15 NOV 1989

TRI-PP-89-8
UM-TH-89-05
Mar 1989

Strong CP Violation and the Neutron Electric Dipole Moment

Jiang Liu

Randall Laboratory of Physics, University of Michigan, Ann Arbor, MI 48109, USA
C. Q. Geng and John N. Ng

TRIUMF Theory Group, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A9

Abstract

We derive an identity that relates the fermion electric dipole moment from weak CP violation effects to the strong CP parameter θ . In the absence of Peccei-Quinn type symmetries, we find that requiring θ naturally small generally implies that, for a large class of models, the dominant contribution to the neutron electric dipole moment, d_n , comes from strong CP violation rather than directly from weak CP violation effects.

(submitted to Physical Review Letters)

CERN LIBRARIES, GENEVA

CM-P00068279

In the twenty five years since the discovery of CP violation¹ little progress has been made in determining where this violation arises in the elementary particle interactions.² The violation of CP invariance has been observed in three decays of the K_L meson also, possibly, in the $K^0 \rightarrow \pi\pi$ decay³ and nowhere else.⁴ In order to determine the source of CP violation and hence distinguish between different theoretical models, it is necessary to have additional experimental information. Among the various possibilities, the possibility of detecting CP violation by measuring the neutron electric dipole moment d_n has long been of special interest.⁵

Model predictions⁶ on the order of magnitude of d_n vary from the neighborhood of the present experimental limit $\leq 10^{-25} e \cdot cm$ to arbitrary values. This arises because a nonzero value of d_n could always be blamed on the "strong CP violation".⁷ Hence, for the class of models, in which a small strong CP parameter θ is realized by means of fine-tuning, the neutron electric dipole moment is fundamentally incalculable. Thus, in general, a nonzero value of d_n actually cannot be unambiguously related to weak CP violation. The most familiar example is the Kobayashi-Maskawa (KM) model.⁸ Although weak interaction effects appears to give a very small contribution⁹ to d_n , one practically has no control¹⁰ on the size of θ . As a result, in the KM model the value of d_n induced by the strong CP parameter θ is essentially arbitrary.

Clearly, in order to be able to predict d_n , we must first consider how to solve the strong CP problem. The best known solution is by introducing Peccei-Quinn type symmetries,¹¹ which allow us to eliminate the θ parameter completely from the theory. However, by doing so either the theory has to have a very light axion, which is still yet to be borned out by experiments,⁷ or we have to make at least one of the light quarks massless which, on the other hand, may not be compatible with

the present view of chiral symmetry of hadrons.¹² Another possibility is to require that the theory violates CP symmetry either spontaneously or through soft breaking terms. We would like to show in this letter that for this class of models,¹³⁻¹⁶ the dominant contribution to d_n *generally*¹⁷ comes from the radiatively induced strong CP θ parameter rather than directly from weak CP violation effects.

We begin by considering the effective θ parameter which characterizes the strength of strong CP violation

$$\theta = \theta_{QCD} + \text{Arg} \text{Det}(M_u M_d) = \theta_{QCD} + i \ln \frac{\text{Det}(M_u M_d)}{\text{Det}(M_u^\dagger M_d^\dagger)}. \quad (1)$$

Here θ_{QCD} comes from a term $(\theta_{QCD}/64\pi^2) \text{Tr} G\tilde{G}$ in the QCD Lagrangian. $M_{u(d)}$ is the up(down)-quark mass matrix. Requiring CP being violated only spontaneously or through soft breaking terms amounts to setting $\theta_{QCD} = 0$. In this case, it is convenient to represent the last term in Eq. (1) by a parameter θ_{free} .¹⁸ In doing so, we have explicitly chosen a basis in which the quark mass matrices are diagonal and positive definite

$$M_q \rightarrow D_q, \quad \text{with} \quad (D_q)_{ij} = m_q^i \delta_{ij}, \quad m_q^i > 0, \quad (2)$$

where q represents up- and down-quarks. Clearly, at higher loop levels, D_q will receive radiative corrections $D_q \rightarrow D'_q$ but if all the corrections are real, they will only contribute to the renormalizations of quark mass and mixing but not to θ_{free} . Thus in the basis in which D'_q remains diagonal and positive definite, θ is unchanged.

Now, suppose at some loop level nonzero complex values of corrections to D_q first show up

$$D_q \rightarrow D'_q + \delta m_q. \quad (3)$$

Here δm_q is the radiative correction to the diagonalized quark mass matrices D'_q .

In general, δm_q is not diagonal. For the consideration of d_n , only the imaginary part of δm_q will be relevant. We will therefore assume that δm_q is much smaller than the *diagonalized* matrices D'_q . Now, using the identities $\text{Det}(D'_q + \delta m_q) = (\text{Det} D'_q) \Pi_i (1 + D'^{-1}_{u,i} \delta m_{u,i}) + \dots$ and $\ln(1 + D'^{-1}_{u,i} \delta m_{u,i}) = D'^{-1}_{u,i} \delta m_{u,i} + \dots$, the ellipses represent terms with higher powers of δm_u , we find to the leading order of δm_q

$$\theta = \theta_{free} + \sum_i \left\{ \frac{\text{Im}(\delta m_u)_{ii}}{(D'_u)_i} + \frac{\text{Im}(\delta m_d)_{ii}}{(D'_d)_i} \right\}. \quad (4)$$

Hence, we see that to this order only the imaginary part of the diagonal elements of δm_q enters the expression of θ . This justifies our earlier assumption on the size of δm_q and shows that perturbation expansion in powers of δm_q is indeed valid as far as computing θ parameter is concerned. When M_q is hermitian, $\theta_{free} = 0$ and thus only the last term in Eq. (4) survives. For this special case, our formula reduces explicitly to a form given by Bég *et al.*¹³

Experimental limits on the size of d_n has yielded⁵ $\theta \leq 10^{-9} - 10^{-10}$. Now we ask: how can we make θ sufficiently small? One possibility is, of course, to introduce arbitrary fine-tunings so that the smallness of the θ parameter arises because of a complete cancellation among the different terms in Eq. (4). This possibility, while viable in principle, cannot however answer the original question of naturalness which is the center of strong CP problem. In what follows we will, therefore, not discuss this possibility any further. Alternatively, we may assume that the reason that θ is small is because every term in Eq. (4) is small

$$\theta_{free} \leq \theta, \quad \frac{\text{Im}(\delta m_q)_{ii}}{(D'_q)_i} \leq \theta \quad (i = 1, 2, 3, \dots). \quad (5)$$

Theoretical models that realize the conditions in Eq. (5) in a natural way are discussed in Refs. 13 to 16. In practice, if $\theta_{free} \neq 0$ one finds that the radiative correction

terms are usually much smaller than θ_{tree} . Among the various relations in Eq. (5), of special interest are those that relate the θ parameter to the first generation quark masses

$$\frac{Im(\delta m_u)}{m_u} \leq \theta, \quad \frac{Im(\delta m_d)}{m_d} \leq \theta, \quad (6)$$

where m_u and m_d represent, respectively, the u - and d -quark masses. Of course, relations given by Eqs. (5) and (6) are valid only if there are no Peccei-Quinn type symmetries. Had we introduced such symmetries, the contribution to θ from Eq. (5) would have been eliminated by a trivial rotation, and it would be unnecessary to require each term in Eq. (5) be small.

Now, we explore the consequences of Eq. (6). First, we realize that in the basis where the quark mass matrices D'_q are diagonal and positive definite, there is a relation between $Im(\delta m_q)$ and the quark electric dipole moment d_q . Indeed, for every effective $q\bar{q}\gamma$ coupling graph shown in Fig. 1 that generates a contribution to d_q , there will be a corresponding diagram illustrated in Fig. 2 contributing to $Im(\delta m_q)$ and hence to θ from Eq. (4). The contributions of the effective $q\bar{q}\gamma$ vertex can be separated into one-particle irreducible (1PI) and reducible diagrams as depicted in Fig. 1a and 1b, respectively. One finds¹⁹ that only 1PI vertex of Fig. 1a contributes to the quark electric dipole moment, d_q , term via

$$\bar{q}(p+k)i f_D(k^2) \sigma_{\mu\nu} k^\nu \gamma_5 q(p) \quad (7)$$

where $d_q = f_D(0)$. From the Gordon decomposition relation

$$\bar{q}(p+k) \sigma_{\mu\nu} k^\nu \gamma_5 q(p) = \bar{q}(p+k)(2p+k)_\mu \gamma_5 q(p), \quad (8)$$

we have

$$\bar{q}(p+k)i f_D(k^2) \sigma_{\mu\nu} k^\nu \gamma_5 q(p) = \bar{q}(p+k)i f_D(k^2)(2p+k)_\mu \gamma_5 q(p). \quad (9)$$

To relate the right-handed side of Eq. (9) to the parameter θ we use the Ward-Takahashi identity

$$\Gamma_\mu^{q\bar{q}}(p+k) = e Q_q \frac{\partial}{\partial p^\mu} D_q(p) + O(k) \quad (10)$$

where $\Gamma_\mu^{q\bar{q}}$ is the effective $q\bar{q}\gamma$ vertex of Fig. 1a and Q_q is the charge of the quark. Putting Eqs. (3), (9) and (10) together, one obtains that

$$\bar{q}(p+k)[e Q_q \frac{\partial}{\partial p^\mu} Im(\delta m_q(p)) \gamma_5 + O(k)] q(p) = \bar{q}(p+k) f_D(k^2)(2p+k)_\mu \gamma_5 q(p), \quad (11)$$

which immediately gives an identity

$$f_D(0) |_{p^2=m_q^2} = e Q_q \frac{\partial}{\partial p^2} Im(\delta m_q(p)) |_{p^2=m_q^2} \quad (12)$$

by taking all the k dependence terms to zero. As a consequence, for arbitrary m_q , we can relate d_q to $Im(\delta m_q)$ as follows:²⁰

$$d_q^{weak} \sim d_q \sim e Q_q \frac{Im(\delta m_q)}{\Lambda^2} \quad (13)$$

where d_q^{weak} represents the contribution to d_q directly from weak CP violation effects, and Λ is a parameter with the dimension of mass. Strictly speaking, the first step of Eq. (13) follows only if long distance contributions to d_q are negligible. Although this is an assumption we will make, for the class of models discussed in Refs. 13 to 16 this turns out to be the case.²¹

It is important to realize from Eq. (12) that in the limit $Im(\delta m_q) = 0$ for arbitrary m_q ,²⁰ contributions to d_q^{weak} at that level must vanish as well. One interesting implication of this observation can be summarized as follows: Theoretical realistic models which have a contribution to d_q at, say, the nth -loop level, must also generate a contribution to θ at the same loop.^{21,22} For example, in the standard KM model, since there is no contribution to θ up to two-loop levels for any m_q ,¹² one concludes

REFERENCES

1. J. Christenson, J. Cronin, V. Fitch, and R. Turlay, *Phys. Rev. Lett.* **13**, 138 (1984).
2. For a recent review on the present status of CP violation see for example L. Wolfenstein, *Ann. Rev. Nucl. Part. Sci.* **36**, 137 (1986).
3. H. Burkhardt et al (NA31 Collaboration), *Phys. Lett.* **206B**, 169 (1988).
4. Possible detections of the neutron electric dipole moment of the order of 10^{-25} e.cm have recently been reported by two different groups: I. S. Altarev et al. *JETP Lett.* **44**, 460 (1986); J. M. Pendlebury, preprint, (to appear in the Proc. of the 9th Symposium on Grand Unification (Aix les Bains, April 1988). However, it is too early to see these as positive signals.
5. For a recent review on electric dipole moments see S. Barr and W. Marciano, to appear in: *CP-Violation*, Ed C. Jorlskog.
6. X. G. He, B. H. J. McKellar, and S. Pakvasa, Melbourne Univ. Rept No. UH-511-666-89 (1989).
7. For recent reviews on the strong CP problem see: J. E. Kim, *Phys. Rep.* **150**, 1 (1987); H. Y. Cheng, *Phys. Rep.* **158**, 1 (1988); R. D. Peccei, DESY-88-109 (1988) (to appear in: *CP-Violation*, Ed C. Jorlskog).
8. M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
9. E. P. Shabalin, *Sov. J. Nucl. Phys.* **28**, 75 (1978); *Sov. Phys. Usp.* **26**, 297 (1983).
10. J. Ellis, and M. K. Gaillard, *Nucl. Phys.* **B150**, 141 (1979).
11. R. D. Peccei and H. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977).
12. A way of avoiding this difficulty has been suggested by: N. V. Krasnikov, V. A. Matveev, *JETP Lett* **35**, 338 (1982); D. B. Kaplan, A. V. Manohar, *Phys. Rev. Lett.* **56**, 2004 (1986).
13. M. A. B. Bég and H.-S. Tsao, *Phys. Rev. Lett.* **41**, 287 (1978); M. A. B. Bég, D. J. Mallof, and H.-S. Tsao, *Phys. Rev.* **D19**, 2221 (1979).
14. H. Georgi, *Hadronic J.* **1**, 155 (1978); G. Segrè and H. A. Weldon, *Phys. Rev. Lett.* **42**, 1191 (1979); S. Barr and P. Langacker, *Phys. Rev. Lett.* **42**, 1654 (1979); R. N. Mohapatra and G. Senjanović, *Phys. Lett.* **79B**, 283 (1978); R. N. Mohapatra and D. Wyler, *Phys. Lett.* **89B**, 181 (1980); A. Masiero, R. N. Mohapatra and R. D. Peccei, *Nucl. Phys.* **B192**, 66 (1981).
15. A. Nelson, *Phys. Lett.* **136B**, 387 (1984); **134B**, 422 (1984); **143B**, 165 (1984); S. Barr, *Phys. Rev. D* **30**, 1805 (1984); *Phys. Rev. Lett.* **53**, 329 (1984).
16. K. Hagiwara, F. Halzen, and K. Hikasa, *Phys. Lett.* **92B**, 103 (1980); **94B**, 557(E) (1980); S. Barr and A. Zec, *Phys. Rev. Lett.* **55**, 2253 (1985).
17. By general we mean our statement will remain true unless arbitrary fine-tunings are allowed when dealing with the θ parameter.
18. Here we assume that θ comes only from the phase of the determinant of the quark mass matrix. However, this may be an incomplete treatment of θ as indicated by H. Georgi, T. Tomaras, and A. Pais, *Phys. Rev.* **D23**, 469 (1981).
19. C. J. Ellis, M. K. Gaillard and P. V. Nanopoulos, *Nucl. Phys.* **B109**, 213 (1976).

20. Here, in the spirit of understanding the strong CP problem without invoking accidental vanishing of Green's functions, we do not consider the cases that (i) $Im(\delta m_q(p))|_{p^2=m_q^2} \neq 0$ and $\frac{\partial}{\partial p^2} Im(\delta m_q(p))|_{p^2=m_q^2} = 0$ or (ii) $Im(\delta m_q(p))|_{p^2=m_q^2} = 0$ and $\frac{\partial}{\partial p^2} Im(\delta m_q(p))|_{p^2=m_q^2} \neq 0$ for some special values of m_q . See Ref.21 for the detailed calculations on various models.

21. Jiang Liu, C. Q. Geng, and John N. Ng, in preparation.

22. To our knowledge, the only occasion that appears to be in conflict with our result is the recent claim of K. S. Babu and R. N. Mohapatra (Phys. Rev. Lett. 62, 1079 (1989)) with $d_h \sim d_d \sim (10^{-26} - 10^{-28}) e.cm$ at the one-loop level, whereas θ is zero at the same level. In their model, instead of θ being zero for the one-loop diagrams, we estimate that, $\theta \sim .01 - .001$ which is too large for d_n .²¹

23. One can make the computation of θ in the KM model meaningful by introducing a cutoff. Since the coefficient of the divergent term of θ (from seven loop diagrams) is very small, even a cutoff of the order of the Planck scale will not disturb the stability of θ once we choose it small.

24. I. B. Khriplovich, Phys. Lett. B173, 193 (1986).

Figure Captions

• Fig.1 The effective $qq\gamma$ vertex where a and b are the one particle irreducible and reducible parts, respectively.

Fig.2 Contributions to δm_q .

Fig. 1.

Fig. 2.