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PROBING THE HADRONIC STRUCTURE OF THE PHOTON AT TRISTAN

M. Drees

CERN - Geneva

and

R.M. Godbole

CERN - Geneva
and

Department of Physics, University of Bombay
Bombay, 400098, India

A B S T R A C T

We show that in the energy range covered by the e^+e^- collider TRISTAN inclusive two photon processes, where both the photons are almost on shell, begin to be dominated by effects of the hadronic structure of the photon. A study of multijet and heavy flavour production at TRISTAN can thus probe the quark and gluon content of the photon, and may provide us with the first clear evidence for hadronically interacting photons in processes other than in deep inelastic scattering.

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1. - INTRODUCTION

The study of two-photon reactions at e^+e^- experiments in order to test the quark-parton model (QPM) through measurements of the photon structure function was suggested [1] long ago. Witten's observation [2] that in the limit of infinite momentum transfer Q^2 the quark and gluon content of the photon is completely calculable in QCD seemed to imply that a measurement of the photon structure function F_2^γ will not just test the QPM, but will serve as an important test of QCD. This calculation of the "asymptotic" part of F_2^γ was later extended [3] to two-loop order by Bardeen and Buras. However, these "asymptotic" solutions of the evolution equations [4] for $\hat{q} \equiv (q_1, G)$ show unphysical divergences as $x \rightarrow 0$, which become worse in higher orders of perturbation theory [3,5]. This problem can be solved [6] by introducing input distributions $\hat{q}(x, Q_0^2)$ at some scale Q_0^2 , such that $F_2^\gamma(x, Q_0^2)$ is finite for all x ; in this case the evolution equations always lead to a well-behaved F_2^γ at different values of Q^2 .

Of course the introduction of unknown input functions makes it impossible to predict the absolute value of F_2^γ , thus destroying much of the original [2] excitement. This led Antoniadis and Grunberg [7] to suggest a regularization procedure, whereby parameters describing the unknown "hadronic" part of $\hat{q}$ are introduced such that the $x \rightarrow 0$ divergences are removed. In second order in perturbation theory this approach only introduces two unknown parameters instead of the three functions of the earlier approach. However, it is not a priori clear that the hadronic part of $\hat{q}(x, Q_0^2)$ is relevant only at small x ; rather, it might affect predictions at median and large x as well. In this context it is interesting to mention that the hadronic part of F_2^γ has recently been calculated [8] from QCD sum rules; it increases with x up to $x \approx 0.7$, where this approach becomes unreliable. Unfortunately it can also not be used for $x < 0.2$ and can thus not contribute to the solution of the small- x problem.

Yet another way [9] of separating the "asymptotic" and hadronic part of F_2^γ involves using the onset of high- p_T jet formation coming from the point-like interaction of the photon. This approach is formally equivalent to that of introducing [6,10] finite input distribution functions at Q_0^2 .

While the debate on theoretical issues involved in the prediction of F_2^γ is not yet closed, two-photon experiments at PEP, PETRA and TRISTAN have measured the photon structure function up to $Q^2 \approx 100 \text{ GeV}^2$. The data [11] can be fitted with different values of Λ_{QCD} depending on the way the hadronic parts of F_2^γ are taken into account. Even the naive QPM prediction [1] alone can fit the data on F_2^γ quite well if one uses reasonable constituent quark masses [11,12], $m_u = m_d = 300 \text{ MeV}$, $m_s = 500$

MeV. In view of this unclear experimental situation and the importance of the study of F_2^γ as a test of perturbative QCD more probes of the hadronic structure of the photon are clearly necessary. A good understanding of the photon structure is also important for other fields of high energy physics, e.g., for a possible explanation [13] of the large number of muons observed [14] in photon-initiated cosmic air showers.

In deep inelastic scattering (DIS) the invariant mass Q^2 of the probing photon is very large, $Q^2 \gg \Lambda^2_{\text{QCD}}$. As a result the cross sections are quite small, $O(\alpha^4_{\text{em}} / Q^2 \ln s/m_e^2)$. Furthermore, in leading order in α_s measurements of F_2^γ only probe the quark content q_1^γ of the photon. Alternative processes at e^+e^- colliders which can yield information on q_1^γ are jet production and heavy flavour production initiated by partons inside the photon. In such processes both photons are nearly on-shell; as a result the cross sections are of order $\alpha^4_{\text{em}} / p_T^2 (\ln s/m_e^2)^2$ and thus higher than in the DIS case. In these processes the parton densities are probed at momentum scales given by the transverse momentum of the jets, which are comparable to the scales that can be probed in DIS.

It has already been shown that in a number of QCD processes such as high- p_T jet production [15,16], heavy flavour production [15,17], direct photon production [18], and lepton pair production [19] at the ep collider HERA the hadronic structure of the photon is very important and in most cases even dominates. It also plays an important role [20,21] in fixed target photoproduction experiments at the highest achievable energies. All these processes can be used to test non-trivial aspects of our knowledge of the hadronic structure of the photon; conversely, an improved knowledge of q_1^γ would help to sharpen theoretical predictions for this large variety of processes.

Some discussion of jet production in e^+e^- collisions to order α^3_{em} and α^4_{em} already exists in the literature [22,23]. However, in these papers rather crude parametrizations for the "asymptotic" parton densities inside the photon were used, and the photon density inside the electron was computed in leading logarithmic approximation only. They also focus on energies either below [22] or above [23] the range relevant for TRISTAN. Finally, heavy quark production has so far not been discussed at all.

There are three different classes of diagrams, shown in Fig. 1, which can give rise to multijet production in two photon reactions. The contribution depicted in Fig. 1a, which we call "direct contribution" because the photons couple directly to the quarks produced in the hard scattering process, is completely calculable from QED and gives rise to dijet events. The diagrams in Figs. 1b and 1c indicate

contributions from processes where one or both of the photons are resolved into their hadronic constituents; we call them "once resolved" and "double resolved", respectively. They give rise to events with three or four jets, as shown in the topological graphs in Figs. 1b and 1c, respectively. Since the spectator jets coming from the fragments of the resolved photons will tend to go down the beam pipes, they might escape detection, in which case all three classes of contributions will give rise to events with similar topology, i.e. two high- p_T jets. It should be stressed that all three classes of diagrams are of the same order in α_{em} and α_s , since $\vec{q} \sim \ln Q^2 \sim 1/\alpha_s(Q^2)$; the resolved processes should therefore not be considered to be higher order QCD corrections to the direct process.

A possible background can arise from the annihilation process with an emission of a photon from any of the initial state fermion legs, as shown in Fig. 2. The cross section for this process is of order $\alpha_{em}^3 \ln s/m_e^2 \ln s' \ln(1-s'/s)$, s' being the invariant mass of the two jets, and can thus easily be comparable to the two-photon cross sections of Fig. 1 in certain regions of phase space. If the spectator jets are detected, this will be a background only for the direct process.

In this paper we concentrate on the situation where both photons are almost on-shell ("no-tag" situation, since the outgoing electron and positron are not seen) and discuss how these processes can be used to extract information about $\vec{q}(x, Q^2)$. Since these processes can probe $\vec{q}$ at values of x smaller than those that can be probed in DIS measurements, they are capable of discriminating between different models [21, 24] of photon structure functions, which differ mostly in the small- x region. Furthermore, processes involving real photons only are better suited to probe $\vec{q}(x, Q^2)$, about which almost nothing is known experimentally. Finally, even if it turns out to be impossible to extract precise quantitative information on $\vec{q}$ from these two photon experiments, the very observation of resolved photons in processes other than in DIS would be a major achievement, comparable to the step from the observation of scaling in DIS off nucleons to the observation of jets and Drell-Yan lepton pairs in nucleon-nucleon collisions.

The rest of this paper is organized as follows. In Section 2 we set up the formalism for calculating jet cross sections in two photon reactions, and discuss the p_T distribution of these jets. We see that in the energy range covered by TRISTAN the resolved contributions begin to dominate the direct one for $p_{T\perp} < 4$ GeV; the step from PETRA to TRISTAN energies thus turns out to be very significant for our purposes. In Section 3 we discuss the invariant mass distribution of the two high p_T jets. Due to a dynamical enhancement of certain double-resolved contributions compared to the direct and once-resolved ones, resolved contributions can dominate for di-jet invariant masses as big as 20 GeV. In Section 4 we show

how the rapidity-distribution of the two high- p_T jets can be used to isolate single and double resolved contributions; the importance of being able to tag on spectator jets is also discussed. In Section 5 we discuss heavy quark production. The rates for $b\bar{b}$ production are probably too low to be useful, but open charm production might prove a useful tool to measure $\vec{q}$. We also find rates of 10 to 30 ev/yr for dileptons from J/ψ production, assuming an integrated luminosity of 100 pb⁻¹ per year; in spite of the somewhat small rate, this process is interesting since the cross section is directly proportional to $\vec{q}$. Finally, in Section 6 we summarize our results and draw some conclusions.

2. TRANSVERSE MOMENTUM DISTRIBUTION OF MULTIJET EVENTS

The general expression for the cross section for jet production by photon-photon collisions in e^+e^- reactions is given by

$$\frac{d\sigma^{jj}}{d\vec{p}_T} = \frac{2}{4\vec{p}_T^2/s} \int d^4x_1 \int d^4x_2 f_{\gamma/e}(\vec{E}, x_1) \int d^4x_3 \int d^4x_4 f_{\gamma/e}(\vec{E}, x_2) \cdot \quad (2.1)$$

$$\cdot \frac{1}{4\vec{p}_T^2} \frac{1}{x_1 x_2} \int d^4x_3 \int d^4x_4 \frac{1}{x_3 x_4} \frac{1}{x_1 x_2 x_3 x_4} \frac{d\sigma'}{d^4p_T}(\vec{s}', \vec{t}', u').$$

The above expression has been obtained in the Weizsäcker-Williams approximation [25] (WWA, also called "equivalent photon approximation"). Here $s=2E$ is the total invariant energy, where E is the beam energy; p_T denotes the transverse momentum of the jets, and $s'=x_1 x_2 x_3 x_4 s$ is the squared invariant energy going into the hard subprocesses which are described by the cross section σ' . The $f_{\gamma/e}(\vec{E}, x_i)$ are the photon flux factors, while $\vec{q}$ are the parton densities inside the photon.

In the given form Eq. (2.1) is valid for double-resolved processes. For the direct contribution, both factors of $\vec{q}(x_i)$ have to be replaced by $\delta(1-x_i)$. The subprocess is now $\gamma\gamma \rightarrow q\bar{q}$; the relevant sub cross section can, e.g., be found in [26]. In the single resolved case one has to replace either $\vec{q}(x_3)$ or $\vec{q}(x_4)$ by a delta function [27]. The subprocesses involved are $\gamma g \rightarrow q\bar{q}$ and $\gamma q \rightarrow gq$; the subprocess cross sections are available, e.g., in Ref. [21]. The eight subprocesses and corresponding cross sections relevant for the double resolved contributions are listed, e.g., in Ref. [28].

Since we do not distinguish between the various partons producing a jet the subprocess cross sections in Eq. (2.1) have to be symmetrized under $t' \leftrightarrow u'$ if the

final state partons are not identical. Finally, s' , t' and p_T^2 are related by

$$t' = -\frac{s'}{2} \left(1 + \sqrt{1 - 4p_T^2/s'} \right) \quad (2.2)$$

For all calculations of high- p_T jet cross sections in this and the following two chapters we assume that four massless flavours contribute, both in α_s and in the photonic parton densities unless stated otherwise. We use p_T^2 as scale in the running coupling constant α_s and the parton distribution functions. We use $\Lambda_{QCD}=0.4$ GeV throughout.

The choice of the photon flux factors and the parton densities inside the photon need to be discussed in some detail. The flux factor $f_{\gamma|e}(\bar{E}, z)$ represents the number of photons carrying a fraction z of the energy of the emitting electron. This function is often used in leading-log approximation, $f_{\gamma|e}(\bar{E}, z) = \alpha_{em}/(4\pi z) [1 + (1-z)^2] \ln \bar{E}/m_e$, with $\bar{E} = E_{beam}$. However, this has been shown [29] to overestimate the direct $\gamma\gamma \rightarrow q\bar{q}$ contribution. One can try to correct for this effect by choosing a smaller scale $\bar{E}$, e.g., half the invariant mass of the two high- p_T jets. Instead, we chose to include [30] non-leading terms in $f_{\gamma|e}$:

$$f_{\gamma|e}(\bar{E}, z) = \frac{\alpha_{em}}{4\pi z} \left\{ \left[1 + (1-z)^2 \right] \left(\ln \frac{\bar{E}}{m_e} - \frac{1}{2} \right) + \frac{z^2}{2} \left[\ln \frac{2(1-z)}{z} + 1 \right] + \frac{(2-z)^2}{2} \ln \frac{2(1-z)}{2-z} \right\} \quad (2.3)$$

where $\bar{E}$ is the beam energy and m_e the electron mass. This form has been shown [29] to give accurate results not only for total (direct) rates, but also for distributions. We had also found [15] this choice to give accurate results (within $\sim 10\%$) for the p_T -distribution of quarks produced in photon-gluon fusion at the ep collider HERA. This does, of course, not guarantee a similar level of accuracy when used to predict resolved contributions, but the errors should not exceed 20-30%; they are thus probably smaller than higher order QCD corrections.

As stated in the introduction there are currently only two parametrizations [21,24] of parton densities inside the photon available. The "asymptotic" distributions of Ref. [21] ("DO" parametrization) are strictly proportional to $\ln Q^2/\Lambda^2$; with our choice of $\Lambda=0.4$ GeV this parametrization alone falls short of existing data on F_2^{γ} , as can be seen from Fig. 3a. It has thus to be supplemented by

a "hadronic" contribution to the parton densities, which is normally estimated by using the vector meson dominance (VMD) principle. This means:

$$F_2^{\gamma}(x, Q^2) \Big|_{\text{hadronic}} = x \frac{4\pi}{f_\pi^2} F_2^{\rho}(x, Q^2), \quad (2.4)$$

where $f_\pi^2/4\pi \approx 2.2$. Usually κ is taken to be one; we find $4\pi\kappa/f_\pi^2 \approx 1$, i.e. [31] $\kappa \approx 2$, by requiring that the data on F_2^{γ} shown in Fig. 3a can be explained by the sum of DO and VMD contributions for our choice of q_{VMD}^{γ} . These we chose to have the same x -dependence as pionic parton densities; we further assumed u, d and s -quarks to contribute equally to the VMD-part of q_i^{γ} . This gives

$$q_{VMD}^{\gamma}(x) = \alpha_{em} \left[\frac{5}{16} \frac{1-x}{\sqrt{x}} + \frac{1}{10} \frac{(1-x)^5}{x} \right]; \quad (2.5)$$

$$G_{VMD}(x) = \alpha_{em} \frac{2(1-x)^3}{x})$$

where the Q^2 -independent (scaling) parametrization of $\bar{q}^{\pi}$ found in Ref. 32 has been used.

Figure 3a shows that $F_2^{\gamma}(\text{DO}) + F_2^{\gamma}(\text{VMD})$ with $\Lambda=0.4$ GeV fits the data reasonably well. However, we must realize that such an addition of these two contributions is completely phenomenological [33]; in particular, since our q_{VMD}^{γ} are taken to be independent of Q^2 we may overestimate its effect at large x and higher Q^2 .

The parametrization of Ref. [24] ("DG") have been obtained by choosing input distributions $\bar{q}^{\gamma}(x, Q_0^2=1\text{GeV}^2)$ such that older data [34] on F_2^{γ} at $Q^2=5$ GeV² were reproduced. As can be seen from Fig. 3a it also fits more recent data [35] quite well.

Obviously the two parametrizations used give similar results for q_i^{γ} over most of the x -region; they do differ, however, quite strongly in the region of small x (<0.05), mainly due to the small- x divergence present in the DO parametrization. They also differ strongly in the gluon content of the photon, as seen in Fig. 3b. Due to the large VMD-contribution to G^{γ} , this difference extends to large values of x . Recall that G^{γ} enters F_2^{γ} only in second order in QCD, so that the differences

shown in Fig. 3b only manifest themselves through the small- x behaviour of F_T^Y .

We are now ready to present some numerical results. In Fig. 4 we show $d\sigma(jj)/dp_T$ at $P_T = 3$ GeV for the DG parametrization as a function of the centre-of-mass energy $\sqrt{s}$. We chose this somewhat high value of p_T in order to make sure that we can neglect the contribution from soft, diffractive processes, which falls off exponentially with p_T . We see that even though the direct process dominates all resolved contributions at PETRA energies the latter add up to about 30 pb/GeV already at $\sqrt{s} = 40$ GeV. This implies that PETRA data should contain several thousand multijet events coming from the hadronic structure of the photon. In this context it is worth remembering [11] that large- p_T jet data of all PETRA experiments showed an excess over the expectations from the direct contribution; this excess was removed by the requirement that the final state contains exactly two jets. The resolved contributions will of course always give rise to additional spectator jets and are thus removed by such a cut.

Notice that resolved processes become more important as $\sqrt{s}$ increases. There are two reasons for this. Firstly, the resolved processes depend [20] more strongly on the available phase space than the direct one, since obviously a parton taken out of a photon will always have less energy than the photon itself; mathematically the cross section formula for the resolved processes contains one or two additional convolutions with more or less soft parton distribution functions. This also explains why for sufficiently large values of $p_T/\sqrt{s}$ single resolved processes dominate over double-resolved ones.

Secondly, the direct and once-resolved processes proceed via the exchange of quarks in the hard scattering, whereas some double-resolved processes get contributions where a gluon is exchanged in the t - or u -channel. This leads to an enhancement factor of roughly $s'/2p_T^2$ for these double resolved processes; this factor obviously tends to increase with increasing s .

As a result, the TRISTAN energy range is ideally suited to study high- p_T jet events in order to get information about the parton content of the photon. Even for the DG-parametrization, double resolved processes become dominant for $\sqrt{s} \gtrsim 60$ GeV. The background (dotted curve) due to the bremsstrahlung process of Fig. 2 has been computed in the leading-log approximation of Ref. [23]. We see that it is very small even at PETRA energies and falls with $\sqrt{s}$; this is due to the cut imposed on this background, $M_{jj} < 30$ GeV, which forces the emitted photon to be increasingly hard. This cut will be justified in more detail in Sect. 3.

In principle it should be possible to "isolate" the resolved contributions in Fig. 4 by subtracting the direct contribution, which is completely calculable from QED. Another possibility is to just focus on the energy-dependence of the cross

section : whereas the direct contribution increases by only 50% when going from $\sqrt{s} = 40$ GeV to 60 GeV, the total cross section is expected to double even if the DG parametrization is used, reaching 130 pb/GeV at $\sqrt{s} = 60$ GeV; for the DO + VMD parametrization this cross section is as big as 220 pb/GeV, only 1/5 of which is due to the direct process !

From Fig. 4 one might be tempted to conclude that the possibility to probe the photon structure via real photon-photon scattering improves with $\sqrt{s}$ forever, so that LEP and SLC would be even better suited than TRISTAN for this kind of investigation. This is, however, not clear. On or close to the Z-pole the most dangerous background comes no longer from bremsstrahlung events, but from annihilation events with multijet final states, some of which are sufficiently close to the beam pipes to lead to a substantial loss of energy. Since the total hadronic cross section is two orders of magnitude above the most optimistic prediction for the two-photon cross section at $P_T = 3$ GeV, it might be very hard to extract the two-photon signal. If forward-backward jets can be detected, these annihilation events will be a background only to the more interesting resolved photon signal.

In Figs. 5a, b, we show the p_T -distribution $d\sigma(jj)/dp_T$ at $\sqrt{s} = 60$ GeV for the DG and DO + VMD parametrization, respectively. We see that for $p_T \lesssim 4$ GeV the expected cross section for DO + VMD is almost a factor of two higher than that for DG, which again reflects the excess of the former parametrizations at low x . The same arguments that explain the faster increase of the resolved distributions with energy for a fixed p_T also explain their steeper p_T spectrum. The bremsstrahlung background has a much flatter p_T -spectrum, as expected for an annihilation process. As a result, the total two-photon signal falls below the background for $p_T \lesssim 8-9$ GeV, depending on the distribution functions used. Note that for $p_T \gtrsim 12$ GeV the signal rates become unobservably low anyway.

We can thus tentatively conclude that the p_T -distribution of dijet-events at TRISTAN energies should be able to give us information about $q^Y(x, Q^2)$ down to rather small x -values ($x \gtrsim 0.01$) for $Q^2 \sim 10-100$ GeV². We now turn to a discussion of the invariant mass distribution of these events, which can be used to enhance [36] the double-resolved contribution.

3. INVARIANT MASS DISTRIBUTION OF HIGH p_T JETS

The invariant mass distribution of the high- p_T jets is given by the general formula

employed in Sect. 2, therefore removes a large fraction of this background, whereas the signal is already very small in the region $M_{jj} > 30$ GeV and therefore hardly changed by such a cut.

Notice that in the region of small M_{jj} , $M_{jj} \lesssim 15$ GeV, the cross sections for both double-resolved and single-resolved processes are quite high, 5-200 pb/GeV and 1-50 pb/GeV, respectively. The event rates should therefore be large enough to get more detailed information on $\vec{q}$. A study of more differential distributions is necessary for that purpose. We discuss this in the following section.

4. RAPIDITY DISTRIBUTION OF MULTIJET EVENTS

The rapidity distribution of two-jet events has been used [37] to extract parton densities inside the proton in $\bar{p}p$ collider experiments. It has been demonstrated [15] that these distributions can also be used to extract information on quark and gluon densities inside a photon at ep colliders. At $\bar{p}p$ colliders the triple differential cross section $d^3\sigma(jj)/dp_T dy_1 dy_2$ is directly proportional to products of parton densities at fixed x-values. In the present case this cross section is given by the general expression

$$\frac{d^3\sigma(jj)}{dp_T dy_1 dy_2} = 2 p_T y_m z_m \int \frac{dy}{y} f_{T1e}(\bar{E}, y) \vec{q} \left(\frac{y_m}{y} \right) \cdot \int \frac{dz}{z} f_{T1e}(\bar{E}, z) \vec{q} \left(\frac{z_m}{z} \right) \frac{d\sigma'}{dt'}(s', t', u'), \quad (4.1)$$

where

$$y_m = \frac{x_T}{2} (e^{y_1} + e^{y_2}), \quad z_m = \frac{x_T}{2} (e^{-y_1} + e^{-y_2}),$$

with

$$x_T = \frac{2 p_T}{\sqrt{s}}; \quad s' = y_m z_m s.$$

y_1 and y_2 are the rapidities of the two high- p_T jets. All other quantities are as in Eq. (2.1). To obtain the once resolved contribution one has to add two terms similar to Eq. (4.1) with either one of the two $\vec{q}$ factors replaced by a δ -function, whereas for the direct contribution both $\vec{q}$ have to be replaced by

$$\frac{d\sigma(jj)}{dM_{jj}} = \frac{2 M_{jj}}{s} \int \frac{dx_1}{M_{jj}^2/s} \int \frac{dx_2}{x_2} f_{T1e}(\bar{E}, x_1) \int \frac{dx_3}{x_3} f_{T1e}(\bar{E}, x_2) \cdot \int \frac{dx_4}{x_4} f_{T1e}(\bar{E}, x_3) \int \frac{dx_5}{x_5} f_{T1e}(\bar{E}, x_4) \cdot \int \frac{dx_6}{x_6} f_{T1e}(\bar{E}, x_5) \int \frac{dx_7}{x_7} f_{T1e}(\bar{E}, x_6) \cdot \int \frac{dx_8}{x_8} f_{T1e}(\bar{E}, x_7) \int \frac{dx_9}{x_9} f_{T1e}(\bar{E}, x_8) \cdot \int \frac{dx_{10}}{x_{10}} f_{T1e}(\bar{E}, x_9) \int \frac{dx_{11}}{x_{11}} f_{T1e}(\bar{E}, x_{10}) \cdot \int \frac{dx_{12}}{x_{12}} f_{T1e}(\bar{E}, x_{11}) \int \frac{dx_{13}}{x_{13}} f_{T1e}(\bar{E}, x_{12}) \cdot \int \frac{dx_{14}}{x_{14}} f_{T1e}(\bar{E}, x_{13}) \int \frac{dx_{15}}{x_{15}} f_{T1e}(\bar{E}, x_{14}) \cdot \int \frac{dx_{16}}{x_{16}} f_{T1e}(\bar{E}, x_{15}) \int \frac{dx_{17}}{x_{17}} f_{T1e}(\bar{E}, x_{16}) \cdot \int \frac{dx_{18}}{x_{18}} f_{T1e}(\bar{E}, x_{17}) \int \frac{dx_{19}}{x_{19}} f_{T1e}(\bar{E}, x_{18}) \cdot \int \frac{dx_{20}}{x_{20}} f_{T1e}(\bar{E}, x_{19}) \int \frac{dx_{21}}{x_{21}} f_{T1e}(\bar{E}, x_{20}) \cdot \int \frac{dx_{22}}{x_{22}} f_{T1e}(\bar{E}, x_{21}) \int \frac{dx_{23}}{x_{23}} f_{T1e}(\bar{E}, x_{22}) \cdot \int \frac{dx_{24}}{x_{24}} f_{T1e}(\bar{E}, x_{23}) \int \frac{dx_{25}}{x_{25}} f_{T1e}(\bar{E}, x_{24}) \cdot \int \frac{dx_{26}}{x_{26}} f_{T1e}(\bar{E}, x_{25}) \int \frac{dx_{27}}{x_{27}} f_{T1e}(\bar{E}, x_{26}) \cdot \int \frac{dx_{28}}{x_{28}} f_{T1e}(\bar{E}, x_{27}) \int \frac{dx_{29}}{x_{29}} f_{T1e}(\bar{E}, x_{28}) \cdot \int \frac{dx_{30}}{x_{30}} f_{T1e}(\bar{E}, x_{29}) \int \frac{dx_{31}}{x_{31}} f_{T1e}(\bar{E}, x_{30}) \cdot \int \frac{dx_{32}}{x_{32}} f_{T1e}(\bar{E}, x_{31}) \int \frac{dx_{33}}{x_{33}} f_{T1e}(\bar{E}, x_{32}) \cdot \int \frac{dx_{34}}{x_{34}} f_{T1e}(\bar{E}, x_{33}) \int \frac{dx_{35}}{x_{35}} f_{T1e}(\bar{E}, x_{34}) \cdot \int \frac{dx_{36}}{x_{36}} f_{T1e}(\bar{E}, x_{35}) \int \frac{dx_{37}}{x_{37}} f_{T1e}(\bar{E}, x_{36}) \cdot \int \frac{dx_{38}}{x_{38}} f_{T1e}(\bar{E}, x_{37}) \int \frac{dx_{39}}{x_{39}} f_{T1e}(\bar{E}, x_{38}) \cdot \int \frac{dx_{40}}{x_{40}} f_{T1e}(\bar{E}, x_{39}) \int \frac{dx_{41}}{x_{41}} f_{T1e}(\bar{E}, x_{40}) \cdot \int \frac{dx_{42}}{x_{42}} f_{T1e}(\bar{E}, x_{41}) \int \frac{dx_{43}}{x_{43}} f_{T1e}(\bar{E}, x_{42}) \cdot \int \frac{dx_{44}}{x_{44}} f_{T1e}(\bar{E}, x_{43}) \int \frac{dx_{45}}{x_{45}} f_{T1e}(\bar{E}, x_{44}) \cdot \int \frac{dx_{46}}{x_{46}} f_{T1e}(\bar{E}, x_{45}) \int \frac{dx_{47}}{x_{47}} f_{T1e}(\bar{E}, x_{46}) \cdot \int \frac{dx_{48}}{x_{48}} f_{T1e}(\bar{E}, x_{47}) \int \frac{dx_{49}}{x_{49}} f_{T1e}(\bar{E}, x_{48}) \cdot \int \frac{dx_{50}}{x_{50}} f_{T1e}(\bar{E}, x_{49}) \int \frac{dx_{51}}{x_{51}} f_{T1e}(\bar{E}, x_{50}) \cdot \int \frac{dx_{52}}{x_{52}} f_{T1e}(\bar{E}, x_{51}) \int \frac{dx_{53}}{x_{53}} f_{T1e}(\bar{E}, x_{52}) \cdot \int \frac{dx_{54}}{x_{54}} f_{T1e}(\bar{E}, x_{53}) \int \frac{dx_{55}}{x_{55}} f_{T1e}(\bar{E}, x_{54}) \cdot \int \frac{dx_{56}}{x_{56}} f_{T1e}(\bar{E}, x_{55}) \int \frac{dx_{57}}{x_{57}} f_{T1e}(\bar{E}, x_{56}) \cdot \int \frac{dx_{58}}{x_{58}} f_{T1e}(\bar{E}, x_{57}) \int \frac{dx_{59}}{x_{59}} f_{T1e}(\bar{E}, x_{58}) \cdot \int \frac{dx_{60}}{x_{60}} f_{T1e}(\bar{E}, x_{59}) \int \frac{dx_{61}}{x_{61}} f_{T1e}(\bar{E}, x_{60}) \cdot \int \frac{dx_{62}}{x_{62}} f_{T1e}(\bar{E}, x_{61}) \int \frac{dx_{63}}{x_{63}} f_{T1e}(\bar{E}, x_{62}) \cdot \int \frac{dx_{64}}{x_{64}} f_{T1e}(\bar{E}, x_{63}) \int \frac{dx_{65}}{x_{65}} f_{T1e}(\bar{E}, x_{64}) \cdot \int \frac{dx_{66}}{x_{66}} f_{T1e}(\bar{E}, x_{65}) \int \frac{dx_{67}}{x_{67}} f_{T1e}(\bar{E}, x_{66}) \cdot \int \frac{dx_{68}}{x_{68}} f_{T1e}(\bar{E}, x_{67}) \int \frac{dx_{69}}{x_{69}} f_{T1e}(\bar{E}, x_{68}) \cdot \int \frac{dx_{70}}{x_{70}} f_{T1e}(\bar{E}, x_{69}) \int \frac{dx_{71}}{x_{71}} f_{T1e}(\bar{E}, x_{70}) \cdot \int \frac{dx_{72}}{x_{72}} f_{T1e}(\bar{E}, x_{71}) \int \frac{dx_{73}}{x_{73}} f_{T1e}(\bar{E}, x_{72}) \cdot \int \frac{dx_{74}}{x_{74}} f_{T1e}(\bar{E}, x_{73}) \int \frac{dx_{75}}{x_{75}} f_{T1e}(\bar{E}, x_{74}) \cdot \int \frac{dx_{76}}{x_{76}} f_{T1e}(\bar{E}, x_{75}) \int \frac{dx_{77}}{x_{77}} f_{T1e}(\bar{E}, x_{76}) \cdot \int \frac{dx_{78}}{x_{78}} f_{T1e}(\bar{E}, x_{77}) \int \frac{dx_{79}}{x_{79}} f_{T1e}(\bar{E}, x_{78}) \cdot \int \frac{dx_{80}}{x_{80}} f_{T1e}(\bar{E}, x_{79}) \int \frac{dx_{81}}{x_{81}} f_{T1e}(\bar{E}, x_{80}) \cdot \int \frac{dx_{82}}{x_{82}} f_{T1e}(\bar{E}, x_{81}) \int \frac{dx_{83}}{x_{83}} f_{T1e}(\bar{E}, x_{82}) \cdot \int \frac{dx_{84}}{x_{84}} f_{T1e}(\bar{E}, x_{83}) \int \frac{dx_{85}}{x_{85}} f_{T1e}(\bar{E}, x_{84}) \cdot \int \frac{dx_{86}}{x_{86}} f_{T1e}(\bar{E}, x_{85}) \int \frac{dx_{87}}{x_{87}} f_{T1e}(\bar{E}, x_{86}) \cdot \int \frac{dx_{88}}{x_{88}} f_{T1e}(\bar{E}, x_{87}) \int \frac{dx_{89}}{x_{89}} f_{T1e}(\bar{E}, x_{88}) \cdot \int \frac{dx_{90}}{x_{90}} f_{T1e}(\bar{E}, x_{89}) \int \frac{dx_{91}}{x_{91}} f_{T1e}(\bar{E}, x_{90}) \cdot \int \frac{dx_{92}}{x_{92}} f_{T1e}(\bar{E}, x_{91}) \int \frac{dx_{93}}{x_{93}} f_{T1e}(\bar{E}, x_{92}) \cdot \int \frac{dx_{94}}{x_{94}} f_{T1e}(\bar{E}, x_{93}) \int \frac{dx_{95}}{x_{95}} f_{T1e}(\bar{E}, x_{94}) \cdot \int \frac{dx_{96}}{x_{96}} f_{T1e}(\bar{E}, x_{95}) \int \frac{dx_{97}}{x_{97}} f_{T1e}(\bar{E}, x_{96}) \cdot \int \frac{dx_{98}}{x_{98}} f_{T1e}(\bar{E}, x_{97}) \int \frac{dx_{99}}{x_{99}} f_{T1e}(\bar{E}, x_{98}) \cdot \int \frac{dx_{100}}{x_{100}} f_{T1e}(\bar{E}, x_{99}) \int \frac{dx_{101}}{x_{101}} f_{T1e}(\bar{E}, x_{100}) \cdot \int \frac{dx_{102}}{x_{102}} f_{T1e}(\bar{E}, x_{101}) \int \frac{dx_{103}}{x_{103}} f_{T1e}(\bar{E}, x_{102}) \cdot \int \frac{dx_{104}}{x_{104}} f_{T1e}(\bar{E}, x_{103}) \int \frac{dx_{105}}{x_{105}} f_{T1e}(\bar{E}, x_{104}) \cdot \int \frac{dx_{106}}{x_{106}} f_{T1e}(\bar{E}, x_{105}) \int \frac{dx_{107}}{x_{107}} f_{T1e}(\bar{E}, x_{106}) \cdot \int \frac{dx_{108}}{x_{108}} f_{T1e}(\bar{E}, x_{107}) \int \frac{dx_{109}}{x_{109}} f_{T1e}(\bar{E}, x_{108}) \cdot \int \frac{dx_{110}}{x_{110}} f_{T1e}(\bar{E}, x_{109}) \int \frac{dx_{111}}{x_{111}} f_{T1e}(\bar{E}, x_{110}) \cdot \int \frac{dx_{112}}{x_{112}} f_{T1e}(\bar{E}, x_{111}) \int \frac{dx_{113}}{x_{113}} f_{T1e}(\bar{E}, x_{112}) \cdot \int \frac{dx_{114}}{x_{114}} f_{T1e}(\bar{E}, x_{113}) \int \frac{dx_{115}}{x_{115}} f_{T1e}(\bar{E}, x_{114}) \cdot \int \frac{dx_{116}}{x_{116}} f_{T1e}(\bar{E}, x_{115}) \int \frac{dx_{117}}{x_{117}} f_{T1e}(\bar{E}, x_{116}) \cdot \int \frac{dx_{118}}{x_{118}} f_{T1e}(\bar{E}, x_{117}) \int \frac{dx_{119}}{x_{119}} f_{T1e}(\bar{E}, x_{118}) \cdot \int \frac{dx_{120}}{x_{120}} f_{T1e}(\bar{E}, x_{119}) \int \frac{dx_{121}}{x_{121}} f_{T1e}(\bar{E}, x_{120}) \cdot \int \frac{dx_{122}}{x_{122}} f_{T1e}(\bar{E}, x_{121}) \int \frac{dx_{123}}{x_{123}} f_{T1e}(\bar{E}, x_{122}) \cdot \int \frac{dx_{124}}{x_{124}} f_{T1e}(\bar{E}, x_{123}) \int \frac{dx_{125}}{x_{125}} f_{T1e}(\bar{E}, x_{124}) \cdot \int \frac{dx_{126}}{x_{126}} f_{T1e}(\bar{E}, x_{125}) \int \frac{dx_{127}}{x_{127}} f_{T1e}(\bar{E}, x_{126}) \cdot \int \frac{dx_{128}}{x_{128}} f_{T1e}(\bar{E}, x_{127}) \int \frac{dx_{129}}{x_{129}} f_{T1e}(\bar{E}, x_{128}) \cdot \int \frac{dx_{130}}{x_{130}} f_{T1e}(\bar{E}, x_{129}) \int \frac{dx_{131}}{x_{131}} f_{T1e}(\bar{E}, x_{130}) \cdot \int \frac{dx_{132}}{x_{132}} f_{T1e}(\bar{E}, x_{131}) \int \frac{dx_{133}}{x_{133}} f_{T1e}(\bar{E}, x_{132}) \cdot \int \frac{dx_{134}}{x_{134}} f_{T1e}(\bar{E}, x_{133}) \int \frac{dx_{135}}{x_{135}} f_{T1e}(\bar{E}, x_{134}) \cdot \int \frac{dx_{136}}{x_{136}} f_{T1e}(\bar{E}, x_{135}) \int \frac{dx_{137}}{x_{137}} f_{T1e}(\bar{E}, x_{136}) \cdot \int \frac{dx_{138}}{x_{138}} f_{T1e}(\bar{E}, x_{137}) \int \frac{dx_{139}}{x_{139}} f_{T1e}(\bar{E}, x_{138}) \cdot \int \frac{dx_{140}}{x_{140}} f_{T1e}(\bar{E}, x_{139}) \int \frac{dx_{141}}{x_{141}} f_{T1e}(\bar{E}, x_{140}) \cdot \int \frac{dx_{142}}{x_{142}} f_{T1e}(\bar{E}, x_{141}) \int \frac{dx_{143}}{x_{143}} f_{T1e}(\bar{E}, x_{142}) \cdot \int \frac{dx_{144}}{x_{144}} f_{T1e}(\bar{E}, x_{143}) \int \frac{dx_{145}}{x_{145}} f_{T1e}(\bar{E}, x_{144}) \cdot \int \frac{dx_{146}}{x_{146}} f_{T1e}(\bar{E}, x_{145}) \int \frac{dx_{147}}{x_{147}} f_{T1e}(\bar{E}, x_{146}) \cdot \int \frac{dx_{148}}{x_{148}} f_{T1e}(\bar{E}, x_{147}) \int \frac{dx_{149}}{x_{149}} f_{T1e}(\bar{E}, x_{148}) \cdot \int \frac{dx_{150}}{x_{150}} f_{T1e}(\bar{E}, x_{149}) \int \frac{dx_{151}}{x_{151}} f_{T1e}(\bar{E}, x_{150}) \cdot \int \frac{dx_{152}}{x_{152}} f_{T1e}(\bar{E}, x_{151}) \int \frac{dx_{153}}{x_{153}} f_{T1e}(\bar{E}, x_{152}) \cdot \int \frac{dx_{154}}{x_{154}} f_{T1e}(\bar{E}, x_{153}) \int \frac{dx_{155}}{x_{155}} f_{T1e}(\bar{E}, x_{154}) \cdot \int \frac{dx_{156}}{x_{156}} f_{T1e}(\bar{E}, x_{155}) \int \frac{dx_{157}}{x_{157}} f_{T1e}(\bar{E}, x_{156}) \cdot \int \frac{dx_{158}}{x_{158}} f_{T1e}(\bar{E}, x_{157}) \int \frac{dx_{159}}{x_{159}} f_{T1e}(\bar{E}, x_{158}) \cdot \int \frac{dx_{160}}{x_{160}} f_{T1e}(\bar{E}, x_{159}) \int \frac{dx_{161}}{x_{161}} f_{T1e}(\bar{E}, x_{160}) \cdot \int \frac{dx_{162}}{x_{162}} f_{T1e}(\bar{E}, x_{161}) \int \frac{dx_{163}}{x_{163}} f_{T1e}(\bar{E}, x_{162}) \cdot \int \frac{dx_{164}}{x_{164}} f_{T1e}(\bar{E}, x_{163}) \int \frac{dx_{165}}{x_{165}} f_{T1e}(\bar{E}, x_{164}) \cdot \int \frac{dx_{166}}{x_{166}} f_{T1e}(\bar{E}, x_{165}) \int \frac{dx_{167}}{x_{167}} f_{T1e}(\bar{E}, x_{166}) \cdot \int \frac{dx_{168}}{x_{168}} f_{T1e}(\bar{E}, x_{167}) \int \frac{dx_{169}}{x_{169}} f_{T1e}(\bar{E}, x_{168}) \cdot \int \frac{dx_{170}}{x_{170}} f_{T1e}(\bar{E}, x_{169}) \int \frac{dx_{171}}{x_{171}} f_{T1e}(\bar{E}, x_{170}) \cdot \int \frac{dx_{172}}{x_{172}} f_{T1e}(\bar{E}, x_{171}) \int \frac{dx_{173}}{x_{173}} f_{T1e}(\bar{E}, x_{172}) \cdot \int \frac{dx_{174}}{x_{174}} f_{T1e}(\bar{E}, x_{173}) \int \frac{dx_{175}}{x_{175}} f_{T1e}(\bar{E}, x_{174}) \cdot \int \frac{dx_{176}}{x_{176}} f_{T1e}(\bar{E}, x_{175}) \int \frac{dx_{177}}{x_{177}} f_{T1e}(\bar{E}, x_{176}) \cdot \int \frac{dx_{178}}{x_{178}} f_{T1e}(\bar{E}, x_{177}) \int \frac{dx_{179}}{x_{179}} f_{T1e}(\bar{E}, x_{178}) \cdot \int \frac{dx_{180}}{x_{180}} f_{T1e}(\bar{E}, x_{179}) \int \frac{dx_{181}}{x_{181}} f_{T1e}(\bar{E}, x_{180}) \cdot \int \frac{dx_{182}}{x_{182}} f_{T1e}(\bar{E}, x_{181}) \int \frac{dx_{183}}{x_{183}} f_{T1e}(\bar{E}, x_{182}) \cdot \int \frac{dx_{184}}{x_{184}} f_{T1e}(\bar{E}, x_{183}) \int \frac{dx_{185}}{x_{185}} f_{T1e}(\bar{E}, x_{184}) \cdot \int \frac{dx_{186}}{x_{186}} f_{T1e}(\bar{E}, x_{185}) \int \frac{dx_{187}}{x_{187}} f_{T1e}(\bar{E}, x_{186}) \cdot \int \frac{dx_{188}}{x_{188}} f_{T1e}(\bar{E}, x_{187}) \int \frac{dx_{189}}{x_{189}} f_{T1e}(\bar{E}, x_{188}) \cdot \int \frac{dx_{190}}{x_{190}} f_{T1e}(\bar{E}, x_{189}) \int \frac{dx_{191}}{x_{191}} f_{T1e}(\bar{E}, x_{190}) \cdot \int \frac{dx_{192}}{x_{192}} f_{T1e}(\bar{E}, x_{191}) \int \frac{dx_{193}}{x_{193}} f_{T1e}(\bar{E}, x_{192}) \cdot \int \frac{dx_{194}}{x_{194}} f_{T1e}(\bar{E}, x_{193}) \int \frac{dx_{195}}{x_{195}} f_{T1e}(\bar{E}, x_{194}) \cdot \int \frac{dx_{196}}{x_{196}} f_{T1e}(\bar{E}, x_{195}) \int \frac{dx_{197}}{x_{197}} f_{T1e}(\bar{E}, x_{196}) \cdot \int \frac{dx_{198}}{x_{198}} f_{T1e}(\bar{E}, x_{197}) \int \frac{dx_{199}}{x_{199}} f_{T1e}(\bar{E}, x_{198}) \cdot \int \frac{dx_{200}}{x_{200}} f_{T1e}(\bar{E}, x_{199}) \int \frac{dx_{201}}{x_{201}} f_{T1e}(\bar{E}, x_{200}) \cdot \int \frac{dx_{202}}{x_{202}} f_{T1e}(\bar{E}, x_{201}) \int \frac{dx_{203}}{x_{203}} f_{T1e}(\bar{E}, x_{202}) \cdot \int \frac{dx_{204}}{x_{204}} f_{T1e}(\bar{E}, x_{203}) \int \frac{dx_{205}}{x_{205}} f_{T1e}(\bar{E}, x_{204}) \cdot \int \frac{dx_{206}}{x_{206}} f_{T1e}(\bar{E}, x_{205}) \int \frac{dx_{207}}{x_{207}} f_{T1e}(\bar{E}, x_{206}) \cdot \int \frac{dx_{208}}{x_{208}} f_{T1e}(\bar{E}, x_{207}) \int \frac{dx_{209}}{x_{209}} f_{T1e}(\bar{E}, x_{208}) \cdot \int \frac{dx_{210}}{x_{210}} f_{T1e}(\bar{E}, x_{209}) \int \frac{dx_{211}}{x_{211}} f_{T1e}(\bar{E}, x_{210}) \cdot \int \frac{dx_{212}}{x_{212}} f_{T1e}(\bar{E}, x_{211}) \int \frac{dx_{213}}{x_{213}} f_{T1e}(\bar{E}, x_{212}) \cdot \int \frac{dx_{214}}{x_{214}} f_{T1e}(\bar{E}, x_{213}) \int \frac{dx_{215}}{x_{215}} f_{T1e}(\bar{E}, x_{214}) \cdot \int \frac{dx_{216}}{x_{216}} f_{T1e}(\bar{E}, x_{215}) \int \frac{dx_{217}}{x_{217}} f_{T1e}(\bar{E}, x_{216}) \cdot \int \frac{dx_{218}}{x_{218}} f_{T1e}(\bar{E}, x_{217}) \int \frac{dx_{219}}{x_{219}} f_{T1e}(\bar{E}, x_{218}) \cdot \int \frac{dx_{220}}{x_{220}} f_{T1e}(\bar{E}, x_{219}) \int \frac{dx_{221}}{x_{221}} f_{T1e}(\bar{E}, x_{220}) \cdot \int \frac{dx_{222}}{x_{222}} f_{T1e}(\bar{E}, x_{221}) \int \frac{dx_{223}}{x_{223}} f_{T1e}(\bar{E}, x_{222}) \cdot \int \frac{dx_{224}}{x_{224}} f_{T1e}(\bar{E}, x_{223}) \int \frac{dx_{225}}{x_{225}} f_{T1e}(\bar{E}, x_{224}) \cdot \int \frac{dx_{226}}{x_{226}} f_{T1e}(\bar{E}, x_{225}) \int \frac{dx_{227}}{x_{227}} f_{T1e}(\bar{E}, x_{226}) \cdot \int \frac{dx_{228}}{x_{228}} f_{T1e}(\bar{E}, x_{227}) \int \frac{dx_{229}}{x_{229}} f_{T1e}(\bar{E}, x_{228}) \cdot \int \frac{dx_{230}}{x_{230}} f_{T1e}(\bar{E}, x_{229}) \int \frac{dx_{231}}{x_{231}} f_{T1e}(\bar{E}, x_{230}) \cdot \int \frac{dx_{232}}{x_{232}} f_{T1e}(\bar{E}, x_{231}) \int \frac{dx_{233}}{x_{233}} f_{T1e}(\bar{E}, x_{232}) \cdot \int \frac{dx_{234}}{x_{234}} f_{T1e}(\bar{E}, x_{233}) \int \frac{dx_{235}}{x_{235}} f_{T1e}(\bar{E}, x_{234}) \cdot \int \frac{dx_{236}}{x_{236}} f_{T1e}(\bar{E}, x_{235}) \int \frac{dx_{237}}{x_{237}} f_{T1e}(\bar{E}, x_{236}) \cdot \int \frac{dx_{238}}{x_{238}} f_{T1e}(\bar{E}, x_{237}) \int \frac{dx_{239}}{x_{239}} f_{T1e}(\bar{E}, x_{238}) \cdot \int \frac{dx_{240}}{x_{240}} f_{T1e}(\bar{E}, x_{239}) \int \frac{dx_{241}}{x_{241}} f_{T1e}(\bar{E}, x_{240}) \cdot \int \frac{dx_{242}}{x_{242}} f_{T1e}(\bar{E}, x_{241}) \int \frac{dx_{243}}{x_{243}} f_{T1e}(\bar{E}, x_{242}) \cdot \int \frac{dx_{244}}{x_{244}} f_{T1e}(\bar{E}, x_{243}) \int \frac{dx_{245}}{x_{245}} f_{T1e}(\bar{E}, x_{244}) \cdot \int \frac{dx_{246}}{x_{246}} f_{T1e}(\bar{E}, x_{245}) \int \frac{dx_{247}}{x_{247}} f_{T1e}(\bar{E}, x_{246}) \cdot \int \frac{dx_{248}}{x_{248}} f_{T1e}(\bar{E}, x_{247}) \int \frac{dx_{249}}{x_{249}} f_{T1e}(\bar{E}, x_{248}) \cdot \int \frac{dx_{250}}{x_{250}} f_{T1e}(\bar{E}, x_{249}) \int \frac{dx_{251}}{x_{251}} f_{T1e}(\bar{E}, x_{250}) \cdot \int \frac{dx_{252}}{x_{252}} f_{T1e}(\bar{E}, x_{251}) \int \frac{dx_{253}}{x_{253}} f_{T1e}(\bar{E}, x_{252}) \cdot \int \frac{dx_{254}}{x_{254}} f_{T1e}(\bar{E}, x_{253}) \int \frac{dx_{255}}{x_{255}} f_{T1e}(\bar{E}, x_{254}) \cdot \int \frac{dx_{256}}{x_{256}} f_{T1e}(\bar{E}, x_{255}) \int \frac{dx_{257}}{x_{257}} f_{T1e}(\bar{E}, x_{256}) \cdot \int \frac{dx_{258}}{x_{258}} f_{T1e}(\bar{E}, x_{257}) \int \frac{dx_{259}}{x_{259}} f_{T1e}(\bar{E}, x_{258}) \cdot \int \frac{dx_{260}}{x_{260}} f_{T1e}(\bar{E}, x_{259}) \int \frac{dx_{261}}{x_{261}} f_{T1e}(\bar{E}, x_{260}) \cdot \int \frac{dx_{262}}{x_{262}} f_{T1e}(\bar{E}, x_{261}) \int \frac{dx_{263}}{x_{263}} f_{T1e}(\bar{E}, x_{262}) \cdot \int \frac{dx_{264}}{x_{264}} f_{T1e}(\bar{E}, x_{263}) \int \frac{dx_{265}}{x_{265}} f_{T1e}(\bar{E}, x_{264}) \cdot \int \frac{dx_{266}}{x_{266}} f_{T1e}(\bar{E}, x_{265}) \int \frac{dx_{267}}{x_{267}} f_{T1e}(\bar{E}, x_{266}) \cdot \int \frac{dx_{268}}{x_{268}} f_{T1e}(\bar{E}, x_{267}) \int \frac{dx_{269}}{x_{269}} f_{T1e}(\bar{E}, x_{268}) \cdot \int \frac{dx_{270}}{x_{270}} f_{T1e}(\bar{E}, x_{269}) \int \frac{dx_{271}}{x_{271}} f_{T1e}(\bar{E}, x_{270}) \cdot \int \frac{dx_{272}}{x_{272}} f_{T1e}(\bar{E}, x_{271}) \int \frac{dx_{273}}{x_{273}} f_{T1e}(\bar{E}, x_{272}) \cdot \int \frac{dx_{274}}{x_{274}} f_{T1e}(\bar{E}, x_{273}) \int \frac{dx_{275}}{x_{275}} f_{T1e}(\bar{E}, x_{274}) \cdot \int \frac{dx_{276}}{x_{276}} f_{T1e}(\bar{E}, x_{275}) \int \frac{dx_{277}}{x_{277}} f_{T1e}(\bar{E}, x_{276}) \cdot \int \frac{dx_{278}}{x_{278}} f_{T1e}(\bar{E}, x_{277}) \int \frac{dx_{279}}{x_{279}} f_{T1e}(\bar{E}, x_{278}) \cdot \int \frac{dx_{280}}{x_{280}} f_{T1e}(\bar{E}, x_{279}) \int \frac{dx_{281}}{x_{281}} f_{T1e}(\bar{E}, x_{280}) \cdot \int \frac{dx_{282}}{x_{282}} f_{T1e}(\bar{E}, x_{281}) \int \frac{dx_{283}}{x_{283}} f_{T1e}(\bar{E}, x_{282}) \cdot \int \frac{dx_{284}}{x_{284}} f_{T1e}(\bar{E}, x_{283}) \int \frac{dx_{285}}{x_{285}} f_{T1e}(\bar{E}, x_{284}) \cdot \int \frac{dx_{286}}{x_{286}} f_{T1e}(\bar{E}, x_{285}) \int \frac{dx_{287}}{x_{287}} f_{T1e}(\bar{E}, x_{286}) \cdot \int \frac{dx_{288}}{x_{288}} f_{T1e}(\bar{E}, x_{287}) \int \frac{dx_{289}}{x_{289}} f_{T1e}(\bar{E}, x_{288}) \cdot \int \frac{dx_{290}}{x_{290}} f_{T1e}(\bar{E}, x_{289}) \int \frac{dx_{291}}{x_{291}} f_{T1e}(\bar{E}, x_{290}) \cdot \int \frac{dx_{292}}{x_{292}} f_{T1e}(\bar{E}, x_{291}) \int \frac{dx_{293}}{x_{293}} f_{T1e}(\bar{E}, x_{292}) \cdot \int \frac{dx_{294}}{x_{294}} f_{T1e}(\bar{E}, x_{293}) \int \frac{dx_{295}}{x_{295}} f_{T1e}(\bar{E}, x_{294}) \cdot \int \frac{dx_{296}}{x_{296}} f_{T1e}(\bar{E}, x_{295}) \int \frac{dx_{297}}{x_{297}} f_{T1e}(\bar{E}, x_{296}) \cdot \int \frac{dx_{298}}{x_{298}} f_{T1e}(\bar{E}, x_{297}) \int \frac{dx_{299}}{x_{299}} f_{T1e}(\bar{E}, x_{298}) \cdot \int \frac{dx_{300}}{x_{300}} f_{T1e}(\bar{E}, x_{299}) \int \frac{dx_{301}}{x_{301}} f_{T1e}(\bar{E}, x_{300}) \cdot \int \frac{dx_{302}}{x_{302}} f_{T1e}(\bar{E}, x_{301}) \int \frac{dx_{303}}{x_{303}} f_{T1e}(\bar{E}, x_{302}) \cdot \int \frac{dx_{304}}{x_{304}} f_{T1e}(\bar{E}, x_{303}) \int \frac{dx_{305}}{x_{305}} f_{T1e}(\bar{E}, x_{304}) \cdot \int \frac{dx_{306}}{x_{306}} f_{T1e}(\bar{E}, x_{305}) \int \frac{dx_{307}}{x_{307}} f_{T1e}(\bar{E}, x_{306}) \cdot \int \frac{dx_{308}}{x_{308}} f_{T1e}(\bar{E}, x_{307}) \int \frac{dx_{309}}{x_{309}} f_{T1e}(\bar{E}, x_{308}) \cdot \int \frac{dx_{310}}{x_{310}} f_{T1e}(\bar{E}, x_{309}) \int \frac{dx_{311}}{x_{311}} f_{T1e}(\bar{E}, x_{310}) \cdot \int \frac{dx_{312}}{x_{312}} f_{T1e}(\bar{E}, x_{311}) \int \frac{dx_{313}}{x_{313}} f_{T1e}(\bar{E}, x_{312}) \cdot \int \frac{dx_{314}}{x_{314}} f_{T1e}(\bar{E}, x_{313}) \int \frac{dx_{315}}{x_{315}} f_{T1e}(\bar{E}, x_{314}) \cdot \int \frac{dx_{316}}{x_{316}} f_{T1e}(\bar{E}, x_{315}) \int \frac{dx_{317}}{x_{317}} f_{T1e}(\bar{E}, x_{316}) \cdot \int \frac{dx_{318}}{x_{318}} f_{T1e}(\bar{E}, x_{317}) \int \frac{dx_{319}}{x_{319}} f_{T1e}(\bar{E}, x_{318}) \cdot \int \frac{dx_{320}}{x_{320}} f_{T1e}(\bar{E}, x_{319}) \int \frac{dx_{321}}{x_{321}} f_{T1e}(\bar{E}, x_{320})$$

δ -functions. For non-identical final state partons q' has again to be symmetrized. We thus see that the triple differential cross section is [22,23] directly proportional to a product of parton densities in the electron, taken at values y_m and z_m , if we consider the photon also to be a parton. Directly one can therefore only measure convolutions of $\vec{q}'$ with $f_{Y|e}$.

In Fig. 7, we show the triple differential cross section at $\sqrt{s} = 60$ GeV, for $p_T = 3$ GeV and the DC parametrization, with $y_1 = y_2 = Y$. For small values of Y , both y_m and z_m are rather small, $y_m = z_m = x_m = x_T$. Hence all the three contributions are quite large. For larger, positive Y , y_m approaches 1 while z_m approaches x_T^2 . The WW function $f_{Y|e}(\vec{q}, x)$ does not vanish as $x \rightarrow 1$; hence the direct and once-resolved contributions remain finite up to the kinematic limit $Y = \ln 1/x_T$. The single resolved contributions actually dominate in the large Y region, since the convolution of $f_{Y|e}$ and $\vec{q}'$ increases more rapidly with decreasing z_m than $f_{Y|e}$ alone. The double-resolved contribution, however, falls off faster than either of these two since the lower limit of one of the convolution integrals approaches one; furthermore, $\vec{q}'(x \rightarrow 1)$ vanishes (see Fig. 3).

These results show us that rapidity distributions offer a good chance to isolate the various two-photon processes, with the help of some information on spectator jets. The direct contribution to the measured cross section at large $Y \approx 2-2.5$ can be isolated from the single resolved one by the requirements of no spectator jets in the former case. Notice that in this configuration the spectator jet has a negative rapidity and is thus well separated from the high- p_T jets.

This can then be used to extract the double-resolved contribution from measurements at small Y . This may be done either by detecting both spectator jets present in the double-resolved contributions, or by triggering on all events with at least one spectator jet and subtracting the once-resolved contribution as measured at large Y ; this is possible since the single resolved contribution is almost constant in Y , at least for both parametrizations of $\vec{q}'$ that we used. The latter method might be preferable if the efficiency of detecting any one spectator jet is low, in which case the efficiency of detecting two of them would be even lower. Notice that in this region of small Y , high- p_T jets and spectator jets are always well separated.

It should be pointed out here that only qualitative information about the existence or absence of spectator jets is necessary. The fact that the excess seen in 2-jet data in PETRA experiments could be removed by requiring the final state not to contain anything but the two high- p_T jets indicates that it is possible to tag against, and therefore also on, the existence of these spectator jets. If this is possible, measurements of triple differential cross sections should be able to

distinguish easily between the two parametrizations of $\vec{q}'$; e.g., the DO + VMD parametrization predicts a three times bigger double-resolved contribution to the cross section at $y_1 = y_2 = 0$!

In Figs. 8a, b, we separate the resolved contributions into different final state configurations; in these figures, " q " denotes a quark or antiquark of any flavour. In the single resolved case (Fig. 8a), the $q\bar{q}$ final state probes $\vec{q}'$, whereas the $q\bar{q}$ final state probes $\vec{q}'$. At larger values of Y , the z -convolution in Eq. (4.1) receives contribution from $\vec{q}'(x)$ at smaller values of x ; since $\vec{q}'$ is strongly peaked at small x , the $q\bar{q}$ contribution rises with Y . On the other hand, the small- x peak of $\vec{q}'(x)$ is not pronounced enough to compensate for the decline of $f_{Y|e}(y_m)$ as $Y \rightarrow 1$, so that the $q\bar{q}$ contribution falls for larger Y . As noted earlier, the sum of the two is almost independent of Y . Notice also that at $Y = 0$ the $q\bar{q}$ final state dominates by a factor of 4 to 5. Therefore this configuration may probe $\vec{q}'$ at $x \gtrsim 0.1$, where data do exist from DIS, so that a relatively firm prediction can be made [38].

In contrast, in the double resolved case (Fig. 8b) almost 2/3 of the cross section at $Y=0$ comes from processes with at least one gluon in the initial state, i.e., the $q\bar{q}$ and $g\bar{g}$ [39] final states. This again shows that a measurement of the double resolved cross section does indeed probe the gluon content of the photon. Recall that the results presented in this figure are conservative estimates, since the DO + VMD parametrization always predicts larger cross sections, due to the low- x peak and the large gluon content.

A further separation between quark and gluon initiated processes may be possible if the spectator jets can be studied in some detail. Since the quark distributions inside the photon are harder than the gluon distribution, spectator jets of quark initiated processes tend to be less energetic than those occurring in gluon initiated reactions. In the former case the average spectator jet energy at $\sqrt{s} = 60$ GeV is 5-7 GeV, and in the latter case 9-12 GeV.

So far we have only discussed the situation $y_1 = y_2$. Since in this case $M_{jj} = 2p_T$, the enhancement factor $M_{jj}^2/2p_T^2$ of the resolved contributions is minimal. One might therefore think that distributions for $y_1 \neq y_2$ should offer a better possibility to isolate the double-resolved contributions. This turns out to be only partly true; the increase of the relative importance of the double-resolved processes is offset by the fact that the shapes of the distributions tend to be quite similar for the three classes of contributions. For the sake of brevity we therefore do not discuss the case $y_1 \neq y_2$ in more detail.

Rapidity distribution can also be used to further reduce the bremsstrahlung

background. As long as only one hard photon is emitted, as is the case for the process depicted in Fig. 2, the transverse momentum and rapidity of the first jet determine [23] the rapidity of the second jet; in principle one can thus remove this background by cutting on the rapidity of the second jet. Since in the region where the signal cross section is big the background is already small and we do not know how precisely jet rapidities can be measured at TRISTAN, we do not try to implement such cuts.

Another possibility to probe the gluonic structure of the photon is heavy quark production, since the once-resolved contribution is directly proportional to G^Y in this case. We discuss this in the next section.

5. HEAVY QUARK PRODUCTION AT TRISTAN

The total cross section for heavy quark production via two photon collision is given by the general formula

$$\sigma(Q\bar{Q}) = \frac{1}{4M_Q^2} \int_0^1 dx_1 f_{\gamma/c}(\bar{E}, x_1) \int_0^1 dx_2 f_{\gamma/c}(\bar{E}, x_2) \cdot \frac{1}{4M_Q^2} \int_0^1 dx_3 \frac{\vec{q}^{\gamma}(x_3)}{x_1 x_2 s} \int_0^1 dx_4 \frac{\vec{q}^{\gamma}(x_4)}{x_1 x_3 s} \sigma_{Q\bar{Q}}^{\gamma}(s', m_Q). \quad (5.1)$$

Here m_Q is the heavy quark mass and $\sigma_{Q\bar{Q}}^{\gamma}$ is the subprocess cross section. The formulae for the once-resolved and direct cases can be derived from eq. (5.1) as in the previous cases. The relevant subprocess cross sections for $\gamma\gamma \rightarrow Q\bar{Q}$ in the direct case, $\gamma g \rightarrow Q\bar{Q}$ in the single resolved case, and $q\bar{q} \rightarrow Q\bar{Q}$ in the double resolved case can be found in Refs. [26], [40] and [41] respectively. We use the bare dynamical quark mass m_Q in evaluating the squared matrix elements, whereas for the determination of the boundaries of the integration region we use the more appropriate mass M_Q of the lowest lying Q -meson. In the case of charm quark production we thus use $m_c = 1.5$ GeV and $M_c = M_D = 1.87$ GeV, whereas for $b\bar{b}$ production we use $m_b = 4.5$ GeV and $M_b = M_B = 5.3$ GeV.

The results of our calculation of charm production are shown in Fig. 9, where we have used the DG parametrization with $N_f=3$ flavours and scale $Q^2 = s'$. While the direct contribution dominates the total cross section even at the highest TRISTAN energies, the cross section for the once-resolved process varies between 15 and 50 pb in the energy range $\sqrt{s} = 40$ –70 GeV; for the $DO + VMD$ parametrization, this cross section is as big as 130 pb at $\sqrt{s} = 70$ GeV.

The best way to tag the heavy quarks would be through their semi-leptonic decay giving a muon inside a jet. Even with the reduction in rates by a factor of five implied by such a muon + jets final state tag, the once resolved contribution should give rise to a measurable number of events, ~ 800 ev/yr at $\sqrt{s} = 60$ GeV for the DG parametrization and about twice as much for the $DO + VMD$ parametrization. Due to the even larger rate for the direct process it will nevertheless be necessary to use the spectator jet present in the resolved case to distinguish between the two contributions experimentally; this jet should have an energy of about 10 GeV in the lab. frame. As stated earlier, the single-resolved heavy quark cross section is a direct measure of the gluon content of the photon; a separation between direct and resolved processes would therefore be particularly interesting in this case. Notice also that the double-resolved contributions are always negligibly small in this case.

Unfortunately, the detection of spectator jets is far from straightforward in this case; the reason is that the c -quarks themselves quite often go in forward-backward direction. This is especially true for the resolved contribution, where the $c\bar{c}$ pair is almost at rest in its centre-of-mass frame (the $c\bar{c}$ invariant mass is less than 6 GeV in $\sim 80\%$ of the resolved events and $\sim 60\%$ of the direct events), but gets quite a strong boost due to the asymmetrical initial state configuration. As a result, requiring the charm quarks to be produced at an angle $\theta > 25^\circ$, which should help to separate them from the spectator jets, reduces the signal by more than a factor of two. And even this cut does not guarantee that hadronic decay products of c -quarks that are almost at rest even in the lab. frame do not "leak" into the forward-backward directions, thereby faking a spectator jet. A full Monte Carlo analysis including the hadronization and decay of the charm quarks will therefore be necessary to really decide whether the resolved contributions can be separated.

For $b\bar{b}$ production we expect the direct background to be smaller, due to the smaller charge of the b -quark. On the other hand the increase of the quark mass affects the resolved contributions more strongly. As a result, we find that resolved contributions again only contribute $\sim 20\%$ of the total cross section. What is worse, the total cross section is now only about 0.5 pb at $\sqrt{s}=60$ GeV. We thus conclude that two-photon processes do not contribute significantly to total beauty production at TRISTAN.

Since the main obstacle to measure G^Y via open flavour heavy quark production comes from the large direct two-photon cross section, it is instructive to look for heavy flavour production processes which are not accessible to a direct $\gamma\gamma$ initial state; J/ψ or ψ' production provides one such heavy flavour production process. It can only be produced if at least one photon is resolved, via $\gamma g \rightarrow J/\psi + \gamma$ or $g g \rightarrow J/\psi + \gamma$. From our experience with open flavour production discussed above we can conclude

that the second of these processes, for which both photons have to be resolved, is negligible. We therefore focus on the photon-gluon fusion process. In this computation we use the model of Ref. [42] for the photoproduction of J/ψ ; essentially this involves using the single resolved version of eq. (5.1) with the expression for $\sigma'(\gamma g \rightarrow J/\psi + g)$ as given in Ref. [42]. We have used $m_{J/\psi} = 3.1$ GeV and [43] $\Gamma_{ee}(J/\psi) = 4.7$ KeV.

Possible sources of direct J/ψ production in $\gamma\gamma$ collisions are $\gamma\gamma \rightarrow \chi_{c0,2} + J/\psi + \gamma$, and $\gamma\gamma \rightarrow b\bar{b}$, $b\bar{b} \rightarrow J/\psi + X$. Even though the $\text{Br}(\chi_{c2} \rightarrow J/\psi + \gamma)$ is sizeable [43], $\approx 14\%$, the $\chi_{c2} \rightarrow \gamma\gamma$ decay width is very small, ~ 2 keV, and hence the production rate is negligible. The same is true for the second background, since $\text{Br}(B \rightarrow J/\psi + X)$ is [43] only $\sim 1\%$ and the $b\bar{b}$ rate is rather low to begin with. We thus see that the J/ψ signal has indeed almost no background.

Our results for J/ψ production for the two parametrizations of G^V are shown in Fig. 10. The contribution coming from $\gamma g \rightarrow \gamma^* + J/\psi + X$ will increase these rates by about 20%. So we may expect a cross section of about one or two pb at TRISTAN energies. Since the J/ψ will have to be detected through its leptonic decay, the visible signal is only 1/7 of the cross section shown in the figure, corresponding to 10 to 30 events per year. Of course, it is now absolutely essential to detect the spectator jet, since otherwise the signal would be swamped by the huge $e^+e^- \rightarrow e^+e^- + \gamma^* \rightarrow e^+e^- + J/\psi$ cross section. In this case, we just have to require that there is some hadronic activity anywhere in the detector; a single pion would be sufficient! We think that this requirement should not reduce the signal any further, while it should get rid of the background. If so, the $J/\psi + \gamma^* \rightarrow J/\psi + \gamma$ signal, though small, can offer a probe into the behaviour of $G^V(x)$ at $x \sim 0.01$ to 0.05.

6. SUMMARY AND CONCLUSIONS

In this paper we studied inclusive two photon processes at the e^+e^- collider TRISTAN. For the sake of larger cross sections we focused on the situation where both photons are almost on-shell ("no tag" situation), with the goal to isolate processes that might allow to determine the quark, and especially gluon, content of real photons. In these "resolved" processes the photon behaves more or less like a hadron.

We found in Sections 2 and 3 that these resolved processes make important contributions to and might even dominate the production of two high- p_T jets if the transverse momentum of the jets is below 5 GeV or so or their invariant mass not larger than 20 GeV. For $p_T \lesssim 3$ GeV or $M_{jj} \lesssim 10$ GeV the rates should be high enough to study more differential distributions, as shown in Section 4. Finally, resolved

processes make a sizeable contribution to the two photon production of charm, and lead to a small but still interesting rate for $e^+e^- \rightarrow J/\psi + X \rightarrow \ell^+ \ell^- + X$.

However, in order to gain insight into the hadronic structure of the photon it turned out to be essential to be able to trigger on or against the existence of spectator jets. As discussed in Section 4, PETRA data indicate that this is indeed possible, at least in certain kinematic regions. In the case of the production of high- p_T jets we found event rates to be sufficiently high to allow us to focus onto configurations where high- p_T jets and spectator jets are well separated, which should facilitate to discriminate experimentally between resolved and direct contributions. In particular, contributions where one photon is resolved while the other interacts in a point-like manner can be isolated by requiring the rapidities of both high- p_T jets to be large and to have the same size; the single spectator jet will then go in the opposite direction. In contrast, events where both photons are resolved may be isolated by requiring both high- p_T jets to be far away from the beam pipes, so that four well-separated jets should be visible.

Unfortunately in the case of open charm production we were not able to isolate a region of phase space where the resolved contribution is sizeable and its spectator jet well isolated. Part of the problem is related to the fact that the produced c-quarks tend to be rather slow, so that their decay products can emerge at any angle. Further Monte Carlo studies of this problem are necessary. On the other hand, the detection of the spectator jet should be very easy in the case of J/ψ production, since there any hadronic activity will do to suppress the fermion-pair background.

Overall we can conclude that TRISTAN might be able to provide us with the first clear evidence for hadronically interacting photons in processes other than deep inelastic scattering. It should also provide us with some, at least semi-quantitative, information about the parton densities inside the photon. Unlike at the HERA collider, we were unfortunately not able to discriminate between the two existing parametrizations of photon structure functions on the basis of the shape of some rapidity distribution. Since we are forced to work at fairly low transverse momenta ($p_T \lesssim 5$ GeV), "k-factors" from higher order QCD corrections can be large, so that our absolute predictions may not be very reliable. However, by experience k-factors are always bigger than one, hence our predictions can serve as lower bounds. A possible way around this problem might be to study the variation of triple-differential cross sections with $\sqrt{s}$; since the scale of the hard scattering cross section remains fixed, k-factors should also stay more or less constant, so that the effects that result from probing different x-regions should be directly visible. This procedure might, however, require to reduce the beam energy again to 25 GeV or less.

What role would resolved photon contributions play at future colliders? As mentioned in Section 2, on or close to the 2-pole they will not play a major role. They will, however, become even more important than at TRISTAN once LEP reaches energies beyond 110 GeV or so; two-photon production will, e.g., dominate total beauty production at $\sqrt{s}=190$ GeV.

Two photon processes, and hence the hadronic structure of the photon, will play an even more important role in more futuristic colliders. It has, e.g., been suggested [44] to transform an e^+e^- collider into an $\gamma\gamma$ collider by letting laser light backscatter from the positrons. Furthermore, the beamstrahlung phenomenon can lead [45] to a vastly enhanced two-photon flux at e^+e^- linacs with energies in the TeV range. At these future machines the majority of hard events will involve the hadronic structure of the photon. It thus seems fair to say that the study of resolved photon reactions at TRISTAN can affect the future of e^+e^- physics for decades to come.

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FIGURE CAPTIONS

Fig. 1 Feynman diagrams (left) and depictions of the final state topology (right) for the three classes of two-photon reactions considered in this paper, i.e. direct (a), single resolved (b) and double resolved (c) processes.

Fig. 2 Feynman diagrams for the hard bremsstrahlung background. The emitted photon is assumed to escape detection.

Fig. 3 Comparison of the two parametrizations of q^2 : DG-full curves; DO+VMD-dashed curves. Figure 3a shows $F_2^{\gamma} = 2x F_1 e_q^2 Y(x)$ at $Q^2 = 5 \text{ GeV}^2$; the data are from the PLUTO collaboration [35]. Figure 3b shows the gluon content of the photon in units of α_{em} for N_f flavours at $Q^2 = 25 \text{ GeV}^2$.

Fig. 4 The dependence of the cross section for the production of two high- p_T jets on the centre-of-mass energy $\sqrt{s}$. We have used $p_T = 3 \text{ GeV}$ and the DG parametrization. The momentum scale Q^2 is chosen as p_T^2 , and $\Lambda_{QCD} = 0.4 \text{ GeV}$. The long dashed, long-short dashed and short dashed curves show the contributions from direct, single resolved and double resolved processes, respectively. The solid line shows the total two photon cross sections while the dotted curve shows the single bremsstrahlung background, where the cut $M_{jj} < 30 \text{ GeV}$ has been applied.

Fig. 5 The transverse momentum spectrum of high- p_T jets at $\sqrt{s} = 60 \text{ GeV}$ for a) the DG and b) the DO+VMD parametrization. Parameters and notation are as in Fig. 4.

Fig. 6 The invariant mass spectrum of high- p_T jets at $\sqrt{s} = 60 \text{ GeV}$ for the DO+VMD parametrizations. We have applied the cuts $p_T > 2 \text{ GeV}$ and $|y| < 2.5$ on the transverse momentum and rapidity of the high- p_T jets. Parameters and notations are as in Fig. 4, except that we have used M_{jj}^2 as scale in α_s and q^2 .

Fig. 7 Rapidity distribution of high- p_T jets produced in two photon reactions at TRISTAN. Notice that for the given case $y_1 = y_2 = Y$, the distribution is symmetric under $Y \rightarrow -Y$. Notations and parameters are as in Fig. 4.

Fig. 8 Different final state contributions to single (a) and double (b) resolved contributions to the rapidity distribution of high- p_T jets. Note that "q" denotes a quark or antiquark of any flavour. Parameters are as in Fig. 4.

Fig. 9 Two-photon contributions to the total $c\bar{c}$ cross section are shown as a function of $\sqrt{s}$, where we have used the DG parametrization with $N_f = 3$ flavours; we have chosen the Q^2 scale to be the squared $c\bar{c}$ invariant mass. The other parameters and notations are as in Fig. 4.

Fig. 10 Single resolved contribution to J/ψ production according to the DG (full curve) and DO+VMD (dashed) parametrizations. We have used $N_f = 3$ flavours; the momentum scale Q^2 is given by the square of the $\gamma\gamma$ invariant mass.

Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 4

Fig. 5b

Fig. 5a

Fig. 6

Fig. 7

Fig. 8b

Fig. 8a

Fig. 9

Fig. 10