CERN-TH-5451/89

Hadronic Matrix Elements: Lessons Learnt from Lattice QCD

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Abstract

I summarise our progress in calculation of matrix elements relevant to non-leptonic Kaon decays. I also present lattice Monte Carlo results for scalar density and axial current matrix elements of the baryon octet.

*Talk presented at the International Workshop on Weak Interactions and Neutrinos
Ginosar, Israel, April 10-14, 1989*

CERN-TH-5451
June 1989

HADRONIC MATRIX ELEMENTS: LESSONS LEARNT FROM LATTICE QCD

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1. Non-leptonic K -decays

Let me begin with a quick overview of the present status of our weak matrix element calculation for non-leptonic K -decays. We are in the midst of a second generation calculation¹ and some results have already been presented during the past year². We are using staggered fermion lattice operators in Landau gauge and quark wall sources in Coulomb gauge. The background gauge field configurations have been generated in the quenched approximation at three values of the gauge coupling $6/g^2 = 5.7, 6.0, 6.2$. So far we have obtained statistically clean signals for the “eight”-diagrams and the “eye”-diagram calculations are in progress.

One of the simplest quantities to measure is the matrix element of the $K^0 - \bar{K}^0$ mixing $\Delta S = 2$ operator:

$$\mathcal{O}_{\Delta S=2} = (\bar{s}^a \gamma_\mu (1 - \gamma_5) d^a) (\bar{s}^b \gamma_\mu (1 - \gamma_5) d^b)$$

In the chiral limit this matrix element should vanish as M_K^2 . It is convenient to parametrise it in terms of the B -parameter which is the ratio of the matrix element to its value in the vacuum saturation approximation, though such a B -parameter is scale dependent. A typical example of our results is shown in fig. 1. The contributions to $B_{\Delta S=2}$ from space and time components in each of the two different contractions are displayed separately. A good test of the desired chiral behaviour is that all four components of the matrix element can be extrapolated to pass through the origin. Slight discrepancies are present, however, and can be attributed to finite-size effects and chiral logarithmic corrections. For a lattice scale of $a^{-1} = 2$ GeV, we quote :

$$B_{\Delta S=2} = 0.70 \pm 0.02(\text{stat.}) \pm 0.02(\text{finite-size}) \\ \pm 0.04(\text{pert. corr.}) \pm ?(\text{quenching}) ,$$

at the physical K -mass (our smallest quark mass point is close to this value). If leading order chiral perturbation theory is used to convert this result into the amplitude for the $\Delta I=3/2$ decay $K^+ \rightarrow \pi^+ \pi^0$, the amplitude comes out about twice the experimental number.

We have also calculated the $K^+ \rightarrow \pi^+$ matrix elements of the electro-penguin operators, which are relevant to the calculation of ϵ'/ϵ :

$$\mathcal{O}_7 = (\bar{s}^a \gamma_\mu (1 - \gamma_5) d^a) \sum_{q=u,d,s} Q_q (\bar{q}^b \gamma_\mu (1 + \gamma_5) q^b) , \\ \mathcal{O}_8 = (\bar{s}^a \gamma_\mu (1 - \gamma_5) d^b) \sum_{q=u,d,s} Q_q (\bar{q}^b \gamma_\mu (1 + \gamma_5) q^a) .$$

These matrix elements are dominated by the “eight”-contractions, and we find (again at $a^{-1} = 2$ GeV)

$$B_8 = 1.10 \pm 0.05(\text{stat.}) , \\ \langle \pi^+ | \mathcal{O}_7 | K^+ \rangle = (0.34 \pm 0.01(\text{stat.})) \langle \pi^+ | \mathcal{O}_8 | K^+ \rangle .$$

2. Non-singlet Matrix Elements of Baryon Octet

I now switch to a different set of matrix elements and a different collaboration. The operators in this case are quark bilinears and the states belong to the baryon octet. The matrix elements of interest are those of scalar densities and axial currents. The former ones parametrise the mass and the s -wave non-leptonic decays, while the latter ones characterise the spin and the semi-leptonic decays. All of them have well-defined static (i.e. $m_q \rightarrow \infty$) limits, which are easily calculated using the non-relativistic quark model. What the lattice calculations allow us to judge is the influence of relativistic corrections on these values as the quark mass is lowered from infinity towards zero.

Let $\mathcal{O}_f^\dagger$ denote the non-singlet quark bilinear with flavour structure λ_c and spin structure Γ . Then in the framework of exact isospin and first order flavour $SU(3)$

Fig. 1 : Different components of the $\Delta S = 2$ matrix element contributing to $K^0 - \bar{K}^0$ mixing.

breaking, the matrix elements can be decomposed into their irreducible parts F_Γ and D_Γ :

$$\langle B_f(s_f) | \mathcal{O}_\Gamma^c | B_i(s_i) \rangle = \bar{\psi}_f(s_f) \Gamma \psi_i(s_i) \cdot \left[F_\Gamma \text{Tr}(B_f^\dagger[\lambda^c, B_i]) + D_\Gamma \text{Tr}(B_f^\dagger\{\lambda^c, B_i\}) \right].$$

Here $B_{f,i}$ represent the final (initial) state octet baryons, $s_{f,i}$ their spins and $\psi_{f,i}$ their spinor wave-functions.

The non-singlet matrix elements get contributions from the valence quarks only, and hence can be consistently evaluated within the quenched approximation. Our results have been obtained on blocked gauge field configurations and with an improved Wilson fermion action³. We extract the desired matrix elements from the ratio of 3-point to 2-point correlation functions:

$$\frac{\sum_{\vec{x}} \langle \mathcal{P}_\alpha(\vec{x}, t) | \sum_z \bar{q}(z) \Gamma q(z) | \mathcal{P}_\beta^\dagger(\vec{0}, 0) \rangle}{\frac{1}{2} \sum_{\delta=1}^2 \sum_{\vec{x}} \langle \mathcal{P}_\alpha(\vec{x}, t) | \mathcal{P}_\beta^\dagger(\vec{0}, 0) \rangle} = \Gamma_{\alpha\beta} R_q^\Gamma(t),$$

where the operator insertion is made on the valence quark q . With $\mathcal{P}_\alpha$ being the proton operator having spin s_α , the slopes of $R_u(R_u - 2R_d)$ vs. t yield $F(D)$.

In the static limit, the scalar density matrix elements are just the flavour quantum numbers of the hadrons. The leading relativistic correction to these values is provided by the hyperfine gluon exchange interaction, proportional to $(\vec{S}_i \cdot \vec{S}_j / m_i m_j)$, between each quark pair. As a result, both F_S and D_S get non-linear corrections

Fig. 2 : g_A and F_A/D_A as functions of m_q . The dotted lines show the static limits and $\otimes$ mark the experimental values⁵. The lattice scale is $a^{-1} \approx 1.15$ GeV.

varying as m_{eff}^{-3} in the $SU(3)$ symmetric case (m_{eff} is the constituent quark mass). Sufficiently close to the chiral limit, F_S and D_S should become constants.

Our results⁴ show the expected non-linear behaviour as m_q is reduced from infinity. We find that F_S approaches the chiral limit smoothly and agrees with the known mass-splittings. (The physical quantity without anomalous dimension is $m_q F_S$.) On the other hand, D_S continues to show highly non-linear behaviour even for $m_q \leq m_s$, and strongly suggests that the first order $SU(3)$ breaking analysis is inadequate to relate it to the mass-splittings. Such a non-linear behaviour does not imply a conflict with the Gell-Mann-Okubo mass relation, as long as the effective interaction between a valence quark pair in an antisymmetric flavour state dominates over the non-linear pieces of all other interactions. This is because the diquark interaction in a flavour antisymmetric state has no overlap with the $\underline{27}$ operator of flavour $SU(3)$ that is responsible for the violation of the Gell-Mann-Okubo mass relation. (For instance, in the hyperfine interaction the flavour antisymmetric part dominates over the flavour symmetric part by a ratio of 3:1.) The fact that D_S becomes more and more negative as one approaches the chiral limit im-

Fig. 3 : M_N/M_ρ as a function of $(M_\pi/M_\rho)^2$.

plies that $\langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle$ are much closer to each other in a proton than the static ratio of 2:1.

For the axial vector matrix elements, the static limits are: $F_A=2/3$, $D_A=1$. The relativistic motion of quarks bound into hadrons provides the dominant correction through non-zero values for the lower components of the Dirac spinors. It gives a suppression factor of $(1 - \langle p_T^2/E(E+m_q) \rangle)$ for both F_A and D_A , where E and p_T are the energy and transverse momentum of a quark. The experimental data on the other hand yields⁵

$$g_A = F_A + D_A = 1.25(1), \quad F_A/D_A = 0.59(5).$$

To measure F_A and D_A on the lattice using Wilson fermions, we had to determine the non-perturbative renormalisation of the axial current⁶ in addition to determining R_q^A . Our final results for the matrix elements⁷ are shown in fig. 2. They clearly show the influence of the relativistic suppression factor and approach the physical values close to the chiral limit. The rather smooth dependence of the matrix elements on the quark mass indicates that the first order flavour $SU(3)$ breaking formalism is a good approximation in this case.

3. Singlet Matrix Elements of Baryon Octet

We have recently also obtained results illustrating the impact of sea quarks on the hadron spectrum⁸. We

designate the quarks appearing in the fermion determinant as the sea quarks and those coupled to external sources as the valence ones. By independently varying the sea and valence quark masses, we find that at $6/g^2 = 5.3$ ($a^{-1} \approx 1.4$ GeV) and for $m_q \approx m_s$,

$$(\partial M_\rho / \partial m_{sea}) = (2.5 \pm 1 \pm ?) (\partial M_\rho / \partial m_{val}),$$

$$(\partial M_N / \partial m_{sea}) = (4 \pm 1 \pm ?) (\partial M_N / \partial m_{val}).$$

Though there are many systematic uncertainties in these results as emphasised by the question marks, the numbers imply that the hadron masses are more sensitive to variation of m_{sea} than to m_{val} in this range. This effect mainly reflects a change in the scale of the theory due to change in vacuum polarisation. It is also connected⁹ with the scale anomaly of QCD providing a sizeable sea quark contribution to the pion-nucleon σ -term.

The effect of sea quarks on dimensionless mass-ratios is exemplified in fig. 3. We observe that at fixed M_π/M_ρ , decreasing m_{sea} decreases M_N/M_ρ . Different influence of the sea quarks on different hadrons is a valuable signal in enhancing our understanding of QCD.

I thank my collaborators for a cohesive effort in producing the results described here and the conference organizers for giving me an opportunity to present them.

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