In the twenty five years since the discovery of CP violation\(^1\) little progress has been made in determining where this violation arises in the elementary particle interactions.\(^2\) The violation of CP invariance has been observed in three decays of the \(K_L\) meson also, possibly, in the \(K^0 \rightarrow \pi \pi\) decay\(^3\) and nowhere else.\(^4\) In order to determine the source of CP violation and hence distinguish between different theoretical models, it is necessary to have additional experimental information. Among the various possibilities, the possibility of detecting CP violation by measuring the neutron electric dipole moment \(d_n\) has long been of special interest.\(^5\)

Model predictions\(^6\) on the order of magnitude of \(d_n\) vary from the neighborhood of the present experimental limit \(\lesssim 10^{-25} e\cdot cm\) to arbitrary values. This arises because a nonzero value of \(d_n\) could always be blamed on the “strong CP violation”.\(^7\) Hence, for the class of models, in which a small strong CP parameter \(\theta\) is realized by means of fine-tuning, the neutron electric dipole moment is fundamentally inescapable. Thus, in general, a nonzero value of \(d_n\) actually cannot be unambiguously related to weak CP violation. The most familiar example is the Kobayashi-Maskawa (KM) model.\(^8\) Although weak interaction effects appears to give a very small contribution\(^9\) to \(d_n\), one practically has no control\(^10\) on the size of \(\theta\). As a result, in the KM model the value of \(d_n\) induced by the strong CP parameter \(\theta\) is essentially arbitrary.

Clearly, in order to be able to predict \(d_n\), we must first consider how to solve the strong CP problem. The best known solution is by introducing Peccei-Quinn type symmetries,\(^11\) which allow us to eliminate the \(\theta\) parameter completely from the theory. However, by doing so either the theory has to have a very light axion, which is still yet to be borned out by experiments,\(^7\) or we have to make at least one of the light quarks massless which, on the other hand, may not be compatible with
Assuming in Eq. 13 to 16, it is possible to reduce the regulations in Eq. (2) in a natural way to also

\[ \theta \geq \frac{\left( D \right)_{\text{max}}}{\left( \theta \right)_{\text{max}}} \quad \theta \geq \theta_{\text{min}} \]

and because every term in Eq. (4) is smaller, this possibility is even more likely. Alternatively, we may assume that the reason that \( \theta \geq \theta_{\text{min}} \) is the case of strong CP violation. In this case, we will therefore only discuss the solution of the original equation of motion with \( \theta = \theta_{\text{min}} \). This is possible when the potential equation contains the different terms in Eq. (4).

Now, suppose that at some loop level we consider cyclic roles of \( \theta \). Then, in the basis in which \( \theta \) takes on values of \( \theta_{\text{min}} \), we may assume that the potentials are not minimized, and only contributes to the potential equation of energy and momentum but not to \( \theta \). This is possible when all the cyclic roles of \( \theta \) are real. Clearly, all higher loop levels will receive potential corrections.

\[ \frac{\left( \theta \right)_{\text{min}}}{\left( \theta \right)_{\text{max}}} \quad \theta \geq \theta_{\text{min}} \]

with \( \left( \theta \right)_{\text{min}} = 0 \) or \( \theta = \theta_{\text{min}} \).

\[ \frac{\left( \theta \right)_{\text{min}}}{\left( \theta \right)_{\text{max}}} \quad \theta \geq \theta_{\text{min}} \]

Experimentally, this is the case of the predicted \( \theta \). Now we

\[ \theta \geq \theta_{\text{min}} \]

to a form given by Eq. (4).

The last term in Eq. (4) survives for the special case only in which \( \theta = \theta_{\text{min}} \). The control parameter \( \theta \) is controlled in the situation described in Eq. (4).

This shows the polynomial solution in powers of \( \theta \) is included and in our analysis.

\[ \theta \geq \theta_{\text{min}} \]

and shows the polynomial solution in powers of \( \theta \) is included and in our analysis.

Here, we see that in this order only the important part of the potential belongs to

\[ \theta \geq \theta_{\text{min}} \]

of string potentials.

We begin by considering the effective potential which determines the strengths

\[ \theta \geq \theta_{\text{min}} \]

of CP potentials rather than directly from weak CP violation effects.

Observation contribution to the gauge group comes from the potential introduced.

Since we would like to show in this letter what this is a model of models in the

\[ \theta \geq \theta_{\text{min}} \]

for the only visible CP symmetric other spontaneously broken symmetry breaking

the present view of cyclic potentials. Another possibility is to consider

\[ \theta \geq \theta_{\text{min}} \]

show up.
terms are usually much smaller than \( \theta_{\text{tree}} \). Among the various relations in Eq. (5), of special interest are those that relate the \( \theta \) parameter to the first generation quark masses

\[
\frac{\text{Im}(\delta m_u)}{m_u} \leq \theta, \quad \frac{\text{Im}(\delta m_d)}{m_d} \leq \theta,
\]

(6)

where \( m_u \) and \( m_d \) represent, respectively, the u- and d-quark masses. Of course, relations given by Eqs. (5) and (6) are valid only if there are no Peccei-Quinn type symmetries. Had we introduced such symmetries, the contribution to \( \theta \) from Eq. (5) would have been eliminated by a trivial rotation, and it would be unnecessary to require each terms in Eq. (5) be small.

Now, we explore the consequences of Eq. (6). First, we realize that in the basis where the quark mass matrices \( D_q \) are diagonal and positive definite, there is a relation between \( \text{Im}(\delta m_q) \) and the quark electric dipole moment \( d_q \). Indeed, for every effective \( q \bar{q} \gamma \gamma \) coupling graph shown in Fig.1 that generates a contribution to \( d_q \), there will be a corresponding diagram illustrated in Fig.2 contributing to \( \text{Im}(\delta m_q) \) and hence to \( \theta \) from Eq. (4). The contributions of the effective \( q \bar{q} \gamma \gamma \) vertex can be separated into one-particle irreducible (1PI) and reducible diagrams as depicted in Fig.1a and 1b, respectively. One finds\(^{19}\) that only 1PI vertex of Fig.1a contributes to the quark electric dipole moment, \( d_q \), term via

\[
\bar{q}(p + k)i f_D(k^2) \sigma_{\mu \nu} k^\nu \gamma_5 q(p)
\]

(7)

where \( d_q = f_D(0) \). From the Gordon decomposition relation

\[
\bar{q}(p + k) \sigma_{\mu \nu} k^\nu \gamma_5 q(p) = \bar{q}(p + k)(2p + k)_\nu \gamma_5 q(p),
\]

(8)

we have

\[
\bar{q}(p + k)i f_D(k^2) \sigma_{\mu \nu} k^\nu \gamma_5 q(p) = \bar{q}(p + k)i f_D(k^2)(2p + k)_\nu \gamma_5 q(p).
\]

(9)

To relate the right-handed side of Eq. (9) to the parameter \( \theta \) we use the Ward-Takahashi identity

\[
\Gamma^{(2)}_\mu(p + k) = c Q_1 \frac{\partial}{\partial p^\mu} D_q(p) + O(k)
\]

(10)

where \( \Gamma^{(2)}_\mu \) is the effective \( q \bar{q} \gamma \) vertex of Fig.1a and \( Q_1 \) is the charge of the quark.

Putting Eqs. (3), (9) and (10) together, one obtains that

\[
\bar{q}(p + k)i c Q_1 \frac{\partial}{\partial p^\mu} \text{Im}(\delta m_q(p)) \gamma_5 + O(k) [q(p)] = \bar{q}(p + k)f_D(k^2)(2p + k)_\nu \gamma_5 q(p),
\]

(11)

which immediately gives an identity

\[
f_D(0) [q^\mu = m_q^2 = c Q_1 \frac{\partial}{\partial p^\mu} \text{Im}(\delta m_q(p))] [q^\mu = m_q^2]
\]

(12)

by taking all the \( k \) dependence terms to zero. As a consequence, for arbitrary \( m_q \), we can relate \( d_q \) to \( \text{Im}(\delta m_q) \) as follows\(^{20}\)

\[
d_n^{\text{weak}} \sim d_n \sim c Q_1 \frac{\text{Im}(\delta m_q)}{\Lambda^2}
\]

(13)

where \( d_n^{\text{weak}} \) represents the contribution to \( d_n \) directly from weak CP violation effects, and \( \Lambda \) is a parameter with the dimension of mass. Strictly speaking, the first step of Eq. (13) follows only if long distance contributions to \( d_q \) are negligible. Although this is an assumption we will make, for the class of models discussed in Refs. 13 to 16 this turns out to be the case.\(^{21}\)

It is important to realize from Eq. (12) that in the limit \( \text{Im}(\delta m_q) = 0 \) for arbitrary \( m_q \), contributions to \( d_n^{\text{weak}} \) at that level must vanish as well. One interesting implication of this observation can be summarized as follows: Theoretical realistic models which have a contribution to \( d_n \) at, say, the \( n \)th-loop level, must also generate a contribution to \( \theta \) at the same loop.\(^{21,22}\) For example, in the standard KM model, since there is no contribution to \( \theta \) up to two-loop levels for any \( m_q \), one concludes
20. Here, in the spirit of understanding the strong CP problem without invoking accidental vanishing of Green's functions, we do not consider the cases that (i) $\text{Im}(\delta m_q(p)) |_{\varphi=\pi^2} \neq 0$ and $\frac{\partial}{\partial p} \text{Im}(\delta m_q(p)) |_{\varphi=\pi^2} = 0$ or (ii) $\text{Im}(\delta m_q(p)) |_{\varphi=\pi^2} = 0$ and $\frac{\partial}{\partial p} \text{Im}(\delta m_q(p)) |_{\varphi=\pi^2} \neq 0$ for some special values of $m_q$. See Ref.21 for the detailed calculations on various models.


22. To our knowledge, the only occasion that appears to be in conflict with our result is the recent claim of K. S. Babu and R. N. Mohapatra (Phys. Rev. Lett. 62, 1079 (1989)) with $d_n \sim d_4 \sim (10^{-26} - 10^{-26})c.m. at the one-loop level, whereas $\theta$ is zero at the same level. In their model, instead of $\theta$ being zero for the one-loop diagrams, we estimate that, $\theta \sim .01 - .001$ which is too large for $d_n$.21

23. One can make the computation of $\theta$ in the KM model meaningful by introducing a cutoff. Since the coefficient of the divergent term of $\theta$ (from seven loop diagrams) is very small, even a cutoff of the order of the Planck scale will not disturb the stability of $\theta$ once we choose it small.


Figure Captions

- Fig.1 The effective $qq\gamma$ vertex where a and b are the one particle irreducible and reducible parts, respectively.

- Fig.2 Contributions to $\delta m_q$. 

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