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IS THERE A QCD "SPIN CRISIS?"

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ABSTRACT

A suitable extension of the Goldberger Treiman relation for the singlet axial current is proved, within the standard QCD resolution of the U(1) problem. No appeal is made to large N_C or quenched approximations. Implications for the so-called "spin crisis", are discussed.

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1. INTRODUCTION

The recent European Muon Collaboration (EMC) data [1] on deep inelastic scattering of polarized muons off polarized protons would imply, at first sight, that the proton spin is not accounted for by its quarks [2].

Let us define, in standard notations:

$$\langle p | \bar{q}_i \gamma_\mu \gamma_5 q_i | p \rangle = G_A^{(i)}(q^2) \bar{p} \gamma_\mu \gamma_5 p + G_P^{(i)}(q^2) q_\mu \bar{p} \gamma_5 p \quad (1)$$

$i=u,d,s$; q = momentum carried by the axial current.

The $q^2 = 0$, non-singlet combinations:

$$\begin{aligned} G_A &= G_A^{(3)} = G_A^{(u)} - G_A^{(d)}, \\ G_A^{(8)} &= G_A^{(u)} + G_A^{(d)} - 2 G_A^{(s)} \end{aligned} \quad (2)$$

have been known experimentally for sometime (possibly with underestimated errors [3,4]):

$$G_A = 1.254 \pm 0.006$$

$$G_A^{(8)} = G_A (3F-D)/(F+D) = 0.685 \pm 0.08 \quad (3)$$

where flavour SU(3) has been used to define and determine F/D [5].

The EMC data [1], together with small QCD evolution effects [6] determine (modulo small-x extrapolations [4]) a third combination:

$$1/9 (4G_A^{(u)} + G_A^{(d)} + G_A^{(s)}) = 0.228 \pm 0.024 \pm 0.052 \quad (4)$$

which, together with (3), implies:

$$G_A^{(1)} = G_A^{(u)} + G_A^{(d)} + G_A^{(s)} = 0.00 \pm 0.24 \quad (5)$$

Eq.(5) is usually referred to as the proton "spin crisis". This naive conclusion has to be reconsidered carefully within the framework of QCD, because of the famous Adler-Bell-Jackiw axial anomaly. It is well known [7] that, precisely because of the anomaly, gauge invariant axial currents do not usually measure spin and/or chirality. Only suitable, anomaly-free combinations do.

Some authors have argued [8-10] that the anomaly still allows one to talk about spin, provided a gluonic component is added to that of the quarks. This interpretation seems to still present problems, however, in particular the gluon "spin" appears to have the wrong sign [11] and to have a large isotriplet component [12]. As we shall argue, these puzzling results simply tell us that it is misleading to talk about spin and to use one's intuition about it in the context of $G_A^{(1)}$ (for other discussions on these delicate issues, see, for instance refs. [13]).

In this note we shall prove that $G_A^{(1)}$, rather than being related to spin, is connected to another "physical" quantity, through a generalization of the well-known Goldberger Treiman (GT) relation. We shall claim, in particular, that eq.(5), if taken at face value, implies the on-shell decoupling from NN of the flavour-singlet Nambu-Goldstone (NG) boson $\eta_1^{(0)}$ associated with the OZI-conserving limit of massless QCD.

This limit, that we shall refer to as OZIQCD, is defined as the gauge-invariant approximation in which no purely gluonic intermediate states are allowed in any channel of the correlation functions of fermionic currents. This is illustrated in fig.1a for two axial currents. In OZIQCD there is no U(1) anomaly and the OZI rule is, of course, exact. Note that this definition does not rely on either a large N_C or a quenched-QCD limit.

In three-flavour (massive) OZIQCD there are nine, unmixed (pseudo) NG bosons π_{ij} (fig1b). The OZI violating diagrams lift up the singlet's mass (if QCD is the correct theory). Our GT-like relation connects the experimental quantity $G_A^{(1)}$ to the coupling of $\eta_1^{(0)} = (3)^{-1/2} \sum_i \pi_{ii}$ to the nucleons by:

$$G_A^{(1)} = (F_\pi/2M_N) G(\eta_1^{(O)} NN) (2N_F)^{1/2} \\ + \text{quark mass/SU(3) breaking corrections, } (N_F = 3). \quad (6)$$

We shall first derive (6) in the massless limit and then argue about the smallness of the corrections. We stress that no large N_C or quenched limit will be assumed in deriving (6). Such limits can be considered [12,14], however, and, as explained at the end of the paper, sharpen the prediction....as much as to (almost) contradict the data.

The fact that experimental observables may get related to "theoretical" ones is not new in QCD. In the large N_C/N_F limit, the η' mass can be related [15,16] to the topological susceptibility of the quarkless (pure glue) theory, a quantity that can only be "measured"...on the computer.

2. THE CHIRAL LIMIT

In the chiral limit the anomalous-divergence equation, together with the absence of a flavour-singlet NG boson, imply:

$$2M_N G_A^{(1)} = 2N_F \langle p | Q | p \rangle, \\ Q = \alpha / 8\pi \text{Tr} (F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}) = \partial_\mu K^\mu = \\ = \text{topological charge density}. \quad (7)$$

In OZIQCD, instead, the GT relation holds for the U(1) current as well and gives:

$$2M_N G_A^{(1)(O)} = F_\pi G^{(O)}(\eta_1^{(O)} NN) (2N_F)^{1/2} \quad (8)$$

where the superscript (O) means that these are OZIQCD quantities.

The relation (6) we want to prove, illustrated in fig.2, is a variant of (8) since it contains $G_A^{(1)}$ and $G(\eta_1^{(O)} NN)$, the coupling of $\eta_1^{(O)}$ to the nucleons including OZI-violations.

The simplest proof we have found of (6) uses a QCD effective action Γ , which includes nine NG bosons fields U_{ij} , the baryon octet B and the field Q (or, better, the ghost [16] associated with K^μ).

Separating out the part which is quadratic in the bosonic fields, we get for Γ the general structure [17]:

$$\begin{aligned} \Gamma = & 1/2 \text{tr}(\partial_\mu U \partial_\mu U^\dagger) + c/6 (\partial_\mu \text{tr} U)(\partial_\mu \text{tr} U^\dagger) + \\ & + i/2 Q \text{tr} \log(U/U^\dagger) + Q^2/(2A(q^2)) + \Gamma_{\text{int}}(U, Q, B) \quad , \\ U U^\dagger = & 1/2 F_\pi^2 \mathbf{1} \quad , \quad F_\pi \cong 95 \text{ MeV}. \end{aligned} \quad (9)$$

Note that OZI violations are included both in the kinetic term of U (term proportional to c) and, implicitly, in $\Gamma_{\text{int}}(U, Q, B)$. $A(0)$ is the one-NG-irreducible topological susceptibility, not necessarily the same, numerically, as the one which enters the small N_F/N_C solution of the $U(1)$ problem [15,16].

We should not regard (9) as just a low energy phenomenological lagrangian, but rather as a fundamental QCD effective action for composite operators [18]. In this optics, Γ is obtained by Legendre transforming the generator $W(J, B)$ of connected Green's functions (for convenience the Legendre transform in B is already implied in W). It is straightforward to show that the quantity on the r.h.s. of (7) can be written as the bosonic propagator, computed from the quadratic part of Γ , times the irreducible vertex computed from Γ_{int} .

The main observation at this point is that the relevant matrix elements of the bosonic propagator at $q=0$ are known from general considerations. This will allow us to establish eq.(6).

Indeed, from (9), the propagator in the flavour singlet channel is a two-by-two matrix ($\eta_1^{(0)}$ and the ghost are in the first and second row/column respectively) reading:

$$K^{-1} = ((1+c)q^2 - 2N_F F_\pi^{-2} A(q^2))^{-1} \begin{pmatrix} 1 & , & - (2N_F)^{1/2} F_\pi^{-1} A \\ - (2N_F)^{1/2} F_\pi^{-1} A & , & (1+c)q^2 A \end{pmatrix} \quad (10)$$

K^{-1} is non-singular at $q=0$, implying the resolution of the U(1) problem. The physical η_1 is easily found by diagonalizing K . Its mass m_1 is given by:

$$m_1^2 = 2N_F F_\pi^{-2} A(m_1^2)/(1+c) \quad (11)$$

and can be related to the topological susceptibility of Yang Mills theory [15,16] only in some limits (large N_C , small N_F) because of quark and/or non-planar loop effects and of the extrapolation needed from $q^2=0$ to $q^2=m_1^2$. One also recovers the well-known fact that the topological susceptibility $\langle Q Q \rangle$ vanishes in QCD owing to the zero in the ghost-ghost propagator in (10) at $q=0$.

What we need in (7) are the matrix elements of the propagator (10) which appear in the second row. The diagonal QQ propagator is zero, while the off-diagonal $Q-\eta_1^{(0)}$ propagator is just $(2N_F)^{-1/2} F_\pi$, independently of A . Both results can also be proved through Ward Identities. This gives immediately:

$$\langle Q | \bar{p}p \rangle = (2N_F)^{-1/2} F_\pi \langle \eta_1^{(0)} | \bar{p}p \rangle \quad (12)$$

Our claim (6) follows after inserting (12) in (7).

Similar results follow if we replace $|\bar{p}p\rangle$ by some other state, e.g. by $\Omega |0\rangle$, where Ω is some local operator. For several operators Ω the coupling to $\eta_1^{(0)}$ is dictated by current algebra to be:

$$(2N_F)^{1/2} F_\pi \langle \eta_1^{(0)} | \Omega | 0 \rangle = \xi_\Omega \langle 0 | \Omega | 0 \rangle \quad (13)$$

where ξ_Ω is the chirality of Ω . In this case we get back an exact, chiral limit result (known, e.g., for $\Omega = \bar{q}_R q_L$, $\xi_\Omega = 2$):

$$i d/d\theta \langle 0 | \Omega | 0 \rangle = \langle Q \Omega | 0 \rangle = \xi_\Omega \langle 0 | \Omega | 0 \rangle (2N_F)^{-1} \quad (14)$$

Reassured by this check, we claim our eq.(6) to be exact in the chiral limit. The reason why this should be so here, unlike in the η' mass case, is that we are interested in a matrix element at $q=0$ and that $\eta_1^{(0)}$ is strictly massless in the chiral limit.

Finally, we note that $\eta_1^{(0)}$ is the properly normalized field of the OZI approximation, while, after addition of OZI violations, the correctly normalized field becomes $\eta_{1R}^{(0)} = (1+c)^{1/2} \eta_1^{(0)}$. If $\eta_{1R}^{(0)}$ is used in eq(6), then F_π should be replaced by $(1+c)^{1/2} F_\pi$, corresponding to a renormalization of the singlet decay constant. Such factor, however, is precisely cancelled by an identical factor relating the coupling of $\eta_{1R}^{(0)}$ to that of $\eta_1^{(0)}$. This is why, in the following, we shall work with OZIQCD-normalized fields and drop c throughout.

3. QUARK MASSES AND SU(3) BREAKING

When quark masses are included, the matrix which mixes Q (the ghost) to the pseudo-NG bosons of OZIQCD becomes a four-by-four matrix reading:

$$K^{-1} = \begin{pmatrix} q^2 - \mu_u^2 & 0 & 0 & F_u^{-1} \\ 0 & q^2 - \mu_d^2 & 0 & F_d^{-1} \\ 0 & 0 & q^2 - \mu_s^2 & F_s^{-1} \\ F_u^{-1} & F_d^{-1} & F_s^{-1} & A^{-1} \end{pmatrix}^{-1} \quad (15)$$

where F_i (resp. μ_i) are the decay constants (resp. masses) of the i^{th} NG boson $\pi_i \equiv \pi_{ji}$ in OZIQCD. We have also set $c=0$ according to the discussion made in the previous section.

In the massive case eq. (7) becomes:

$$2M_N G_A^{(1)} = \langle p | 2N_F Q + \sum 2i m_i \bar{q}_i \gamma_5 q_i | p \rangle \quad (16)$$

The various terms in (16) can be computed separately by first coupling the operator to a state α (the ghost for Q, the corresponding OZI NG boson for each pseudoscalar density), by then inserting the propagator matrix $K^{-1}(q)_{\alpha\beta}$ obtained from (15) and, finally, by coupling the state β to the proton. The generic outcome of the calculation is complicated, but simplifies in the limit $\mu_{u,d}^2 \ll \mu_s^2$, $F^{-2}A$, where it gives ($q=0$, $N_F=3$, $F_u=F_d=1/2 F=2^{-1/2} F_\pi$):

$$\langle p | 2N_F Q | p \rangle = 3 F (\mu_u^2 + \mu_d^2)^{-1} \{ \mu_d^2 G(\pi_{u}pp) + \mu_u^2 G(\pi_{d}pp) \} \quad (17)$$

$$\langle p | 2i m_u \bar{u} \gamma_5 u | p \rangle = F (\mu_u^2 + \mu_d^2)^{-1} \{ \mu_u^2 G(\pi_{u}pp) - \mu_u^2 G(\pi_{d}pp) \} \quad (18)$$

$$\langle p | 2i m_d \bar{d} \gamma_5 d | p \rangle = F (\mu_u^2 + \mu_d^2)^{-1} \{ -\mu_d^2 G(\pi_{u}pp) + \mu_d^2 G(\pi_{d}pp) \} \quad (19)$$

$$\langle p | 2i m_s \bar{s} \gamma_5 s | p \rangle = - F (\mu_u^2 + \mu_d^2)^{-1} \{ \mu_d^2 G(\pi_{u}pp) + \mu_u^2 G(\pi_{d}pp) \} + F_s G(\pi_s pp) \quad (20)$$

It is a simple matter to check that (18-20) give back the correct GT relations for the non-singlet axial currents. On the other hand, each one of the four contributions is clearly very sensitive to the ratios of quark masses even when these are small. This is why, if we call (17) the "spin" of the gluon and (18-20) the spin of each quark flavour inside the proton we get unintuitive results, including a large isospin violation [12] for the "spin" of the gluon (or of each quark flavour) in the nucleon.

If, instead, we stick to what we really measure, the sum of (17) through (20), the result simplifies considerably and becomes much less sensitive to the ratio of quark masses. For $F_s = F_{u,d}$ we get the particularly simple result:

$$\begin{aligned} \langle p | 2N_F Q + \sum 2i m_i \bar{q}_i \gamma_5 q_i | p \rangle &= 2^{1/2} F_\pi \{ G(\pi_u pp) + G(\pi_d pp) + G(\pi_s pp) \} \\ &= (2N_F)^{1/2} F_\pi G(\eta_1^{(0)} pp) \end{aligned} \quad (21)$$

i.e. the same as the massless result (6). Which corrections to (21) should we expect?

First of all, because of $F_s \neq F_{u,d}$ there will be a small correction to the, already small, strange quark contribution. The second source of error is the use of pole dominance (PCAC) for the strange-quark axial current. It is harder to follow carefully how these corrections propagate through our derivation since they do not, in general, factorize. Still we believe the most important part of the error to be in the strange current matrix elements, hence not very important.

We conclude that, in order to explain the EMC data, we have to invoke a non-negligible contribution to $G(\eta_1^{(0)}NN)$ from the OZI-rule violating diagrams which contribute just to the singlet coupling. Since the OZI-conserving couplings must satisfy:

$$G^{(0)}(\eta_1^{(0)}NN) = 2^{1/2} G(\eta_8 NN) \quad (22)$$

we find:

$$G(\eta_1^{(0)}NN)_{\text{OZIviol.}} \cong -0.44 G(\pi NN) \quad (23)$$

which is perhaps not unreasonable for a channel which, notoriously, has large OZI-rule violations.

It is now a simple exercise to insert (23) back into eqs.(17-20). One finds the announced "wrong" sign and large isospin violations for the artificially defined gluon and quark "spins". Here we give the numbers (divided by an overall $2M_N$ factor), putting in parenthesis the corresponding couplings to the neutron:

$$\begin{aligned} \langle p | 2i m_u \bar{u} \gamma_5 u | p \rangle &\cong 0.45 \ (-0.45) \ , \quad \langle p | 2i m_d \bar{d} \gamma_5 d | p \rangle \cong -0.80 \ (0.80) \\ \langle p | 2i m_s \bar{s} \gamma_5 s | p \rangle &\cong -0.52 \ (-0.17) \ , \quad \langle p | 2N_F Q | p \rangle \cong 0.87 \ (-0.18) \end{aligned} \quad (24)$$

These are to be contrasted with the perfectly isospin conserving physical couplings:

$$G(\pi_{upp}) : G(\pi_{dpp}) : G(\pi_{spp}) = 1 : -0.69 : -0.31$$

$$G(\pi_{upp})=G(\pi_{dnn}), G(\pi_{dpp})=G(\pi_{unn}), G(\pi_{spp})=G(\pi_{snn}) \quad (25)$$

4. LARGE N_C AND QUENCHED LIMITS

What happens if we take a more drastic approximation to QCD? The large N_C approximation is the one in which the resolution of the $U(1)$ problem has been discussed at greater length [15-17]. The difficulty is that, by consistency, one should also take the large N_C limit of the proton (a proton consisting of N_C quarks). Although this limit has been studied as well [19], it is far from obvious that the large N_C predictions can be used for the physical 3 -valence-quark proton.

A better limit seems to be that of quenched QCD, i.e. the quark-loop expansion or the small N_F limit. The necessity of a light $U(1)$ particle can be also proved in that limit [16] since, after all, the anomaly and the η_1 mass scale like N_F/N_C . In this limit one can prove even more directly eq. (6) by a Witten-like [15] argument.

Indeed, eq (7) would seem to imply that $G_A^{(1)}$ is $O(N_F)$ relative to $G_A^{(3)}$. At small N_F (quenched limit) this would mean a very small $G_A^{(1)}/G_A^{(3)}$ which is, of course, what we would like to have! This argument has, however, a loophole, precisely because of the η_1 pole contributing to the right-hand side of eq.(7). Thus all we can prove in the small N_F limit is our relation (6).

The small N_F limit (quenched approximation), however, makes a further prediction, eq. (22), since OZI-violating diagrams are excluded. Thus we get the result (which could have been guessed a priori):

$$G_A^{(8)} = G_A^{(1)} \quad (26)$$

We do not see, actually, how such a conclusion can be avoided in

any conceivable large N_C limit, since any OZI-violation appears to be down by $1/N_C$. This conclusion is supported by direct, large N_C effective Lagrangian calculations [20,14].

Equation (25) appears to contradict the data by several standard deviations. It is, however, interesting to mention that the quenched approximation value for F/D has been recently found [21] by lattice calculations to be smaller than the experimental value ($F/D(\text{quenched}) = 0.46 \pm 0.06$). A small quenched F/D is also suggested by QCD sum rules [22]. If this is confirmed, the quenched approximation would give both $G_A^{(8)}$ and $G_A^{(1)}$ to be 0.30 ± 0.10 , hence only one or two standard deviations off. It would then be up to quark loops and OZI-violations to correct the wrong common value in opposite directions.

5. CONCLUSIONS

Far from establishing a "spin crisis", the EMC data imply, within our present understanding of the $U(1)$ problem in QCD, an OZI-violating coupling of $\eta_1^{(0)}$ to the nucleons as given by eq.(23). This induces the approximate decoupling of the would-be $U(1)$ -NG-boson from the nucleon. This result should be exact in the chiral limit: it should have at least the same degree of validity as PCAC for kaons.

Our results do not rely on large N_C or quenched approximations, which inevitably lead to the "wrong" prediction (25). Thus, if we trust the experimental number (5), we have to find a dynamical reason for the suppression of the coupling of $\eta_1^{(0)}$ to the nucleons, possibly along the skyrmion model ideas suggested in ref.[2], but in the physical "limit" $N_C = N_F = 3$.

In any case, there is no reason to panic about a spin crisis... unless spin is incorrectly defined.

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FIGURE CAPTIONS

Fig.1. a) Typical diagram occurring in OZIQCD.

b) Pole dominance of the nine axial currents in OZIQCD

Fig.2. Illustration of our modified GT relation for the singlet current, eq. (6). The state $\Sigma_i \pi_{ii}$ is the singlet constructed out of the states of fig. 1b and it is massless in the chiral limit. Notice that OZI-violations are included in the coupling.

Fig. 1

Fig. 2