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CHERN-SIMONS INDUCED MASSIVE GAUGE FIELDS IN FOUR DIMENSIONS

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A B S T R A C T

A phenomenon, reminiscent of topological mass generation in 2+1 dimensions, is extracted out of the theory of new minimal supergravity coupled to a linear multiplet with superfield Chern-Simons terms in four dimensions. A physical, massive Abelian gauge field, originally not in the theory, is induced by the addition of a specific Chern-Simons term to the Lagrangian.

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It is by now well known that, in 2+1 dimensions, if one starts with the Lagrangian for a physical, propagating, massless Abelian or non-Abelian gauge field, and adds to it the associated Chern-Simons term, that a massive pole is induced in the propagator [1]. In this paper, we will extract out of a much larger physical theory, new minimal supergravity [2] coupled to a linear supermultiplet [3,4] with superfield Chern-Simons terms [5,6], an Abelian gauge field sector whose behaviour is reminiscent of topological mass generation in 2+1 dimensions, only in reverse. In our case, one starts with a Lagrangian that contains a mass term for an auxiliary Abelian vector field, one of the auxiliary fields of new minimal supergravity, and adds to it the appropriately supersymmetrized associated Chern-Simons term. We show that this induces a kinetic energy term for this Abelian vector field which, therefore, is no longer auxiliary. Hence, the theory describes a physical, propagating, massive Abelian gauge field.

Consider a U(1) gauge connection a_m and a scalar field C' which transform under U(1) gauge transformations as

$$\begin{aligned}\delta a_m &= \partial_m \Lambda \\ \delta C' &= \Lambda\end{aligned}\tag{1}$$

where Λ is a finite parameter. Note that C' can be transformed to zero. Now construct the gauge invariant Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{mn})^2 - \frac{M^2}{2}(a_m - \frac{1}{2}\partial_m C')^2\tag{2}$$

Clearly, $\mathcal{L}$ describes a propagating, massive vector field of mass M , despite the fact that $\mathcal{L}$ is gauge invariant. It is informative to cast this Lagrangian into an alternative form. Let G_m be a real vector field, and consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{mn})^2 + 2a_m G^m + \frac{2}{M^2}G^m G_m - (\partial_m C')G^m\tag{3}$$

Since G_m has no kinetic term, it is an auxiliary field which can be eliminated using its equation of motion

$$G_m = -\frac{M^2}{2}(a_m - \frac{1}{2}\partial_m C')\tag{4}$$

Substituting (4) into (3) yields Lagrangian (2). Hence, at least at the classical level, Lagrangians (2) and (3) are physically equivalent. As we will presently show, a Lagrangian similar to (3) will naturally appear within the context of new minimal supergravity.

It is well known that there are four different $N=1$ supergravity multiplets in four dimensions; namely, old minimal, new minimal, non-minimal and 16-16 supergravity. The new minimal supergravity multiplet contains, in addition to the

graviton e_m^a and gravitino ψ_m^α common to all supergravity theories, two auxiliary fields; a real vector field a_m and a real antisymmetric tensor field t_{mn} . New minimal supergravity, and its couplings to matter, also exhibits a local U(1) invariance for which auxiliary field a_m plays the role of the gauge connection. A complete description of the superspace approach to new minimal supergravity can be found in [4]. Here we simply give, without proof, the relevant results. The structure group of the superspace is taken to be $SL(2, C) \times U(1)$. Therefore, two Lie algebra valued one-form supergauge connections, $\Phi_B^A = dz^M \Phi_{MB}^A$ and $A = dz^M A_M$ corresponding to the Lorentz and U(1) groups respectively, are introduced. In addition, one introduces the supervielbein E_M^A and a two-form supergauge potential $B = \frac{1}{2} dz^M dz^N B_{MN}$. The associated torsion, curvature and field strength superfields are then subjected to a set of constraints, which greatly reduce the number of independent fields.

Solving the Bianchi identities subject to an initial set of constraints, one finds that all quantities can be expressed in terms of superfields E_M^A , A_M , B_{MN} , W_α , R , G_a and U . A gauge can be chosen so that

$$E_M^A| = \begin{pmatrix} e_m^a & \frac{1}{2} \psi_m^\alpha & \frac{1}{2} \bar{\psi}_{m\dot{\alpha}} \\ 0 & \delta_m^\alpha & 0 \\ 0 & 0 & \delta_{m\dot{\alpha}}^{\dot{\alpha}} \end{pmatrix} \quad (5)$$

$$A_M| = (ia_m, 0, 0)$$

$$B_{MN}| = \begin{pmatrix} t_{mn} & 0 \\ 0 & 0 \end{pmatrix}$$

where $|$ means the $\theta = \bar{\theta} = 0$ component, thus defining the graviton, gravitino, U(1) gauge connection and the antisymmetric tensor field. The remaining component fields are defined as

$$W_\alpha| = -i\lambda_\alpha \quad (6)$$

$$D^\alpha W_\alpha| = -2D$$

$$R| = -\frac{1}{6}M$$

$$G_a| = -\frac{1}{3}b_a$$

$$U| = c \quad (6)_{\text{ctd}}$$

$$\mathcal{D}_\alpha U| = \phi_\alpha$$

The supermultiplet thus described is a reducible 20+20 multiplet

$$(e_m^a, \psi_m^\alpha, a_m, t_{mn}, \lambda_\alpha, D, M, b_a, C, \phi_\alpha) \quad (7)$$

It is reducible since it contains, among other things, the irreducible Abelian vector supermultiplet (a_m, λ_α, D) . The irreducible new minimal supergravity multiplet can be obtained from (7) by imposing the additional constraint

$$U = 0 \quad (8)$$

It follows from the Bianchi identities that M , C , and ϕ_α vanish, and b_a becomes a function of e_m^a , ψ_m^α and t_{mn} . Furthermore

$$\lambda_\alpha = -\frac{i}{2} (G^{mn} \psi_{mn})_\alpha - \frac{3}{8} \psi_{m\alpha} \tilde{G}^m| \quad (9)$$

$$D = +\frac{1}{4} R_{ab}{}^{ab}| - \frac{3}{8} \tilde{G}^m \tilde{G}_m|$$

where $\phi_{mn}^\alpha = \mathcal{D}_m \psi_n^\alpha - \mathcal{D}_n \psi_m^\alpha$,

$$\tilde{G}^m| = \cancel{\psi}^m + \frac{i}{2} \epsilon^{mnkl} \psi_n G_k \bar{\psi}_l \quad (10)$$

and

$$\cancel{\psi}^m = \frac{1}{2} \epsilon^{mnkl} \partial_n t_{kl} \quad (11)$$

Hence, λ_α and D are no longer independent fields. We are left with the irreducible 12+12 new minimal supergravity multiplet

$$(e_m^a, \psi_m^\alpha, a_m, t_{mn}) \quad (12)$$

The superfield action for the new minimal supergravity multiplet is most easily expressed in terms of a prepotential Abelian vector superfield V , defined by

$$A_\alpha = \mathcal{D}_\alpha V \quad (13)$$

where A_α is the spinorial component of the $U(1)$ supergauge connection. It can be shown that, in Wess-Zumino gauge, V corresponds to the component field supermultiplet

$$(a_m, \lambda_\alpha, D) \quad (14)$$

where λ_α and D are given in (9). Furthermore

$$W_\alpha = -\frac{1}{4} \bar{\mathcal{D}}^2 \mathcal{D}_\alpha V \quad (15)$$

and, hence, W_α is the field strength for this vector supermultiplet. The new minimal supergravity Lagrangian is given by

$$\mathcal{L}_{SG} = \frac{1}{2} \int d^2\theta \mathcal{E} \bar{\mathcal{D}}^2 V + h.c. \quad (16)$$

where $\mathcal{E}$ is a density superfield. In terms of component fields, $\mathcal{L}_{SG}$ is found to be

$$\mathcal{L}_{SG}/e = -\frac{1}{2} R + \frac{1}{4} \epsilon^{\ell mn} (\bar{\Psi}_\ell \bar{\mathcal{E}}_m \psi_{mn} - \psi_\ell \mathcal{E}_m \bar{\Psi}_{mn}) + 2a_m \not{x}^m + \frac{3}{4} \tilde{G}^m \tilde{G}_m \quad (17)$$

where $\tilde{G}^m$ and $\not{x}^m$ are defined in (10) and (11) respectively. It is convenient to elevate $\not{x}^m$ to the status of an independent field. This can be accomplished by adding to $\mathcal{L}_{SG}/e$ the term

$$\mathcal{L}_{LM}/e = -(\partial_m C') \not{x}^m \quad (18)$$

where C' is a Lagrange multiplier scalar field. Since $\partial_m \not{x}^m$ no longer vanishes, $\mathcal{L}_{SG} + \mathcal{L}_{LM}$ will be $U(1)$ gauge invariant if and only if C' transforms as

$$\delta C' = 2\Lambda \quad (19)$$

under $U(1)$ gauge transformations. Putting everything together, the complete Lagrangian for the new minimal supergravity multiplet can be written as

$$\frac{\mathcal{L}_{SG} + \mathcal{L}_{LM}}{e} = -\frac{1}{2} R + \frac{1}{4} \epsilon^{\ell m n} (\bar{\psi}_\ell \bar{\epsilon}_L \psi_{mn}^{C'} - \psi_\ell \epsilon_L \bar{\psi}_{mn}^{C'})$$

$$+ 2a_m \tilde{G}^m + \frac{3}{4} \tilde{G}^m \tilde{G}_m - (\partial_m C') \tilde{G}^m \quad (20)$$

where the superscript C' means that the U(1) gauge connection a_m has been replaced by $\frac{1}{2} \partial_m C'$ in the covariant derivatives in ψ_{mn} . Note that the auxiliary field structure in the Lagrangian of new minimal supergravity, the last three terms in (20), is identical to the auxiliary field structure for a massive Abelian vector field, the last three terms in (3). However, unlike Lagrangian (3), the new minimal supergravity Lagrangian does not contain a kinetic energy term for a_m . That is, in Lagrangian (20), a_m is an auxiliary field. The a_m equation of motion is

$$\tilde{G}^m = 0 \quad (21)$$

and, therefore, the last three terms in (20) vanish on-shell. If, for some reason, a kinetic energy for a_m was generated in the new minimal supergravity Lagrangian, then a_m would be massive. This is precisely what happens with the addition of the Chern-Simons term associated with a_m to the theory, as we will presently show. First, however, we must couple a linear multiplet to new minimal supergravity.

A linear supermultiplet can be added to the theory by introducing a new two-form supergauge potential $L = \frac{1}{2} dz^M dz^N L_{NM}$. The associated field strength is defined by

$$H = dL \quad (22)$$

The number of independent fields can be greatly reduced by subjecting H to a set of constraints, one of which is

$$H_{\gamma\beta\alpha} = 2i(\sigma_\alpha)_{\gamma\beta} \ell \quad (23)$$

where ℓ is an hermitian superfield of vanishing U(1) weight. Solving the Bianchi identities subject to these constraints, one finds, among other things, that

$$\bar{D}^2 \ell = D^2 \ell = 0 \quad (24)$$

and, hence, ℓ is a linear superfield. A gauge can be chosen so that

$$L_{MN}| = \begin{pmatrix} b_{mn} & 0 \\ 0 & 0 \end{pmatrix} \quad (25)$$

thus defining an antisymmetric tensor field. The remaining component fields are defined by

$$\ell| = B \quad (26)$$

$$\partial_\alpha \ell| = t_\alpha$$

The supermultiplet thus described is an irreducible linear multiplet

$$(B, b_{mn}, t_\alpha) \quad (27)$$

The superfield Lagrangian for the coupling of this linear supermultiplet to new minimal supergravity is given by

$$\mathcal{L}_L = \int d^2\theta d\bar{\theta}^2 f(\ell) + h.c. \quad (28)$$

where f is an arbitrary real function. The component field expression for $\mathcal{L}_L$ can be found [3,4] and added to $\mathcal{L}_{SG} + \mathcal{L}_{LM}$. However, as far as Abelian vector field a_m is concerned, nothing new is learned since a_m and $\tilde{G}^m$ remain auxiliary fields. We are now in a position to add the Chern-Simons term associated with a_m to the theory.

From the $U(1)$ supergauge connection A , one can define its field strength

$$F = dA \quad (29)$$

The superfield Chern-Simons term associated with A is the three-form

$$\Omega = AF \quad (30)$$

Now reconsider the supergauge potential L and its field strength H given in (22). We modify H by adding to it the Chern-Simons three-form. The new expression for H is

$$H = dL + k\Omega \quad (31)$$

where k is, in general, an arbitrary constant. One imposes on the modified field strength H the same constraints as imposed before adding the Chern-Simons term, for example

$$H_{\gamma\dot{\beta}\alpha} = 2i(\epsilon_{\alpha})_{\gamma\dot{\beta}} \ell \quad (32)$$

We find, however, that the addition of the Chern-Simons term greatly modifies the solution of the Bianchi identities. One such modification is that Eq. (24) now becomes

$$\bar{D}^2 \ell = 2k W^\alpha W_\alpha \quad (33)$$

$$D^2 \ell = 2k \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$$

where W^α is defined in (15). Hence, ℓ is no longer a linear superfield. Be that as it may, the superfield Lagrangian for the coupling of the linear supermultiplet to new minimal supergravity is still given by

$$\mathcal{L}_L^{c-s} = \int d^2\theta \mathcal{E} \bar{D}^2 f(\ell) + h.c. \quad (34)$$

since ℓ has vanishing $U(1)$ weight. It follows from (24), (28), and (33) that

$$\mathcal{L}_L^{c-s} = \mathcal{L}_L + [2k \int d^2\theta \mathcal{E} f^{(1)}(\ell) W^\alpha W_\alpha + h.c.] + \dots \quad (35)$$

The second term on the right-hand side of (35) is immediately recognized as the superfield kinetic energy for the Abelian vector supermultiplet V defined in (13). In terms of component fields

$$\mathcal{L}_L^{c-s}/e = \dots - 4k f^{(1)}(0) \left[\frac{1}{2} (F_{mn})^2 + i(\lambda \epsilon^m \partial_m \bar{\lambda} - \partial_m \lambda \epsilon^m \bar{\lambda}) - D^2 \right] + \dots \quad (36)$$

Note that the $(F_{mn})^2$ term is a ghost free kinetic energy for the real vector field a_m . Therefore, among other things, the addition of the superfield Chern-Simons term associated with A to the theory metamorphizes the auxiliary field a_m into a physical, propagating field. Hence, a_m can no longer be eliminated from the theory. The reason for this is the following. In component fields, one could try to add the bosonic Chern-Simons term $A_{[k} F_{mn]}$ to $\partial_{[k} b_{mn]}$. This, however, would violate local supersymmetry unless appropriate fermionic and bosonic "superpartners" of $A_{[k} F_{mn]}$ are simultaneously introduced into the theory. The simplest way to determine the structure of these extra terms is to use the superfield formalism, as we have done. In this paper, we have demonstrated that the $(F_{mn})^2$ term in (35) is one such superpartner of $A_{[k} F_{mn]}$. That is, the kinetic energy term for a_m in $\mathcal{L}_L^{c-s}$ is simply part of the component field, supersymmetric Chern-Simons term.

Adding (35) to (20), one finds that

$$\begin{aligned} \frac{\mathcal{L}_{\text{SE}} + \mathcal{L}_{\text{LM}} + \mathcal{L}_L^{c-s}}{e} = & -\frac{1}{4} (8k f^{(1)}) (F_{mn})^2 + 2\alpha_m \tilde{G}^m | \\ & + \frac{3}{4} \tilde{G}^m \tilde{G}_m | - (\partial_m c') \tilde{G}^m | + \dots \end{aligned} \quad (37)$$

Comparing this expression to (3), we conclude that Lagrangian (36) describes, among other things, a physical, propagating real vector field a_m with mass

$$M = \frac{1}{\sqrt{3k f^{(1)} \langle B \rangle}} \quad (38)$$

in Planck units, where $\langle B \rangle$ denotes the vacuum expectation value of B . The situation is reminiscent of topological mass generation in 2+1 dimensions. In that case, one starts with the Lagrangian, $-1/4(F_{mn})^2$, for a physical, propagating Abelian or non-Abelian massless gauge field, and adds to it the associated Chern-Simons term. It is then shown that this induces a massive pole in the gauge field propagator. In the 3+1 dimensional new minimal supergravity theory discussed in this paper, the situation is reversed. One starts with a Lagrangian which contains a mass term for an auxiliary Abelian vector field and adds to it the appropriately supersymmetrized associated Chern-Simons term. It is then shown that this induces a kinetic energy term proportional to $(F_{mn})^2$ in the Lagrangian. Hence, the theory describes a physical, propagating, massive Abelian gauge field.

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