$ depends, through $(\Delta r)_w$, on the t-quark mass, m_t and on the Higgs boson mass, m_H .

Using the results of ref.(16), we have computed preliminarily the values of $\Gamma_e = \Gamma_\mu$, Γ_ν and $\sin^2\theta_w$ for some representative values of m_t and m_H , see Table 1. The numbers in parenthesis are taken from the similar calculation of ref.(14). In ref.(19), the corresponding results obtained by Yu. Bardin, T. Riemann et al. are reported. In some cases, we show also the latter results, with a BR mark. All results are in excellent agreement among themselves. For the convenience of the reader, we have included the values of the total Z-width in Table 1, taken from ref.(14).

The predictions for R and $\mathcal{J}$ in the Standard Theory are given in Table 2, for the same set of values of m_t and m_H as in Table 1. At the points:

$$s_\pm \simeq M^2 \pm M\Gamma \quad (3.4)$$

the computed value of R corresponds to a shift of the cross-section:

$$\frac{\Delta\sigma_0(s_{\pm})}{\sigma_0(s_{\pm})} \simeq \pm 1.5 \cdot 10^{-3} \quad (3.5)$$

while the value of $\mathcal{J}$ renormalizes the peak cross-section:

$$\left(\frac{\Delta\sigma_0}{\sigma_0}\right)_{\text{peak}} \simeq (0.3 \div 0.6) \cdot 10^{-3} \quad (3.6)$$

Both effects are quite small, at the limit of visibility. This is an important result: the one-loop corrections in the Standard Theory affect the cross-section mostly through their influence on the values of M , Γ and B .

The value of R predicted by Standard Theory is affected by a considerable error, due to the influence of corrections of still higher order than those considered here. This is because the zeroth-order contribution to R is strongly suppressed, being proportional to the electron vector coupling squared, v_e^2 eq.(2.18), which is close to zero. The next order correction is, therefore, quite important and, in addition, it makes a sensible difference whether in the latter contribution we use the fully corrected value of $\sin^2 \theta_w$ or the uncorrected one. The resulting values of R in the two alternatives, which differ by higher order terms in perturbation theory, are shown in Fig.2, versus m_t . Differences of the order of 40% are observed, which gives an estimate of the systematic error due to the lack of knowledge of the higher order corrections.

A similar systematic error affects the parameter $\mathcal{J}$. To the present order in perturbation theory, one could use the uncorrected value of $\sin^2 \theta_w$ which, being m_t -independent, would give a constant $\mathcal{J}$, with respect to m_t . The variation of $\mathcal{J}$ seen from Table 2, where the fully corrected value of $\sin^2 \theta_w$ has been used, gives an estimate of the error associated with the higher order corrections.

Also in Table 2, we give our values of $\sigma_0 + \sigma_{0\gamma}$ at the peak, of the peak position and of the energy values where the cross-section is 1/2 of the peak value. The one photon exchange cross-section has been added to compare with the numerical results of ref.(19), reported in parenthesis. An excellent agreement is found, with respect to ref.(19), for the peak position and for the apparent width. A small systematic discrepancy in the peak cross-section may be present.

4. The Observable Cross-section.

The folding with the initial state radiation has been computed by numerical integration, with the results reported in Table 3. To compare with ref.(19), we have folded the one-photon cross-section with the bremsstrahlung spectrum, down to the minimum threshold, $s_0 = 4m_\mu^2$. We find an excellent agreement, both in shape and normalization.

Our preferred result is reported in Table 4, where we have folded the one-photon cross-section only down to the value $\sqrt{s_0} = 1\text{GeV}$, closer to the minimum invariant mass for which the muon pair can be experimentally detected. In parenthesis, we give in Table 4 the values obtained with the analytic expression, eq.(1.15), the one-photon contribution being always computed numerically. The two results are in good agreement.

When all corrections are included, the peak of the cross-section is shifted by an amount ΔE with respect to the Z mass. From Table 4 we read:

$$\Delta E \cong +95 \text{ MeV} \quad (4.1)$$

ΔE can be estimated directly from eq.(1.15). The approximate expression is:

$$\Delta E = \frac{\pi\beta\Gamma}{8} - \frac{\Gamma^2}{4M}(1-R) + O(\beta^2\Gamma, \frac{\beta\Gamma^2}{M}, \frac{\Gamma^3}{M^2}) \cong 95 \text{ MeV} \quad (4.2)$$

Without the s-dependence of the width, one would get instead:

$$\Delta E = \frac{\pi\beta\Gamma}{8} + \frac{\Gamma^2}{4M}(1+R) + O(\beta^2\Gamma, \frac{\beta\Gamma^2}{M}, \frac{\Gamma^3}{M^2}) \cong 130 \text{ MeV} \quad (4.3)$$

a difference of about 35MeV, as noticed in refs.(9,20).

The difference between $\sqrt{s_+}$ and $\sqrt{s_-}$ in Table 4 is about 410 MeV larger than the real width Γ . Of course, this is due mostly to the effect of the initial state radiation. Some part is also due to the effect of σ_γ , however.

In fact $\sqrt{s_\pm}$ are given by the equation:

$$\sigma_Z(s_\pm) + \sigma_\gamma = \frac{1}{2} [\sigma_Z + \sigma_\gamma]_{\text{peak}}$$

i.e.

$$\sigma_z(s_{\pm}) = \frac{1}{2} (\sigma_z)_{\text{peak}} - \frac{1}{2} \sigma_\gamma$$

From these equations, one estimates that the presence of a constant σ_γ increases the apparent width by about:

$$\Delta\Gamma = \left(\frac{\sigma_\gamma}{\sigma_z}\right)_{\text{peak}} \frac{\Gamma}{2} (A_+ + A_-) \quad (4.4)$$

$$A_{\pm} = \left| \frac{2}{\Gamma(\partial \ln \sigma / \partial \sqrt{s})_{\pm}} \right|$$

where $A_{\pm}$ are normalized so as to be unity if σ has a pure Breit-Wigner shape. Numerically we find:

$$A_+ \simeq 1.5 \quad (4.5)$$

$$A_- \simeq 1.0$$

reflecting the consistent decrease in the slope of σ after the resonance. Correspondingly:

$$\Delta\Gamma(\mu) \simeq +41 \text{ MeV} \quad (4.6)$$

We shall come back to this point in Sect. 6.

5. Sensitivity to the Experimental Errors.

To assess the sensitivity of our parametrization to the experimental errors, we have generated a set of "data points", adding a Gaussian error to the result of eq.(1.8), for given values of the parameters. 16 points, spaced in energy by 0.2 GeV, between 90.5 and 93.5 GeV or 5 points spaced by 1 GeV between 89.7 and 93.7 GeV have been considered. The data points have then been fitted, with MINUIT, to find the best value of the parameters.

We give in Table 5 the result in correspondence to two sets of values for m_t and m_H (all other cases we have tried are quite similar). In the first two columns of the Table, we give the values of m_t and m_H which, via Tables 1 and 2, identify the input values of Γ , $\Gamma_e = \Gamma_\mu$ and R . M is fixed to 92.00 GeV and, for simplicity, we have taken $\beta=0$ (non vanishing

$\mathcal{J}$ corresponds to a redefinition of the leptonic width). In the Table, we give the best values of the fitted parameters, M , Γ , $\Gamma_e = \Gamma_\mu$ and R , in correspondence to a Gaussian error:

$$\Delta\sigma = \rho \sigma_{\text{peak}}$$

with $\rho = 10^{-2}$ and 10^{-3} . With 16 points, we get very good values of M , Γ , and to a minor extent, of Γ_e even for $\rho = 10^{-2}$. On the other hand, $\rho = 10^{-3}$ is needed to reobtain the parameter R with a precision of the order of the theoretical uncertainty discussed in Sect.3. Interestingly, the situation does not get worse when only 5 points are considered. R is well determined even for $\rho = 10^{-2}$, probably because we have chosen in this case a larger energy range, and the effect of R is more important on the tail of the cross-section.

6. The Hadronic Line Shape.

We can easily extend our considerations to the Z-line shape in the fully inclusive hadronic channel, eq.(1.2), described as the decay of the Z into all available quark pairs.

The parameter B , eq.(1.3), is replaced in this case by:

$$B^{(h)} = \left[\frac{\Gamma_e \Gamma^{(h)}}{\Gamma^2} \right]^{1/2} \simeq 0.15 \simeq 4.6B \quad (6.1)$$

The resonant amplitude is correspondingly larger than in the case of muon pairs. We may therefore neglect the interference of all the electroweak corrections with the resonant amplitude (which gave not very important contributions also in the muon pair, see the discussion at the end of Sect.2) but for the one-photon exchange, taken to lowest order plus leading logarithmic and leading m_t corrections. In particular this means neglecting the very small term proportional to $\mathcal{J}^{(h)}$, and writing:

$$\sigma_0^{(h)} = 12\pi \frac{\Gamma_e \Gamma^{(h)}}{|D(s)|^2} \left[\frac{s}{M^2} + R^{(h)} \frac{s-M^2}{M^2} \right] \quad (6.2)$$

For a single, massless, $q\bar{q}$ channel, we use eq.(2.11) which gives, in this case:

$$R^{(q)} = \frac{2Q_e Q_q v'_e v'_q}{(v'_e{}^2 + a_e{}^2)(v'_q{}^2 + a_q{}^2)} \frac{\alpha(M)}{\alpha(0)} \frac{(37.2805 \text{ GeV})^2}{(1 + \delta\rho)M^2} \quad (6.3)$$

Summing over u, d, c, s and b channels, we obtain:

$$R^{(h)} = \sum \frac{\Gamma^{(q)}}{\Gamma^{(h)}} R^{(q)} = 2 \frac{\alpha(M)M}{\Gamma^{(h)}} \frac{-v'e}{v'e^2 + a_e^2} \left[\frac{7}{12} - \frac{11}{9}(s_W^2)' \right] \quad (6.4)$$

Remarkably, δp disappears from $R^{(h)}$. All the m_t dependence is contained in M , $\Gamma^{(h)}$ and $(s_W^2)'$. Numerically, $R^{(h)}$ is about a factor of two larger than in the muon case.

After folding $\sigma_0^{(h)}$ eq.(6.2) with the bremsstrahlung spectrum, and adding the one-photon exchange cross-section, $\sigma_\gamma^{(h)}$, we find the results reported in Table 6, where the values of $R^{(h)}$ are also reported.

As expected, the peak position is the same as in Tables 3 and 4. On the other hand, the apparent width in the hadronic channel is reduced, with respect to the apparent width in the muon channel, of about:

$$\delta\Gamma^{(h)} = \Gamma_{\text{eff}}^{(h)} - \Gamma_{\text{eff}}^{(\mu)} = -32 \text{ MeV} \quad (6.5)$$

This is due to the reduced effect of the one-photon cross section in the hadron, with respect to the muon case. With reference to eq.(4.4), we expect

$$\begin{aligned} \delta\Gamma^{(h)} &= \Delta\Gamma^{(h)} - \Delta\Gamma^{(\mu)} = \left\{ \left[\frac{\sigma_\gamma^{(h)}}{\sigma_Z^{(h)}} \right]_{\text{peak}} - \left[\frac{\sigma_\gamma}{\sigma_Z} \right]_{\text{peak}} \right\} \frac{\Gamma}{2} (A_+ + A_-) \\ &\simeq -0.83 \Delta\Gamma^{(\mu)} \simeq -34 \text{ MeV} \end{aligned} \quad (6.6)$$

in good agreement with (6.5).

7. The Effect of A New Vector Boson.

The presence of a new vector boson coupled to electrons and muons (or quarks) modifies the tree level amplitude. Within the Z-line shape, the most important effect is an additional contribution to R , with respect to the Standard Theory value. There is also a contribution to $\mathcal{I}$, when we take the absorptive part of $N(M^2)$ in the resonating amplitude, eq.(2.10), and consider the interference of this term with the vector boson exchange. However, the effect on R is larger and we restrict ourselves to it.

The appropriate generalization of eq.(2.11) in the case of muon-pair production is:

$$\Delta R = \frac{2(V_e v'_e + A_e a_e)(V_\mu v'_\mu + A_\mu a_\mu)}{(v'^2_e + a^2_e)(v'^2_\mu + a^2_\mu)} \left(\frac{g'^2}{e^2} \right) \frac{(37.2805 \text{ GeV})^2}{(1 + \delta\rho)(M^2 - M'^2)} \quad (7.1)$$

where M' is the mass of the new boson, and V , A and g' are defined as:

$$L_{\text{int}} = g' Z'_\mu [\bar{e}\gamma_\mu(V_e + A_e\gamma_5)e + \dots] \quad (7.2)$$

The variation of R can be sizeable, particularly if the vector boson has a non negligible axial coupling.

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Appendix

We give in this Appendix the detailed expressions for the parameters R and $\mathcal{I}$ defined in eq.(1.4). We specialize to the $\mu^+\mu^-$ final state. However, since we leave indicated the vector (v) and axial (a) couplings of the electron and muon, the extension to the quark case is straightforward.

$$R = R^{(1)} + R^{(2)} + R^{(3)} + R^{\text{box}} \quad (\text{A.1})$$

where ($\mu=37.2805$ GeV)

$$R^{(1)} = \frac{4}{(v^2+a^2)} \left\{ v \operatorname{Re} \frac{dA_Z}{ds} \Big|_{s=M^2} + a \operatorname{Re} \frac{dB_Z}{ds} \Big|_{s=M^2} \right\} \quad (\text{A.2})$$

$$R^{(2)} = 2 v^2 \frac{\mu^2 \alpha(M)}{M^2 \alpha(0)} \frac{1}{P} \frac{1}{[|\bar{A}_Z|^2 + |\bar{B}_Z|^2]^2} \quad (\text{A.3})$$

$$R^{(3)} = \frac{4v}{(v^2+a^2)} \frac{\mu^2}{M^2} \left\{ \operatorname{Re}(\bar{A}_Z - v) + v \operatorname{Re}(\bar{A}_\gamma - 1) + a \operatorname{Re} \bar{B}_\gamma \right\} \quad (\text{A.4})$$

$$R^{\text{box}} = R_{\gamma\gamma} + R_{\gamma Z} + R_{ZZ} + R_{WW} \quad (\text{A.5})$$

$$R_{\gamma\gamma} = \frac{3}{2} \frac{\alpha}{\pi} \frac{a^2}{(v^2+a^2)^2} \frac{\mu^2}{M^2}$$

$$R_{\gamma Z} = \frac{6\alpha}{\pi} \frac{v^2 a^2}{(v^2+a^2)^2} \left\{ -3 \left(1 + 2 \ln \frac{\Gamma}{M} \right) + \frac{2}{3} \pi^2 - 3 \right\}$$

$$R_{ZZ} = \frac{3\alpha}{2\pi} \frac{1}{(v^2+a^2)^2} \frac{a^2 (a^2 + 3v^2)^2}{\mu^2} \int_{-1}^1 dz \frac{s^2}{4} (1+z)^2 I_{ZZ}(s, q^2, M^2)$$

$$R_{WW} = \frac{3\alpha}{32\pi} \frac{(v+a)^2}{(v^2+a^2)^2} \frac{1}{s_w^2} \frac{1}{M_w^2} \int_{-1}^1 dz \frac{s^2}{4} (1+z)^2 I_1(s, q^2, M_w^2)$$

with $s = M^2$ and $q^2 = \frac{M^2}{2} (1-z)$.

The various quantities introduced in the above expressions are defined as follows.

1. P (introduced in eq.A3) is given by

$$\begin{aligned}
 P = 1 + f_{zz}^{(b)}(M^2) + f_{zz}^{(H)}(M^2) + \frac{\alpha}{4\pi s_w^2} \frac{4}{c_w^2} (c_w^2 - s_w^2 + \frac{8}{3} s_w^4) + \frac{\alpha}{4\pi s_w^2} (R_1^w + R_1^H) \\
 + \frac{\alpha}{4\pi s_w^2} \left\{ \frac{1}{4c_w^2} \left[\left(1 - \frac{8}{3} s_w^2\right)^2 + 1 \right] [\delta I_t^1 + \right. \\
 \left. \frac{2 m_t^2}{M^2} (1 + \delta I_t^1 - \bar{I}_t)] + \frac{3}{2} \frac{m_t^2}{c_w^2 M^2} (\bar{I}_t - 1 - \delta I_t^1) \right\} + \delta\rho
 \end{aligned} \tag{A.6}$$

where we assume $M < 2m_t$ and:

$f_{zz}^{(b)}(M^2)$ is defined in eq.(B.6) pag. 497 of ref.(18)

$f_{zz}^{(H)}(M^2)$ in eq.(B.7) pag. 498, ibidem

R_1^w in eq.(A.3) pag.494, ibidem

R_1^H in eq.(A.8) pag. 495, ibidem

$$\delta I_t^1 = - \frac{4 m_t^2}{M^2} \frac{1}{\sqrt{\frac{4 m_t^2}{M^2} - 1}} \operatorname{tg}^{-1} \left(\frac{1}{\sqrt{\frac{4 m_t^2}{M^2} - 1}} \right) \tag{A.7}$$

$$\bar{I}_t = I_t(M^2) \tag{A.8}$$

$$I_t(s) = -2 + 2 \sqrt{\frac{4 m_t^2}{s} - 1} \operatorname{tg}^{-1} \left(\frac{1}{\sqrt{\frac{4 m_t^2}{s} - 1}} \right) \tag{A.9}$$

$$\delta\rho = \frac{3 m_t^2}{8\pi^2} \frac{G}{\sqrt{2}} \tag{A.10}$$

2. $A_Z(s)$, $B_Z(s)$, $A_\gamma(s)$ and $B_\gamma(s)$ denote the vector (A_Z and A_γ) and axial vector (B_Z and B_γ) coupling of the electron to the Z and γ , at a scale fixed by s , with all one-loop corrections except the purely electromagnetic ones. A_Z to B_γ include the (non-electromagnetic) wave function renormalizations. A bar over A or B indicates that $s=M^2$.

$$\begin{aligned}
A_Z(s) = & v + \frac{\alpha}{4\pi s_w^2} \left\{ \frac{1}{4} \left[c_w^2 \left(U\left(\frac{s}{M_w^2}\right) - \frac{1}{2} \right) - \frac{1}{2} \left(V\left(\frac{s}{M_w^2}\right) - \frac{1}{2} \right) \right] \right. \\
& - v \frac{(v^2 + 3a^2)}{c_w^2} \left(V\left(\frac{s}{M^2}\right) - \frac{1}{2} \right) + \\
& + s_w^2 \left[2 \left(3 c_w^2 + \frac{1}{6} + \frac{4M_w^2}{s} (c_w^2 + \frac{1}{3}) \right) \left(\sqrt{\frac{4M_w^2}{s} - 1} \operatorname{tg}^{-1} \frac{1}{(\frac{4M_t^2}{s} - 1)^{1/2}} - 1 \right) - \frac{1}{9} \right] - \\
& - 4 s_w^2 \left(1 - \frac{8}{3} s_w^2 \right) \left(\ln \frac{s}{M^2} - i\pi \right) - \\
& - \frac{2}{3} s_w^2 \left(1 - \frac{8}{3} s_w^2 \right) \operatorname{Re} (t(s) - t(M^2)) + \\
& + c_w^2 \left[4 \ln c_w^2 + \Pi_w^0 + \frac{1}{c_w^2} H(M) - \Pi_w^+ - H(M_w) \right] \} + \\
& + c_w^2 \delta\rho + c_w^2 (F_{ww}(M_w^2) - F_{33}(M^2)) + \\
& + 2 \frac{c_w^2 s_w^2}{s_w^2 - c_w^2} (F_{33}(M^2) - 1/s_w F_{3\gamma}(M^2)) + \\
& + \frac{\alpha}{4\pi s_w^2} \frac{c_w^2 s_w^2}{s_w^2 - c_w^2} \operatorname{Re} \left(\frac{1}{6c_w^2} t(M) + \frac{3}{2} \frac{m_t^2}{M_w^2} I_t(M) \right)
\end{aligned} \tag{A.11}$$

with

$F_{ww}(M_w^2) - F_{33}(M^2)$ is given in (53) pag. 484, ref.(18)

$F_{33}(M^2) - \frac{1}{s_w} F_{3\gamma}(M^2)$ in eq.(54) pag. 484, ibidem

Π_w^0 in eq.(A.13), pag.495, ibidem

Π_w^+ in eq.(A.6), pag.495, ibidem

$H(M)$ in eq.(A.9), pag. 495, ibidem

$$t(s) = I_t(s) - I_t(s)|_{m_t=0} + \frac{2m_t^2}{s} I_t(s) \quad (A.12)$$

and $I_t(s)$ is defined above, eq.(A.8) and:

$$I_t(s)|_{m_t=0} = \ln\left(\frac{s}{M^2}\right) - 2$$

The form factors $U(\frac{s}{M^2})$ and $V(\frac{s}{M^2})$ are defined as ($s < 4M^2$):

$$\begin{aligned} U\left(\frac{s}{M^2}\right) &= \frac{4M^2}{s} \left(1 + \frac{M^2}{2s}\right) \left[\cos^{-1}\left(1 - \frac{s}{2M^2}\right)\right]^2 - 2\left(1 + \frac{M^2}{s}\right) - \\ &- 2\left(1 + \frac{2M^2}{s}\right) \left(\frac{4M^2}{s} - 1\right)^{1/2} \operatorname{tg}^{-1} \left[\frac{1}{\left(\frac{4M^2}{s} - 1\right)^{1/2}} \right] \end{aligned} \quad (A.13)$$

$$0 \leq \cos^{-1} \leq \pi$$

$$\begin{aligned} V\left(\frac{s}{M^2}\right) &= \left(1 + \frac{M^2}{s}\right)^2 \left[2 \operatorname{Li}_2\left(\frac{M^2}{M^2+s}\right) + \ln^2\left(\frac{M^2+s}{M^2}\right) - \frac{\pi^2}{3} \right] + \\ &+ \left(3 + 2\frac{M^2}{s}\right) \ln \frac{M^2}{s} + 2\left(2 + \frac{M^2}{s}\right) + \\ &+ i\pi \left[3 + \frac{2M^2}{s} - 2\left(1 + \frac{M^2}{s}\right)^2 \ln \frac{M^2+s}{M^2} \right] \end{aligned} \quad (A.14)$$

In terms of the same functions we find:

$$\begin{aligned}
A_\gamma(s) = 1 - \frac{\alpha}{4\pi s_w^2} \left\{ \frac{1}{4} \left(U\left(\frac{s}{M_w^2}\right) - 1/2 \right) + \frac{v^2 + a^2}{c_w^2} \left(V\left(\frac{s}{M^2}\right) - \frac{1}{2} \right) + \right. \\
\left. + s_w^2 \left(3 + 4 \frac{M_w^2}{s} \right) \left[\left(4 \frac{M_w^2}{s} - 1 \right)^{1/2} \operatorname{tg}^{-1} \left[\frac{1}{\left(\frac{4 M_w^2}{s} - 1 \right)^{1/2}} \right] - 1 \right] + \frac{s_w^2}{3} \right\} \quad (A.15)
\end{aligned}$$

$$\begin{aligned}
B_Z(s) = a + \frac{\alpha}{4\pi s_w^2} \left\{ \frac{1}{4} \left[c_w^2 \left(U\left(\frac{s}{M_w^2}\right) - \frac{1}{2} \right) - \frac{1}{2} \left(V\left(\frac{s}{M_w^2}\right) - \frac{1}{2} \right) \right] - \right. \\
\left. - \frac{a(a^2 + 3v^2)}{c_w^2} \left(V\left(\frac{s}{M^2}\right) - \frac{1}{2} \right) \right\} \quad (A.16)
\end{aligned}$$

$$B_\gamma(s) = - \frac{\alpha}{4\pi s_w^2} \left\{ \frac{1}{4} \left(U\left(\frac{s}{M_w^2}\right) - \frac{1}{2} \right) + \frac{2va}{c_w^2} \left(V\left(\frac{s}{M_z^2}\right) - \frac{1}{2} \right) \right\} \quad (A.17)$$

Finally the two form factors $I_{ZZ}(s, q^2, M^2)$ and $I_1(s, q^2, M_w^2)$, coming from the weak box diagrams and appearing in eq.(A.5) are given in the Appendix of ref.(18).

The parameter $\mathfrak{J}$ related to the imaginary parts is given by

$$\mathfrak{J} = \mathfrak{J}_1 + \mathfrak{J}_2 + \mathfrak{J}^{\text{box}} (\gamma - Z) \quad (A.18)$$

$$\mathfrak{J}_1 = \frac{40\alpha}{9} \frac{v^2}{(v^2 + a^2)^2} \frac{\mu^2}{M^2} - \frac{4v}{(v^2 + a^2)^2} \frac{\mu^2}{M^2} \left[v \mathfrak{J}_m(\bar{A}_\gamma) + a \mathfrak{J}_m(\bar{B}_\gamma) \right] \quad (A.19)$$

$$\mathfrak{J}_2 = \frac{4v}{(v^2 + a^2)^2} \frac{\mu^2}{M^2} \mathfrak{J}_m(\bar{A}_Z) \quad (A.20)$$

$$\mathfrak{J}^{\text{box}} (\gamma - Z) = - \frac{36\alpha}{\pi} \frac{v^2 a^2}{(v^2 + a^2)^2} \left(\frac{\pi}{2} + \operatorname{tg}^{-1} \frac{s - M^2}{M\Gamma} \right) \quad s \geq M^2 \quad (A.21)$$

$$\mathfrak{J}^{\text{box}} (\gamma - Z) = 0 \quad s < M^2$$

Table 1

m_t (GeV)	m_H (GeV)	$\sin^2\theta_w$	Γ_e (MeV)	Γ_ν (MeV)	Γ_x ref.(14) (GeV)
60	100	0.2297 (0.2296)	85.8 (85.8)	170.5 (170.5)	2.562 2.559 BR
90	100	0.2258 (0.2258)	86.0 (86.0) 85.9 BR	170.8 (170.8) 170.7 BR	2.567 2.565 BR
90	1000	0.2286 (0.2289)	86.0 (85.7)	170.5 (170.5) 170.4 BR	2.559 2.557 BR
150	100	0.2187 (0.2189) 0.2187 BR	86.44 (86.4)	171.5 (171.5)	2.580 2.578 BR
200	100	0.2111 (0.2120) 0.2109 BR	86.95 (87.0) 86.9 BR	172.5 (172.5) 172.4 BR	2.596 2.594 BR
230	100	0.2056 (0.2070) 0.2052 BR	87.30 (87.4) 87.2 BR	173.1 (173.1) 173.2 BR	2.607 2.606 BR
230	1000	0.2084 (0.2100) 0.2086 BR	87.16 (87.2) 87.1 BR	172.9 (172.9) 172.8 BR	2.599 2.598 BR

Table 2

m_t (GeV)	m_H (GeV)	$\sigma_{\text{peak}}^0 + \sigma_{\text{oy}}$ (nb)	$\sqrt{s}_+$ (GeV)	$\sqrt{s}_-$ (GeV)	$\sqrt{s}_{\text{peak}}$ (GeV)	$R \cdot 10^2$	$\delta \cdot 10^2$
60	100	1.957 (1.950)	93.280 (93.280)	90.704 (90.704)	91.983 (91.982)	4.5	-1.0
90	100	1.958 (1.950)	93.283 (93.283)	90.702 (90.701)	91.983 (91.982)	4.8	-1.2
90	1000	1.957 (1.952)	93.279 (93.279)	90.706 (90.705)	91.983 (91.982)	4.4	-1.1
150	100	1.956 (1.952)	93.290 (93.289)	90.695 (90.695)	91.983 (91.982)	5.2	-1.6
200	100	1.959 (1.955)	93.298 (93.297)	90.688 (90.688)	91.983 (91.982)	5.7	-1.9
230	100	1.960 (1.958)	93.303 (93.303)	90.682 (90.682)	91.983 (91.982)	6.2	-2.2
230	1000	1.963 (1.959)	93.299 (93.299)	90.686 (90.687)	91.983 (91.982)	5.9	-2.1

Table 3

m_t (GeV)	m_H (GeV)	σ_{peak} (nb)	$\sqrt{s_+}$ (GeV)	$\sqrt{s_-}$ (GeV)	$\sqrt{s_{\text{peak}}}$ (GeV)
60	100	1.453 (1.452)	93.726 (93.725)	90.752 (90.753)	92.093 (92.094)
90	100	1.455 (1.453)	93.729 (93.728)	90.750 (90.751)	92.094 (92.094)
90	1000	1.453 (1.454)	93.724 (93.723)	90.754 (90.755)	92.094 (92.094)
150	100	1.454 (1.455)	93.738 (93.737)	90.744 (90.745)	92.094 (92.095)
200	100	1.457 (1.458)	93.749 (93.748)	90.736 (90.738)	92.094 (92.095)
230	100	1.458 (1.461)	93.756 (93.755)	90.731 (90.732)	92.095 (92.096)
230	1000	1.460 (1.461)	93.751 (93.750)	90.735 (90.736)	92.095 (92.095)

Table 4

m_t (GeV)	m_H (GeV)	σ_{peak} (nb)	$\sqrt{s_+}$ (GeV)	$\sqrt{s_-}$ (GeV)	$\sqrt{s_{\text{peak}}}$ (GeV)
60	100	1.451 (1.452)	93.723 (93.724)	90.755 (90.756)	92.094 (92.094)
90	100	1.452 (1.453)	93.726 (93.727)	90.752 (90.754)	92.094 (92.095)
90	1000	1.451 (1.452)	93.721 (93.722)	90.756 (90.758)	92.093 (92.094)
150	100	1.451 (1.453)	93.735 (93.736)	90.746 (90.748)	92.094 (92.095)
200	100	1.454 (1.456)	93.746 (93.747)	90.738 (90.740)	92.095 (92.096)
230	100	1.456 (1.457)	93.753 (93.754)	90.733 (90.735)	92.095 (92.096)
230	1000	1.458 (1.459)	93.748 (93.749)	90.737 (90.739)	92.095 (92.096)

Table 5

m_t (GeV)	m_H (GeV)	$\frac{\Delta\sigma}{\sigma_{\text{peak}}}$	Energy Range	N. of steps	M_Z (GeV)	Γ_Z (GeV)	$\Gamma_{e^+e^-}$ (MeV)	R
60	100	10^{-2}	90.5/93.5	16	91.996	2.565	86.0	$3.5 \cdot 10^{-1}$
			89.7/93.7	5	92.002	2.571	85.9	$4.1 \cdot 10^{-2}$
		10^{-3}	90.5/93.5	16	92.000	2.562	85.8	$7.3 \cdot 10^{-2}$
			89.7/93.7	5	92.000	2.563	85.8	$4.4 \cdot 10^{-2}$
230	1000	10^{-2}	90.5/93.5	16	91.995	2.605	87.4	$4.0 \cdot 10^{-1}$
			89.7/93.7	5	92.002	2.608	87.3	$5.2 \cdot 10^{-2}$
		10^{-3}	90.5/93.5	16	92.000	2.599	87.2	$9.5 \cdot 10^{-2}$
			89.7/93.7	5	92.000	2.600	87.2	$5.8 \cdot 10^{-2}$

Table 6

m_t (GeV)	m_H (GeV)	$R \cdot 10^2$	σ_{peak} (nb)	$\sqrt{s_+}$ (GeV)	$\sqrt{s_-}$ (GeV)	$\sqrt{s_{\text{peak}}}$ (GeV)
60	100	7.4	30.02	93.705	90.770	92.093
90	100	8.6	30.04	93.709	90.768	92.095
90	1000	7.4	30.00	93.703	90.772	92.094
150	100	9.9	30.05	93.718	90.762	92.094
200	100	11	30.09	93.729	90.755	92.096
230	100	12	30.12	93.737	90.750	92.096
230	1000	11	30.12	93.731	90.754	92.096

References

- 1) S.L.Glashow, Nucl.Phys. **22** (1961) 579;
S.Weinberg, Phys.Rev.Lett. **19** (1967) 1264;
A.Salam, in Proc. 8th Nobel Symp., N.Svartholm ed. (Almqvist and Wiksell, Stockholm, 1968) p.367;
S.L.Glashow, J.Iliopoulos and L.Maiani, Phys.Rev. **D2** (1970) 1285;
G.'t Hooft, Nucl.Phys. **B33** (1971) 173; **B35** (1971) 167;
C.Bouchiat, J.Iliopoulos and Ph.Meyer, Phys.Lett. **B38** (1972) 519.
- 2) G.Passarino and M.Veltman, Nucl.Phys. **B160** (1979) 151.
- 3) M.Greco, G.Pancheri and Y.Srivastava, Nucl.Phys. **B171** (1980); E: **B197** (1982) 543.
- 4) F.A.Berends, R.Kleiss and S.Jadach, Nucl.Phys. **B202** (1982) 63.
- 5) M.Böhm and W.Hollik, Nucl.Phys. **B204** (1982) 45; Z.Phys. **C23** (1984) 31;
M.Böhm and W.Hollik, Phys.Lett. **139B** (1984) 213.
- 6) W.Wetzel, Nucl.Phys. **B227** (1982) 1.
- 7) R.W.Brown, R.Decker and E.A.Paschos, Phys.Rev.Lett. **52** (1984) 1192.
- 8) B.W.Lynn and R.G.Stuart, Nucl.Phys. **B253** (1985) 216.
- 9) D.Yu.Bardin, A.Leike, T.Riemann and M.Sachwitz, Phys.Lett. **B206** (1988) 539.
- 10) E.A.Kuraev and V.S.Fadin, Sov.J.Nucl.Phys. **41** (1985) 446.
- 11) G.Altarelli and G.Martinelli, in "Physics at LEP", eds. J.Ellis and R.Peccei, CERN Yellow Report 86 (02), 1986, pag.47.
- 12) F.A.Berends, G.Burgers and W.L.van Neerven, Phys.Lett. **B185** (1987) 395;
Nucl.Phys. **B247** (1988) 429.
- 13) O.Nicrosini and L.Trentadue, Phys.Lett. **B196** (1987) 551.
- 14) G.Burgers, in "Polarization at LEP", eds. G.Alexander et al., CERN Yellow Report 88-06, 1988.
- 15) R.N.Cahn, Phys.Rev. **D36** (1987) 2666.
- 16) M.Consoli, S.Lo Presti and L.Maiani, Nucl.Phys. **B223** (1983) 474.
- 17) A.Sirlin, Phys.Rev. **D22** (1980) 971.
- 16) M.Consoli and A.Sirlin, in "Physics at LEP", J.Ellis and R.Peccei eds., CERN Yellow Report 86 (02), 1986, pag.63.
- 19) F.A.Berends et al, private communication.
- 20) F.A.Berends, G.Burgers, W.Hollik and W.L.van Neerven, Phys.Lett. **B203** (1988) 177.

Table captions

- Tab.1 The value of $\sin^2\theta_w$, of the partial leptonic widths and of the total width at one loop level, as functions of m_t and m_H . Values in parenthesis and total widths are taken from ref.(14).
BR: Bardin and Riemann, ref.(19).
- Tab.2 Leptonic cross-section, without initial state bremsstrahlung: peak value, peak and semimaxima positions. In the last two columns, we report the values of R and $\mathcal{J}$, determined by m_t and m_H .
- Tab.3 Observable leptonic cross-section: peak value, peak and semimaxima positions, all evaluated numerically with $\sqrt{s_0}=2m_\mu$. Values in parenthesis are taken from ref.(14), R and $\mathcal{J}$ are the same as in Table 2.
- Tab.4 Observable leptonic cross-section, with $\sqrt{s_0}=1$ GeV. Values in parenthesis have been obtained by approximate analytical integration.
- Tab.5 Comparison between the best-fit values of M , Γ , Γ_e and R obtained for two different energy ranges and number of points. The input values are determined by m_t and m_H via Tables 1 and 2. $\Delta\sigma/\sigma_{\text{peak}}$ is the value of the Gaussian error included in the data points.
- Tab.6 Observable hadronic cross-section evaluated with $\sqrt{s_0}=1$ GeV. We have taken $\mathcal{J}^{(h)}=0$ and the values of $R^{(h)}$ reported in the Table and obtained from eq.(7.1) of text.

Figure captions

- Fig.1 Diagrammatic expansion of $\Pi(s)$ defined in Sect.2. The blobs denote one-particle irreducible diagrams, with respect to both Z and γ lines. The imaginary parts of two-loop diagrams are needed to reproduce the corrections of order α^3 to the Z physical widths.
- Fig.2 R as a function of m_t , ($m_H=100$ GeV). The one-loop corrected $\sin^2\theta_w$ has been used also in higher order terms, curve (a), or only in the lowest order ones, curve (b).

fig.1

fig. 2