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Superstring models from the second realization of Yau's three generation manifold

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Abstract

We classify the models obtained upon compactification of the $E_8 \times E_8$ heterotic superstring on the second realization of Yau's three generation manifold according to their discrete symmetries and study the most symmetric case. We find that the presence of R-symmetries, in spite of the absence of acceptable matter parities, leads to considerable improvement, compared with most models based on the first realization of the three generation manifold, as far as the proton decay problem is concerned.

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The much celebrated uniqueness of the $E_8 \times E_8$ heterotic superstring [1] is lost once the ten dimensional theory is compactified in order to yield the macroscopically well known four dimensional Minkowski space-time. Due to the lack of a dynamical procedure for choosing the vacuum state phenomenological restrictions can prove useful in reducing the number of choices. The requirement that the number of light chiral fermion generations be equal to three, applied to the Calabi-Yau (C-Y) compactification [2], has led to only one multiply connected candidate [3] in the large class of complete intersection C-Y manifolds modded out by projectively inherited freely acting discrete groups. This is the space originally constructed by Yau [4] and extensively studied by many authors. It is defined by the complete intersection in $CP^3 \times CP^3$ of bidegree (3,0), (0,3) and (1,1) homogeneous polynomials, modded out by a freely acting Z_3 discrete symmetry group.

In addition to this smooth three generation C-Y manifold there are another two multiply connected such spaces whose construction involves division of a simply connected space by a non-freely acting discrete symmetry group and subsequent resolution of the resulting singularities. The one of these constructions is again due to Yau [4] and the second to Schimmrigk [5]. Both these manifolds were subsequently shown to be diffeomorphic to the original one [6]. This proof of ‘equivalence’ combined with the technical difficulties associated with the more complicated construction hampered extensive phenomenological studies of these alternatives to the original three generation space.

Very recently the resolution of the quotient singularities and the computation of the properties of the resulting blow up modes became possible for both the second realization of Yau’s manifold [7] and the Schimmrigk space [8]. The latter has an exactly soluble minimal superconformal model counterpart for a particular value of the moduli [9]. A comparison of the corresponding spectra provided some

confidence concerning the singularity resolving techniques employed.

The purpose of the present paper is to study in some detail the phenomenology of the second realization of Yau's three generation manifold. This is by no means a pointless exercise. Although diffeomorphic to the first, the second realization almost certainly leads to inequivalent physics since the discrete symmetries of the two realizations are different. [The models built on the second realization differ from the ones built on the first by a change of the complex structure [6].]

The second realization of Yau's three generation manifold K is defined as a bidegree (3,3) hypersurface K_0 in $CP^2 \times CP^2$ modded out by a non-freely acting order 27 symmetry group G_{27} with the resulting fixed points resolved. The bicubic K_0 has the form

$$M_1(\mathbf{x})M_1(\mathbf{y}) + (c_1 + c_2)M_2(\mathbf{x})M_3(\mathbf{y}) + (c_1 - c_2)M_3(\mathbf{x})M_2(\mathbf{y}) + \\ + c_3M_{10}(\mathbf{x})M_{10}(\mathbf{y}) + c_4M_1(\mathbf{x})M_{10}(\mathbf{y}) + c_5M_{10}(\mathbf{x})M_1(\mathbf{y}) = 0$$

for almost any choice of the complex parameters c_1, c_2, c_3, c_4, c_5 . [The polynomials $M_i(\mathbf{z})$ with $i = 1, 2, \dots, 10$ and $\mathbf{z} = (z_0, z_1, z_2)$ are defined in table 1.] This is the most general bicubic invariant under the group G_{27} generated by:

$$\sigma_1: (x_0, x_1, x_2) \times (y_0, y_1, y_2) \rightarrow (x_1, x_2, x_0) \times (y_1, y_2, y_0), \\ \sigma_2: (x_0, x_1, x_2) \times (y_0, y_1, y_2) \rightarrow (x_0, \alpha x_1, \alpha^2 x_2) \times (y_0, \alpha y_1, \alpha^2 y_2), \\ \sigma_3: (x_0, x_1, x_2) \times (y_0, y_1, y_2) \rightarrow (x_0, x_1, x_2) \times (y_0, \alpha y_1, \alpha^2 y_2),$$

with $\mathbf{x} \times \mathbf{y}$ the homogeneous coordinates of $CP^2 \times CP^2$ and $\alpha = \exp(2\pi i/3)$.

The simply connected space K_0 has $h^{2,1} = 83$ and $h^{1,1} = 2$. The (2,1) harmonic forms correspond to the 83 independent deformations of the complex structure represented by the 83 polynomials of the form $M_i(\mathbf{x})M_j(\mathbf{y})$ with $i, j \in \{2, 3, \dots, 10\}$, $M_1(\mathbf{x})M_{10}(\mathbf{y})$ and $M_{10}(\mathbf{x})M_1(\mathbf{y})$. The (1,1) harmonic forms correspond to the

Kaehler forms of the two CP^2 spaces. Modding out by G_{27} (and subsequent resolution of the fixed point singularities) leads to a space whose cohomology can be split into a sum of that derived from the non-singular part of the space with that derived from blowing up the singularities. The first contributes the 2 (1,1) harmonic forms of K_0 and 5 (2,1) harmonic forms corresponding to the 5 deformations represented by the polynomials $M_2(\mathbf{x})M_3(\mathbf{y})$, $M_3(\mathbf{x})M_2(\mathbf{y})$, $M_{10}(\mathbf{x})M_{10}(\mathbf{y})$, $M_1(\mathbf{x})M_{10}(\mathbf{y})$ and $M_{10}(\mathbf{x})M_1(\mathbf{y})$. Since the second realization of Yau's space is diffeomorphic to the first, for which $h^{2,1} = 9$ and $h^{1,1} = 6$, there should be a contribution of 4 to both $h^{2,1}$ and $h^{1,1}$ from the blow up modes. The fundamental group $\pi_1(K) = Z_3$ is associated with the freely acting symmetry generated by σ_1 .

Flux breaking [2] of the gauge symmetry E_6 depends on the choice of an embedding of σ_1 into E_6 defined by a homomorphism $g \rightarrow U_g$. The unbroken gauge group is the subgroup H of E_6 commuting with the image of σ_1 under U_g . As far as flux breaking is concerned there is a complete analogy to the first realization of Yau's manifold [10, 11]. The possible proper subgroups H are $SU(3)^3$ and $SU(6) \times U(1)$. The 'standard' embedding, which we will adopt in the following, employs the homomorphism $g \rightarrow U_g \equiv (I_3, \alpha I_3, \alpha I_3) \in SU(3)_c \times SU(3)_L \times SU(3)_R$ and $H \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$. Under H the left-handed lepton (λ), quark (Q) and antiquark (Q^c) fields in a 27 of E_6 transform as follows: $\lambda = (1, \bar{3}, 3) = \begin{pmatrix} H^1 & H^2 & l \\ e^c & \nu^c & N \end{pmatrix}$, $Q = (3, 3, 1) = \begin{pmatrix} q \\ g \end{pmatrix}$, $Q^c = (\bar{3}, 1, \bar{3}) = (u^c, d^c, g^c)$. There are 9 λ lepton fields, 7 Q (Q^c) quark (antiquark) fields, 6 $\bar{\lambda}$ mirror lepton fields and 4 $\bar{Q}$ ($\bar{Q}^c$) mirror quark (antiquark) fields. Four of the fields in each category are blow up modes.

The maximal symmetry of K_0 [7] is generated by $\sigma_1, \sigma_2, \sigma_3$ and

$$P: (x_0, x_1, x_2) \times (y_0, y_1, y_2) \rightarrow (x_1, x_0, x_2) \times (y_1, y_0, y_2),$$

$$S: x_i \rightarrow y_i, y_i \rightarrow x_i \quad (i = 0, 1, 2),$$

$$G: x_0 \rightarrow \alpha x_0.$$

On modding out by G_{27} the discrete symmetries d surviving on K , before flux breaking (i.e. assuming $U_g \equiv I$), are the ones satisfying the relation $d^{-1}gd = g'$, with $g, g' \in G_{27}$. G satisfies this relation with $g' = g$ for $g = \sigma_2$ or $g = \sigma_3$ and with $g' = \sigma_1\sigma_2\sigma_3^2$ for $g = \sigma_1$. For the permutation symmetries P and S we have $P\sigma_iP = \sigma_i^2$ ($i = 0, 1, 2$), $S\sigma_1S = \sigma_1$, $S\sigma_2S = \sigma_2$ and $S\sigma_3S = \sigma_3^2\sigma_2$. After flux breaking (i.e. $U_g \neq I$) the scaling symmetry G and the permutation symmetry S (for which $g' = g''g$ for elements $g'' \in G_{27}$ mapped to the identity under U_g) survive as honest symmetries on K for all possible homomorphisms U_g of σ_1 into E_6 . The survival of P depends on the existence of an E_6 element J with $J^2 = 1$ satisfying $JU_{\sigma_1}J = U_{\sigma_1^2}$. Such an element J exists only for the ‘standard’ embedding, leading to an unbroken $SU(3)^3$ gauge symmetry, and for the three embeddings characterized by the breaking directions $[00aa00]$, $[-aaa0a0]$ and $[-2a2aa2a2a0]$ (in the dual basis) in E_6 root space (a is identified with the generator of $\pi_1(K)$), leading to an unbroken $SU(6) \times U(1)$ gauge group [10, 11]. For the rest of the possible embeddings P is a pseudosymmetry.

The classification of the possible models according to their discrete symmetries for the embeddings of σ_1 into E_6 for which P survives as a honest symmetry is given in table 2. For the rest of the embeddings it is given in table 3. In these tables the reader can find the non-zero parameters c_i in the bicubic K_0 defining the complex structure, the generators of the discrete symmetry group and its order.

Table 4 contains the transformation properties of the ‘massless’ (compared to the compactification scale M_c) non gauge singlet superfields under the discrete symmetry generators P , S and G for the ‘standard’ embedding. For the rest of the embeddings it is trivial to obtain the corresponding tables from table 4. The last four fields of each type are blow up modes.

It is important to notice that the scaling symmetry G is an R-symmetry [7] (it does not leave invariant the harmonic (3,0) form ω and the covariantly constant spinor of the C-Y space and therefore does not commute with supersymmetry). The restrictions imposed by an R-symmetry on the superpotential is a somewhat subtle issue [12]. Let $27 \rightarrow \beta 27$ and $\bar{27} \rightarrow \beta' \bar{27}$ be the ‘naive’ (i.e. obtained without realizing that the discrete symmetry is an R-symmetry) transformation property of (a component of) a 27 and a $\bar{27}$ respectively under the R-symmetry (table 4 gives the ‘naive’ transformations). Also assume that under the R-symmetry $\omega \rightarrow \gamma^{-1} \omega$. Let us now redefine the R-symmetry such that $27 \rightarrow \gamma^{-1/3} \beta 27$ and $\bar{27} \rightarrow \gamma^{1/3} \beta' \bar{27}$. Gauge singlets need no such redefinition. The allowed superpotential terms are the ones transforming like γ under the so redefined R-symmetry. For the R-symmetry G under discussion $\gamma = \alpha^2$.

An important phenomenological question is the existence of matter parities [13] which solve the rapid proton decay problem in the case in which the gauge group is broken, in one step, down to the standard group at an intermediate scale. It is easy to see (using table 4) that there is no such a satisfactory matter parity. The Z_3 symmetry generated by GSG^2S (appropriately combined with a Z_3 subgroup of the gauge symmetry) could be regarded as a matter parity but it does not allow for an intermediate vacuum expectation value (vev) in both N and ν^c directions. If we, nevertheless, follow this matter parity scenario we find an unacceptable spectrum at low energies (for each light chiral quark there is another vector-like one which is degenerate in mass with it).

Here we concentrate on the phenomenology of the most symmetric model on space K with gauge symmetry group $SU(3)^3$, namely the model (1) in table 2 with discrete symmetry the 36 element group generated by P , S and G . There is a good reason for this choice. In ref. [7] has been pointed out that due to the combined

effect of the R-symmetries generated by G and SGS there exist (modulo some uncertainties concerning the unknown behavior of possible blow up gauge singlets) exact flat directions for the standard gauge group singlets contained in λ_1 and λ_2 (together with anyone of the mirror lepton fields) which could break E_6 down to $SO(10)$ or even $SU(5)$ at M_c . Since this is a very interesting novel possibility it is worth exploring its consequences. The only models in table 2 possessing both of these R-symmetries are model (1) and (2). This already points towards the most symmetric models. We found the presence of the permutation symmetry P in model (1) very useful in connection with the gauge hierarchy problem and light fermion masses.

The absence of acceptable matter parities immediately eliminates the possibility of breaking E_6 down to $SU(5)$ or, taking into account flux breaking, to the standard group through large vevs in both N and ν^c directions at M_c . We will therefore assume that the vev in, say, the ν^c direction is at most of the order of the supersymmetry breaking scale M_s ($\sim 10^4 \text{ GeV}$) in the visible sector. We also found favorable to keep the dominant vev in the N direction slightly below M_c .

We will choose a large vev $\sim 10^{-1} M_c$ for N_2 and for one combination, which we will call $\overline{N}_4'$, out of the set $\{\overline{N}_4, \overline{N}_6\}$, namely the combination appearing at most linearly in a coupling of the form $\overline{N}\overline{N}N_8N_6$. The orthogonal combination will be called $\overline{N}_6'$. It is easy to see that, without any intermediate vevs for N 's transforming non-trivially under the R-symmetries G and SGS , very few states acquire any mass at all. To remedy this clearly unacceptable situation we choose an intermediate vev $\sim 10^{14} \text{ GeV}$ for one combination of the form $N_4 + b_1 N_8 + b_2 N_5$ (with $b_1 \sim 10^{-2.5}$ and $b_2 \sim 10^{-7}$) and $\overline{N}_2'$. [$\overline{N}_2'$ is the combination of $\overline{N}_2$ and $\overline{N}_5$ which does not have a coupling of the form $\overline{N}\overline{N}_4'N_8N_7$. The orthogonal combination will be called $\overline{N}_5'$.] This choice was made after a careful examination of the

gauge hierarchy problem, the proton lifetime and the light fermion spectrum. To obtain the vevs mentioned above some higher order superpotential couplings must be small. For instance (assuming $M_c \sim 10^{17}\text{GeV}$) the couplings of the terms $\overline{N}_2'^2 N_8 N_6$, $\overline{N}_2' \overline{N}_4'^2 N_8 N_4^2$, $\overline{N}_4'^3 N_9 N_4^2$ and $\overline{N}_2' \overline{N}_4'^3 N_8 N_4^2 N_2$ must be of the order of $10^{-4.5}$, 10^{-5} , 10^{-7} and 10^{-3} respectively. Notice that, in the absence of any other vev, the superlarge one ($\sim 10^{-1} M_c$) does not need any arrangements.

Before we embark on the evaluation of the physical spectrum we will quote some useful mathematical facts concerning mass matrices. The mass matrices M we deal with are, in general, non-hermitian ones which are diagonalizable through left and right multiplication with two different unitary matrices. The physical masses are not their eigenvalues but are equal, in absolute value, to the square root of the eigenvalues of the hermitian matrix $MM^\dagger$. In ref. [11] we proved that for an arbitrary square matrix M of order n , the sum of the products of the eigenvalues of $MM^\dagger$ in groups of $l \leq n$ is equal to the sum of the squares of the absolute values of all order l minor determinants of M . An immediate consequence of this proposition is that the order of magnitude of the product of the l largest physical masses associated with a general mass matrix M is equal to the order of magnitude of the largest order l minor determinant of M .

In the following we will work with fermion mass matrices since we are not interested in fermion-boson mass splittings of order M_c or smaller.

The mass matrix of the $SU(2)_L$ and $SU(2)_R$ doublet leptons is a 15×15 matrix with rows labelled by the 9 H^2 's and the 6 $\overline{H}^1$'s and columns labelled by the 9 H^1 's and the 6 $\overline{H}^2$'s. These are the states (actually their supersymmetric partners) with the correct quantum numbers to play the role of the electroweak Higgs doublets. $\overline{H}^1_4$ ($\overline{H}^2_4$) and $\overline{H}^1_6$ ($\overline{H}^2_6$) pair up mainly with 2 combinations out of the set $\{ \overline{H}^2_1, \overline{H}^2_2, \overline{H}^2_3, \overline{H}^2_5 \}$ ($\{ \overline{H}^1_1, \overline{H}^1_2, \overline{H}^1_3, \overline{H}^1_5 \}$) to form 2 supermassive states with mass

$\langle \bar{N}_4' \rangle \sim 10^{-1} M_c$. The remaining 2 combinations out of the set $\{ \bar{H}^2_1, \bar{H}^2_2, \bar{H}^2_3, \bar{H}^2_5 \}$ pair up with the remaining 2 combinations out of the set $\{ \bar{H}^1_1, \bar{H}^1_2, \bar{H}^1_3, \bar{H}^1_5 \}$ to make 2 states with mass $\sim \langle \bar{N}_2' \rangle$. H^2_2 and H^2_3 pair up mainly with H^1_3 and H^1_2 to make 2 superheavy states with mass $\sim \langle N_2 \rangle \sim 10^{-1} M_c$. H^2_1 (H^2_5) pairs up mainly with H^1_5 (H^1_1) to form a state with mass $\sim \langle N_4 \rangle$. H^2_6 (H^2_7) pairs up mainly with H^1_6 (H^1_7) to form a state with mass $\sim \langle N_5 \rangle$ and H^2_8 (H^2_9) pairs up mainly with H^1_8 (H^1_9) to form a state with mass $\sim \langle N_4 \rangle$. Finally H^2_4 pairs up mainly with H^1_4 into a state with mass $\sim \langle N_8 \rangle^2 / \langle N_4 \rangle$. This is the typical physical mass spectrum with coupling constants of order unity. Clearly, this does not have to be the case. Assuming that the coupling constant of the superpotential term $\lambda_1 \lambda_8 \lambda_8 + \lambda_5 \lambda_6 \lambda_6$ is $\sim 10^{-5}$ we get two pairs (H^1_4, H^1_6 and H^2_4, H^2_6) having the quantum numbers of electroweak Higgs doublets with masses $\sim M_*$ or smaller. We see that, in spite of the existence of vevs as large as $10^{-1} M_c$, we do not need couplings as small as $10 M_*/M_c$ in order to obtain acceptably light electroweak Higgs doublets. Clearly, this should be regarded as a positive feature.

The mass matrix of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons is a 6×9 matrix whose rows are labelled by the 6 $\bar{l}$'s and its columns by the 9 l 's. $\bar{l}_2'$ and $\bar{l}_6'$ pair up mainly with l_8 and l_6 into states with mass $\sim 10^{-5} \langle \bar{N}_4' \rangle^2 \langle N_4 \rangle^2 M_c^{-3}$ and $\langle \bar{N}_4' \rangle \langle N_8 \rangle M_c^{-1}$ respectively. $\bar{l}_1'$ pairs up with one combination out of the set $\{l_1, l_9\}$ to form a state with mass $\sim 10^2 - 10^3 \text{ GeV}$. Finally $\bar{l}_1, \bar{l}_3$ and $\bar{l}_5'$ pair up with 3 combinations out of the set $\{l_2, l_3, l_4, l_8\}$ to form 3 states with mass $\sim 10^2 - 10^4 \text{ GeV}$. We will assume that the light combination from this set is mainly l_3 with an order 10^{-2} contribution from l_4 and suppressed mixing with l_2 . The other 2 light leptons are l_5 and one combination out of the set $\{l_1, l_9\}$.

The mass matrix of the $SU(2)_L$ singlet, $SU(2)_R$ doublet leptons (ν^c, e^c) is

identical to the one just discussed.

We now turn to the quark sectors. The $SU(2)_L$ and $SU(2)_R$ singlet quark mass matrix has rows labelled by the 7 g 's and the 4 $\bar{g}^c$'s and columns labelled by the 7 g^c 's and the 4 $\bar{g}$'s. The $\bar{g}^c$'s pair up with the $\bar{g}$'s into 4 states of mass $\sim \langle \bar{N}_4' \rangle \sim 10^{-1} M_c$. Also g_1 and g_2 pair up mainly with g^c_2 and g^c_1 to form 2 states of mass $\sim \langle N_2 \rangle \sim 10^{-1} M_c$. We will assume the coupling $g_5 g^c_5 \lambda_4$ to be strong (will be used later to create third generation quark masses) and the couplings $g_i g^c_j \lambda_4$, $i, j \in \{5, 7\}$ with $i = j = 5$ excluded, to be weak. Then, g_5 pairs up mainly with g^c_5 into a state with mass $\sim \langle N_1 \rangle \sim 10^{11} \text{GeV}$ and g_3, g_7 pair up mainly with g^c_7, g^c_3 into 2 states of mass $\langle \bar{N}_4' \rangle \langle N_8 \rangle \langle N_4 \rangle M_c^{-2} \sim 10^7 - 10^8 \text{GeV}$. Finally g_4 and g_6 pair up with g^c_4 and g^c_6 to form 2 states with mass of the order of $\langle N_5 \rangle \sim 10^7 \text{GeV}$ and $\langle \bar{N}_4' \rangle \langle N_8 \rangle^2 M_c^{-2} \sim 10^5 \text{GeV}$ respectively.

In the $SU(2)_L$ doublet, $SU(2)_R$ singlet quark sector 2 out of the 4 $\bar{q}$'s pair up with q_4 and q_6 to form 2 states of mass $\langle \bar{N}_4' \rangle \langle N_8 \rangle M_c^{-1} \sim 3 \times 10^{10} \text{GeV}$ and the other 2 $\bar{q}$'s pair up mainly with 2 combinations out of the set $\{q_1, q_5, q_7\}$. We will assume that (through small couplings of some sixth order superpotential terms) the light quark out of this set is q_5 (which will later be identified with the third generation). The mass of q_1 is $\sim \langle \bar{N}_4' \rangle^2 \langle N_4 \rangle \langle N_8 \rangle M_c^{-3} \sim 10^7 \text{GeV}$ and of q_7 is $\sim \langle \bar{N}_4' \rangle^2 \langle N_4 \rangle^2 M_c^{-3} \sim 10^9 \text{GeV}$. Therefore the 3 light quarks (corresponding to the ordinary families) are q_2, q_3 and q_5 .

The mass matrix of the $SU(2)_L$ singlet, $SU(2)_R$ doublet antiquarks (u^c, d^c) is identical to the one just discussed.

To summarize, the light leptons are contained in λ_3 (with an 10^{-2} admixture with λ_4), λ_5 and one combination out of $\{\lambda_1, \lambda_9\}$. The light quarks (antiquarks) are contained in Q_2, Q_3 and Q_5 (Q^c_2, Q^c_3 and Q^c_5). The candidate electroweak Higgs doublets are H^1_1, H^2_4, H^1_6 and H^2_6 .

With the above light spectrum we can readily check that there exist Yukawa couplings leading to tree level masses for at least the third generation of quarks and leptons. For the third generation quark masses we make use of the term $Q_5 Q_5^c \lambda_4$, while for the third generation lepton masses of the term $\lambda_3 \lambda_4 \lambda_6$. Therefore the $t(b)$ quark is contained in Q_5 and the τ lepton in the light combination of λ_3 and λ_4 . Notice that the electroweak Higgs doublets giving masses to quarks and leptons are different. This is very satisfactory since otherwise (taking into account the $SU(2)_R$ symmetry of the light spectrum) Dirac neutrino masses would be of the same order of magnitude as up quark masses. In this event we would have to invent some sort of a see-saw mechanism involving the enormous (and to some extent unknown) gauge singlet sector.

The most dangerous proton decay processes involve the exchange of a g (g^c) quark (antiquark) between vertices corresponding to superpotential couplings qqg ($u^c d^c g^c$) and $gu^c e^c$ ($g^c q l$). The R-symmetries G and SGS severely restrict the allowed baryon number violating couplings. The only allowed such couplings are the following three: $f_1(Q_1 Q_2 Q_3 + Q_1^c Q_2^c Q_3^c)$, $f_2(Q_1 Q_5 Q_5 + Q_1^c Q_5^c Q_5^c + Q_2 Q_4 Q_4 + Q_2^c Q_4^c Q_4^c)$ and $f_3(Q_1 Q_5 Q_7 + Q_1^c Q_5^c Q_7^c + Q_2 Q_4 Q_6 + Q_2^c Q_4^c Q_6^c)$. The least suppressed process involves the exchange of g_1^c between the vertices $u_2^c d_3^c g_1^c$ and $g_1^c q_2 l_2$ (or $g_1^c q_3 l_5$). Its amplitude has a suppression by one power of the large scale $10^{-1} M_c \sim 10^{16} \text{ GeV}$ (the mass of g_1^c), one power of $M_s \sim 10^4 \text{ GeV}$ and a loop factor. The rest of the suppression has to be provided by the coupling constants associated with two vertices. [The large coupling of the term $Q_1^c Q_2 \lambda_2$ is compensated by the fact that l_2 has a suppressed contribution to a light lepton state.] With a value of $\sim 10^{-5}$ for these couplings, the proton decay rate is acceptably small. For other processes occurring through an exchange of g_1 or g_1^c the baryon number violating vertex involved has no light (i.e. first or second gener-

ation) quarks. The exchange of g_2, g_2^c, g_5, g_5^c is associated with baryon number violating vertices involving at least another heavy quark (antiquark). The danger from processes occurring through an exchange of the relatively light g_3, g_3^c, g_7, g_7^c is only apparent since the lightness of the exchanged state is compensated by the additional suppression provided by the heavy Q_1, Q_1^c . For g_7 and g_7^c there is even more suppression from the presence of a Q_5 or Q_5^c , both of which contain only third generation quarks. Finally, the exchange of the light (mass $\sim 10^7 \text{GeV}$) g_4, g_4^c or the even lighter (mass $\sim 10^5 \text{GeV}$) g_6, g_6^c is compensated by the presence of one additional heavy state in both the baryon and lepton number violating vertices involved. We see that the most severely constrained baryon number violating coupling is the first among the ones listed above. The bounds on f_2 and f_3 are certainly much more loose. The situation concerning proton decay is clearly better than in most other three generation C-Y superstring models.

We conclude by summarizing our results. We studied the second realization of Yau's three generation manifold obtained by modding out a simply connected C-Y space by a non-freely acting discrete group and subsequent resolution of the fixed point singularities. We classified the possible models according to their discrete symmetries and wrote down the transformation properties of the light fields under these symmetries. We then constructed a specific model based on the maximal discrete symmetry group of this C-Y space. We found that, in spite of the absence of symmetries acting as matter parities, the presence of R-symmetries resulted in a clear improvement concerning the proton decay problem.

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$M_1(\mathbf{z}) = z_0^3 + z_1^3 + z_2^3$	$M_6(\mathbf{z}) = z_0^2 z_1 + \alpha z_1^2 z_2 + \alpha^2 z_2^2 z_0$
$M_2(\mathbf{z}) = z_0^3 + \alpha^2 z_1^3 + \alpha z_2^3$	$M_7(\mathbf{z}) = z_0^2 z_2 + z_1^2 z_0 + z_2^2 z_1$
$M_3(\mathbf{z}) = z_0^3 + \alpha z_1^3 + \alpha^2 z_2^3$	$M_8(\mathbf{z}) = z_0^2 z_2 + \alpha^2 z_1^2 z_0 + \alpha z_2^2 z_1$
$M_4(\mathbf{z}) = z_0^2 z_1 + z_1^2 z_2 + z_2^2 z_0$	$M_9(\mathbf{z}) = z_0^2 z_2 + \alpha z_1^2 z_0 + \alpha^2 z_2^2 z_1$
$M_5(\mathbf{z}) = z_0^2 z_1 + \alpha^2 z_1^2 z_2 + \alpha z_2^2 z_0$	$M_{10}(\mathbf{z}) = z_0 z_1 z_2$

Table 1: Definition of the polynomials $M_i(\mathbf{z})$, $i = 1, 2, \dots, 10$.

Model	Non-zero parameters	Symmetry generators	Order
(1)	c_1	G, P, S	36
(2)	c_1, c_2	G, PS	18
(3)	c_1, c_3	GSG^2S, P, S	12
(4)	c_1, c_4	G, P	6
(5)	c_1, c_5	SGS, P	6
(6)	c_1, c_2, c_3	GSG^2S, PS	6
(7)	c_1, c_2, c_4	G	3
(8)	c_1, c_2, c_5	SGS	3
(9)	$c_1, c_3, c_4 = c_5$	P, S	4
(10)	c_1, c_3, c_4, c_5	P	2
(11)	$c_1, c_2, c_3, c_4 = c_5$	PS	2
(12)	c_1, c_2, c_3, c_4, c_5		1

Table 2: Classification of models according to their discrete symmetries for flux breakings leading to an unbroken P symmetry generator.

Model	Non-zero parameters	Symmetry generators	Order	Pseudosymmetry generators
(1)	c_1	G, S	18	P
(2)	c_1, c_2	G, SGS	9	PS
(3)	c_1, c_3	GSG^2S, S	6	P
(4)	c_1, c_2, c_3	GSG^2S	3	PS
(5)	c_1, c_2, c_4	G	3	
(6)	c_1, c_2, c_5	SGS	3	
(7)	$c_1, c_3, c_4 = c_5$	S	2	P
(8)	c_1, c_2, c_3, c_4, c_5		1	

Table 3: Classification of models according to their discrete symmetries for flux breakings leading to a broken P symmetry generator.

	G	P	S		G	P	S
λ_1	1	1	λ_1	$\bar{\lambda}_1$	1	1	$\bar{\lambda}_1$
λ_2	1	-1	$-\lambda_2$	$\bar{\lambda}_2$	1	1	$-\bar{\lambda}_2$
λ_3	α	1	λ_3	$\bar{\lambda}_3$	1	1	$\bar{\lambda}_3$
λ_4	1	1	λ_5	$\bar{\lambda}_4$	1	-1	$\bar{\lambda}_1$
λ_5	α	1	λ_4	$\bar{\lambda}_5$	1	1	$-\bar{\lambda}_5$
λ_6	1	1	λ_8	$\bar{\lambda}_6$	1	-1	$-\bar{\lambda}_6$
λ_7	1	-1	λ_9				
λ_8	α^2	1	λ_6				
λ_9	α^2	-1	λ_7				
Q_1	1	Q_1^c	Q_2	$\bar{Q}_1$	1	$\bar{Q}_1^c$	$\bar{Q}_1$
Q_2	α	Q_2^c	Q_1	$\bar{Q}_2$	1	$\bar{Q}_2^c$	$-\bar{Q}_2$
Q_3	1	Q_3^c	Q_3	$\bar{Q}_3$	1	$\bar{Q}_3^c$	$\bar{Q}_3$
Q_4	1	Q_4^c	Q_5	$\bar{Q}_4$	1	$\bar{Q}_4^c$	$-\bar{Q}_4$
Q_5	α^2	Q_5^c	Q_4				
Q_6	1	Q_6^c	Q_7				
Q_7	α^2	Q_7^c	Q_6				
Q_1^c	1	Q_1	Q_2^c	$\bar{Q}_1^c$	1	$\bar{Q}_1$	$\bar{Q}_1^c$
Q_2^c	α	Q_2	Q_1^c	$\bar{Q}_2^c$	1	$\bar{Q}_2$	$-\bar{Q}_2^c$
Q_3^c	1	Q_3	Q_3^c	$\bar{Q}_3^c$	1	$\bar{Q}_3$	$\bar{Q}_3^c$
Q_4^c	1	Q_4	Q_5^c	$\bar{Q}_4^c$	1	$\bar{Q}_4$	$-\bar{Q}_4^c$
Q_5^c	α^2	Q_5	Q_4^c				
Q_6^c	1	Q_6	Q_7^c				
Q_7^c	α^2	Q_7	Q_6^c				

Table 4: Transformation properties of the ‘massless’ superfields under the symmetries generated by G , P , and S .