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EXTENDED SUPERSYMMETRY IN FOUR-DIMENSIONAL
TYPE II STRINGS

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ABSTRACT. We construct models with $N = 3, 4, 5, 6$ and 8 spacetime supersymmetry in type II four-dimensional superstrings and analyze their moduli space. These models are obtained in the fermionic formulation and most of them can be reformulated as asymmetric orbifold compactifications. The massless sector of these theories is shown to be compatible with the symmetries of the extended supergravity Lagrangians.

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Introduction.

In four spacetime dimensions there is a plethora of consistent string models [1-19]. Such models can be classified according to the superparameterization properties of the world sheet and the number of spacetime supersymmetries [19]. We shall restrict ourselves to string models which contain spacetime fermions in their physical spectrum. As a result, we must consider theories with analytic local world sheet supersymmetry $(1,0)$ or with both analytic and antianalytic world sheet supersymmetries $(1,1)$ [19].

The maximal spacetime supersymmetry in the $(1,0)$ case (heterotic models) [20] is $N = 4$, while in the $(1,1)$ case (type II models) [21] it is $N = 8$. The purpose of this work is to show that in the framework of $(1,1)$ superconformal theories we can easily obtain superstring models with all possible $N \leq 8$ spacetime supersymmetries. Although there are some examples in the literature with $N = 8, 4, 2, 1, 0$ supersymmetry, constructed either by using fermionic world sheet variables [4,5,8], lattice constructions [3,6], orbifold compactifications [2,9], sigma-model techniques [13,14] or by using exactly solvable superconformal blocks [12], no string model based on $N = 6, 5, 3$ supersymmetry has been constructed yet. Very recently in Ref. [22] by using a classification based on exceptional groups, a minimal content of massless multiplets of $N = 6, 5$ and 3 superstrings has been given. However, no explicit model has been presented.

In this work we shall concentrate on the construction of $N \geq 3$ superstrings based on $(1,1)$ superconformal theory. It is clear that to get $N > 4$ superstrings one has to work in the framework of $(1,1)$ theories while a string solution with $N = 3$ could be obtained in principle, from either $(1,0)$ or $(1,1)$ theories. Nevertheless, it seems impossible [22] to construct a $N = 3$ superstring from $(1,0)$ superconformal theory and so we must resort to

its (1,1) extension for a possible solution. Thus all exotic supersymmetries with $N \geq 3$ will result from type II strings, with the exception of $N = 4$ which can also be obtained through heterotic constructions. Moreover, the $N = 7$ case cannot exist since we can show, using supergravity arguments, that it is always extended to $N = 8$ [23].

Even though the physical relevance of extended supersymmetries ($N > 1$) is questionable, their study is of main interest especially on string framework. Indeed, as was already observed in the literature, physically relevant models with $N \leq 1$ supersymmetry are strongly related to those with extended supersymmetry ($N > 1$) [4,24]: a subsector of the spectrum of less symmetric solution is obtained by projecting out consistently some states of a more symmetric solution.

An additional motivation of our interest in extended supersymmetric solutions is that of string universality. If, as it is commonly believed all string models are solutions of a universal string field theory, then it is of main importance to examine some special phases of the theory and namely those with a different number of spacetime supersymmetries.

This paper is organized as follows. In Sections 2, 3 and 4, using the fermionic formulation [4,5], we construct examples of $N = 8, 6$ and 5 superstring solutions and we analyze their moduli space. We also comment on the string induced effective supergravities. From the string Kac-Moody symmetries we shall be able to classify the massless scalar states as coset representatives of manifolds which occur in nonlinear sigma models coupled to N-extended supergravity ($N \geq 5$). The massless sector of $N \geq 5$ superstrings is fully specified by N as expected from supersymmetry algebra; (all massless states belong to the graviton supermultiplet). Their massive content, however, depends on the particular string solution.

The $N = 4$ and $N = 3$ cases are examined in Sections 5 and 6, respectively. There also, we shall be able to classify the massless scalars as coset representatives of manifolds which occur in nonlinear sigma models coupled to N-extended supergravity ($N \leq 4$). For these cases the massless states are not fully specified by N and this is due to the presence of vector supermultiplets whose number depends also on the particular string solution. All possible constructions through fermionic formulation, with periodic and antiperiodic world sheet fermions are given. We also analyze the moduli space of the models obtained and we comment on the induced effective supergravities.

The last section (7) is devoted to a discussion of our results together with conclusions.

1. Notation and Conventions.

For our constructions we shall use the fermionic formulation where all internal string degrees of freedom are parametrized by free periodic and antiperiodic world sheet fermions [4,5]. Being in the framework of (1,1) local superconformal theory the analytic world sheet degrees of freedom (left movers) are [4]:

(i) String spacetime supercoordinates

$$(\partial_z x^\mu(z, \bar{z}), \psi^\mu(z)) ; \quad \mu = 1, 2, 3, 4 . \quad (1.1a)$$

(ii) Fermionized internal supercoordinates [4]

$$(x^I(z), y^I(z), w^I(z)) ; \quad I = 1, 2, \dots, 6 \quad (1.1b)$$

where ψ^μ, x^I, y^I, w^I are two-dimensional Weyl majorana fermions.

The anti-analytic degrees of freedom (right movers) are similarly given by:

$$(i) \quad (\partial_{\bar{z}} x^\mu(z, \bar{z}), \bar{\psi}^\mu(\bar{z})) ; \quad \mu = 1, 2, 3, 4 \quad (1.2a)$$

$$(ii) \quad (\bar{x}^I(\bar{z}), \bar{y}^I(\bar{z}), \bar{w}^I(\bar{z})) ; \quad I = 1, 2, \dots, 6 . \quad (1.2b)$$

The Weyl-Majorana fermions $(x^I, y^I, w^I), I = 1, 2, \dots, 6$ are in the adjoint of $H_L = \prod_{I=1}^6 SO(3)(I)$ and similarly the $(\bar{x}^I, \bar{y}^I, \bar{w}^I)$ are in the adjoint of $H_R = \prod_{I=1}^6 SO(3)(I')$ [4]. Since in all models we shall construct, the boundary conditions on the world sheet fermions are restricted to be periodic or anti-periodic, the choice $H_L = \prod_{I=1}^6 SO(3)(I)$ and $H_R = \prod_{I'=1}^6 SO(3)(I')$ is

essentially unique [4,5]. If H_L and H_R are chosen differently the maximal number of spacetime supersymmetries is $N \leq 4$ and $N \neq 3$. The case $N = 4$ with $H_L = \prod_{I=1}^6 SO(3)(I)$ and $H_R = \prod_{I'=1}^6 SO(3)(I')$ will be analyzed in Section 5 since by itself it is of main interest. In all other cases $H_{L,R}$ must be taken to be $\prod_{I=1}^6 SO(3)(I)$.

All constructed models must be well defined globally (25,26,4,5), namely, we request:

- (i) The modular invariance of the vacuum to vacuum amplitude at genus one and higher genus world sheet surfaces.
- (ii) The factorization of the vacuum to vacuum amplitude on the boundaries of the moduli space. (Unitary requirement).
- (iii) The global existence of left and right supercurrents [4].

$$\begin{aligned} T_F(z) &= i\partial_z x^\mu \psi_\mu + \sum_{I=1}^6 x^I y^I \bar{w}^I \\ \bar{T}_F(\bar{z}) &= i\partial_{\bar{z}} x^\mu \bar{\psi}_\mu + \sum_{I=1}^6 \bar{x}^I \bar{y}^I \bar{w}^I . \end{aligned} \quad (1.3)$$

We refer the reader to Ref. [4] where the general solution of these global requirements is given. Here we use the results, notation and conventions of Ref. [4] to construct and analyze the desired superstring solutions.

2. N = 8 Superstring Solutions.

In order to obtain string solutions with eight physical massless spin-3/2 states, we must introduce (Ramond)-(Neveu-Schwarz) sector in which the Ramond spinors must transform non-trivially under the action of spacetime γ -matrices, e.g. the zero modes of ψ^μ and $\bar{\psi}^\mu$ world sheet fermions. This requirement is met if one uses, at least, three generalized GSO projections induced by the following base of fermion sets $\{F, S, \bar{S}\}$, where F is the set of all world sheet fermions in the light-cone gauge [4]

$$F = \{\psi^{3,4}, x^{1,2}, \dots, 6, y^{1,2}, \dots, 6, w^{1,2}, \dots, 6, \bar{\psi}^{-3,4}, \bar{x}^{1,2}, \dots, 6, \bar{y}^{1,2}, \dots, 6, \bar{w}^{1,2}, \dots, 6\}, \quad (2.1)$$

containing the 20 left and 20 right-moving fermionic degrees of freedom.

S is a set of 8 left-moving fermions

$$S = \{\psi^{3,4}, x^{1,2}, \dots, 6\} \quad (2.2)$$

and $\bar{S}$ is a set of 8 right-moving fermions

$$\bar{S} = \{\bar{\psi}^{3,4}, \bar{x}^{1,2}, \dots, 6\}. \quad (2.3)$$

The $(F, S, \bar{S})$ model defines the minimal $N = 8$ superstring solution since it contains the minimal number of generalized GSO projections which are necessary for the presence of 8 massless gravitinos. Namely $(-)^F = +1$, $(-)^S = -1$ and $(-)^{\bar{S}} = -1$.

Still following Ref. [4], the Hilbert space of this model is given by eight sectors of the type $R^A N^{F,A}$, where the set of fermions A is one of the

following sets:

$$A \in \{\emptyset, S, \bar{S}, S\bar{S}, F, S\bar{F}, \bar{S}\bar{F}, S\bar{S}\bar{F}\}. \quad (2.4)$$

The set multiplication is defined by [4]

$$A \cdot B = A \cup B - A \cap B \quad (2.5)$$

(the union minus the intersection).

By $R^A N^{F,A}$ we mean that the fermions on the set A are with periodic boundary conditions (Ramond) and the rest, $F \cdot A$, with antiperiodic ones (Neveu-Schwarz). So the empty set, $\emptyset$, defines the pure Neveu-Schwarz sector and F the pure Ramond sector.

The F as well as the $S\bar{F}$, $\bar{S}F$ and $S\bar{S}F$ sectors do not contain massless states since the lower $(\text{mass})^2$ of those sectors is (in our normalization) equal to

$$\begin{aligned} \frac{1}{2} m_F^2 &= \frac{(\# \text{ of left-fermions on } F)}{16} - \frac{1}{2} \\ &= \frac{(\# \text{ of right-fermions on } F)}{16} - \frac{1}{2} = \frac{3}{4}, \end{aligned} \quad (2.6)$$

$$(m_{S\bar{F}}^2 = m_{\bar{S}F}^2 = m_{S\bar{S}F}^2 = m_F^2).$$

Therefore only the $\emptyset$, S , $\bar{S}$ and $S\bar{S}$ sectors contain massless states. For later discussion it is convenient to be more specific about the massless states of the $(F, S, \bar{S})$ minimal model.

(i) Pure Neveu-Schwarz massless states (ϕ -sector).

They are the states which satisfy the $(-)^F = +1$, $(-)^S = -1$ and $(-)^{\bar{S}} = -1$ GSO projections and which are constructed from the tachyonic $\left[-\frac{1}{2}, -\frac{1}{2} \right]$ left- and right-vacuum $|0\rangle$ acting with N-S fermionic oscillators. Explicitly:

$$\begin{aligned} \psi^{\mu\nu} |0\rangle & \text{ graviton, dilaton, pseudoscalar (anti-symmetric tensor).} \\ x^{\mu\nu} |0\rangle & \text{ 6 gauge bosons (graviphotons) in } \underline{6} \text{ of } SO(6)_L \approx SU(4)_L. \\ \psi^{\mu I} |0\rangle & \text{ 6 gauge bosons (graviphotons) in } \underline{6} \text{ of } SO(6)_R \approx SU(4)_R. \\ x^{\mu I} |0\rangle & \text{ 36 scalars in } (\underline{5}, \underline{6}) \text{ of } SO(6)_L \times SO(6)_R. \end{aligned} \quad (2.7)$$

In total there are $2+36 = 38$ scalars from the ϕ -sector. For simplicity of the notation we use $\psi^{\mu I}, x^{\mu I}, \dots, \bar{\psi}^{\mu I}, \bar{x}^{\mu I}, \dots$ to denote the fermionic oscillators $\psi^{\mu}_{-1/2}, x^{\mu}_{-1/2}, \dots, \bar{\psi}^{\mu}_{-1/2}, \bar{x}^{\mu}_{-1/2}, \dots$.

(ii) Massless states in the S-sector.

They are the states which are constructed by the action of the N-S fermionic oscillators on the Ramond-S vacuum and satisfy the $(-)^F, (-)^S$ and $(-)^{\bar{S}}$ GSO projections. The Ramond-S vacuum with $(-)$ chirality is obtained by the action of the spin field, $\text{spin}(\psi^3, \psi^4, \psi^1, \psi^2, \dots, \psi^6) = \text{spin}(S)$, on the N-S vacuum $|0\rangle$. Explicitly

$$\begin{aligned} \bar{\psi}^{\mu\nu} |0\rangle & \text{ 4 gravitinos and 4 spin-1/2 fermions in } \underline{4} \text{ of } SU(4)_L \\ \bar{x}^{\mu I} |0\rangle & \text{ 24 spin-1/2 fermions in } (\underline{4}, \underline{6}) \text{ of } SU(4)_L \times SU(4)_R. \end{aligned} \quad (2.8)$$

(iii) Massless states in the $\bar{S}$ -sector.

They are obtained by the action of the N-S fermionic oscillators on the Ramond- $\bar{S}$ vacuum. Explicitly:

$$\begin{aligned} \psi^{\mu} |0\rangle & \text{ 4 gravitinos and 4 spin-1/2 fermions in } \underline{4} \text{ of } SU(4)_R. \\ x^{\mu I} |0\rangle & \text{ 24 spin-1/2 fermions in } (\underline{6}, \underline{4}) \text{ of } SU(4)_L \times SU(4)_R. \end{aligned} \quad (2.9)$$

(iv) Massless states in $\bar{S}\bar{S}$ -sector.

They are the states of the Ramond- $\bar{S}\bar{S}$ massless vacuum.

$$\begin{aligned} \text{spin}(S) |0\rangle & \text{ 16 complex scalars and 16 gauge bosons in } (\underline{4}, \underline{4}) \\ & \text{ of } SU(4)_L \times SU(4)_R. \end{aligned} \quad (2.10)$$

From Eqs. (2.7)-(2.10) we see that all massless states of the $(F, S, \bar{S})$ N = 8 superstring fit naturally in representations of $SU(4)_L \times SU(4)_R \times U(1)$, where $U(1)$ stands for the complexification of the 16 scalars of $\bar{S}\bar{S}$ sector and the (scalar, pseudoscalar) of the ϕ -sector. Embedding $SU(4)_L \times SU(4)_R \times U(1) \subset SU(8)$ group the degrees of freedom with $\text{spin}(-2, 3/2, 1, 1/2, 0)$ transform like the $(\underline{1}, \underline{8}, \underline{28}, \underline{56}, \underline{70})$ representations of $SU(8)$ as expected from the N = 8 supersymmetry algebra [27].

Any continuous or discrete deformations of the minimal $(F, S, \bar{S})$ model which preserve the number of massless spin-3/2 states has precisely the same massless spectrum. The massive spectrum however, depends on the particular string solution. For instance, the $[SO(6, 6)]/[SO(6) \times SO(6)]$ lattice deformations [3] induced by the six left and six right periodic currents

$$J_L^I = \bar{\psi}^I \psi, \quad J_R^I = \bar{\psi}^I \bar{\psi}^I; \quad (I = 1, 2, \dots, 6) \quad (2.11)$$

correspond to the 36 possible vacuum expectation values (v.e.v.) of the N-S scalars coming from ϕ -sector (Eq. (2.7)). $(J_L^I \text{ and } J_R^I)$ are the two-dimensional superpartners of x^I and $\bar{x}^I$ Weyl-Majorana fermions). Since all massless states

are neutral under J_L^I and J_R^I only massive states change their masses. Due to this neutrality of the massless sector the $N = 8$ effective supergravity appears to possess an $SU(4)_L \times SU(4)_R$ global symmetry which we use for the classification of the massless states.

It is easy to see that the genus one partition function of the minimal $(F, S, \bar{S})$ model factorizes in four modular functions.

$$Z_{F, S, \bar{S}}(\tau, \bar{\tau}) = Z_B(\tau, \bar{\tau}) O_-^S(\tau) O_-^{\bar{S}}(\tau) O_+^{FSS}(\tau, \bar{\tau}) \quad (2.12a)$$

where

$$Z_B(\tau, \bar{\tau}) = (|\text{Im } \tau|)^{-1/2} |\eta(\tau)|^{-2}, \quad (2.12b)$$

is the contribution of the string coordinates $(x^{\mu})^2$,

$$\begin{aligned} O_-^S(\tau) &= \frac{1}{2} \left\{ \left(\frac{\theta_1}{\eta} \right)^4 - \left(\frac{\theta_2}{\eta} \right)^4 + \left(\frac{\theta_3}{\eta} \right)^4 - \left(\frac{\theta_4}{\eta} \right)^4 \right\} \\ O_-^{\bar{S}}(\tau) &= \frac{1}{2} \left\{ \left(\frac{\bar{\theta}_1}{\eta} \right)^4 - \left(\frac{\bar{\theta}_2}{\eta} \right)^4 + \left(\frac{\bar{\theta}_3}{\eta} \right)^4 - \left(\frac{\bar{\theta}_4}{\eta} \right)^4 \right\}, \end{aligned} \quad (2.12c)$$

with $O_-^S(\tau)$ and $O_-^{\bar{S}}(\tau)$ are the contribution of the left and right moving fermions in the S and $\bar{S}$ sets respectively, and finally $O_+^{FSS}(\tau, \bar{\tau})$ is the contribution of the remaining world sheet fermions $(\psi^I, \bar{\psi}^I; \bar{\psi}^I, \bar{\psi}^I)$, $I = 1, 2, \dots, 6$.

$$O_+^{FSS}(\tau, \bar{\tau}) = \frac{1}{2} \sum_{i=1}^4 \left| \frac{\theta_i(\tau)}{\eta(\tau)} \right|^{12}. \quad (2.12d)$$

In Eqs. (12b-d) η denotes the Dedekind η -function and $\theta_i(\tau)$ are the four

Jacobi θ -functions [28].

The $O_+^{FSS}(\tau, \bar{\tau})$ modular orbit corresponds to a particular point of the $\Gamma(6, 6)$ Lorentzian lattice. The general $N = 8$ superstring after $SO(6, 6)/SO(6) \times SO(6)$ deformations depends on 36 continuous parameters corresponding to the 36 v.e.v.'s of the scalars of the ϕ -sector. Any additional fermion number projection corresponding to basis sets ξ_i with $S \cap \xi_i$ and $\bar{S} \cap \xi_i = \emptyset$ changes only the O_+^{FSS} to a O_+^{FSS} which correspond to a different point of the $\Gamma(6, 6)$ Lorentzian lattice. Because of $(-)^S = -1$ and $(-)^{\bar{S}} = -1$ GSO projections, all lattice states are massive which shows that all $N = 8$ superstrings have common massless sector. This is in agreement with the uniqueness of $N = 8$ supergravity.

The remaining scalars, namely, the dilaton and pseudoscalar and the 32 Ramond-Ramond scalars of the $\bar{S}\bar{S}$ sector form respectively the $SU(1, 1)/U(1)$ and $SU(4, 4)/SU(4)_L \times SU(4)_R \times U(1)$ submanifolds. It is interesting to notice here, that the only non-trivial induced deformation of the theory by giving v.e.v.'s to these scalars, is that of the dilaton. All other scalars appear with spacetime derivative couplings and so the theory remains invariant under constant field shiftings [13]. The dilaton shifting, however, changes the value of the string coupling constant and in that sense is not trivial.

It is also interesting to notice that the massless scalars associated with the non-trivial deformations and those which correspond to "trivial" ones are combined together to form the maximally symmetric scalar manifold $E_7^+/SU(8)$ found in $N = 8$ supergravity by Cremmer and Julia [27]. Indeed, the sub-scalar manifolds $SO(6, 6)/SO(6)_L \times SO(6)_R$, $SU(4, 4)/SU(4)_L \times SU(4)_R \times U(1)$ and $SU(1, 1)/U(1)$ are those obtained by means of coset decomposition corresponding to $SU(8) \rightarrow SU(4)_L \times SU(4)_R \times U(1)$ breaking. $(SO(6)_L, R \approx SU(4)_L, R)$. Furthermore, from the point of view of extended supergravity, the three cosets

$SO(6,6)/SO(6)_L \times SO(6)_R$, $SU(4,4)/SU(4)_L \times SU(4)_R \times U(1)$ and $SU(1,1)/U(1)$ correspond to the scalar fields contained in the $N = 4$ supermultiplets of maximal helicities, $\lambda_{\max} = 1, 3/2$ and 2 respectively, contained in $N = 8$ supergravity respectively [29]. This fact explains from a point of view of supergravity why the Ramond scalars ($\bar{S}\bar{S}$ -sector) are coset representatives of $SU(4,4)/SU(4) \times SU(4) \times U(1)$ submanifold (e.g., the scalars of $\lambda_{\max} = 3/2$ $N = 4$ supermultiplet [29]). It explains also why the N -S and Ramond gauge bosons have the quantum numbers of Eqs. (2.7) and (2.10), and also why the fermionic states have the quantum number of Eqs. (2.8) and (2.9).

At this point we stop our discussion on $N = 8$ superstrings, and we shall study the $N = 6$ case.

3. $N = 6$ Superstring Solutions.

In this section we shall first construct $N = 6$ superstring solutions using the fermionic formulation and then we shall examine the possible deformations of the obtained models.

The minimal number of necessary GSO projections in order to construct $N = 6$ solutions is four. The three are those of the $N = 8$ minimal model $(-)^F = +1$, $(-)^{\bar{S}} = -1$, $(-)^{\bar{S}} = -1$ and the fourth $(-)^{b_1} = -1$ is necessary to reduce the number of gravitinos from 8 to 6. So the "minimal" $N = 6$ solutions are defined by the basis sets $\{F, \bar{S}, \bar{S}, b_1\}$ where b_1 has the properties

$$S \cap b_1 = \{\psi^{3,4}, x^{1,2}\} \quad \text{and} \quad \bar{S} \cap b_1 = \emptyset. \quad (3.1)$$

Indeed, if Eqs. (3.1) are valid then, b_1 will generate a Z_2 projection acting non-trivially on spin (S) and therefore, it will reduce by a factor of 2 the gravitinos coming from the S -sector. In the choice of b_1 care must be taken not to reintroduce new massless gravitino states coming from the b_1 sector. Taking this into account, the general form of b_1 is then

$$b_1 = \{\psi^{3,4}, x^{1,2}, y^{3,4,5,6}\} \cup \Delta,$$

$$\Delta \subset \{y^1, w^1, \bar{y}^2, w^2, \bar{y}^I, \bar{w}^I, I = 1, 2, \dots, 6\}. \quad (3.2)$$

Δ must have the property $\#(\text{fermions})_L - \#(\text{fermions})_R = 0 \pmod{8}$ [4], and also $(-)^{\bar{A}}$ has to commute with the left and right supercurrents [4]

$$\begin{pmatrix} (-) \\ T_F, (-)^{\bar{A}} \end{pmatrix} = 0. \quad (3.3)$$

Since we want the b_1 sector not to regenerate new massless spin-3/2 states we must impose $\Delta \neq \phi$. Under these circumstances there are just two independent choices for Δ .

$$\Delta^{(1)} = \{y^1, w^1, \bar{y}^1, \bar{w}^1\}$$

$$\Delta^{(2)} = \{y^1, w^1, \bar{y}^2, \bar{w}^2, \bar{y}^1, \bar{w}^1, \bar{y}^2, \bar{w}^2\}; \quad (3.4)$$

all other choices give equivalent models.

In the language of orbifold compactification the $(F, S, \bar{S}, b_1)$ $N = 6$ model is obtained by an asymmetric $\tau^{6/2}$ orbifold compactification [9], in which Z_2 acts non-trivially on the four internal supercoordinates $(\phi^I \rightarrow -\phi^I, I = 3, 4, 5, 6)$ and shifts symmetrically the left and right momenta of the $\Gamma(2, 2)$ lattice of the two other internal coordinates $(\phi_{L,R}^I \rightarrow \phi_{L,R}^I + \delta^I, I = 1, 2)$.

What is general for all $N = 6$ superstrings is that they have identical massless spectrum as was the case for the $N = 8$ models. This fact will be clear in our later discussion.

The minimal $N = 6$ models contain sixteen sectors, namely:

$$\begin{array}{ll} \phi, S, \bar{S}, \bar{S}\bar{S} & \text{with lower mass states } \frac{1}{2} m^2 = 0 \\ F, SF, \bar{S}F, \bar{S}\bar{S}F & " \quad \frac{1}{2} m^2 = \frac{3}{4} \\ b_1, Sb_1, \bar{S}b_1, \bar{S}\bar{S}b_1 & " \quad \left\{ \begin{array}{l} \frac{1}{2} m^2(\Delta^{(1)}) = \frac{1}{8} \\ \frac{1}{2} m^2(\Delta^{(2)}) = \frac{1}{4} \end{array} \right. \\ Fb_1, SFb_1, \bar{S}Fb_1, \bar{S}\bar{S}Fb_1 & " \quad \left\{ \begin{array}{l} \frac{1}{2} m^2(\Delta^{(1)}) = \frac{1}{8} \\ \frac{1}{2} m^2(\Delta^{(2)}) = \frac{1}{2} \end{array} \right. \end{array}$$

Only the sectors $\phi, S, \bar{S}$ and $\bar{S}\bar{S}$ contain massless states. This implies that the massless $N = 6$ superstring states are those of the $N = 8$ which are

invariant under $Z_2^{(b_1)}$ fermion number projection. Explicitly

(i) ϕ -sector.

$$\begin{array}{ll} \psi^\mu, \bar{\psi}^\mu |0\rangle & \text{graviton, dilaton, pseudoscalar.} \\ \psi^{\mu, 1, 2, \dots, 6} |0\rangle & 6 \text{ gauge bosons in } \underline{6} \text{ of } SO(6)_R \approx SU(4)_R. \\ x^{1, 2} \bar{\psi} |0\rangle & 2 \text{ gauge bosons in } \underline{2} \text{ of } SO(2)_L. \\ x^{1, 2} \bar{x}^{1, 2, \dots, 6} |0\rangle & 12 \text{ scalars in } (\underline{2}, \underline{6}) \text{ of } SO(2)_L \times SO(6)_R. \quad (3.6) \end{array}$$

(ii) $\bar{S}$ -sector.

$$\begin{array}{ll} \bar{\psi}^\mu \text{ spin}(S)_{-+} |0\rangle & 2 \text{ gravitinos and 2 spin-1/2 fermions in } (\underline{2}, \underline{1}) \\ & \text{of } SU(2)_L \times SU(2)'_L = SO(4)_L. \\ \bar{x}^{1, 2, \dots, 6} \text{ spin}(S)_{-+} |0\rangle & 12 \text{ spin-1/2 fermions in } (\underline{2}, \underline{6}) \text{ of} \\ & SU(2)_L \times SO(6)_R. \end{array}$$

where

$$\text{spin}(S)_{-+} = \text{spin}(\psi^3, \psi^4, \psi^{1, 2})_{-} \text{spin}(x^3, x^4, x^5, x^6)_{+}. \quad (3.7)$$

(iii) $\bar{S}$ -sector.

$$\begin{array}{ll} \psi^\mu \text{ spin}(\bar{S})_{-} |0\rangle & 4 \text{ gravitinos and 4 spin-1/2 fermions in } \underline{4} \text{ of} \\ & SO(6)_R \approx SU(4)_R. \\ x^{1, 2} \text{ spin}(\bar{S})_{-} |0\rangle & 8 \text{ spin-1/2 fermions in } (\underline{2}, \underline{4}) \text{ of } SO(2)_L \times \\ & SU(4)_R. \quad (3.8) \end{array}$$

(iv) $\bar{S}\bar{S}$ -sector.

$$\begin{array}{ll} \text{spin}(S)_{-+} \text{ spin}(\bar{S})_{-} |0\rangle & 8 \text{ complex scalars and 8 gauge bosons in } (\underline{2}, \underline{4}) \\ & \text{of } SU(2)_L \times SU(4)_R. \quad (3.9) \end{array}$$

From the spectrum of Eqs. (3.6)-(3.9), one can see that the several degrees of freedom with spin $(2, 3/2, 1, 1/2, 0)$ come with helicity multiplicities $(2, 12, 32, 52, 30)$ as is expected from $N = 6$ supersymmetry algebra. These states are classified in representations of $SU(6) \times U(1)$ which appear in our construction as the factor group

$$SU(6) \times U(1) \supset U(1)_L \times SU(2)_L \times SU(4)_R \times U(1). \quad (3.10)$$

In fact the z_2 projection generated by $(-)^{b_1}$ breaks down the $SO(6)_L \rightarrow SO(4)_L \times SO(2)_L$. It is easy then to check that in terms of the factor group [Eq. (3.10)] the 30 scalars of $N = 6$ model are classified in the following manifolds:

$$\frac{SU(1,1)}{U(1)}, \quad \frac{SO(2,6)}{SO(2) \times SO(6)} \quad \text{and} \quad \frac{SU(2,4)}{SU(2)_L \times SU(4)_R \times U(1)}. \quad (3.11)$$

Again, one can show that this three sub-manifolds correspond to the coset of scalars of $N = 4$ supermultiplets contained in $N = 6$ with maximal helicities $\lambda_{\max} = 2, 1$ and $3/2$ respectively [29].

The first two manifolds of Eq. (3.11) correspond to the (dilaton, pseudo-scalar) and to the 12 extra scalars of the $N=8$ sector while the third one corresponds to the 16 scalars of the Ramond $\bar{SS}$ sector. All scalars together form the unique scalar manifold of $N = 6$ supergravity, namely the $SO(12)^*/SU(6) \times U(1)$ [27,29].

The non-trivial deformations of the $N = 6$ superstring are induced by giving v.e.v.'s to the scalars of the $SO(2,6)/SO(2) \times SO(6)$ manifold. These deformations correspond to boostings of the Lorentzian lattice $\Gamma(2,6)$. Any such deformation modifies the massive spectrum of the theory and keeps all states of Eqs. (3.9) massless, since these states are neutral under the action of

$J_L^{1,2}$ and $J_R^{1,2,\dots,6}$ (the $z_2^{(b_1)}$ invariant currents). Defining $z_2^{(b_1)}$ by its action on $\bar{g}$ of $SU(8)$ as is dictated from $(-)^{b_1}$ fermion number projection,

$$z_2^{(b_1)}: (1,1,\alpha,1,1,1,1) \quad \text{with} \quad \alpha^2 = 1, \quad (3.12)$$

we may easily obtain the effective $N = 6$ supergravity from the $N = 8$ one by truncating out the $z_2^{(b_1)}$ transforming states. This operation breaks $SU(8) \rightarrow SU(6) \times U(1)$ as expected.

We shall now consider the $N = 5$ superstring solutions.

4. $N = 5$ Superstring Solutions.

In order to obtain them through fermionic construction, it is necessary to introduce an extra z_2 projection which is defined by a fermion number $(-)^{b_2}$ where b_2 is an extra fermion set with the property

$$b_2 \cap S = \{\psi^{3,4}, x^{3,4}\} \quad \text{and} \quad b_2 \cap \bar{S} = \emptyset \quad (4.1)$$

and so, $(-)^{b_2}$ will act non-trivially on $\text{spin}(S)_{-+}$ reducing further the number of S -gravitinos. In terms of $N = 8$ supergravity the $z_2^{(b_1)} \times z_2^{(b_2)}$ will act on $\underline{8}$ of $SU(8)$ as

$$z_2^{(b_1)} \times z_2^{(b_2)}: \{1, \beta, \alpha, \alpha\beta, 1, 1, 1, 1\}, \text{ with } \alpha^2 = \beta^2 = 1, \quad (4.2)$$

$$\text{and } \alpha \in z_2^{(b_1)}, \quad \beta \in z_2^{(b_2)}.$$

This action transforms three of the 8 gravitinos and breaks

$$N = 4 + 4 \rightarrow N = 1 + 4. \quad (4.3)$$

In principle $N = 5$ superstring solutions can also be constructed using asymmetric orbifold compactification. For instance, the equivalent of the $z_2^{(b_1)} \times z_2^{(b_2)}$ fermionic models correspond to a $\tau^6/z_2 \times z_2$ asymmetric orbifold compactification where the $z_2 \times z_2$ acts non-trivially on the six left-moving supercoordinates. Another possibility is through τ^6/z_3 asymmetric orbifold compactification in which z_3 will act non-trivially on $\underline{4}$ of $SU(4)_L = SO(6)_L$.

Using the fermionic construction with periodic and antiperiodic world sheet fermions, we can construct only four independent "minimal" $N = 5$ superstring solutions, $(F, S, \bar{S}, b_1, b_2)$, which correspond to four independent choices of b_1 and b_2 we display below:

$$(i) \quad b_1 = (\psi^{3,4}, x^{1,2}, x^{3,4}, y^{5,6}, y^1, w^1, \bar{y}^1, \bar{w}^1)$$

$$b_2 = (\psi^{3,4}, y^{1,2}, x^{3,4}, x^{5,6}, y^3, y^4, y^5, y^6, \bar{y}^2, \bar{w}^2)$$

(ii) b_1 as in (i)

$$b_2 = (\psi^{3,4}, y^{1,2}, x^{3,4}, y^{5,6}, y^3, y^4, y^5, y^6, \bar{y}^1, \bar{w}^1, \bar{y}^2, \bar{w}^2)$$

(iii) $b_1 = (\psi^{3,4}, x^{1,2}, y^{3,4}, y^{5,6}, y^1, w^1, \bar{y}^1, \bar{w}^1)$

$$b_2 = (\psi^{3,4}, y^{1,2}, x^{3,4}, x^{5,6}, y^3, y^4, y^5, y^6, \bar{y}^1, \bar{w}^1, \bar{y}^2, \bar{w}^2)$$

(iv) b_1 as in (iii)

$$b_2 = (\psi^{3,4}, y^{1,2}, x^{3,4}, y^{5,6}, y^3, y^4, y^5, y^6, \bar{y}^1, \bar{w}^1). \quad (4.4)$$

All other choices give equivalent models or are inconsistent.

The four independent minimal $N = 5$ solutions $(F, S, \bar{S}, b_1, b_2)$ have identical massless states which are coming from the $\Phi, S, \bar{S}$, and $\bar{S}\bar{S}$ sectors and which are invariant under $(-)^F = 1$, $(-)^S = -1$, $(-)^{\bar{S}} = -1$, $(-)^{b_1} = -1$, $(-)^{b_2} = -1$ GSO projections. There are in total 32 sectors, however all other sectors, $(\Phi, S, \bar{S}, \bar{S}\bar{S}) \cdot (F, b_1, b_2, F \cdot b_1 \cdot b_2, F \cdot b_1 \cdot b_2)$, are always massive for all cases. This fact was expected since $N = 5$ supergravity theory is unique [27].

Sector by sector we have the following massless states:

(i) ϕ -sector.

$$\begin{aligned} & \psi^{\mu\nu} |0\rangle && \text{graviton, dilaton, pseudoscalar} \\ & \psi^{\mu} \bar{x}^1, 2, \dots, 6 |0\rangle && 6 \text{ gauge bosons in } \underline{6} \text{ of } SO(6)_R. \end{aligned} \quad (4.5a)$$

(ii) $\bar{S}$ -sector.

$$\begin{aligned} & \bar{\psi}^{\mu} \text{spin}(S)_{-++} |0\rangle && 1 \text{ gravitino and } 1 \text{ spin-1/2 fermion} \\ & \bar{x}^1, 2, \dots, 6 \text{ spin}(S)_{-++} |0\rangle && 6 \text{ fermions in } \underline{6} \text{ of } SO(6)_R. \end{aligned}$$

$$(4.5b)$$

where

$$\begin{aligned} \text{spin}(S)_{-++} = & \text{spin}(\psi^{3,4})_{-} \text{spin}(x^{1,2})_{+} \text{spin}(x^{3,4})_{+} \text{spin}(x^{5,6})_{+} \\ & + \text{spin}(\psi^{3,4})_{+} \text{spin}(x^{1,2})_{-} \text{spin}(x^{3,4})_{-} \text{spin}(x^{5,6})_{-}. \end{aligned}$$

(iii) $\bar{S}$ -sector.

$$\begin{aligned} & \bar{\psi}^{\mu} \text{spin}(\bar{S})_{-} |0\rangle && 4 \text{ gravitinos and } 4 \text{ spin-1/2 fermions} \\ & && \text{in } \underline{4} \text{ of } SO(6)_R \cong SU(4)_R. \end{aligned} \quad (4.5c)$$

(iv) $\bar{S}\bar{S}$ -sector.

$$\begin{aligned} & \text{spin}(S)_{-++} \text{spin}(\bar{S})_{-} |0\rangle && 4 \text{ complex scalars and } 4 \text{ gauge bosons} \\ & && \text{in } \underline{4} \text{ of } SO(6)_R \cong SU(4)_R. \end{aligned} \quad (4.5d)$$

Like in the $N = 8$ and $N = 6$ cases the dilaton and pseudoscalar of the ϕ -sector parametrize the $SU(1,1)/U(1)$ sub-manifold. The other eight scalars comes from the $\bar{S}\bar{S}$ sector and parametrize a $SU(1,4)/U(1) \times SU(4)$ manifold. We observe that we cannot perform any non-trivial continuous deformation based on $J_L^I = \bar{\psi}^I \gamma^I$ and $J_R^I = \bar{\psi}^I \gamma^I$ currents. In fact all J_L^I currents transform non-trivially under $Z_2^{(b_1)} \times Z_2^{(b_2)}$ while all J_R^I currents are invariant. Only discrete

deformations exist and some of those correspond to the four minimal $N = 5$ superstring solutions of Eq. (4.4).

The massless states of the $N = 5$ superstring solutions with spin $\left[2, \frac{3}{2}, 1, \frac{1}{2}, 0 \right]$ appear with helicity multiplicities $(2, 10, 20, 22, 10)$ respectively as expected from supersymmetry algebra [27,29]. Also, the 10 scalars together, parametrize the $N = 5$ supergravity manifold $\frac{SU(1,5)}{U(1) \times SU(5)}$ which appear in our construction as $\frac{SU(1,1)}{U(1)}$ and $\frac{SU(1,4)}{U(1) \times SU(4)}$ sub-manifolds. These two sub-manifolds correspond to the $N = 4$ decomposition of $N = 5$; they are the cosets of scalars of $N = 4$ supermultiplets with maximal helicities $\lambda_{\max} = 2$ and $3/2$ respectively.

Up to now we have considered $N > 4$ superstring solutions and we have shown that massless states are uniquely defined. All deformations which respect the number of spacetime supersymmetries (continuous or discrete), maintain the massless spectrum of the corresponding theory. In the $N = 8$ case, there are continuous lattice deformations $\frac{SO(6,6)}{SO(6) \times SO(6)}$, in $N = 6$ these deformations are reduced to $\frac{SO(2,6)}{SO(2) \times SO(6)}$, while in the $N = 5$ case there are only discrete deformations.

The trivial deformations correspond to $(\text{Ramond})_L - (\text{Ramond})_R$ scalars and parametrize in general $SU(m,n)/SU(m) \times SU(n) \times U(1)$ type manifolds with $SU(n) \times SU(m) \times U(1) \subset SU(8)$, $SU(6) \times U(1)$ and $SU(5) \times U(1)$ for $N = 8, 6$ and 5 respectively. All these results have a simple interpretation in terms of $N = 4$ supergravity multiplets embedded in $N > 4$ theories.

As we shall see in the following, this uniqueness of higher supersymmetries ($N \geq 5$) will not be respected after we consider string solutions with $N \leq 4$. As expected from supersymmetry algebra with $N \leq 4$, the massless states of the theory are now classified in different types of supermultiplets: namely, the graviton supermultiplet as well as some matter supermultiplets. The uniqueness

of $N \geq 5$ follows from the fact that all massless states are members of the graviton supermultiplet. This will not be the case for $N \leq 4$. So in general we expect that the effective $N \leq 4$ supergravities obtained from strings and also their corresponding moduli space will strongly depend on the choice of GSO projections and on the structure of other possible deformations. This phenomenon will be clear in the explicit examples that we shall consider in the following. We shall first start with some $N = 4$ examples.

5. $N = 4$ Superstring Solutions (Type II Construction).

There are two general ways of constructing $N = 4$ type II superstring solutions. In the first way, which has also a compactification interpretation, two of the gravitinos come from the S-sector and the other two from the $\bar{S}$ -sector ($N = 2_L + 2_R$). In the other way the obtained $N = 4$ models do not have an obvious compactification interpretation, and all gravitinos come from the S-sector. ($N = 4_L + 0_R$.) In the second case the $\bar{S}$ -sector is not present in the theory. The obtained $N = 4_L + 0_R$ theories are very similar to the heterotic-like models rather than to the type II ones. Some authors referred to them as type III superstrings [5]. The $N = 4_L + 0_R$ theories are interesting on their own, so we start our analysis with them.

(a) $N = 4_L + 0_R$ Superstring Solutions.

The "minimal" $N = 4_L + 0_R$ model is constructed by introducing two GSO projections, $(-)^F = +1$ and $(-)^{\bar{S}} = -1$, so it is defined by two basis sets $\{F, S\}$ and has four sectors:

$$(\phi, S, F, SF) \quad (5.1)$$

Using the rules of Ref. [4], it is easy to show that this model has a very simple factorized partition function of the form:

$$(N^S - R^S)(N^{\bar{S}} - R^{\bar{S}}) \quad (5.2)$$

It is also easy to check that the massless states come only from the ϕ - and S -sector. The other two sectors are massive. Sector by sector we have the following massless spectrum:

ϕ -sector.

$$\psi_{\bar{\psi}}^{I\bar{V}} |0\rangle$$

graviton, dilaton pseudoscalar

18 gauge bosons in the adjoint of $H_R = [SO(3)]^6$,

$$\bar{\phi}^{\alpha} \in \{\bar{x}^I, \bar{y}^I, \bar{w}^I\}, \quad I = 1, 2, \dots, 6$$

6 gauge bosons in $\underline{6}$ of $SO(6)_L$ (the

graviphoton of $N = 4$)

$6 \times 18 = 108$ scalars in $\underline{6}$ of $SO(6)_L$ and

in the adjoint of H_R

(5.3)

 S -sector.

$$\bar{\psi}^{\bar{V}} \text{ spin}(S)_{-10}$$

4 gravitinos and 4 spin-1/2 fermions in

$\underline{4}$ of $SO(6)_L$

$4 \times 18 = 72$ gauginos in $\underline{4}$ of $SO(6)_L$ and

in the adjoint of H_R

(5.4)

The above spectrum corresponds to $N = 4$ supergravity spectrum based on matter gauge group $H_R = [SO(3)]^6$. The gravitational multiplet is formed by the graviton, the dilaton, the pseudoscalar, the $\underline{6}$ graviphotons, the $\underline{4}$ gravitinos and the $\underline{4}$ spin-1/2 fermions. The matter multiplet corresponds to the 18 gauge bosons and the 18 gauginos in the adjoint of H_R .

What is important here is that for the minimal $N = 4_L + 0_R$ model the choice of $H_R = [SO(3)]^6$ is not uniquely fixed. One can choose any H_R with the property $\dim H_R = 18$, provided that H_R does not contain any $U(1)$ factors [4]. There are in fact two other possible choices [4,5,10],

$$H_R = SU(4) \times SO(3), \quad H_R = SU(3) \times SO(5). \quad (5.5)$$

The moduli space and the possible continuous deformations of the minimal $N = 4 + 0$ model can be easily derived. First, the dilaton and pseudoscalar parametrize as usual the $SU(1,1)/U(1)$ manifold. The other permitted continuous deformations are associated with the scalars of the ϕ sector which form an $SO(6,n)/SO(6)_L \times SO(n)$ scalar manifold ($n = \dim H_R = 18$.) All scalars together form the $N = 4$ scalar manifold $[SU(1,1)/U(1)] \times [SO(6,n)/SO(6) \times SO(n)]$ as expected from $N = 4$ supergravity.

The dimension of the moduli space however is not $6 \cdot n$ but much smaller. The reason is that only $r = \text{rank}(H_R)$ right currents out of $n = \dim H_R$ are mutually commuting; namely, those which correspond to the Cartan sub-algebra of H_R . Therefore the dimension of the moduli space is $6 \cdot r$ and the corresponding scalars associated with those deformations are on the $SO(6,r)/SO(6) \times SO(r)$ manifold.

The $6 \cdot r$ parameters of the possible deformations are those which correspond to $\Gamma(6,r)$ Lorentzian lattice boostings. Since r depends on the choice of H_R we have different dimensionality of the moduli space: 36 for $H_R = [SO(3)]^6$ and 24 for $H_R = SU(4) \times SO(3)$ or $H_R = SU(3) \times SO(5)$.

Notice that the $N = 4_L + 0_R$ superstring solutions do not have a simple asymmetric orbifold or other compactification interpretation.

(b) $N = 2_L + 2_R$ Superstring Solutions.

The $N = 2_L + 2_R$ models are obtained by introducing some projections on $(F, S, \bar{S})$ $N = 8$ superstring which eliminate the $2_L + 2_R$ out of $4_L + 4_R$ gravitinos. In general those models have orbifold compactification interpretation.

The "minimal" $N = 2_L + 2_R$ models are defined by four basis sets $\{F, S, \bar{S}, b_{11}\}$ where b_{11} is chosen such as to define a $z_{11}^{(b_{11})}$ symmetric projection acting non-trivially on S and $\bar{S}$ sectors:

$$S \cap b_{11} = \{\psi^{3,4}, x^{1,2}\}, \quad \bar{S} \cap b_{11} = \{\bar{\psi}^{3,4}, \bar{x}^{1,2}\}. \quad (5.6)$$

Following the rules of Ref. [4], the general structure of b_{11} must be:

$$b_{11}^{(\alpha)} = \left\{ \begin{array}{l} \psi^{3,4}, x^{1,2}, y^{3,4}, \dots, 6 \\ \bar{\psi}^{3,4}, \bar{x}^{1,2}, \bar{y}^{3,4}, \dots, 6 \end{array} \right\} \cup \Delta^{(\alpha)} \quad (5.7a)$$

with

$$\begin{aligned} \Delta^{(\alpha)} &= \phi, \\ \Delta^{(1)} &= \{y^1, \bar{y}^1, \bar{y}^1, \bar{y}^1\}, \\ \Delta^{(2)} &= \{y^1, \bar{y}^1, y^2, \bar{y}^2, \bar{y}^1, \bar{y}^1, \bar{y}^2, \bar{y}^2\}. \end{aligned} \quad (5.7b)$$

There are 16 sectors in the minimal $N = 2_L + 2_R$ models and their lower mass states per sector depend on the choice of $\Delta^{(\alpha)}$,

Sectors	$\frac{1}{2} \frac{(\text{mass})^2}{(\text{mass})^2}$
$\phi, \bar{S}, \bar{S}, \bar{S}$	lower (mass) ²
$F \cdot (\phi, \bar{S}, \bar{S}, \bar{S})$	0
$b_{11} \cdot (\phi, \bar{S}, \bar{S}, \bar{S})$	3/4
$F \cdot b_{11} (\phi, \bar{S}, \bar{S}, \bar{S})$	$\alpha/8$
$F \cdot b_{11} (\phi, \bar{S}, \bar{S}, \bar{S})$	$(2-\alpha)/8$

From Eq. (5.8) we see that the super-sector $\hat{F} [= F \cdot (\phi, \bar{S}, \bar{S}, \bar{S})]$ is always massive while the super-sectors $\hat{b}_{11}^{(\alpha)}$ and $\hat{F} \cdot b_{11}^{(\alpha)}$ contain massless states depending on the value of α . The $\alpha = 0$ and $\alpha = 2$ cases give equivalent models; they are related by a basis change $b_{11}^{(\alpha)} \rightarrow \bar{S} F b_{11}^{(2)}$ and $y^i \leftrightarrow \bar{y}^i$. So only two independent choices for $\Delta^{(\alpha)}$ exist, $\alpha = 0$ and $\alpha = 1$.

When $\alpha = 1$ only the $(\phi, \bar{S}, \bar{S}, \bar{S})$ sectors contain massless physical states. Those states are nothing but the $z_2^{(b_{11})}$ invariant $N = 8$ supergravity states. It is easy to see that the action of $z_2^{(b_{11})}$ on $\underline{8}$ of $SU(8)$ is given by,

$$z_2^{(b_{11})}: (1, 1, \alpha, 1, 1, \alpha, \alpha): \alpha^2 = 1, \quad \alpha \in z_2^{(b_{11})}, \quad (5.9)$$

and so projects out the 4 out of the 8 gravitinos and breaks

$$SU(8) \rightarrow SU(4)_1 \times SU(4)_2 \times U(1)_1 \times U(1)_2. \quad (5.10)$$

At the string level, however, due to the left-right separation, we can only see manifestly the $(SU(2) \times SU(2) \times U(1))_{L,R}$ sub-groups of $SU(4)_1 \times SU(4)_2 \times U(1)_1 \times U(1)_2$. Below we give the massless states and their manifest quantum numbers for the $(F, \bar{S}, \bar{S}, b_{11}^{(1)})$ model.

(i) $\underline{\phi}$ -sector.

$\psi^{\mu} \bar{\psi} 0\rangle$	graviton, dilaton, pseudoscalar.
$x^{1,2} \bar{\psi} 0\rangle$	2 gauge bosons in $\underline{2}$ of $SO(2)_L$.
$\bar{\psi}^{\mu} \bar{x}^{1,2} 0\rangle$	2 gauge bosons in $\underline{2}$ of $SO(2)_R$.
$x^{1,2} \bar{x}^{1,2} 0\rangle$	4 scalars in $(\underline{2}, \underline{2})$ of $SO(2)_L \times SO(2)_R$.
$x^3, \dots, x^6 0\rangle$	16 scalars in $(\underline{4}, \underline{4})$ of $SO(4)_L \times SO(4)_R$.

(5.11a)

(ii) $\underline{S}$ -sector.

$\bar{\psi} \text{ spin}(S)_{-+} 0\rangle$	2 gravitinos and 2 spin-1/2 fermions
$\bar{x}^{1,2} \text{ spin}(S)_{-+} 0\rangle$	in $(\underline{2}, \underline{1})$ of $SU(2)_L \times SU(2)_R \simeq SO(4)_L$.
	4 spin-1/2 fermions in $(\underline{2}, \underline{1}; \underline{2})$ of $SU(2)_L \times SU(2)_R \simeq SO(4)_R$.

$$\bar{x}^{3,4,5,6} \text{ spin}(S)_{+-} |0\rangle \text{ 8 spin-1/2 fermions in } (\underline{1}, \underline{2}; \underline{2}, \underline{2}) \text{ of } SU(2)_L \times SU(2)'_L \times SU(2)_R \times SU(2)'_R.$$

(5.11b)

where

$$\text{spin}(S)_{+-} = \text{spin}(\psi^{3,4} x^{1,2})_{-} \text{spin}(x^3, \dots, x^6)_{+}$$

and

$$\text{spin}(S)_{+-} = \text{spin}(\psi^{3,4} x^{1,2})_{+} \text{spin}(x^3, \dots, x^6)_{-}.$$

(iii) $\bar{S}$ -sector as in S sector by changing $L \leftrightarrow R$.(iv) $\bar{S}\bar{S}$ -sector

$$\begin{aligned} & \text{spin}(S)_{+-} \text{ spin}(\bar{S})_{-+} |0\rangle \quad \text{4 complex scalars and 4 gauge bosons} \\ & \quad \text{in } (\underline{2}, \underline{1}; \underline{2}, \underline{1}) \text{ of } SU(2)_L \times SU(2)'_L \times SU(2)_R \times SU(2)'_R. \\ & \text{spin}(S)_{+-} \text{ spin}(\bar{S})_{+-} |0\rangle \quad \text{4 complex scalars and 4 gauge bosons} \\ & \quad \text{in } (\underline{1}, \underline{2}; \underline{1}, \underline{2}) \text{ of } SU(2)_L \times SU(2)'_L \times SU(2)_R \times SU(2)'_R. \end{aligned} \quad (5.11c)$$

The scalars of the theory parametrize the following sub-manifolds

$$(i) \quad \frac{SU(1,1)}{U(1)_1}, \text{ dilaton, pseudoscalar} \quad (\text{from } \phi\text{-sector})$$

$$(ii) \quad \frac{SO(2,2)}{SO(2)_L \times SO(2)_R} = \frac{SU(1,1)}{U(1)_0} \times \frac{SU(1,1)}{U(1)_2}, \quad (\text{N-S scalars from } \phi\text{-sector})$$

$$(iii) \quad \frac{SO(4,4)}{SO(4)_L \times SO(4)_R}, \quad (\text{N-S scalars from } \phi\text{-sector})$$

$$(iv) \quad \frac{SU(2,2)}{SU(2)_L \times SU(2)_R \times U(1)_1}, \quad (\text{Ramond scalars from } \bar{S}\bar{S}\text{-sector})$$

$$(v) \quad \frac{SU(2,2)}{SU(2)'_L \times SU(2)'_R \times U(1)_2}, \quad (\text{Ramond scalars from } \bar{S}\bar{S}\text{-sector}) \quad (5.12)$$

Altogether these scalars parametrize the $[SU(1,1)/U(1)_0] \times [SO(6,6)/SO(6)_1 \times SO(6)_2]$ N = 4 manifold based on $U(1)^6$ matter gauge group. Indeed, considering the decompositions,

$$\begin{aligned} SO(6)_1 & \supset SU(2)_L \times SU(2)_R \times U(1)_1 \\ SO(6)_2 & \supset SU(2)'_L \times SU(2)'_R \times U(1)_2, \end{aligned} \quad (5.13)$$

one can see from the scalar field quantum numbers that this embedding is unique. Notice also, that the two scalars of the N = 4 graviton supermultiplet are not the dilaton and pseudoscalar but two of the N-S scalars of the $SO(2,2)/SO(2) \times SO(2)$ manifold with $U(1)_0$ quantum numbers (see Eq. (5.12(ii))). [30]. So, in N = $2_L + 2_R$ models the dilaton and pseudoscalar are members of the matter scalar manifold. This is not true for the N = 4 heterotic models neither for the N = $4_L + 0_R$ type II models.

The cosets of Eq. (5.12) can be derived also in supergravity by considering N = 2 supermultiplets, with maximal helicities $\lambda_{\max} = 1$ and $1/2$, contained in N = 4 theory [29].

The non-trivial moduli of the theory corresponding to lattice deformations are associated with the scalars of the $SO(2,2)/SO(2) \times SO(2)$ and $SO(4,4)/SO(4) \times SO(4)$ manifolds. The first correspond to $J_L^{1,2}$ and $J_R^{1,2}$ z_{b11} invariant currents and the second to $J_L^{3,\dots,6}$, $J_R^{3,\dots,6}$ z_{b11} transforming currents.

Any such deformation only modifies the massive spectrum of the model, since all massless states are neutral under $J_{L,R}^{1,2,\dots,6}$. One however, must be careful with this statement because it is possible for some massive states to become massless after a suitably large deformation and as a result of this, the massless sector of the theory would be enlarged. In order to see how this enlargement takes place we shall consider the $N = 2_L + 2_R$ model with $\alpha = 0$.

In fact, in terms of orbifolds both $\alpha = 0$ and $\alpha = 1$ models correspond to a $T^{6/2}$ symmetric orbifold compactification but with different shiftings of the Lorentzian $\Gamma(2,2)$ lattice momenta. In the $\alpha = 1$ case we have seen that the only massless states are coming from the $\hat{\phi}$ -supersector. In the $\alpha = 0$ case some other extra massless states exist namely those which are coming from the $\hat{b}_{11}^{(o)} = b_{11}^{(o)} \cdot (\hat{\phi}, S, \bar{S}, \bar{S})$ supersector. We have therefore the following extra massless states:

$$(i) \quad \underline{b_{11}^{(o)}} \text{-sector.} \\ \text{spin } (\bar{\psi}^{3,4}, x^{1,2})_{-} \text{ spin } (\bar{\psi}^{3,4}, \bar{x}^{1,2})_{-} \text{ spin } (y^3, \dots, y^6, \bar{y}^3, \dots, \bar{y}^6)_{+} |0\rangle \\ 8 \text{ complex scalars and 8 gauge bosons in } \underline{8} \text{ of } SO(8)_{(y\bar{y})}. \quad (5.14a)$$

$$(ii) \quad \underline{Sb_{11}^{(o)}} \text{-sector.} \\ \text{spin } (x^3, \dots, x^6)_{-} \text{ spin } (\bar{\psi}^{3,4}, \bar{x}^{1,2})_{-} \text{ spin } (y^3, \dots, y^6, \bar{y}^3, \dots, \bar{y}^6)_{+} |0\rangle \\ 16 \text{ fermions in } (\underline{2}, \underline{8}) \text{ of } SU(2)_L \times SO(8)_{(y\bar{y})}. \quad (5.14b)$$

$$(iii) \quad \underline{Sb_{11}^{(o)}} \text{-sector.} \\ \text{spin } (\bar{\psi}^{3,4}, x^{1,2})_{-} \text{ spin } (\bar{x}^2, \dots, \bar{x}^6)_{-} \text{ spin } (y^3, \dots, y^6, \bar{y}^3, \dots, \bar{y}^6)_{+} |0\rangle \\ 16 \text{ fermions in } (\underline{2}, \underline{8}) \text{ of } SU(2)_R \times SO(8)_{(y\bar{y})}. \quad (5.14c)$$

$$(iv) \quad \underline{S\bar{S}b_{11}^{(o)}} \text{-sector.} \\ \text{spin } (x^3, \dots, x^6)_{-} \text{ spin } (\bar{x}^3, \dots, \bar{x}^6)_{-} \text{ spin } (y^3, \dots, y^6, \bar{y}^3, \dots, \bar{y}^6)_{+} |0\rangle \\ 32 \text{ scalars in } (\underline{2}, \underline{2}, \underline{8}) \text{ of } SU(2)_L \times SU(2)_R \times SO(8)_{(y\bar{y})}. \quad (5.14d)$$

It is easy to check that the $b_{11}^{(o)}$ sector scalars parametrize an $SO(2,8)/U(1)_1 \times SO(8)$ manifold while those of $\bar{S}b_{11}^{(o)}$ sectors parametrize the $SO(4,8)/SU(2)_L \times SU(2)_R \times SO(8)$ manifold. Those of $b_{11}^{(o)}$ sector are "trivial" moduli since they are (spacetime Ramond left)-(spacetime Ramond right) and so, they appear in the theory only through derivative couplings. On the contrary, the $\bar{S}\bar{S}b_{11}^{(o)}$ sector scalars correspond to non-trivial moduli. It is interesting to observe that the "trivial" and non-trivial moduli are combined together in a larger manifold, the $SO(6,8)/SO(6)_1 \times SO(8)$, where $SO(6)_1$ is defined by its decomposition $SO(6)_1 \supset SU(2)_L \times SU(2)_R \times U(1)_1$. Also, as expected, the $(\hat{\phi}, \bar{S}\bar{S})$ scalars with those of $b_{11}^{(o)}$ and $\bar{S}\bar{S}b_{11}^{(o)}$ together, parametrize the $N = 4$ supergravity manifold, $[SU(1,1)/U(1)_O] \times [SO(6,6+8)/SO(6)_1 \times SO(6+8)]$, with a $U(1)^{14}$ matter gauge group.

Many other $N = 4$ models can be constructed with a larger or even smaller number of massless states. Indeed by introducing the basis set

$$\xi = (y^1, w^1, \bar{y}^2, w^2, \bar{y}^1, \bar{w}^1, \bar{y}^2, \bar{w}^2), \quad (5.15)$$

the $N = 2_L + 2_R$ model which is based on $(F, S, \bar{S}, b_{11}^{(o)}, \bar{\xi})$ basis sets has the property to factorize in the partition function the $\Gamma(2,2)$ lattice; e.g., the lattice which corresponds to the $J_L^{1,2}$ and $J_R^{1,2}$ invariant currents. In the presence of ξ , all sectors of Eq. (5.8) exist together with 12 extra sectors, namely:

$$\begin{aligned}
& \xi \cdot F \cdot (\phi, \bar{S}, \bar{S}, \bar{S}) \quad \text{with lower } \frac{1}{2} m^2 = \frac{1}{2} \\
& \xi \cdot b_{11}^{(o)} \cdot (\phi, \bar{S}, \bar{S}, \bar{S}) \quad " \quad " \quad = \frac{1}{4} \\
& \xi \cdot F \cdot b_{11}^{(o)} (\phi, \bar{S}, \bar{S}, \bar{S}) \quad " \quad " \quad = 0 \quad . \quad (5.16)
\end{aligned}$$

The super-sector $\xi \cdot F \cdot b_{11}^{(o)}$ introduces 8 more vector supermultiplets in a way very similar to that of $b_{11}^{(o)}$ in Eq. (5.14), and consequently the matter gauge group is extended to $U(1)^{22}$. The scalars of the theory now parametrize the $[SU(1,1)/U(1)_o] \times [SO(6,22)/SO(6)_1 \times SO(22)]$ $N = 4$ supergravity manifold.

As we already mentioned, in this model, the $\Gamma(2,2)$ lattice is factorized. Because of this factorization, one may define the limit in which the lattice deformation associated with the $(g_{11}^{(1)} J_L^{(1)} J_R^{(1)} + g_{22}^{(2)} J_L^{(2)} J_R^{(2)})$ Thirring interaction becomes large ($g_{11}, g_{22} \rightarrow \infty$), so that the two internal dimensions will be decompactified. In this limit one obtains a six-dimensional theory

$[D = 6, (F, \bar{S}, \bar{S}, b_{11}^{(o)})]$ which has two physical massless spin-3/2 states and contains $4+16 = 20$ matter vector multiplets $[U(1)]^{20}$. The scalars of the obtained six-dimensional model parametrize an $SO(4,20)/SO(4) \times SO(20)$ scalar manifold. In the orbifold language this model may be obtained by T^4/Z_2 compactification from a ten-dimensional superstring. Following the statement of Seiberg, the cancellation of the gravitational anomaly in six dimensions implies a matter gauge group $U(1)^n$, with $n = 20$ [13]. (The orbifold T^4/Z_2 is just a limiting case of the smooth compact manifold K_3 , the unique 4-dimensional Calabi-Yau space). So, any four-dimensional $N = 4$ model obtained by means of a two-dimensional compactification manifold $M^{(2)}$ from the above six-dimensional model, must be based on a rank 22 gauge group [13].

This statement, however, does not imply that all $N = 4$ models must be based on a $U(1)^{22}$ gauge group. As we already saw in the $(F, \bar{S}, \bar{S}, b_{11}^{(o)})$, $\alpha = 0, 1$ examples that the gauge group was $U(1)^{14}$ and $U(1)^6$ respectively. This is not in

contradiction with the six-dimensional anomaly conjecture since the $\alpha = 0$ and $\alpha = 1$ models are based on more general $M^{(6)} \times M^{(2)}$ compactification manifold. Indeed the $(F, \bar{S}, \bar{S}, b_{11}^{(o)})$, $\alpha = 0, 1$ models are equivalent to a T^6/Z_2 orbifold compactification where Z_2 acts non-trivially in all internal coordinates. The $(F, \bar{S}, \bar{S}, b_{11}^{(o)})$, ξ on the other hand, is a $(T^4/Z_2) \times T^2$ orbifold model with Z_2 acting non-trivially only on the four internal coordinates. The first two have rank 6 + (1- α)8, $\alpha = 0, 1$, while the last one 6 + 16 = 22.

Up to this point we show the existence of $N = 2_L + 2_R$ models with rank 6 + K 8, $K = 0, 1, 2$. All of them have a Z_2 symmetric orbifold interpretation. The action of $Z_2^{b_{11}}$ in $\mathfrak{g}$ of $SU(8)$ is for all models:

$$Z_2^{b_{11}}: (1, 1, \alpha, 1, 1, \alpha, \alpha), \quad \alpha^2 = 1, \quad \alpha \in Z_2^{b_{11}} \quad (5.17)$$

which breaks the $SU(8)$ to $SU(4)_1 \times SU(4)_2 \times U(1)$'s.

Another class of models based on two asymmetric Z_2 projections can also be constructed through fermionic formulation. For instance, consider the model $(F, \bar{S}, \bar{S}, b_1, \bar{b}_1)$ with:

$$\begin{aligned}
b_1 \cap S &= (\bar{\psi}^{3,4}, \bar{x}^{1,2}), \quad b_1 \cap \bar{S} = \phi \\
\bar{b}_1 \cap \bar{S} &= (\bar{\psi}^{3,4}, \bar{x}^{1,2}), \quad \bar{b}_1 \cap S = \phi.
\end{aligned} \quad (5.18)$$

The b_1 and $\bar{b}_1$ induce $Z_2^{(b_1)}$ and $Z_2^{(\bar{b}_1)}$ projections which reduce $N = 4_L + 4_R$ to $2_L + 2_R$. The action of $Z_2^{(b_1)} \times Z_2^{(\bar{b}_1)}$ in $\mathfrak{g}$ of $SU(8)$ will be:

$$\begin{aligned}
Z_2^{(b_1)} \times Z_2^{(\bar{b}_1)}: (1, 1, \alpha, \alpha, 1, 1, \beta, \beta), \quad \alpha^2 = \beta^2 = 1, \\
\alpha \in Z_2^{(b_1)}, \quad \beta \in Z_2^{(\bar{b}_1)}.
\end{aligned} \quad (5.19)$$

corresponds to $N = 2_L + 2_R$ superstring solution with a minimal number of massless states.

Other $N = 2_L + 2_R$ models based on $SO(6, 2+4K)/SO(6)_1 \times SO(2+4K)$ matter scalar manifold can also be constructed. Indeed, one can start with a generic $(F, S, \bar{S}, b_{11}, \xi)$ type model which is based on $SO(6, 6+8K)/SO(6)_1 \times SO(6+8K)$ matter scalar manifold, then, the $(F, S, \bar{S}, b_{11}^{(\alpha)}, \xi, b_1)$ model is again a $N = 2_L + 2_R$ superstring but now with scalars in a $SO(6, 2+4K)/SO(6)_1 \times SO(2+4K)$ type manifold. In fact the b_1 action breaks the $SO(6+8K) \rightarrow SO(2+4K) \times SO(4+4K)$. This shows at least the existence of $N = 2_L + 2_R$ superstring solutions with $U(1)^n$ gauge group where $n = 2, 6, 10, 14$ or 22 .

Here we stop our discussion on $N = 4$ type II solutions and we shall consider the $N = 3$ case.

A possible consistent choice for b_1 and $\bar{b}_1$ is

$$\begin{aligned} b_1 &= \{\psi^{3,4}, x^{1,2}, y^{3,4,5,6}, \bar{y}^{1,2}, w^1, \bar{w}^1, \bar{y}^2, \bar{w}^2\} \\ \bar{b}_1 &= \{\bar{\psi}^{3,4}, \bar{x}^{1,2}, \bar{y}^{3,4,5,6}, \bar{y}^2, w^2, \bar{w}^2\}. \end{aligned} \quad (5.20)$$

The $(F, S, \bar{S}, b_1, \bar{b}_1)$ model is equivalent to $(F, S, S, b_{11}^{(1)}, b_1)$ one, as one can easily check by considering the induced sectors. The choices of $y^i, w^i, \bar{y}^i, \bar{w}^i$ in b_1 and $\bar{b}_1$ are such that only the $(\phi, S, \bar{S}, SS)$ sectors contain massless states. The other 28 sectors of the model give only massive states. So, the massless states of $(F, S, \bar{S}, b_1, \bar{b}_1)$ model are those of $(F, S, \bar{S}, b_{11}^{(1)})$ of Eq. (5.11) which are invariant under $Z_2^{(b_1)}$. Namely,

$$\begin{aligned} \phi\text{-sector: } & \psi^{\mu\nu} |0\rangle, \quad x^{1,2} \bar{\psi}^{\nu} |0\rangle, \\ & \psi^{\mu 1,2} |0\rangle, \quad x^{1,2} \bar{x}^{1,2} |0\rangle \end{aligned}$$

$$S\text{-sector: } \left. \begin{array}{l} \bar{\psi}^{\nu} \\ x^{1,2} \end{array} \right\} \text{spin}(S)_{-+} |0\rangle$$

$$\bar{S}\text{-sector: } \left. \begin{array}{l} \bar{\psi}^{\nu} \\ x^{1,2} \end{array} \right\} \text{spin}(\bar{S})_{-+} |0\rangle$$

$$SS\text{-sector: } \text{spin}(S)_{-+} \text{spin}(\bar{S})_{-+} |0\rangle \quad (5.21)$$

The states of Eq. (5.21) are in number the half of those of Eqs. (5.11) due to the extra $Z_2^{(b_1)}$ projection. In total the 14 $Z_2^{(b_1)}$ invariant scalars are those given by Eq. (5.12) (i), (ii), and (iv). All together they form the $[SU(1,1)/U(1)_0] \times [SO(6,2)/SO(6)_1 \times SO(2)] \times N = 4$ supergravity manifold with matter gauge group $U(1)^2$. The non-trivial deformation of this model is reduced to the $N=3$ scalars of the $SO(2,2)/SO(2) \times SO(2)$ manifold. This model

6. N = 3 Superstring Solutions.

A minimal N = 3 solution must at least contain the $(F, S, \bar{S}, b_{11}, b_2)$ as basis sets, where b_{11} defines a Z_{11} projection $(-)^{b_{11}} = 1$ which breaks symmetrically $N = 4_L + 4_R$ to $2_L + 2_R$ and $(-)^{b_2} = -1$ which defines an extra Z_2 which breaks further the $N = 2_L + 2_R$ to $N = 1_L + 2_R$. As was the case of N = 4 we expect matter vector supermultiplets, the number of which will depend on b_{11} and b_2 . Consider for instance

$$b_{11}^{(1)} = \begin{cases} \psi^{3,4}, x^{1,2}, y^3, \dots, 6^1, y^1, w^1 \\ \bar{\psi}^{3,4}, \bar{x}^{1,2}, \bar{y}^3, \dots, 6^1, \bar{y}^1, \bar{w}^1 \end{cases} \quad (6.1)$$

$$b_2 = \{\psi^{3,4}, y^{1,2}, x^{3,4}, y^5, w^6, y^3, \bar{w}^3, \bar{y}^3\},$$

which is a choice consistent with the overlapping conditions of Ref. [4] and respects the global properties of left and right supercurrents. This minimal N = 1+2 superstring solution contains in total 32 sectors (8 super-sectors): the 16 sectors of N = 2+2 model $(F, S, \bar{S}, b_{11}^{(1)})$ given in Eqs. (5.8) with $\alpha = 1$ plus 16 new ones due to the b_2 set of fermions:

$$\begin{array}{ll} b_2 \cdot (\phi, S, \bar{S}, \bar{S}\bar{S}) & \text{with lower } 1/2 \text{ } m^2 \text{ states} = 1/8 \\ F \cdot b_2 \cdot (\phi, S, \bar{S}, \bar{S}\bar{S}) & " " " = 5/8 \\ b_{11}^{(1)} \cdot b_2 \cdot (\phi, S, \bar{S}, \bar{S}\bar{S}) & " " " = 1/8 \\ F \cdot b_{11}^{(1)} \cdot b_2 \cdot (\phi, S, \bar{S}, \bar{S}\bar{S}) & " " " = 1/8. \end{array} \quad (6.2)$$

Therefore, in this model all new sectors contain only massive states, and so the massless states are those of N = 4, $(F, S, \bar{S}, b_{11}^{(1)})$, given in Eqs. (5.11), which are invariant under Z_2 (b_2). Namely:

(i) $\bar{\psi}$ -sector.

$$\begin{array}{ll} \bar{\psi}^{\mu\nu} |0\rangle & \text{graviton, dilaton, pseudoscalar.} \\ \bar{\psi}^{\mu} \bar{x}^{1,2} |0\rangle & 2 \text{ gauge bosons in } \underline{2} \text{ of } SO(2)_R. \\ \bar{x}^{3,4}, \bar{y}^3, \dots, 6^1 |0\rangle & 8 \text{ scalars in } (\underline{2}, \underline{4}) \text{ of } SO(2)_L \times SO(4)_R. \end{array} \quad (6.3a)$$

(ii) $\bar{S}$ -sector.

$$\begin{array}{ll} \bar{\psi}^{\mu} \text{spin}(S)_{-++} |0\rangle & 1 \text{ gravitino, } 1 \text{ spin-1/2 fermion.} \\ \bar{x}^{1,2} \text{spin}(S)_{-++} |0\rangle & 2 \text{ spin-1/2 fermions in } \underline{2} \text{ of } SO(2)_R. \\ \bar{x}^{3,4}, 5, 6 \text{spin}(S)_{-++} |0\rangle & 4 \text{ spin-1/2 fermions in } (\underline{2}, \underline{2}) \text{ of } \\ & SU(2)_R \times SU(2)'_R \approx SO(4)_R. \end{array} \quad (6.3b)$$

where

$$\begin{aligned} \text{spin}(S)_{-++} = & \text{spin}(\psi^{3,4})_{-} \text{spin}(x^{1,2})_{+} \text{spin}(x^{3,4})_{+} \text{spin}(x^{5,6})_{+} \\ & + \text{spin}(\psi^{3,4})_{+} \text{spin}(x^{1,2})_{-} \text{spin}(x^{3,4})_{-} \text{spin}(x^{5,6})_{-}, \end{aligned}$$

and

$$\begin{aligned} \text{spin}(S)_{-++} = & \text{spin}(\psi^{3,4})_{-} \text{spin}(x^{1,2})_{-} \text{spin}(x^{3,4})_{+} \text{spin}(x^{5,6})_{-} \\ & + \text{spin}(\psi^{3,4})_{+} \text{spin}(x^{1,2})_{+} \text{spin}(x^{3,4})_{-} \text{spin}(x^{5,6})_{+}. \end{aligned}$$

(iii) $\bar{S}$ -sector.

$$\begin{array}{ll} \bar{\psi}^{\mu} \text{spin}(\bar{S})_{-} |0\rangle & 2 \text{ gravitinos, } 2 \text{ spin-1/2 fermions in } (\underline{2}, \underline{1}) \\ & \text{of } SU(2)_R \times SU(2)'_R \approx SO(4)_R. \\ \bar{x}^{3,4} \text{spin}(\bar{S})_{-} |0\rangle & 4 \text{ spin-1/2 fermions in } (\underline{1}, \underline{2}; \underline{2}) \text{ of } \\ & SU(2)_R \times SU(2)'_R \times SO(2)_L. \end{array} \quad (6.3c)$$

(iv) $\bar{S}\bar{S}$ -sector.

$\text{spin}(S)_{-++} \text{spin}(\bar{S})_{-+} |0\rangle$ 2 gauge bosons and 2 complex scalars in

$(\bar{2}, \bar{1})$ of $SU(2)_R \times SU(2)'_R$.

$\text{spin}(S)_{+-+} \text{spin}(\bar{S})_{+-} |0\rangle$ 2 gauge bosons and 2 complex scalars in

$(\bar{1}, \bar{2})$ of $SU(2)_R \times SU(2)'_R$.

(6.3d)

From Eqs. (6.3a) and (6.3d), we conclude that the massless scalars of the theory parametrize the following manifolds:

$$(i) \frac{SU(1,1)}{U(1)}, \quad \text{dilaton and pseudoscalar}$$

$$(ii) \frac{SO(2,4)}{SO(2)_L \times SO(4)_R} = \frac{SU(2,2)}{SU(2)_L \times SU(2)_R \times U(1)}, \quad \text{from } \phi\text{-sector}$$

$$(iii) \frac{SU(2,1)}{SU(2)_R \times U(1)}, \quad \text{from } \bar{S}\bar{S}\text{-sector}$$

$$(iv) \frac{SU(1,2)}{U(1) \times SU(2)'_R}, \quad \text{from } \bar{S}\bar{S}\text{-sector}$$

(6.4)

The totality of them (9 complex scalars) parametrize the irreducible $SU(3,3)/SU(3) \times SU(3) \times U(1)$ manifold of the $N = 3$ supergravity [29] coupled to 3 vector multiplets. In this theory there are 3 gauge bosons and 3 graviphotons; there are 12 matter spin-1/2 fermions and the spin-1/2 fermion of the graviton supermultiplet.

In our construction, because of the left-right separation, the $SU(3)$ which rotates the three gravitinos appears in a decomposed form

$$SU(3) \rightarrow U(1) \times SU(2)_R \quad (6.5a)$$

while the second $SU(3)'$ appears as

$$SU(3)' \rightarrow SU(2)'_R \times U(1). \quad (6.5b)$$

The massless sector and the effective interactions of this model, can be easily obtained by a $Z_2^{b_{11}} \times Z_2^{b_2}$ truncation of the $N = 8$ supergravity Lagrangian. The action of $Z_2^{b_{11}} \times Z_2^{b_2}$ on $\underline{8}$ of $SU(8)$ is given by:

$$Z_2^{b_{11}} \times Z_2^{b_2}: (1, \beta, \alpha, \alpha\beta, 1, 1, \alpha, \alpha), \quad (6.6)$$

$$\alpha \in Z_2^{b_{11}}, \quad \beta \in Z_2^{b_2}, \quad \alpha^2 = \beta^2 = 1.$$

This breaks $SU(8) \rightarrow SU(3) \times SU(3)' \times U(1)^3$.

We observe that the dilaton and pseudoscalar are in a vector multiplet and so, they are coordinates of the $SU(3,3)/SU(3) \times SU(3) \times U(1)$ manifold. Since this theory is obtained by a $Z_2^{b_2}$ truncation of the $N = 4$ $SO(6,6)/SO(6) \times SO(6)$ model, it shows that the dilaton and pseudoscalar in $N = 2_L + 2_R$ superstrings are not the superpartners of the graviton; they are members of a matter vector multiplet [30]. This is in contrast with the $N = 4_L + 0_R$ and $N = 4$ heterotic superstring solutions; it confirms also our analysis for $N = 2_L + 2_R$ of the previous section.

The non-trivial deformations of the above $N = 3$ model are induced by the marginal $(1,1)$ operators $J_{L,3,4} J_{R,3,4,5,6}$ and they correspond to $\Gamma(2,4)$ Lorentzian lattice deformations. The associated left and right currents are $Z_2^{b_2}$ invariant but $Z_2^{b_{11}}$ transforming currents. Any sufficiently small (continuous) lattice deformation preserves the massless spectrum of the theory; only the massive states change their masses. This is because all massless

states of the model have zero $J_L^{3,4}$ and $J_R^{3,4,5,6}$ charges.

As was the case of $N = 2_L + 2_R$ we may construct $N = 1_L + 2_R$ solutions with larger or even smaller moduli. For instance, a model with larger moduli is obtained by choosing $b_{11}^{(o)}$ instead of $b_{11}^{(1)}$ so one has extra massless states coming from the $b_{11}^{(o)}$ supersector. Once $b_{11}^{(o)}$ is chosen the b_2 of Eq. (6.1) must also be modified in order to satisfy the overlapping conditions of Ref. [4]. A consistent choice is:

$$b'_2 = \{\psi^{3,4}_{1,2}, \psi^{3,4}_{3,4}, \psi^{5,6}_{3,4}, \psi^{3,3}_{3,4}, \psi^{3,3}_{w,w}, \psi^{3,3}_{w,w}\} \quad (6.7)$$

$b_{11}^{(o)}$ and $b_{11}^{(o)}$ as in Eq. (5.7a). The $\{F, S, \bar{S}, b_{11}^{(o)}, b_2^{(o)}\}$ model has 32 sectors. Those of $N = 2_L + 2_R \{F, S, \bar{S}, b_{11}^{(o)}\}$ with $\alpha = 0$ of Eq. (5.8) plus some extra sectors due to be $b_2^{(o)}$:

$b'_2 \cdot (\phi, \bar{S}, \bar{S})$	with lower $1/2 m^2$	$= 1/8$
$F \cdot b'_2 \cdot (\phi, \bar{S}, \bar{S})$	"	$= 5/8$
$b'_{11}(\phi) \cdot b'_2 \cdot (\phi, \bar{S}, \bar{S})$	"	$= 0$
$F \cdot b'_{11}(\phi) \cdot b'_2 \cdot (\phi, \bar{S}, \bar{S})$	"	$= 1/4$

(6.8)

Therefore, in this model we have massless states coming from $(\bar{\phi}, S, \bar{S}, \bar{S})$ as well as from $(\phi, S, S, \bar{S})$ sectors. The massless states of $b_{11}^{(o)} \cdot (\phi, S, \bar{S}, \bar{S})$ are the states of $N = 2_L + 2_R$ in Eqs. (5.14) which are $2_L^{b'2}$ invariant. Namely:

$$\begin{array}{ccc} \mathrm{SO}(6) \times \mathrm{SO}(8) & \xrightarrow{b_2} & \mathrm{SU}(3) \times \mathrm{SU}(4) \times \mathrm{U}(1) \quad . \\ & & (6, 10) \end{array}$$

$$b_{11}^{(0)} \text{ -sector.}$$

$$\text{spin}(\bar{\psi}^{-3,4}, \bar{x}^{1,2})_{-} \text{spin}(\psi^{3,4}, x^{1,2})_{-} \text{spin}(\bar{y}^3, \dots, \bar{y}^6, \dots, \bar{y}^6)_{+}, \bar{b}_2 |0\rangle$$

sb11(a) -sector.

$$\text{spin}(\psi^{3,4}, \bar{x}^{1,2})_{-} \text{spin}(x^3, \dots, 6)_{-} \text{spin}(y^3, \dots, 6 \bar{y}^3, \dots, 6)_{+} b_2^{10}$$

$\text{Sb}_{11}(\text{o})$ -sector.

$$8 \text{ fermions in } (\underline{2}, \underline{4}) \text{ of } \text{SU}(2)_R \times \text{SU}(4)_{(\overline{\mathbf{V}})}.$$

$$\overline{SSb}_{11}^{(0)}\text{-sector.}$$

$$\text{spin}(x^3, \dots, x^5)_{-} \text{spin}(y^3, \dots, y^6)_{+} \text{spin}(z^3, \dots, z^6)_{+} b_2 |0\rangle$$

The scalars of $\bar{b}_{11}^{(o)}$ sector form a $SU(1,4)/U(1) \times SU(4)_{(y\bar{y})}$ manifold and their associated deformations are trivial. (Spacetime R_L - R_R scalars [13]). The non-trivial moduli are associated with the scalars of $\bar{S}\bar{b}_{11}^{(o)}$ -sector which form a $SU(2,4)/SU(2)_R \times SU(4)_{(y\bar{y})} \times U(1)_L$ manifold. We observe that the totality of scalars from the $\bar{b}_{11}^{(o)}$ supersector together parametrize a more symmetric manifold, the $SU(3,4)/SU(3) \times SU(4) \times U(1)$. Comparing with that of $\bar{b}_{11}^{(o)}$ supersector of $N = 2_L + 2_R$ theory, ($SO(6,8)/SO(6) \times SO(8)$), we observe the breaking rule:

$$\begin{array}{ccc} \mathrm{SO}(6) \times \mathrm{SO}(8) & \xrightarrow{b_2} & \mathrm{SU}(3) \times \mathrm{SU}(4) \times \mathrm{U}(1) \quad . \\ & & (6, 10) \end{array}$$

In a very similar way, the massless scalars of $b_{11}^{(o)}$, b'_{12} -supersector parametrize again a new $SU(3,4)/SU(3) \times SU(4)' \times U(1)$ manifold, which is decomposed into $SU(1,4)/U(1) \times SU(4)'$ and $SU(2,4)/SU(2) \times SU(4)' \times U(1)$ associated with the trivial and non-trivial moduli. In total, the $(F, S, \bar{S}, b_{11}^{(o)}, b'_{12})$ model contains 33 complex scalars which parametrize successfully the scalar manifold, $SU(3,11)/SU(3) \times SU(11) \times U(1)$, of $N = 3$ supergravity coupled to 11 massless vector multiplets.

All possible $N = 3$ solutions, with minimal number of GSO projections, can be classified according to their induced effective supergravity. As we shall show below, the induced $N = 3$ supergravity is based on a scalar manifold of the form $SU(3, 3+4K)/U(1) \times SU(3) \times SU(3+4K)$ with $K = 0, 1, 2$. We shall prove that statement by explicit construction of all independent $(F, S, \bar{S}, b_{11}, b_{12})$ $N = 1_L + 2_R$ solutions.

Up to base changes, there are two possible choices for $b_{11}^{(1)}$; the $b_{11}^{(0)}$ and $b_{11}^{(1)}$ of Eq. (5.7a). Once b_{11} is fixed then we have the following eleven possibilities (up to base changes).

1. $b_{11}^{(0)}; b_2^{(1)} = \{w^{3,4,1,2,3,4,5,6,3,3,4,4,3,3,4,4}, y, w, y, w, y, w, y, w, y, w, y, w, y, w, y\}$
2. $b_{11}^{(0)}; b_2^{(2)} = \{w^{3,4,1,2,3,4,5,6,3,3,3,3,3,3,3,3}, y, y, w, y, w, y, w, y, w, y, w, y, w, y, w, y\}$
3. $b_{11}^{(0)}; b_2^{(3)} = \{w^{3,4,1,2,3,4,5,6,3,3,3,3,3,3,3,3}, y, w, y, w, y, w, y, w, y, w, y, w, y, w, y, w, y\}$
4. $b_{11}^{(0)}; b_2^{(4)} = \{w^{3,4,1,2,3,4,5,6,3,3,3,4,4,4,1,3,3,3}, y, w, y, w, y, w, y, w, y, w, y, w, y, w, y, w, y\}$
5. $b_{11}^{(0)}; b_2^{(5)} = \{w^{3,4,1,2,3,4,5,6,3,3,4,4,4,4,1,1,2,2}, y, w, y, w, y, w, y, w, y, w, y, w, y, w, y, w, y\}$

- $$\begin{aligned}
6. \quad b_{11}^{(1)}; b_2^{(6)} &= [\psi^{3,4,1,2,3,4}_{,y,x}, y^{5,6,3,3,4}_{,w,y}, \bar{y}^1] \\
7. \quad b_{11}^{(1)}; b_2^{(7)} &= [\psi^{3,4,1,2,3,4}_{,y,x}, y^{5,6,3,3,4}_{,w,y}, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1] \\
8. \quad b_{11}^{(1)}; b_2^{(8)} &= [\psi^{3,4,1,2,3,4}_{,y,x}, y^{5,6,3,3,4}_{,w,y}, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1] \\
9. \quad b_{11}^{(1)}; b_2^{(9)} &= [\psi^{3,4,1,2,3,4}_{,y,x}, y^{5,6,3,3,3}_{,w,y}, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1] \\
10. \quad b_{11}^{(1)}; b_2^{(10)} &= [\psi^{3,4,1,2,3,4}_{,y,x}, w^{5,6,3,3,2}_{,w,y}, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1] \\
11. \quad b_{11}^{(1)}; b_2^{(11)} &= [\psi^{3,4,1,2,3,4}_{,y,x}, w^{5,6,3,3,4}_{,w,y}, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1, \bar{y}^1]
\end{aligned}
\tag{6.11}$$

In all the above models there is a common massless supersector, $\hat{\phi}$, which contains the $N = 3$ graviton supermultiplet and 3 massless vector multiplets. The scalars of the $\hat{\phi}$ supersector parametrize the $SU(3,3)/SU(3) \times U(1)$ $N = 3$ supergravity manifold.

The $\{b_{11}^{(o)}, b_2^{(1)}\}$ and $\{b_{11}^{(o)}, b_2^{(2)}\}$ models have two extra massless supersectors, that of $\hat{b}_{11}^{(o)}$ and $b_{11}^{(o)-}(i)$, $(i = 1, 2)$. Per massless supersector we have four extra massless vector supermultiplets ($K = 2$).

The $(b_{11}^{(o)}, b_2^{(3)})$ and $(b_{11}^{(o)}, b_2^{(4)})$ models have one extra supersector, $\hat{b}_{11}^{(o)}$ which gives four vector supermultiplets ($K = 1$).

The $\{b_{11}^{(o)}, b_2^{(5)}\}$ has two massless supersectors, that of $b_{11}^{(o)}$ and $\{b_{11}^{(o)}, b_2^{(5)}\}$, with four massless vector supermultiplets per super-sector ($K = 2$).

The $b_{11}^{(1)}, b_2^{(6)}$ and $b_{11}^{(1)}, b_2^{(7)}$ models have one extra supersector, that of $b_{11}^{(1)}, b_2^{(i)}, i = 6, 7 (K = 1)$.

The $b_{11}^{(1)}$, $b_2^{(8)}$ and $(b_{11}^{(1)}, b_2^{(9)})$ models have no extra supersectors ($K = 0$).

Finally, the $(b_{11}^{(1)}, b_2^{(10)})$ and $(b_{11}^{(1)}, b_2^{(11)})$ models have one massless extra supersector, that of $F \cdot b_{11}^{(1)} \cdot b_2^{(i)}$, $i = 10, 11$, with four vector supermultiplets ($K = 1$).

We observe that the maximal moduli for the $(F, S, \bar{S}, b_{11}, b_2)$ solutions corresponds to models based on $SU(3, 11)/SU(3) \times SU(11) \times U(1)$ manifolds while the minimal moduli corresponds to $SU(3, 3)/SU(3) \times SU(3) \times U(1)$ ones.

$N = 3$ superstring solutions with even lower moduli can be also constructed after we consider asymmetric projections. We shall present as an example an $N = 3$ solution with the lowest possible moduli, which will be based on the $SU(3, 1)/SU(3) \times U(1)$ $N = 3$ supergravity scalar manifold. In order to obtain such a model we must consider at least the basis sets $(F, S, \bar{S}, b_1, b_2, \bar{b}_1)$ with b_1, b_2 and $\bar{b}_1$ suitably chosen to break the

$$N = 4_L + 4_R \xrightarrow{b_1} 2_L + 4_R \xrightarrow{b_2} 1_L + 4_R \xrightarrow{\bar{b}_1} 1_L + 2_R. \quad (6.12)$$

A consistent choice is:

$$\begin{aligned} b_1 &= [\psi^{3,4}, x^{1,2}, y^{3,4,5,6}, \bar{y}^{1,1,5,5}, \bar{w}] \\ b_2 &= [\psi^{3,4}, y^{1,2}, x^{3,4,5,6}, \bar{y}^{3,3,6,6}, \bar{w}^6] \\ \bar{b}_1 &= [\bar{\psi}^{3,4}, \bar{x}^{1,2}, \bar{y}^{3,4,5,6}, \bar{w}^1, \bar{w}^5, \bar{w}] \end{aligned} \quad (6.13)$$

One can check the validity of the overlapping conditions of Ref. [4] and the global consistency of left and right supercurrents [4]. This model has 64 sectors (16 supersectors) which are all massive except the $\hat{\phi} \Rightarrow (\phi, S, \bar{S}, \bar{S}\bar{S})$. We give below the 16 possible supersectors and their lower $1/2 m^2$ in parenthesis:

$$\begin{aligned} \hat{\phi}(0), \hat{F}\left(\frac{3}{4}\right), \hat{b}_1\left(\frac{1}{8}\right), \hat{b}_2\left(\frac{1}{8}\right), \hat{b}_1\left(\frac{1}{8}\right), b_1\bar{b}_2\left(\frac{1}{4}\right), b_1\bar{b}_1\left(\frac{1}{8}\right), \\ b_2\bar{b}_1\left(\frac{1}{8}\right), b_1\bar{b}_2\bar{b}_1\left(\frac{1}{8}\right), \hat{F}\bar{b}_1\left(\frac{5}{8}\right), \hat{F}\bar{b}_2\left(\frac{5}{8}\right), \hat{F}\bar{b}_1\left(\frac{5}{8}\right), \hat{F}\bar{b}_2\left(\frac{5}{8}\right), \\ \hat{F}\bar{b}_1\bar{b}_2\left(\frac{1}{8}\right), \hat{F}\bar{b}_2\bar{b}_1\left(\frac{1}{8}\right), \hat{F}\bar{b}_1\bar{b}_2\bar{b}_1\left(\frac{1}{8}\right). \end{aligned} \quad (6.14)$$

Since all the supersectors except $\hat{\phi}$ are massive, the massless states of the theory are those of $N = 8$ which are invariant under $(-)^{b_1} = (-)^{b_2} = (-)^{\bar{b}_1} = -1$ GSO projections; explicitly:

$$\begin{aligned} \underline{\hat{\phi}\text{-sector.}} \\ \psi^{\mu-\nu} \psi |0\rangle \\ \psi^{\mu-1,2} |0\rangle \\ \underline{S\text{-sector.}} \\ \bar{\psi}^{\nu} \{ \text{spin } (\psi^{3,4}, x^{1,2})_- \text{ spin } (x^{3,4}, x^{5,6})_+ b_2 |0\rangle \\ \bar{x}^{1,2} \{ \text{spin } (\psi^{3,4}, x^{1,2})_- \text{ spin } (x^{3,4}, x^{5,6})_+ b_2 |0\rangle \end{aligned}$$

$$\begin{aligned} \underline{\bar{S}\text{-sector.}} \\ \bar{\psi}^{\nu} \text{ spin } (\bar{S})_- |0\rangle \\ \underline{\bar{S}\bar{S}\text{-sector.}} \\ \{ \text{spin } (\psi^{3,4}, x^{1,2})_- \text{ spin } (x^{3,4}, x^{5,6})_+ b_2 \text{ spin } (\bar{S})_- |0\rangle \end{aligned} \quad (6.15)$$

The states of Eq. (6.15) compared to those of Eqs. (6.3) are half in number due to the extra $z_2^{(b_1)}$ projection. The massless scalars of the model are only six:

- (i) dilaton, pseudoscalar of the ϕ sector, on $SU(1,1)/U(1)$ manifold;
(ii) 4 Ramond-Ramond scalars of $\bar{S}\bar{S}$ -sector on $SU(2,1)/SU(2)_R \times U(1)$ manifold.

Together they parametrize the $N = 3$ supergravity manifold $SU(3,1)/SU(3) \times U(1)$. In this model there is only one massless vector multiplet.

The induced effective supergravity theory corresponding to this model can be easily obtained from $N = 8$ supergravity theory after $z_2^{b_1} \times z_2^{b_2} \times z_2^{\bar{b}_1}$ projections. The action of $z_2^{b_1} \times z_2^{b_2} \times z_2^{\bar{b}_1}$ in $\bar{g}$ of $SU(8)$ is

$$z_2^{b_1} \times z_2^{b_2} \times z_2^{\bar{b}_1} : (1, \beta, \alpha, \alpha\bar{\beta}, 1, 1, \gamma, \gamma) ; \quad (6.16)$$

$$\alpha \in z_2^{b_1}, \quad \beta \in z_2^{b_2}, \quad \gamma \in z_2^{\bar{b}_1}, \quad \alpha^2 = \beta^2 = \gamma^2 = 1.$$

What is particular in this model is the absence of any non-trivial continuous deformations. There are only discrete ones as was the case of the $N = 5$ superstring models. One can easily check that there is no $J_L \cdot J_R$ marginal operator which is invariant under $z_2^{b_1} \times z_2^{b_2} \times z_2^{\bar{b}_1}$.

Changing the base $(b_1, b_2, \bar{b}_1)$ to (b_{11}, b_2, b_1) with $b_{11} = b_1 \cdot \bar{b}_1$ we can see that the asymmetric $N = 3$ theories are obtained by introducing one more fermion number projection $(-)^{b_1}$ acting on the states of (F, b_{11}, b_2) theories. As we have shown previously the $N = 3$ (F, b_{11}, b_2) models are based on $SU(3, 3+4K)/SU(3) \times SU(3+4K) \times U(1)$ scalar manifolds. After the new projection induced by $(-)^{b_1}$ in the asymmetric (b_{11}, b_2, b_1) theories, this manifold is reduced to $SU(3, 1+2K)/SU(3) \times SU(1+2K) \times U(1)$ type manifolds with $K = 0, 1, 2$. Indeed, the $SU(3, 3)/SU(3) \times SU(3)$ manifold of the ϕ supersector is reduced to $SU(3, 1) \times SU(3) \times U(1)$ as we already saw by examining the minimal moduli $N = 3$ solution. For any extra supersector of the (b_{11}, b_2) models we saw that the corresponding scalars form additional $SU(3, 4)/SU(3) \times SU(4)$ submanifolds. The

action of the extra $(-)^{b_1}$ projection maintains only the half of the states in any massless supersector, breaks down the $SU(4)' \rightarrow SU(2)'$ and reduces the additional scalar submanifold to $SU(3, 2)/SU(3) \times SU(2)' \times U(1)$ type cosets (per massless supersector). This shows that the $(F, S, \bar{S}, b_1, b_2, \bar{b}_1)$ $N = 3$ solutions have massless scalars parametrizing a $SU(3, 1+2K)/SU(3) \times SU(1+2K) \times U(1)$ type manifolds.

Many other $N = 3$ superstring solutions can be constructed by using different approaches, e.g., fermionic constructions with generalized boundary conditions, asymmetric orbifold compactifications or else. For instance, a $N = 3$ solution based on T^6/Z_3 asymmetric orbifold can be easily constructed once we choose Z_3 such that to act on $(4, 4)$ representation of $SU(4)_L \times SU(4)_R$ like:

$$Z_3: (\underline{4}, \underline{4}), \quad (1, \alpha, \alpha, \alpha; 1, 1, \alpha, \alpha^{-1}) \quad (6.17)$$

with $\alpha \in Z_3; \alpha^3 = 1$. This action transforms three left and two right gravitinos and so it leaves invariant the remaining three. The consistency of this orbifold model was shown by Narain [31]. The trivial and non-trivial moduli correspond to massless scalars of $N = 3$ supergravity with three matter vector supermultiplets. The associated scalar manifold is the $SU(3, 3)/SU(3) \times SU(3)$ coset space. The induced effective supergravity Lagrangian can be obtained easily from the $N = 8$ supergravity theory by a Z_3 truncation. In that case Z_3 is defined by its action on $\bar{g}$ of $SU(8)$ as:

$$Z_3: \bar{g}, \quad (1, \alpha, \alpha, \alpha, 1, 1, \alpha, \alpha^{-1}); \quad \alpha \in Z_3, \quad \alpha^3 = 1.$$

7. Conclusion.

In this work we have examined superstring models based on the fermionic construction, with $N = 8, 6, 5, 4$ and 3 spacetime supersymmetry in four spacetime dimensions.

Those models are all based on type II construction with 0, 1, 2 or 4 supersymmetries coming from the left or right moving sector. The following possibilities exist: $N = 8$ ($4_L + 4_R$), $N = 6$ ($2_L + 4_R$), $N = 5$ ($1_L + 4_R$), $N = 4$ ($4_L + 0_R$) or ($2_L + 2_R$), $N = 3$ ($1_L + 2_R$).

From the string Kac-Moody symmetries, we are able to classify the scalar states as coset representatives of manifolds which occur in nonlinear sigma models coupled to N-extended supergravity. For $N \geq 5$ there are spacetime R_L - R_R scalars. Those states have only spacetime derivative couplings and the associated deformations are trivial at the perturbative level. We show that these R_L - R_R states are the coset representatives of $SU(n, m)/SU(n) \times SU(m) \times U(1)$ type manifolds. From supergravity point of view they are the spin-0 states of $\lambda_{\max} = 3/2$ $N = 4$ gravitino supermultiplet.

The moduli space of the "internal" degrees of freedom correspond to lattice deformation and parametrize a coset space of the type $\Pi \times [SO(n, m)/SO(n) \times SO(m)]$. For $N = 5$ only trivial R_L - R_R deformations exist which show that the full scalar manifold is $SU(5, 1)/SU(5) \times U(1)$.

For $N \leq 4$ there are two types of R_L - R_R scalars; (i) those in which the R_L and R_R are spacetime spinors and so the corresponding states have only spacetime derivative couplings, then the induced deformations are perturbatively trivial; (ii) those which are not R_L or R_R spacetime spinors but spinors with internal indices. The deformations associated with them are non-trivial and they enlarge the moduli space of the "internal" degrees of freedom.

For $N = 4_L + 0_R$ the "internal" moduli space is of the type $SO(6, n)/SO(6) \times SO(n)$, while for the $N = 2_L + 2_R$ the "internal" moduli space is of the type $\frac{SO(2, 2)}{SO(2) \times SO(2)} \times \frac{SO(4, n)}{SO(4) \times SO(n)}$. In the second version of $N = 4$, the moduli space is enlarged to $\frac{SO(2, 2)}{SO(6) \times SO(n)}$ only when we include the spacetime R_L - R_R and dilaton and pseudoscalar while (by themselves) they parametrize a $SO(2, n_R)/SO(2) \times SO(n_R)$ and $SU(1, 1)/U(1)$ manifolds, respectively.

For $N = 3$, the internal moduli space is of the form $SU(2, n)/SU(2) \times SU(n) \times U(1)$. The spacetime R_L - R_R scalars with the dilaton and pseudoscalar give together a $SU(1, n)/U(1) \times SU(n)$. The entire sigma model manifold of the effective $N = 3$ supergravity [32] is then enlarged to $SU(3, n)/SU(3) \times SU(n) \times U(1)$.

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