Superstring models from the second realization of Yau's three generation manifold

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Abstract

We classify the models obtained upon compactification of the $E_8 \times E_8$ heterotic superstring on the second realization of Yau's three generation manifold according to their discrete symmetries and study the most symmetric case. We find that the presence of $R$-symmetries, in spite of the absence of acceptable matter parities, leads to considerable improvement, compared with most models based on the first realization of the three generation manifold, as far as the proton decay problem is concerned.

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The much celebrated uniqueness of the $E_8 \times E_8$ heterotic superstring [1] is lost once the ten dimensional theory is compactified in order to yield the macroscopically well known four dimensional Minkowski space-time. Due to the lack of a dynamical procedure for choosing the vacuum state phenomenological restrictions can prove useful in reducing the number of choices. The requirement that the number of light chiral fermion generations be equal to three, applied to the Calabi-Yau (C-Y) compactification [2], has led to only one multiply connected candidate [3] in the large class of complete intersection C-Y manifolds modded out by projectively inherited freely acting discrete groups. This is the space originally constructed by Yau [4] and extensively studied by many authors. It is defined by the complete intersection in $CP^3 \times CP^3$ of bidegree $(3,0)$, $(0,3)$ and $(1,1)$ homogeneous polynomials, modded out by a freely acting $Z_3$ discrete symmetry group.

In addition to this smooth three generation C-Y manifold there are another two multiply connected such spaces whose construction involves division of a simply connected space by a non-freeley acting discrete symmetry group and subsequent resolution of the resulting singularities. The one of these constructions is again due to Yau [4] and the second to Schimmrigk [5]. Both these manifolds were subsequently shown to be diffeomorphic to the original one [6]. This proof of 'equivalence' combined with the technical difficulties associated with the more complicated construction hampered extensive phenomenological studies of these alternatives to the original three generation space.

Very recently the resolution of the quotient singularities and the computation of the properties of the resulting blow up modes became possible for both the second realization of Yau's manifold [7] and the Schimmrigk space [8]. The latter has an exactly soluble minimal superconformal model counterpart for a particular value of the moduli [9]. A comparison of the corresponding spectra provided some
confidence concerning the singularity resolving techniques employed.

The purpose of the present paper is to study in some detail the phenomenology of the second realization of Yau’s three generation manifold. This is by no means a pointless exercise. Although diffeomorphic to the first, the second realization almost certainly leads to inequivalent physics since the discrete symmetries of the two realizations are different. [The models built on the second realization differ from the ones built on the first by a change of the complex structure [6].]

The second realization of Yau’s three generation manifold \( K \) is defined as a bidegree \((3,3)\) hypersurface \( K_0 \) in \( CP^2 \times CP^2 \) modded out by a non-freely acting order 27 symmetry group \( G_{27} \) with the resulting fixed points resolved. The bicubic \( K_0 \) has the form

\[
M_1(x)M_1(y) + (c_1 + c_2)M_2(x)M_3(y) + (c_1 - c_2)M_3(x)M_2(y) + 
+ c_3M_{10}(x)M_{10}(y) + c_4M_1(x)M_{10}(y) + c_5M_{10}(x)M_1(y) = 0
\]

for almost any choice of the complex parameters \( c_1, c_2, c_3, c_4, c_5 \). [The polynomials \( M_i(z) \) with \( i = 1, 2, \ldots, 10 \) and \( z = (z_0, z_1, z_2) \) are defined in table 1.] This is the most general bicubic invariant under the group \( G_{27} \) generated by:

\[
\sigma_1: (x_0, x_1, x_2) \times (y_0, y_1, y_2) \rightarrow (x_1, x_2, x_0) \times (y_1, y_2, y_0),
\]

\[
\sigma_2: (x_0, x_1, x_2) \times (y_0, y_1, y_2) \rightarrow (x_0, \alpha x_1, \alpha^2 x_2) \times (y_0, \alpha y_1, \alpha^2 y_2),
\]

\[
\sigma_3: (x_0, x_1, x_2) \times (y_0, y_1, y_2) \rightarrow (x_0, x_1, x_2) \times (y_0, \alpha y_1, \alpha^2 y_2),
\]

where \( x \times y \) the homogeneous coordinates of \( CP^2 \times CP^2 \) and \( \alpha = \exp(2\pi i/3) \).

The simply connected space \( K_0 \) has \( h^{2,1} = 83 \) and \( h^{1,1} = 2 \). The \( (2,1) \) harmonic forms correspond to the 83 independent deformations of the complex structure represented by the 83 polynomials of the form \( M_i(x)M_j(y) \) with \( i, j \in \{2, 3, \ldots, 10\} \), \( M_1(x)M_{10}(y) \) and \( M_{10}(x)M_1(y) \). The \( (1,1) \) harmonic forms correspond to the
Kaehler forms of the two $CP^2$ spaces. Modding out by $G_{27}$ (and subsequent resolution of the fixed point singularities) leads to a space whose cohomology can be split into a sum of that derived from the non-singular part of the space with that derived from blowing up the singularities. The first contributes the 2 (1,1) harmonic forms of $K_0$ and 5 (2,1) harmonic forms corresponding to the 5 deformations represented by the polynomials $M_2(x)M_3(y)$, $M_3(x)M_2(y)$, $M_{10}(x)M_{10}(y)$, $M_1(x)M_{10}(y)$ and $M_{10}(x)M_1(y)$. Since the second realization of Yau’s space is diffeomorphic to the first, for which $h^{2,1} = 9$ and $h^{1,1} = 6$, there should be a contribution of 4 to both $h^{2,1}$ and $h^{1,1}$ from the blow up modes. The fundamental group $\pi_1(K) = \mathbb{Z}_3$ is associated with the freely acting symmetry generated by $\sigma_1$.

Flux breaking [2] of the gauge symmetry $E_6$ depends on the choice of an embedding of $\sigma_1$ into $E_6$ defined by a homomorphism $g \rightarrow U_g$. The unbroken gauge group is the subgroup $H$ of $E_6$ commuting with the image of $\sigma_1$ under $U_g$. As far as flux breaking is concerned there is a complete analogy to the first realization of Yau’s manifold [10, 11]. The possible proper subgroups $H$ are $SU(3)^3$ and $SU(6) \times U(1)$. The ‘standard’ embedding, which we will adopt in the following, employs the homomorphism $g \rightarrow U_g \equiv (I_3, a_1I_3, a_1I_3) \in SU(3)_c \times SU(3)_b \times SU(3)_f$ and $H \equiv SU(3)_c \times SU(3)_b \times SU(3)_f$. Under $H$ the left-handed lepton ($\lambda$), quark ($Q$) and antiquark ($Q^c$) fields in a 27 of $E_6$ transform as follows: $\lambda = (1, \overline{3}, 3) = \begin{pmatrix} H^1 \\ \nu^c \\ N \end{pmatrix}$, $Q = (3, 3, 1) = \begin{pmatrix} q \\ g \end{pmatrix}$, $Q^c = (\overline{3}, 1, \overline{3}) = (w^c, d^c, g^c)$. There are 9 $\lambda$ lepton fields, 7 $Q$ ($Q^c$) quark (antiquark) fields, 6 $\overline{\lambda}$ mirror lepton fields and 4 $\overline{Q}$ ($\overline{Q^c}$) mirror quark (antiquark) fields. Four of the fields in each category are blow up modes.

The maximal symmetry of $K_0$ [7] is generated by $\sigma_1, \sigma_2, \sigma_3$ and

\[ P_i: (x_0, x_1, x_2) \times (y_0, y_1, y_2) \rightarrow (x_i, x_0, x_2) \times (y_1, y_0, y_2); \]

\[ S: x_i \rightarrow y_i, y_i \rightarrow x_i; \quad (i = 0, 1, 2), \]

3
\( G \cdot x_0 \rightarrow \alpha x_0. \)

On modding out by \( G_{27} \) the discrete symmetries \( d \) surviving on \( K \), before flux breaking (i.e. assuming \( U_\alpha \equiv 1 \)), are the ones satisfying the relation \( d^{-1} g d = g' \), with \( g, g' \in G_{27} \). \( G \) satisfies this relation with \( g' = g \) for \( g = \sigma_2 \) or \( g = \sigma_3 \) and with \( g' = \sigma_1 \sigma_2 \sigma_3 \) for \( g = \sigma_1 \). For the permutation symmetries \( P \) and \( S \) we have \( P \sigma_i P = \sigma_i^2 (i = 0, 1, 2), S \sigma_1 S = \sigma_1, S \sigma_2 S = \sigma_2 \) and \( S \sigma_3 S = \sigma_3^2 \sigma_2 \). After flux breaking (i.e. \( U_\alpha \neq 1 \)) the scaling symmetry \( G \) and the permutation symmetry \( S \) (for which \( g' = g''g \) for elements \( g'' \in G_{27} \) mapped to the identity under \( U_\alpha \)) survive as honest symmetries on \( K \) for all possible homomorphisms \( U_\alpha \) of \( \sigma_1 \) into \( E_6 \). The survival of \( P \) depends on the existence of an \( E_6 \) element \( J \) with \( J^2 = 1 \) satisfying \( J U_\alpha J = U_\alpha^{-1} \). Such an element \( J \) exists only for the 'standard' embedding, leading to an unbroken \( SU(3)^3 \) gauge symmetry, and for the three embeddings characterized by the breaking directions \([000a000], [aaa00a0] \) and \([2a2aa2a2a0] \) (in the dual basis) in \( E_6 \) root space (\( a \) is identified with the generator of \( \pi_1(K) \)), leading to an unbroken \( SU(6) \times U(1) \) gauge group [10, 11]. For the rest of the possible embeddings \( P \) is a pseudosymmetry.

The classification of the possible models according to their discrete symmetries for the embeddings of \( \sigma_1 \) into \( E_6 \) for which \( P \) survives as a honest symmetry is given in table 2. For the rest of the embeddings it is given in table 3. In these tables the reader can find the non-zero parameters \( c_i \) in the bicubic \( K_0 \) defining the complex structure, the generators of the discrete symmetry group and its order.

Table 4 contains the transformation properties of the 'massless' (compared to the compactification scale \( M_* \)) non gauge singlet superfields under the discrete symmetry generators \( P, S \) and \( G \) for the 'standard' embedding. For the rest of the embeddings it is trivial to obtain the corresponding tables from table 4. The last four fields of each type are blow up modes.
It is important to notice that the scaling symmetry $G$ is an R-symmetry [7] (it does not leave invariant the harmonic $(3,0)$ form $\omega$ and the covariantly constant spinor of the C-Y space and therefore does not commute with supersymmetry). The restrictions imposed by an R-symmetry on the superpotential is a somewhat subtle issue [12]. Let $27 \rightarrow \beta 27$ and $\bar{27} \rightarrow \beta^* \bar{27}$ be the 'naive' (i.e. obtained without realizing that the discrete symmetry is an R-symmetry) transformation property of (a component of) a $27$ and a $\bar{27}$ respectively under the R-symmetry (table 4 gives the 'naive' transformations). Also assume that under the R-symmetry $\omega \rightarrow \gamma^{-1} \omega$. Let us now redefine the R-symmetry such that $27 \rightarrow \gamma^{-1/2} \beta 27$ and $\bar{27} \rightarrow \gamma^{1/2} \beta^* \bar{27}$. Gauge singlets need no such redefinition. The allowed superpotential terms are the ones transforming like $\gamma$ under the so redefined R-symmetry. For the R-symmetry $G$ under discussion $\gamma = \alpha^2$.

An important phenomenological question is the existence of matter parities [13] which solve the rapid proton decay problem in the case in which the gauge group is broken, in one step, down to the standard group at an intermediate scale. It is easy to see (using table 4) that there is no such a satisfactory matter parity. The $Z_3$ symmetry generated by $GSG^2S$ (appropriately combined with a $Z_3$ subgroup of the gauge symmetry) could be regarded as a matter parity but it does not allow for an intermediate vacuum expectation value (vev) in both $N$ and $\nu^c$ directions. If we, nevertheless, follow this matter parity scenario we find an unacceptable spectrum at low energies (for each light chiral quark there is another vector-like one which is degenerate in mass with it).

Here we concentrate on the phenomenology of the most symmetric model on space $K$ with gauge symmetry group $SU(3)^3$, namely the model (1) in table 2 with discrete symmetry the 36 element group generated by $P$, $S$ and $G$. There is a good reason for this choice. In ref. [7] has been pointed out that due to the combined
effect of the R-symmetries generated by \( G \) and \( SGS \) there exist (modulo some uncertainties concerning the unknown behavior of possible blow up gauge singlets) exact flat directions for the standard gauge group singlets contained in \( \lambda_1 \) and \( \lambda_2 \) (together with anyone of the mirror lepton fields) which could break \( E_6 \) down to \( SO(10) \) or even \( SU(5) \) at \( M_c \). Since this is a very interesting novel possibility it is worth exploring its consequences. The only models in table 2 possessing both of these R-symmetries are model (1) and (2). This already points towards the most symmetric models. We found the presence of the permutation symmetry \( P \) in model (1) very useful in connection with the gauge hierarchy problem and light fermion masses.

The absence of acceptable matter parities immediately eliminates the possibility of breaking \( E_6 \) down to \( SU(5) \) or, taking into account flux breaking, to the standard group through large vevs in both \( N \) and \( \nu^c \) directions at \( M_c \). We will therefore assume that the vev in, say, the \( \nu^c \) direction is at most of the order of the supersymmetry breaking scale \( M_s (~10^{14}\text{GeV}) \) in the visible sector. We also found favorable to keep the dominant vev in the \( N \) direction slightly below \( M_c \).

We will choose a large vev \( \sim 10^{-1}M_c \) for \( N_2 \) and for one combination, which we will call \( \overline{N}_1' \), out of the set \( \{ \overline{N}_1, \overline{N}_6 \} \), namely the combination appearing at most linearly in a coupling of the form \( \overline{N}N_NN_NN \). The orthogonal combination will be called \( \overline{N}_6' \). It is easy to see that, without any intermediate vevs for \( N \)'s transforming non-trivially under the R-symmetries \( G \) and \( SGS \), very few states acquire any mass at all. To remedy this clearly unacceptable situation we choose an intermediate vev \( \sim 10^{14}\text{GeV} \) for one combination of the form \( N_1 + b_1 N_6 + b_2 N_6 \) (with \( b_1 \sim 10^{-2.5} \) and \( b_2 \sim 10^{-7} \)) and \( \overline{N}_2' \). \( |\overline{N}_2' \rangle \) is the combination of \( \overline{N}_2 \) and \( \overline{N}_6 \) which does not have a coupling of the form \( \overline{N}N_NN_NN \). The orthogonal combination will be called \( \overline{N}_5' \). This choice was made after a careful examination of the
gauge hierarchy problem, the proton lifetime and the light fermion spectrum. To obtain the vevs mentioned above some higher order superpotential couplings must be small. For instance (assuming $M_c \sim 10^{17}$ GeV) the couplings of the terms $\overline{N}_2 \overline{N}_6$, $\overline{N}_2 \overline{N}_1$, $\overline{N}_6 N_1$, $\overline{N}_1 \overline{N}_6$, $N_6 N_1^2$, $\overline{N}_1 N_6^2$, $\overline{N}_6 N_1^2$, $\overline{N}_1 N_6^2$, and $\overline{N}_1 \overline{N}_6 N_1^2$ must be of the order of $10^{-1.5}$, $10^{-5}$, $10^{-7}$ and $10^{-3}$ respectively. Notice that, in the absence of any other vev, the superlarge one ($\sim 10^{-1}$ GeV) does not need any arrangements.

Before we embark on the evaluation of the physical spectrum we will quote some useful mathematical facts concerning mass matrices. The mass matrices $M$ we deal with are, in general, non-hermitian ones which are diagonalizable through left and right multiplication with two different unitary matrices. The physical masses are not their eigenvalues but are equal, in absolute value, to the square root of the eigenvalues of the hermitian matrix $MM^\dagger$. In ref. [11] we proved that for an arbitrary square matrix $M$ of order $n$, the sum of the products of the eigenvalues of $MM^\dagger$ in groups of $l \leq n$ is equal to the sum of the squares of the absolute values of all order $l$ minor determinants of $M$. An immediate consequence of this proposition is that the order of magnitude of the product of the $l$ largest physical masses associated with a general mass matrix $M$ is equal to the order of magnitude of the largest order $l$ minor determinant of $M$.

In the following we will work with fermion mass matrices since we are not interested in fermion-boson mass splittings of order $M_s$ or smaller.

The mass matrix of the $SU(2)_L$ and $SU(2)_R$ doublet leptons is a $15 \times 15$ matrix with rows labelled by the $9 \ H^2$'s and the $6 \ \overline{H}^1$'s and columns labelled by the $9 \ H^1$'s and the $6 \ \overline{H}^2$'s. These are the states (actually their supersymmetric partners) with the correct quantum numbers to play the role of the electroweak Higgs doublets. $\overline{H}^1_4 (\overline{H}^2_4)$ and $\overline{H}^1_5 (\overline{H}^2_5)$ pair up mainly with 2 combinations out of the set $\{ \overline{H}^1_1, \overline{H}^2_2, \overline{H}^2_3, \overline{H}^2_5 \}$ $\{ \overline{H}^1_1, \overline{H}^2_1, \overline{H}^2_3, \overline{H}^2_5 \}$ to form 2 supermassive states with mass
\(< \overline{N}_4' > \sim 10^{-1} M_e. \) The remaining 2 combinations out of the set \{ \overline{H}^2_1, \overline{H}^2_2, \overline{H}^2_3, \overline{H}^2_5 \} pair up with the remaining 2 combinations out of the set \{ \overline{H}^1_1, \overline{H}^1_2, \overline{H}^1_3, \overline{H}^1_5 \} to make 2 states with mass \(< \overline{N}_2 >. \) \(H^2_2\) and \(H^2_3\) pair up mainly with \(H^1_3\) and \(H^1_2\) to make 2 superheavy states with mass \(< N_2 > \sim 10^{-1} M_e. \) \(H^2_4\) \((H^2_5)\) pairs up mainly with \(H^1_5\) \((H^1_4)\) to form a state with mass \(< N_4 >. \) \(H^2_6\) \((H^2_7)\) pairs up mainly with \(H^1_6\) \((H^1_7)\) to form a state with mass \(< N_8 >\) and \(H^2_8\) \((H^2_9)\) pairs up mainly with \(H^1_8\) \((H^1_9)\) to form a state with mass \(< N_4 >. \) Finally \(H^2_4\) pairs up mainly with \(H^1_4\) into a state with mass \(< N_8 >^2 / < N_4 >. \)\n
This is the typical physical mass spectrum with coupling constants of order unity. Clearly, this does not have to be the case. Assuming that the coupling constant of the superpotential term \(\lambda_4 \lambda_8 \lambda_8 + \lambda_5 \lambda_6 \lambda_6\) is \(< 10^{-5}\) we get two pairs \(\langle H^1_4, H^1_5, H^2_4, H^2_5\rangle\) having the quantum numbers of electroweak Higgs doublets with masses \(< M_e\) or smaller. We see that, in spite of the existence of vevs as large as \(< 10^{-1} M_e,\) we do not need couplings as small as \(< 10 M_e / M_e\) in order to obtain acceptably light electroweak Higgs doublets. Clearly, this should be regarded as a positive feature.

The mass matrix of the \(SU(2)_L\) doublet, \(SU(2)_R\) singlet leptons is a \(6 \times 9\) matrix whose rows are labelled by the 6 \(\tilde{e}\)'s and its columns by the 9 \(\ell\)'s. \(\overline{\ell}_2'\) and \(\overline{\ell}_6'\) pair up mainly with \(\ell_8\) and \(\ell_6\) into states with mass \(< 10^{-3} \sim \overline{N}_4 >^2 < N_4 >^2 M_e^{-3}\) and \(< \overline{N}_4 >^2 < N_8 > M_e^{-1}\) respectively. \(\overline{\ell}_4'\) pairs up with one combination out of the set \{\(l_1, l_5\)\} to form a state with mass \(< 10^2 - 10^3\) GeV. Finally \(\overline{l}_3'\) and \(\overline{l}_5'\) pair up with 3 combinations out of the set \{\(l_2, l_3, l_4, l_6\)\} to form 3 states with mass \(< 10^2 - 10^3\) GeV. We will assume that the light combination from this set is mainly \(l_3\) with an order \(< 10^{-2}\) contribution from \(l_1\) and suppressed mixing with \(l_2.\) The other 2 light leptons are \(l_5\) and one combination out of the set \{\(l_1, l_6\)\}.

The mass matrix of the \(SU(2)_L\) singlet, \(SU(2)_R\) doublet leptons \((\nu^c, e^c)\) is
identical to the one just discussed.

We now turn to the quark sectors. The \( SU(2)_L \) and \( SU(2)_R \) singlet quark mass matrix has rows labelled by the 7 \( q \)'s and the 4 \( \tilde{q} \)'s and columns labelled by the 7 \( q \)'s and the 4 \( \tilde{q} \)'s. The \( q \)'s pair up with the \( \tilde{q} \)'s into 4 states of mass \( \sim< \tilde{N}_4' > \sim 10^{-1} M_c \). Also \( g_1 \) and \( g_2 \) pair up mainly with \( g_5' \) and \( g_6' \) to form 2 states of mass \( \sim< N_2 > \sim 10^{-1} M_c \). We will assume the coupling \( g_5 g_5' \lambda_4 \) to be strong (will be used later to create third generation quark masses) and the couplings \( g_i g_j' \lambda_4 \), \( i, j \in \{5, 7\} \) with \( i = j = 5 \) excluded, to be weak. Then, \( g_5 \) pairs up mainly with \( g_5' \) into a state with mass \( \sim< N_1 > \sim 10^{11} \text{GeV} \) and \( g_7 \), \( g_7' \) pair up mainly with \( g_7' \), \( g_7' \) into 2 states of mass \( \sim< 
abla_1' > < N_8 > < N_1 > M_c^{-2} \sim 10^7 - 10^8 \text{GeV} \). Finally \( g_4 \) and \( g_6 \) pair up with \( g_2 \) and \( g_3 \) to form 2 states with mass of the order of \( < N_5 > \sim 10^7 \text{GeV} \) and \( < \nabla_4' > < N_8 > M_c^{-2} \sim 10^5 \text{GeV} \) respectively.

In the \( SU(2)_L \) doublet, \( SU(2)_R \) singlet quark sector 2 out of the 4 \( q \)'s pair up with \( q_4 \) and \( q_6 \) to form 2 states of mass \( < \nabla_1' > < N_8 > M_c^{-1} \sim 3 \times 10^{10} \text{GeV} \) and the other 2 \( q \)'s pair up mainly with 2 combinations out of the set \( \{ q_1, q_5; q_7 \} \).

We will assume that (through small couplings of some sixth order superpotential terms) the light quark out of this set is \( q_5 \) (which will later be identified with the third generation). The mass of \( q_1 \) is \( \sim< \nabla_1' > \sim< N_1 > < N_8 > M_c^{-3} \sim 10^7 \text{GeV} \) and of \( q_7 \) is \( \sim< \nabla_1' > < N_4 > \sim< N_8 >^2 M_c^{-3} \sim 10^6 \text{GeV} \). Therefore the 3 light quarks (corresponding to the ordinary families) are \( q_2, q_3 \) and \( q_5 \).

The mass matrix of the \( SU(2)_L \) singlet, \( SU(2)_R \) doublet antiquarks \( (u^c, d^c) \) is identical to the one just discussed.

To summarize, the light leptons are contained in \( \lambda_3 \) (with an \( 10^{-2} \) admixture with \( \lambda_4 \)), \( \lambda_5 \) and one combination out of \( \{ \lambda_1, \lambda_5 \} \). The light quarks (antiquarks) are contained in \( Q_2, Q_3 \) and \( Q_5 \) \( (Q_2', Q_3', Q_5') \). The candidate electroweak Higgs doublets are \( H^1_a, H^2_a, H^3_a \) and \( H^2_c \).
With the above light spectrum we can readily check that there exist Yukawa
couplings leading to tree level masses for at least the third generation of quarks and
leptons. For the third generation quark masses we make use of the term \( Q_3 \bar{Q}^c_3 \lambda_4 \),
while for the third generation lepton masses of the term \( \lambda_3 \lambda_4 \lambda_6 \). Therefore the \( t(b) \)
quark is contained in \( Q_3 \) and the \( \tau \) lepton in the light combination of \( \lambda_3 \) and \( \lambda_4 \).
Notice that the electroweak Higgs doublets giving masses to quarks and leptons
are different. This is very satisfactory since otherwise (taking into account the
\( SU(2)_R \) symmetry of the light spectrum) Dirac neutrino masses would be of the
same order of magnitude as up quark masses. In this event we would have to
invent some sort of a see-saw mechanism involving the enormous (and to some
extent unknown) gauge singlet sector.

The most dangerous proton decay processes involve the exchange of a \( g \) (\( g^c \))
quark (antiquark) between vertices corresponding to superpotential couplings \( qqq \)
\( (u^c d^c g^c) \) and \( g\mu^c e^c (g^c q l) \). The R-symmetries \( G \) and \( SGS \) severely restrict the
allowed baryon number violating couplings. The only allowed such couplings are the
following three: \( f_1(Q_1 Q_2 Q_3 + Q^c_1 Q^c_2 Q^c_3) \), \( f_2(Q_1 Q_2 Q_3 + Q^c_1 Q^c_2 Q^c_3 + Q_2 Q_1 Q_4 +
Q^c_2 Q^c_4 Q^c_4) \) and \( f_3(Q_1 Q_5 Q_7 + Q^c_1 Q^c_5 Q^c_7 + Q_2 Q_6 Q_8 + Q^c_2 Q^c_6 Q^c_8) \). The least
suppressed process involves the exchange of \( g^c \), between the vertices \( u^c_2 d^c_2 g^c_1 \) and
\( g^c_1 q_2 l_2 \) (or \( g^c_1 q_2 l_2 \)). Its amplitude has a suppression by one power of the large
scale \( 10^{-1} M_s \sim 10^{19} \text{GeV} \) (the mass of \( g^c_1 \)), one power of \( M_s \sim 10^4 \text{GeV} \) and
a loop factor. The rest of the suppression has to be provided by the coupling
constants associated with two vertices. [The large coupling of the term \( Q^c_1 Q_2 \lambda_2 \) is
compensated by the fact that \( l_2 \) has a suppressed contribution to a light lepton
state.] With a value of \( \sim 10^{-5} \) for these couplings, the proton decay rate is ac-
ceptably small. For other processes occurring through an exchange of \( g_1 \) or \( g^c_1 \) the
baryon number violating vertex involved has no light (i.e. first or second gener-
ation) quarks. The exchange of \( g_2, g'_2, g_3, g'_3 \) is associated with baryon number violating vertices involving at least another heavy quark (antiquark). The danger from processes occurring through an exchange of the relatively light \( g_5, g'_5, g_7, g'_7 \) is only apparent since the lightness of the exchanged state is compensated by the additional suppression provided by the heavy \( Q_1, Q'_1 \). For \( g_7 \) and \( g'_7 \) there is even more suppression from the presence of a \( Q_5 \) or \( Q'_5 \), both of which contain only third generation quarks. Finally, the exchange of the light (mass \( \sim 10^7 \text{GeV} \)) \( g_4, g'_4 \) or the even lighter (mass \( \sim 10^5 \text{GeV} \)) \( g_6, g'_6 \) is compensated by the presence of one additional heavy state in both the baryon and lepton number violating vertices involved. We see that the most severely constrained baryon number violating coupling is the first among the ones listed above. The bounds on \( f_2 \) and \( f_3 \) are certainly much more loose. The situation concerning proton decay is clearly better than in most other three generation C-Y superstring models.

We conclude by summarizing our results. We studied the second realization of Yau's three generation manifold obtained by modding out a simply connected C-Y space by a non-freely acting discrete group and subsequent resolution of the fixed point singularities. We classified the possible models according to their discrete symmetries and wrote down the transformation properties of the light fields under these symmetries. We then constructed a specific model based on the maximal discrete symmetry group of this C-Y space. We found that, in spite of the absence of symmetries acting as matter parities, the presence of R-symmetries resulted in a clear improvement concerning the proton decay problem.
References


\[ M_i(z) = z_0^i + z_1^i + z_2^i \]
\[ M_i(z) = z_0^i + \alpha z_1^i + \alpha z_2^i \]
\[ M_i(z) = z_0^i + \alpha z_1^i + \alpha z_2^i \]
\[ M_i(z) = z_0^2 z_1 + z_1^2 z_2 + z_2^2 z_0 \]
\[ M_i(z) = z_0^2 z_1 + \alpha z_1^2 z_2 + \alpha z_2^2 z_0 \]

Table 1: Definition of the polynomials \( M_i(z) \), \( i = 1, 2, \ldots, 10 \).

<table>
<thead>
<tr>
<th>Model</th>
<th>Non-zero parameters</th>
<th>Symmetry generators</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>( c_1 )</td>
<td>( G, P, S )</td>
<td>36</td>
</tr>
<tr>
<td>(2)</td>
<td>( c_1, c_2 )</td>
<td>( G, PS )</td>
<td>18</td>
</tr>
<tr>
<td>(3)</td>
<td>( c_1, c_3 )</td>
<td>( GS G^2 S, P, S )</td>
<td>12</td>
</tr>
<tr>
<td>(4)</td>
<td>( c_1, c_4 )</td>
<td>( G, P )</td>
<td>6</td>
</tr>
<tr>
<td>(5)</td>
<td>( c_1, c_5 )</td>
<td>( SGS, P )</td>
<td>6</td>
</tr>
<tr>
<td>(6)</td>
<td>( c_1, c_2, c_3 )</td>
<td>( GS G^2 S, PS )</td>
<td>6</td>
</tr>
<tr>
<td>(7)</td>
<td>( c_1, c_2, c_4 )</td>
<td>( G )</td>
<td>3</td>
</tr>
<tr>
<td>(8)</td>
<td>( c_1, c_2, c_5 )</td>
<td>( SGS )</td>
<td>3</td>
</tr>
<tr>
<td>(9)</td>
<td>( c_1, c_3, c_1 = c_5 )</td>
<td>( P, S )</td>
<td>1</td>
</tr>
<tr>
<td>(10)</td>
<td>( c_1, c_3, c_1, c_5 )</td>
<td>( P )</td>
<td>2</td>
</tr>
<tr>
<td>(11)</td>
<td>( c_1, c_2, c_3, c_1 = c_5 )</td>
<td>( PS )</td>
<td>2</td>
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<tr>
<td>(12)</td>
<td>( c_1, c_2, c_3, c_4, c_5 )</td>
<td>( P )</td>
<td>1</td>
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</table>

Table 2: Classification of models according to their discrete symmetries for flux breakings leading to an unbroken \( P \) symmetry generator.

<table>
<thead>
<tr>
<th>Model</th>
<th>Non-zero parameters</th>
<th>Symmetry generators</th>
<th>Order</th>
<th>Pseudosymmetry generators</th>
</tr>
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<tbody>
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<td>( G, S )</td>
<td>18</td>
<td>( P )</td>
</tr>
<tr>
<td>(2)</td>
<td>( c_1, c_2 )</td>
<td>( G, SGS )</td>
<td>9</td>
<td>( PS )</td>
</tr>
<tr>
<td>(3)</td>
<td>( c_1, c_3 )</td>
<td>( GS G^2 S, S )</td>
<td>6</td>
<td>( P )</td>
</tr>
<tr>
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<td>( PS )</td>
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<tr>
<td>(5)</td>
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<td>( G )</td>
<td>3</td>
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<tr>
<td>(6)</td>
<td>( c_1, c_2, c_5 )</td>
<td>( SGS )</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td>( c_1, c_3, c_1 = c_5 )</td>
<td>( S )</td>
<td>2</td>
<td>( P )</td>
</tr>
<tr>
<td>(8)</td>
<td>( c_1, c_2, c_3, c_4, c_5 )</td>
<td>( P )</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Classification of models according to their discrete symmetries for flux breakings leading to a broken \( P \) symmetry generator.
| $\lambda_1$ | 1 | 1 | $\lambda_1$ | 1 | 1 | $\lambda_1$ |
| $\lambda_2$ | 1 | -1 | $-\lambda_2$ | 1 | 1 | $-\lambda_2$ |
| $\lambda_3$ | $\alpha$ | 1 | $\lambda_3$ | $\lambda_3$ | 1 | $\lambda_3$ |
| $\lambda_4$ | 1 | 1 | $\lambda_4$ | $\lambda_4$ | 1 | $\lambda_4$ |
| $\lambda_5$ | $\alpha$ | 1 | $\lambda_5$ | $\lambda_5$ | 1 | $\lambda_5$ |
| $\lambda_6$ | 1 | -1 | $\lambda_6$ | $\lambda_6$ | 1 | $\lambda_6$ |
| $\lambda_7$ | $\alpha^2$ | 1 | $\lambda_7$ | $\lambda_7$ | 1 | $\lambda_7$ |
| $\lambda_8$ | $\alpha^2$ | -1 | $\lambda_8$ | $\lambda_8$ | 1 | $\lambda_8$ |
| $\lambda_9$ | $\alpha^2$ | 1 | $\lambda_9$ | $\lambda_9$ | 1 | $\lambda_9$ |

| $Q_1$ | 1 | $Q^c_1$ | $Q_1$ | 1 | $Q^c_1$ | $Q_1$ |
| $Q_2$ | $\alpha$ | $Q^c_2$ | $Q_2$ | 1 | $Q^c_2$ | $\frac{Q_2}{Q_2}$ |
| $Q_3$ | 1 | $Q^c_3$ | $Q_3$ | 1 | $Q^c_3$ | $\frac{Q_3}{Q_3}$ |
| $Q_4$ | $\alpha^2$ | $Q^c_4$ | $Q_4$ | 1 | $Q^c_4$ | $\frac{Q_4}{Q_4}$ |
| $Q_5$ | 1 | $Q^c_5$ | $Q_5$ | 1 | $Q^c_5$ | $\frac{Q_5}{Q_5}$ |
| $Q_6$ | $\alpha^2$ | $Q^c_6$ | $Q_6$ | 1 | $Q^c_6$ | $\frac{Q_6}{Q_6}$ |
| $Q_7$ | 1 | $Q^c_7$ | $Q_7$ | 1 | $Q^c_7$ | $\frac{Q_7}{Q_7}$ |

Table 4: Transformation properties of the 'massless' superfields under the symmetries generated by $G$, $P$, and $S$. 