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Comparative study of Final State Interaction contributions to the dynamic response of quantum fluids

A.S. Rinat* and M.N. Butler

TRIUMF, 4004 Wesbrook Mall, Vancouver BC, Canada V6T 2A9

Abstract

The expression for the final state interaction part of the dynamic response $S(q, \omega)$ as given by Silver is derived from a multiple scattering theory. We first discuss the single scattering term which has the same form in the analyses of Silver and Rinat. Next, we indicate the approximations which permit a summation of the full scattering series in the form given by Silver, but we argue that under currently prevailing conditions the approximated sum for S is dominated by the first two terms. In order to determine the better approach, we suggest as a test a comparison of the part of S odd in the variable y .

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*Permanent address: Weizmann Institute of Science, 76100 Rehovot, Israel

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Two approaches have recently been put forward for the description of final state interaction (FSI) corrections to the impulse approximation (IA) for the linear dynamic response of quantum fluids (Refs. 1 and 2, hereafter called I and II respectively). Both emphasize i) the short range repulsion in the inter-atomic interaction v which brings about modifications to the standard IA result and ii) the FSI correction known for regular forces. The techniques used and the final, formal results look totally unrelated, yet produce similar results for the same systems under identical conditions.^{3,4} The present note serves to establish and elucidate the relation between these two descriptions.

Consider the dynamic structure function, or linear response, $S(q, \omega)$ which is a function of the momentum q and energy transfer ω . Whenever convenient we shall replace ω by an alternative variable, e.g.,

$$y_0 \simeq \frac{1-A}{A}q + \left(\frac{A-1}{A} \left(2m\omega - \frac{q^2}{A} \right) \right)^{\frac{1}{2}} \quad (1)$$

or

$$y_W = \frac{m}{q} \left(\omega - \frac{q^2}{2m} \right)$$

For regular forces, S tends in the limit $q \rightarrow \infty$ and fixed y (notation $\Rightarrow$) to the IA result

$$S(q, y) \Rightarrow \frac{m}{q} F_0(y) = \frac{1}{4\pi^2} \frac{m}{q} \int_{|y|}^{\infty} dpp n(p), \quad (2)$$

with $n(p)$ the single particle momentum distribution. Instead of S consider the dynamic polarizability per particle χ for a system of A particles in a volume Ω^5

$$\begin{aligned} S(q, y) &= -\pi^{-1} \text{Im} \chi(q, y) \\ \chi(q, y) &= A^{-1} (\Phi_A^0 | \rho_q^\dagger G_A(\omega(q, y) + \epsilon_A^0 + iy) \rho_q | \Phi_A^0) \end{aligned} \quad (3)$$

with $\rho_q = \sum_j \rho_{qj} = \sum_j e^{iq \cdot r_j}$, a fluctuation of the density $\rho = A/\Omega$. Φ_A^0 and ϵ_A^0 are, respectively, the ground state wavefunction and energy of the A -body Hamiltonian H_A . $G_A(z) \equiv (z - H_A)^{-1}$ is the Green's function corresponding to H_A .

The polarizability χ can be split into incoherent and coherent parts, according to contributions where the same, respectively different, ρ_{qj} act on Φ_A^0 in (1). Thus

$$\chi^{inc} = (\Phi_A^0 | \rho_{q1}^\dagger G_A \rho_{q1} | \Phi_A^0) \quad (4a)$$

$$\chi^{coh} = (A-1) (\Phi_A^0 | \rho_{q1}^\dagger G_A \rho_{q2} | \Phi_A^0) \quad (4b)$$

Both decrease with increasing q but χ^{coh} does so much faster.⁶

One may decompose $H_A = H_A^0 + \sum_{j \neq 1} v_j$, $H_A^0 = H_{A-1} + h_1$, with h_1 the kinetic energy of particle 1, $v_j \equiv v_{1j}$. Introducing $G_A^0 \equiv (z - H_A^0)^{-1}$, an expression for G_A reads

$$G_A = G_A^0 + G_A^0 T_1 G_A^0 \quad (5)$$

where T_1 describes the scattering of '1' from the remaining $(A-1)$ -particle core. Standard multiple scattering expansions for T_1 read⁷

$$T_1(z) = \sum_{j \geq 2} v_j + \sum_{j \geq 2} v_j G_A^0(z) \sum_{k \geq 2} v_k + \quad (6a)$$

$$= \sum_{j \geq 2} t_j(z) + \sum_{j \geq 2} t_j(z) G_A^0(z) \sum_{j \neq k \geq 2} t_k(z) + \dots \quad (6b)$$

Eq. (6a) suffices if v is regular and not too strong. In that case an exact asymptotic series in q^{-1} may be derived for χ or S .^{2,8} If, however, v is either singular or strong, only the expansion (6b) in terms of the scattering matrix $t = v + v G^0 v$ converges. In that case, successive terms of S or χ produced by Eq. (6b) no longer lead to a power series in q^{-1} . One then has

$$\chi = \sum_{n=0}^{\infty} \chi^{(n)} \quad (7)$$

$$= \langle \Phi_A^0 | \rho_{q1}^{\dagger} G^0 [1 + (A-1)t_2 G^0 + (A-1)(A-2)t_2 G^0 t_3 G^0 + (A-1)(A-2)\{(A-3)t_2 G^0 t_3 G^0 t_4 G^0 + t_2 G^0 t_3 G^0 t_2 G^0\} + \dots] \rho_{q1} | \Phi_A^0 \rangle$$

where A -dependent factors replace the sums in (6b). $\chi^{(n)}$ is seen from (3) to be ($\vec{q} = \vec{1}, z$)

$$\chi^{(0)}(q, y) = \frac{m}{q} \int \frac{d^3 p}{(2\pi)^3} n(p)(y - p_z + i\eta)^{-1} \quad (8)$$

and we thus focus on $\chi^{(1)}$, linear in t :

$$\chi^{(1)}(q, y) \Rightarrow A^{-1} \left(\frac{m}{q} \right)^2 \iiint \frac{d^3 p_1 d^3 p_2 d^3 p_1' d^3 p_2'}{(2\pi)^{12}} \frac{\rho_2(\vec{p}_1, \vec{p}_2; \vec{p}_1', \vec{p}_2')}{(y - p_{1z} + i\eta)(y - p_{1'z} + i\eta)} (2\pi)^3 \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \left\langle \frac{\vec{p}_1 - \vec{p}_2 + \vec{q}}{2} \left| t \left(\frac{q^2}{4m} \right) \right| \frac{\vec{p}_1' - \vec{p}_2'}{2} \right\rangle \quad (9)$$

ρ_2 above is a specific case of the general n -particle density matrix

$$\rho_n(1 \dots n, 1' \dots n') = (A(A-1) \dots (A-n+1))^{-1} \left(\prod_{l=n+1}^A \int dt_l \right) \Phi_A^0(1 \dots n, l \dots A) \Phi_A^{0*}(1' \dots n', l \dots A) \quad (10)$$

In Eq. (9) we have already taken the $q \rightarrow \infty$ limit in G^0 as it appears in (7). In the same limit the energy argument of t in (9) is $q^2/4m + \mathcal{O}(q^{-1})$, but care is needed in applying the limit to the momentum dependence of t . For regular but strong forces the high- q limit is simple and t will be dominated by on-shell forward scattering in the direction of $\vec{q}$. However, for a pure hard core potential the scattering for $q \rightarrow \infty$ is not exclusively limited to the forward direction, but is instead concentrated in an angular range^{1,9}

$$\theta \approx 2|\vec{p}_1 \perp - \vec{p}_1' \perp|/q + \mathcal{O}(q^{-2})$$

$$\left\langle \frac{\vec{p}_1 - \vec{p}_2 + \vec{q}}{2} \left| t \left(\frac{q^2}{4m} \right) \right| \frac{\vec{p}_1' - \vec{p}_2' + \vec{q}}{2} \right\rangle \Rightarrow t \left(\frac{q^2}{4m}, \cos \theta \right) \quad (11)$$

This will also be the angular range in the $q \rightarrow \infty$ limit for any force with a strong short-range repulsion exceeding in magnitude the scattering energy. Whatever the case, the large q limit of t will be (11), essentially independent of $\vec{p}_2$.

Finally we need in (9) the off-diagonal ρ_2 , which is best studied in the coordinate representation. To approximate ρ_2 , it is useful to introduce the pair distribution function $g(r)$ and the off-diagonal $\rho_1(\vec{r}, 0)$ which (up to the density ρ) is the Fourier transform of $n(p)$ (cf. (2)). Some approximations suggested in the literature are ($\vec{r} = \vec{r}_1 - \vec{r}_2$; $\vec{r}' = \vec{r}_1' - \vec{r}_2'$)

$$\rho_2(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') \approx \rho \rho_1(|\vec{r} - \vec{r}'|, 0) \times \begin{cases} g(|\vec{r} + \vec{r}'|/2) & (a) \\ g(r) & (b) \\ g(r') & (c) \end{cases} \quad (12)$$

All of the options in (12) satisfy some consistency requirements (cf. II(4.4)), but only the choice (12a) is manifestly symmetric in $\vec{r}$ and $\vec{r}'$. It has moreover been found to lead to satisfactory numerical results. Although in II the coordinate representation was used throughout, we shall, for the sake of the comparison, stay here with the momentum representation (9). With t there dependent only on $\vec{p}_1 - \vec{p}_1'$ (cf. Eq. (11)), one may perform the $\vec{p}_2$ and $\vec{p}_2'$ integrations to find

$$\int \frac{d^3 p_2}{(2\pi)^3} \rho_2(\vec{p}_1, \vec{p}_2; \vec{p}_1', \vec{p}_2 + \vec{p}_2 - \vec{p}_1') = A \rho \bar{g}(|\vec{p}_1 - \vec{p}_1'|) \times \begin{cases} n(|\vec{p}_1 + \vec{p}_1'|/2) & (a) \\ n(p_1) & (b) \\ n(p_1') & (c) \end{cases} \quad (13)$$

where $\bar{g}(k)$, the static structure function, is the Fourier transform of $g(r)$.¹⁰ The results (11) and (13), substituted in (9), yield

$$\chi^{(1)}(q, y) = \rho \left(\frac{m}{q} \right)^2 \iint \frac{d^3 p d^3 p'}{(2\pi)^6} \frac{K(q, \vec{p} - \vec{p}') n(\vec{p}, \vec{p}')}{(y - p_z + i\eta)(y - p_z' + i\eta)} \quad (14)$$

with

$$K(q, \vec{p} - \vec{p}') = t(q^2/4m, |\vec{p}_1 - \vec{p}_1'|/q) \bar{g}(|\vec{p} - \vec{p}'|)$$

and the $(\vec{p}, \vec{p}')$ argument in n (Eq. 14) standing for the 3 options of Eq. (13).

Since an expression for the summed series $\sum_n \chi^{(n)}$ is Silver's ultimate result, we now attempt a generalization and illustrate the procedure for some terms in (7) which correspond to $\chi^{(2)}$. If at all, only expressions for collisions of particle 1 from distinct partners allow such a generalization. Others, like the second term in curly brackets for $\chi^{(2)}$, do not. Some of these (e.g. the collision chain (1→2), (1→3), (1→2)) require large angle scattering and are strongly suppressed in multiple diffraction scattering theory.¹¹ Leaving out arguments where not necessary, one has $(\rho_{\vec{q}_1}^\dagger \cdot \rho_{\vec{q}_1} = 1)$

$$\begin{aligned} \chi^{(2)} &\cong (A-1)(A-2) \langle \Phi_A^0 | \rho_1^\dagger G^0 \rho_1^\dagger t_2 \rho_1 \rho_1^\dagger G^0 \rho_1 t_3 \rho_1 \rho_1^\dagger G^0 \rho_1 | \Phi_A^0 \rangle \\ &\Rightarrow \left(\frac{m}{q} \right)^3 (A-1)(A-2) \left(\prod_{i,j=1}^3 \int \int \frac{d^3 p_i}{(2\pi)^3} \frac{d^3 p_j}{(2\pi)^3} \right) \\ &\quad \times \frac{\langle \Phi_A^0 | \vec{p}_1, \vec{p}_2, \vec{p}_3 \rangle \langle \vec{p}_1 + \vec{q}, \vec{p}_2 | t_2(q^2/4m) | \vec{p}_1'' + \vec{q}, \vec{p}_2' \rangle}{(y - p_{1z} + i\eta)(y - p_{1z}' + i\eta)} \\ &\quad \times \frac{\langle \vec{p}_1'' + \vec{q}, \vec{p}_3 | t_3(q^2/4m) | \vec{p}_1' + \vec{q}, \vec{p}_3' \rangle \langle \vec{p}_1', \vec{p}_2', \vec{p}_3' | \Phi_A^0 \rangle}{y - p_{1z} + i\eta} \end{aligned} \quad (15)$$

$\chi^{(2)}$ manifestly features a 3-particle density distribution.

Consider next the following sequence of approximations on a product of two 2-particle densities, most conveniently expressed in second quantization:

$$\begin{aligned} \text{for } \langle \Phi_A^0 | a_{\vec{p}_1}^\dagger a_{\vec{p}_2}^\dagger a_{\vec{p}_3}^\dagger a_{\vec{p}_2} a_{\vec{p}_1} | \Phi_A^0 \rangle \langle \Phi_A^0 | a_{\vec{p}_1'}^\dagger a_{\vec{p}_2'}^\dagger a_{\vec{p}_3'}^\dagger a_{\vec{p}_2'} a_{\vec{p}_1'} | \Phi_A^0 \rangle, \\ \text{i) replace } P_0 = |\Phi_A^0\rangle \langle \Phi_A^0| \longrightarrow \sum_n |\Phi_A^n\rangle \langle \Phi_A^n| = 1 \\ \text{ii) replace ('contract')} \quad a_{\vec{p}_1''}^\dagger a_{\vec{p}_1'}^\dagger \implies \frac{\langle \Phi_A^0 | a_{\vec{p}_1''}^\dagger a_{\vec{p}_1'}^\dagger | \Phi_A^0 \rangle}{n(p_1'')} \end{aligned}$$

We can reverse these arguments which, neglecting in addition some commutators $\mathcal{O}(1/A)$, lead to

$$\begin{aligned} \langle \Phi_A^0 | a_{\vec{p}_1}^\dagger a_{\vec{p}_2}^\dagger a_{\vec{p}_3}^\dagger a_{\vec{p}_3} a_{\vec{p}_2} a_{\vec{p}_1} | \Phi_A^0 \rangle &\approx \\ \langle \Phi_A^0 | a_{\vec{p}_1}^\dagger a_{\vec{p}_2}^\dagger a_{\vec{p}_3}^\dagger a_{\vec{p}_2} a_{\vec{p}_1} | \Phi_A^0 \rangle \frac{1}{n(p_1'')} \langle \Phi_A^0 | a_{\vec{p}_1'}^\dagger a_{\vec{p}_2'}^\dagger a_{\vec{p}_3'}^\dagger a_{\vec{p}_3'} a_{\vec{p}_2'} a_{\vec{p}_1'} | \Phi_A^0 \rangle. \end{aligned} \quad (16)$$

Substituting (16) into (15) leads by means of Eqs. (11)-(14) to

$$\begin{aligned} \chi^{(2)}(q, y) &\sim \rho^2 \left(\frac{m}{q} \right)^3 \iiint \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} \frac{d^3 p''}{(2\pi)^3} \\ &\quad \frac{u(\vec{p}_1, \vec{p}_2, \vec{p}_3) u(\vec{p}_1'', \vec{p}_2'', \vec{p}_3'') K(q, \vec{p} - \vec{p}'') K(q, \vec{p}'' - \vec{p}')}{n(\vec{p}'') (y - p_z + i\eta) (y - p_z'' + i\eta) (y - p_z' + i\eta)} \end{aligned} \quad (17)$$

The 'procedure' above can clearly be generalized to any $\chi^{(n)}$. Choosing in addition option (13b), (i.e. $n(\vec{p}, \vec{p}') K(q, \vec{p} - \vec{p}') \longrightarrow n(p) K(q, \vec{p} - \vec{p}')$) we can write (cf. Eq. (2))

$$\begin{aligned} \chi(q, y) &= \frac{m}{q} \int \frac{d^3 p}{(2\pi)^3} n(p) \xi(q, y - p_z) \\ &= \frac{m}{q} \int_{-\infty}^{\infty} dp_z F_0(p_z) \xi(q, y - p_z). \end{aligned} \quad (18)$$

ξ is easily shown to satisfy the one-dimensional integral equation

$$\begin{aligned} (y - p_z) \xi(q, y - p_z) &= 1 + \int_{-\infty}^{\infty} \frac{dp_z'}{2\pi} \Gamma(q, p_z - p_z') \xi(q, y - p_z') \\ \Gamma(q, p_z) &= \rho \frac{m}{q} \int \frac{d^2 p_\perp}{(2\pi)^2} K(q, \vec{p}). \end{aligned} \quad (19)$$

The option (13b) (or alternatively (13c)) implies a convolution of Γ and ξ in (19), which permits the solution¹¹

$$\xi(q, y) = -i \int_0^\infty dz e^{iyz} e^{i \int_0^z dz' \tilde{\Gamma}(q, z')}, \quad (20)$$

where $\tilde{\Gamma}(q, z)$ is the Fourier transform of $\Gamma(q, p_z)$. In the stated approximation one can thus write for the sum of the (incoherent!) IA and FSI contributions¹²

$$\begin{aligned} S(q, y) &\approx S^{\text{inc}}(q, y) = \frac{m}{q} \int_{-\infty}^{\infty} dy' F_0(y') R(q, y - y') \\ R(q, y - y') &= -\frac{1}{\pi} \text{Im} \xi(q, y - y'). \end{aligned} \quad (21)$$

Both asymmetric ansatzes (12b, 12c) and (13b, 13c) for an intrinsically symmetric ρ_2 lead to the same $\chi^{(1)}$, and so does the symmetric combination $((g(r) + g(r')))/2$.¹⁴ It remains true that, again, either (12b) or (12c) generate the same $\chi^{(n)}$, $n > 2$, but this is not the case for the symmetric combination above.¹⁵

Eqs. (19-21) constitute Silver's solution with one important difference: the large q scattering matrix in the kernels Γ and K , (Eqs. (14) and (19)) is $t(\theta)$ (Eq. (11)) and not the symmetrized $t^B = t(\theta) + t(\pi - \theta)$ as suggested by Silver.¹ The former, we recall, comes from the incoherent FSI part (cf. Eq. (4a)) which describes the case where one particle absorbs, and then later emits, the momentum transfer q . In multiple scattering theory, $t(\pi - \theta)$ appears only in coherent terms, where one particle absorbs the momentum q , but a different particle emits it (cf. Eq. (4b)).

At first sight it seems that use of t_B instead of the unsymmetrized scattering matrix t implies inclusion of some coherent correction to χ . However, this is not the case. Eqs. (4a) and (4b) can be shown to lead to $t(\theta)$ and $t(\pi - \theta)$, each accompanied by different factors, 1 and $e^{i\vec{q}(\vec{r}_1 - \vec{r}_2)}$ respectively, but not to their sum t_B . (see for

instance Ref. 8, Eqs. (36-37)) It is the large q behaviour of the exponent above which causes coherent contributions to decrease faster than incoherent ones.⁶

Direct evidence of the relative insignificance of coherent contributions comes from a comparison between the oscillations in the (anti-)symmetrized $\lim(q^2/4m, 0^\circ)$ for elastic ^3He - ^3He and ^4He - ^4He scattering with the smooth behaviour of the same for ^3He - ^4He . It is clear that the former are exclusively due to symmetrization.¹⁸ However, the amplitude of these, otherwise marked oscillations is only a few percent of the unsymmetrized ones or, differently stated, $|\chi^{\text{coh}}/\chi^{\text{inc}}| \ll 1$.

Till here, we have dealt with the response of a general system interacting through strong forces which may have a hard core. Terms in the discussed series for the response are of order $(\rho(m/q)K)^n$ and, although there is additional q -dependence in K , consecutive terms generally decrease as $1/q$. It is therefore likely that the dominant correction to the IA is $\mathcal{O}(\rho(m/q)K)$ and thus $|\chi^{(1)}| < \chi^{(0)}$. This is, of course, just the condition under which the series can be summed.

An exceptional case is the response of liquid ^4He below the superfluid transition temperature T_λ . For it, the momentum distribution appearing in Eqs. (13) and (14) reads

$$n(p, T) = (1 - n_0(T))n(p, T) + n_0(T)(2\pi)^3 \delta^3(\vec{p}), \quad (22)$$

with $n_n(p, T)$ the momentum distribution of normal atoms, and $n_0(T)$ the fraction of atoms in the Bose condensate at temperature T . With (22), F_0 and all $\chi^{(n)}$ are singular, but the formal sum (21) is not.

Of course, any experimental outcome contains the effect of finite instrumental resolution, for instance through folding with a resolution function which is characteristic for the apparatus. In the special case discussed above the experimental resolution function restores meaning to the individual, otherwise singular terms discussed above, and it is clearly of interest to calculate their relative magnitude in this case. The point has been addressed² in an analysis of data for $T = 1.2^\circ\text{K}$ at $q = 10 \text{ \AA}^{-1}$.²¹ After instrument broadening, it remains true that the first two terms dominate. The same has been shown to be the case for data with $q \geq 7 \text{ \AA}^{-1}$.⁴

Next we return to the non-diagonal 2-particle density matrix ρ_2 . One is particularly not far from a numerical determination of that quantity (see the contribution of Ristig and Clark in Ref. 4 and also Ref. 22) but for the moment only approximations of the form (12) are in use. In II the symmetric form (12a) has been employed and we reconstructed above that Silver's approximation amounts to the asymmetric (time-invariance violating) choice (12b) or (12c). In addition to the requirement of symmetry, we propose a test which can be performed if high- q data are accurate enough to yield a separation of S in parts even and odd in y i.e. $S = S^{\text{even}} + S^{\text{odd}}$. The data on ^4He for $q = 10 \text{ \AA}^{-1}$ at $T = 1.2^\circ\text{K}$ ²¹ meet this requirement. In II it was shown that with the choice (12a) only a narrow range of a_c values in agreement with expectation produces agreement for $S^{\text{odd}} \propto y^{(1)}$. We suspect sensitivity of $y^{(1)}$ to all input elements²³ and it would be instructive to see Silver's choice (12b), as well as the use of the symmetrized t_B , applied to it and compared with the same for (12a). Such a test is presently under way in an analysis of a representative set of data on $^3,4\text{He}$, where we also study numerically differences in the two approaches, for instance as due to instrumental resolution.²⁴

In summary we have presented above a derivation of Silver's expression for the response $S(q, \omega)$ for a quantum system in which the interaction between the constituents is strongly repulsive. Standard multiple scattering theory has been shown to be an adequate and efficient tool to reach that result.²⁵ The approach gives better insight into the physical processes which contribute to the response than do formal super-operator techniques and the Liouville equation. Moreover, manipulations with the multiple scattering series give clearer insight into the approximations involved in producing a sum to all orders, as Silver has done.^{1,3}

We thank our colleagues who have participated in discussions on the subject matter of this note.

REFERENCES

- ¹R.N. Silver, Condensed Matter theory **3**, 131 (1987); Phys. Rev. **B37**, 3794 (1988); Phys. Rev. **B38**, 2283 (1988); Condensed Matter Theory **4**, to be published.
- ²A.S. Rinat, WIS preprint 88/31.
- ³R.N. Silver, preprint LA-UR-88-2097; T.R. Sosnick, W.M. Snow, P.E. Sokol and R.N. Silver, preprint LA-UR-88-055.
- ⁴A.S. Rinat, Proceedings of the Workshop on Momentum distributions, ANL 1988, Plenum Press, to be published.
- ⁵See for instance, J. Hubbard, Proc. Royal Soc. **240A**, 539 (1957); A.J. Glick, Ann. Phys.(NY) **17**, 61 (1961).
- ⁶G.B. West, Phys. Rep. **18**, 263 (1975).
- ⁷e.g. M.L. Goldberger and K.M. Watson, Collision Theory, J. Wiley & Sons 1965, p. 750.
- ⁸H.A. Gersch, L.J. Rodriguez and Phil N. Smith, Phys. Rev. **A5**, 1547 (1972); V.F. Sears, Phys. Rev. **B30**, 1266 (1984).
- ⁹S.A. Gurvitz, A.S. Rinat and R. Rosenfelder, PSI preprint 88-21
- ¹⁰An alternative proposal for ρ_2 ⁸ leads to a more complicated treatment of $g(r, r')$ in (12) and (13) and is not discussed in the following.
- ¹¹e.g. J.M. Eisenberg and D.S. Kolton, Theory of Meson Interactions With Nuclei, J. Wiley and Sons 1980, p. 184.
- ¹²Using a Wigner representation for $S(q, \omega)$, a similar form for χ has been derived in Ref. 13 for non-singular interactions and may presumably be generalized for singular interactions. The dominant FSI correction there is the same as (14).
- ¹³R. Rosenfelder, Nucl. Phys. **A459**, 452 (1986).
- ¹⁴Using (13a) will simply change the limits on the exponential integral, from $(0, z)$ to $(-z/2, z/2)$.
- ¹⁵Notice that by construction χ or S is a sum of multiple scattering terms. S in the alternative form (21) has been discussed before by Gersch and Rodriguez¹⁶

and the evaluation of R there is complicated in general. However, in the sketched approximation and using options (12b) or (12c), the simple expression (21) for R results. No such form can readily be derived from (12a).

¹⁶H.A. Gersch and L.J. Rodriguez, Phys. Rev. **A8**, 905 (1973).

¹⁷B. Tanatar, E.F. Talbot and H.R. Glyde, Phys. Rev. **B36**, 8376 (1987).

¹⁸The oscillations in the ^4He - ^4He , ^3He - ^3He total cross sections as function of energy are frequently referred to as 'glory oscillations.' To our knowledge that expression is usually reserved for oscillations in a differential cross section when (in a semi-classical treatment) paths around a target conspire to produce these oscillations.¹⁹ From a comparison of the elastic cross sections for the processes above with $d\sigma/d\Omega$ for ^3He - ^4He ²⁰ it transpires that there is some structure for small θ due to dynamics. However, most of the oscillations are due to statistics.

¹⁹R.G. Newton, Scattering theory of particles and waves, McGraw Hill (1966), p.580.

²⁰P.E. Siska, J.M. Parson, T.O. Schafer and Y.T. Lee, J. Chem. Phys. **55**, 5762 (1971); R. Feltgen, H. Kirst, K.A. Kohler, H. Pauly and F. Torello, *ibid* **76**, 2360 (1982).

²¹P. Martel, E.C. Svensson, A.D.B. Woods, V.F. Sears and R.A. Cowley, J. Low Temp. Phys. **23**, 285 (1976).

²²D.M. Ceperley, V.R. Pandharipande, A. Belic, private communication.

²³Notice that as long as $|\chi^{(1)}| \ll |\chi^{(0)}|$, the sensitive quantity is S^{odd} and not S^{tot} which is dominated by $\chi^{(0)}$.

²⁴A.S. Rinat and M. Taragin, in preparation.

²⁵Refs. 1 and 2 are not the only ones which link FSI to t . In Ref. 17 one finds references and discussions on χ related to the random phase approximation⁵ with optionally dressed particles and holes. In Ref. 26 a non-perturbative Green's functions method²⁷ is used for a calculation of S . One easily shows that a particular contraction in ρ_2 (Eq. (9)) leads to the basic results, Eqs. (16) and (17) of Ref. 26.

²⁶A.S. Rinat (Rinat), Phys. Rev. **138**, B389 (1965).

²⁷P.C. Martin and J. Schwinger, Phys. Rev. **115**, 1342 (1959).

FIGURE CAPTIONS

Figure 1

a) Representation of the double-scattering contribution $\chi^{(2)}$. The A particle system emits 3 particles, one of which absorbs the supplied momentum transfer $\vec{q}$ before scattering from '2.' The scattered '1' has a subsequent momentum transfer $\vec{q}'$ whereupon all 3 return to the interacting ground state. b) Silver's approximation to (a) where '1' and '2' produce again Φ_A^0 , which re-emits $\vec{q}$, which re-emits to (a) where Notice that particle 1 after the first, and before the second, scattering is assumed to have the same momentum.

Fig. 1.