Experimental predictions from a minimal version of the supersymmetric standard model *

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Abstract

We make predictions for particle masses and couplings on the basis of the minimal supersymmetric standard model. We assume that supersymmetry breaking is seeded by a universal gaugino mass, and we perform a full one-loop analysis of the electroweak gauge symmetry breaking, imposing phenomenological and theoretical constraints on the model's input parameters. We find $76 \text{ GeV} < m_t < 90 \text{ GeV}$, $13 \text{ GeV} < m_H < 72 \text{ GeV}$, $4 \text{ GeV} < m_{\tilde{N}_1} < 20 \text{ GeV}$, $160 \text{ GeV} < m_{\tilde{g}} < 400 \text{ GeV}$, $140 \text{ GeV} < m_{\tilde{q}} < 350 \text{ GeV}$, $m_{\tilde{C}_1} < 65 \text{ GeV}$, $55 \text{ GeV} < m_{\tilde{e}_L} < 120 \text{ GeV}$, $40 \text{ GeV} < m_{\tilde{e}_R} < 75 \text{ GeV}$, and $0.4 < R_{inv} \equiv [\Gamma(Z \rightarrow \tilde{N}_1 \tilde{N}_1) + \sum_{i=1}^3 \Gamma(Z \rightarrow \tilde{\nu}_i \tilde{\nu}_i)]/\Gamma(Z \rightarrow \bar{\nu} \nu) < 1.4$, where H , $\tilde{C}_1$ and $\tilde{N}_1$ are the lightest neutral Higgs scalar, spin- $\frac{1}{2}$ chargino and spin- $\frac{1}{2}$ neutralino, respectively, and the upper limits on the masses (lower limit on R_{inv}) hold if $\tilde{N}_1$ is to solve the dark matter problem. Therefore, several new particle signatures could be detectable at SLC, LEP I or II: $Z \rightarrow H \bar{f} f$, $e^+ e^- \rightarrow Z H$, $e^+ e^- \rightarrow \tilde{l}^+ \tilde{l}^-$, $e^+ e^- \rightarrow \tilde{C}_1^+ \tilde{C}_1^-$ and an apparently non-integer number of neutrinos from $Z \rightarrow \text{missing neutrals}$ ($\tilde{N}_1 \tilde{N}_1, \tilde{\nu} \tilde{\nu}$).

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U. S. Department of Energy under Contract DE-AC03-76SF00098, and in part by NSF under grant PHY-8515857.

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1 Introduction

There has been intense theoretical interest in the minimal supersymmetric version of the Standard Model (MSSM) with two Higgs doublets, and no other additional supermultiplets (for reviews and references see, for example, ref.[1]). In the most popular variants, the MSSM is thought to originate at some high unification scale M_U with equal $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge couplings, and some combination of small supersymmetry-breaking parameters: universal gaugino ($m_{1/2}$) and scalar (m_0) masses, and scalar couplings of the same form as the superpotential couplings. These soft supersymmetry-breaking parameters are then subject to radiative corrections analogous to those for the gauge couplings in a grand unified theory. Gauge symmetry breaking at the weak interaction scale is made possible by a destabilization of the origin in the space of the Higgs fields, which originates from the radiative corrections due to a heavy top quark [2]. Within this general framework, some authors [3] have favoured the possibility that the dominant seed of supersymmetry breaking be a gaugino mass $m_{1/2}$: this ‘minimal’ version of the MSSM will be considered in the present work.

Many papers have investigated this and other versions of the MSSM [2] (as well as non-minimal supersymmetric models [4]), and shown that it is possible to break the $SU(2)_L \times U(1)_Y$ gauge symmetry with a suitably chosen top quark mass. Representative top quark, Higgs boson and sparticle spectra have also been given. However, as shown recently in ref.[5], the theoretical approximations and constraints used in most of the previous analyses [2,4,6,7] can often be misleading. There are now also important phenomenological constraints, not only from particle physics [8-13], but also from astrophysics [14,15] and from cosmology [16,17], which must be obeyed.

In this paper we break no new theoretical ground, but we do take systematically into account all the known theoretical and phenomenological constraints. Remarkably, we find that many unknown particle and sparticle masses are tightly restricted, whilst several may lie in

ranges accessible to LEP. For example, we find that

$$76 \text{ GeV} < m_t < 90 \text{ GeV}, \quad (1.1)$$

(where the upper limit holds if the lightest spin- $\frac{1}{2}$ neutralino is to account for the dark matter in the Universe), whilst m_H , $m_{\tilde{e}_L}$, $m_{\tilde{e}_R}$ and $m_{\tilde{Z}_1}$ could be as light as 13 GeV, 55 GeV, 40 GeV and the present experimental limit of approximately 26 GeV, respectively. We also find that the lightest neutralino $\tilde{N}_1$ could be pair-produced, as well as the sneutrinos $\tilde{\nu}_i$ ($i = 1, 2, 3$), in Z^0 decays at a rate which is a significant fraction of $Z^0 \rightarrow \bar{\nu}\nu$. This offers neutrino-counting experiments the possibility of measuring an apparently non-integer number of neutrino species!

The structure of this paper is as follows. In the three parts of section 2 we introduce respectively the MSSM, theoretical constraints, which mainly relate to the existence of the desired asymmetric electroweak vacuum, and the phenomenological constraints, which are mainly from unsuccessful accelerator searches for sparticles, but also include astrophysical and cosmological constraints like the density of dark matter in the Universe. In section 3 we discuss the interplay of these constraints in restricting the input parameters of the MSSM, which are, within our assumptions, the top quark Yukawa coupling at the unification scale, a Higgs mass-mixing parameter and the primordial gaugino mass $m_{1/2}$. As a result, we find allowed ranges of variation for the unknown particle masses and couplings. Finally, in section 4 we review the possible experimental signatures of this version of the MSSM at SLC, LEP and other accelerators, and discuss how it may best be tested.

2 Model Ingredients

2.1 The MSSM

In this model, all the left-handed fermion fields of the Standard Model are promoted to chiral superfields: $[Q \equiv (U, D), U^c, D^c, L \equiv (N, E), E^c]$ for each generation. There are two Higgs doublet superfields: (H_1^0, H_1^-) , whose neutral vacuum expectation value (v.e.v.)

v_1 gives masses to the charge- $\frac{1}{3}$ quarks and to charged leptons, and (H_2^+, H_2^0) , whose neutral v.e.v. v_2 gives masses to the charge- $\frac{2}{3}$ quarks. The trilinear superpotential is given in terms of the standard Yukawa couplings. As usual, we will neglect intergenerational mixing. However, we will not neglect the τ -lepton and b-quark Yukawa couplings (h_τ, h_b) with respect to the t-quark Yukawa coupling (h_t) , since for some acceptable values of $\tan\beta \equiv v_2/v_1$ and m_t they can be comparable in magnitude. In order to avoid a phenomenologically unacceptable axion, and to make sure that both v_1 and v_2 are non-zero, there must be a mixing term between H_1 and H_2 in the effective potential. A possible origin for such a term is a mixing term $\mu H_1 H_2$ in the superpotential, which can only be bilinear in the absence of any other superfields. The most important terms in the superpotential of the MSSM involve the superfields associated to the Higgs doublets and to the quarks and leptons of the third generation,

$$W = h_t(UU^c H_2^0 - DU^c H_2^+) + h_b(DD^c H_1^0 - UD^c H_1^-) + h_\tau(LE^c H_1^0 - NE^c H_1^-) + \mu(H_1^0 H_2^0 - H_1^- H_2^+). \quad (2.1)$$

Given the experimental values of the b-quark and τ -lepton masses, $m_b \simeq 5$ GeV and $m_\tau = 1.784$ GeV, the adjustable parameters in the superpotential of eq.(2.1) are only h_t and μ , whose values at the unification scale M_U will be denoted by $h_U \equiv h_t(M_U)$ and $\mu_U \equiv \mu(M_U)$, respectively.

The superpotential W of eq.(2.1) fixes all the supersymmetric terms in the tree-level potential, whose F - and D -terms we will not write out explicitly as they are well-known. There are also related soft supersymmetry breaking terms, which we parametrize as

$$V_{soft} = [h_t A_t(UU^c H_2^0 - DU^c H_2^+) + \text{h.c.}] + [h_b A_b(DD^c H_1^0 - UD^c H_1^-) + \text{h.c.}] + [h_\tau A_\tau(EE^c H_1^0 - NE^c H_1^-) + \text{h.c.}] + [B\mu(H_1^0 H_2^0 - H_1^- H_2^+) + \text{h.c.}] + m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_L^2 (|N|^2 + |E|^2) + m_{E^c}^2 |E^c|^2 + m_Q^2 (|U|^2 + |D|^2) + m_{U^c}^2 |U^c|^2 + m_{D^c}^2 |D^c|^2. \quad (2.2)$$

The resulting tree-level potential for the neutral Higgs components,

whose v.e.v.'s v_1, v_2 we wish to be non-zero, is

$$V_0 = m_1^2 v_1^2 + m_2^2 v_2^2 + 2m_3^2 v_1 v_2 + \frac{g_2^2 + g'^2}{8} (v_1^2 - v_2^2)^2, \quad (2.3)$$

where g_2 and $g' = \sqrt{3/5} g_1$ are the usual $SU(2)_L$ and $U(1)_Y$ gauge couplings,

$$m_i^2 \equiv \mu^2 + m_{H_i}^2 \quad (i=1,2), \quad m_3^2 \equiv B\mu, \quad (2.4)$$

and, to be able to work with real and positive v_1 and v_2 , we adopt phase conventions on the fields such that $m_3^2 < 0$.

The values of h_t, h_b, h_τ, μ and of the soft supersymmetry breaking parameters at low renormalization scales Q are given by the renormalization group equations [18]

$$\frac{dh_t}{dt} = \frac{h_t}{8\pi^2} \left(-\frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{13}{18}g'^2 + 3h_t^2 + \frac{1}{2}h_b^2 \right), \quad (2.5)$$

$$\frac{dh_b}{dt} = \frac{h_b}{8\pi^2} \left(-\frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{7}{18}g'^2 + \frac{1}{2}h_t^2 + 3h_b^2 + \frac{1}{2}h_\tau^2 \right), \quad (2.6)$$

$$\frac{dh_\tau}{dt} = \frac{h_\tau}{8\pi^2} \left(-\frac{3}{2}g_2^2 - \frac{3}{2}g'^2 + \frac{3}{2}h_b^2 + 2h_\tau^2 \right), \quad (2.7)$$

$$\frac{d\mu}{dt} = \frac{\mu}{8\pi^2} \left(-\frac{3}{2}g_2^2 - \frac{1}{2}g'^2 + \frac{3}{2}h_t^2 + \frac{3}{2}h_b^2 + \frac{1}{2}h_\tau^2 \right), \quad (2.8)$$

$$\frac{dA_t}{dt} = \frac{1}{8\pi^2} \left(\frac{16}{3}g_3^2 M_3 + 3g_2^2 M_2 + \frac{13}{9}g'^2 M_1 + 6h_t^2 A_t + h_b^2 A_b \right), \quad (2.9)$$

$$\frac{dA_b}{dt} = \frac{1}{8\pi^2} \left(\frac{16}{3}g_3^2 M_3 + 3g_2^2 M_2 + \frac{7}{9}g'^2 M_1 + h_t^2 A_t + 6h_b^2 A_b + h_\tau^2 A_\tau \right), \quad (2.10)$$

$$\frac{dA_\tau}{dt} = \frac{1}{8\pi^2} \left(3g_2^2 M_2 + 3g'^2 M_1 + 3h_b^2 A_b + 4h_\tau^2 A_\tau \right), \quad (2.11)$$

$$\frac{dB}{dt} = \frac{1}{8\pi^2} \left(3g_2^2 M_2 + g'^2 M_1 + 3h_t^2 A_t + 3h_b^2 A_b + h_\tau^2 A_\tau \right), \quad (2.12)$$

$$\frac{dm_{H_1}^2}{dt} = \frac{1}{8\pi^2} \left(-3g_2^2 M_2^2 - g'^2 M_1^2 + 3h_b^2 F_b + h_\tau^2 F_\tau \right), \quad (2.13)$$

$$\frac{dm_{H_2}^2}{dt} = \frac{1}{8\pi^2} \left(-3g_2^2 M_2^2 - g'^2 M_1^2 + 3h_t^2 F_t \right), \quad (2.14)$$

where $t \equiv \log Q$ and

$$F_t \equiv m_Q^2 + m_{U^c}^2 + m_{H_2}^2 + A_t^2, \quad (2.15)$$

$$F_b \equiv m_Q^2 + m_{D^c}^2 + m_{H_1}^2 + A_b^2, \quad (2.16)$$

$$F_\tau \equiv m_L^2 + m_{E^c}^2 + m_{H_1}^2 + A_\tau^2. \quad (2.17)$$

Here m_Q^2 , $m_{U^c}^2$, $m_{D^c}^2$, m_L^2 and $m_{E^c}^2$ are the supersymmetry-breaking masses-squared of the third-generation left-handed squark and sleptons, and obey the renormalization group equations

$$\frac{dm_Q^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{16}{3} g_3^2 M_3^2 - 3g_2^2 M_2^2 - \frac{1}{9} g'^2 M_1^2 + h_t^2 F_t + h_b^2 F_b \right) \quad (2.18)$$

$$\frac{dm_{U^c}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{16}{3} g_3^2 M_3^2 - \frac{16}{9} g'^2 M_1^2 + 2h_t^2 F_t \right), \quad (2.19)$$

$$\frac{dm_{D^c}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{16}{3} g_3^2 M_3^2 - \frac{4}{9} g'^2 M_1^2 + 2h_b^2 F_b \right), \quad (2.20)$$

$$\frac{dm_L^2}{dt} = \frac{1}{8\pi^2} \left(-3g_2^2 M_2^2 - g'^2 M_1^2 + h_\tau^2 F_\tau \right), \quad (2.21)$$

$$\frac{dm_{E^c}^2}{dt} = \frac{1}{8\pi^2} \left(-4g'^2 M_1^2 + 2h_\tau^2 F_\tau \right). \quad (2.22)$$

The supersymmetry-breaking masses-squared of the first two squark and slepton generations evolve according to analogous expressions in which the terms proportional to h_t^2 , h_b^2 , h_τ^2 are dropped.

The determination of the v.e.v.'s v_1 and v_2 will be discussed in the following subsection. Here we summarize the tree-level expressions for the various particle masses, beginning with the gauge bosons masses,

$$m_W^2 = \frac{g_2^2}{2} (v_1^2 + v_2^2), \quad m_Z^2 = \frac{g_2^2 + g'^2}{2} (v_1^2 + v_2^2), \quad (2.23)$$

which set the scale of electroweak symmetry breaking, and with the t-quark mass,

$$m_t = |h_t v_2|, \quad (2.24)$$

which plays a crucial role in our analysis. Given m_W and m_Z , the Higgs bosons' masses and mixing angles are completely determined by only two parameters, e.g.

$$\tan \beta \equiv \frac{v_2}{v_1} \quad (2.25)$$

and the mass of the physical CP-odd neutral Higgs boson P ,

$$m_P^2 = -m_3^2 \left(\tan \beta + \frac{1}{\tan \beta} \right), \quad (2.26)$$

so that for the physical charged Higgs boson one has

$$m_{H^\pm}^2 = m_W^2 + m_P^2, \quad (2.27)$$

and for the CP-even neutral Higgs bosons H , H' :

$$m_{H,H'}^2 = \frac{1}{2} \left\{ [m_{(22)}^2 + m_{(11)}^2] \mp \sqrt{[m_{(22)}^2 - m_{(11)}^2]^2 + 4m_{(21)}^4} \right\}, \quad (2.28)$$

where

$$m_{(22)}^2 = m_P^2 \frac{1}{1+\tan^2 \beta} + m_Z^2 \frac{\tan^2 \beta}{1+\tan^2 \beta}, \quad (2.29)$$

$$m_{(11)}^2 = m_P^2 \frac{\tan^2 \beta}{1+\tan^2 \beta} + m_Z^2 \frac{1}{1+\tan^2 \beta},$$

$$m_{(21)}^2 = -m_P^2 \frac{\tan \beta}{1+\tan^2 \beta} - m_Z^2 \frac{\tan \beta}{1+\tan^2 \beta}.$$

Notice that $m_H < m_Z < m_{H'}$. The above formulae for the Higgs masses, eqs.(2.26-29), have been derived assuming that the v.e.v.'s (v_1, v_2) minimize the tree-level potential of eq.(2.3): we will discuss later under which circumstances they give sensible results. The sneutrino masses are

$$\begin{aligned} m_{\tilde{\nu}}^2 = & m_0^2 + m_W^2 \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} \left(-\frac{1}{2} \tan^2 \theta_W - \frac{1}{2} \right) \\ & + m_{1/2}^2 \left[\frac{3}{2b_2} \left(1 - \frac{1}{F_2^2} \right) + \frac{3}{5} \frac{1}{2b_1} \left(1 - \frac{1}{F_1^2} \right) \right], \end{aligned} \quad (2.30)$$

where

$$b_3 = -3, \quad b_2 = 1, \quad b_1 = \frac{33}{5}, \quad (2.31)$$

and

$$F_2 = 1 + b_2 \frac{g_U^2}{8\pi^2} \log \frac{M_U}{Q_0}, \quad F_1 = 1 + b_1 \frac{g_U^2}{8\pi^2} \log \frac{M_U}{Q_0}. \quad (2.32)$$

Here g_U is the common value of the properly normalized gauge couplings at the unification scale M_U , $g_3(M_U) = g_2(M_U) = g_1(M_U) = g_U$, and Q_0 is a low-energy threshold renormalization scale at which we cease integration of the supersymmetric renormalization group equations, to be specified later¹. The analogous expressions for the left- and right-handed selectron masses-squared are

$$\begin{aligned} m_{\tilde{e}_L}^2 &= m_0^2 + m_W^2 \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} \left(-\frac{1}{2} \tan^2 \theta_W + \frac{1}{2} \right) \\ &+ m_{1/2}^2 \left[\frac{3}{2b_2} \left(1 - \frac{1}{F_2^2} \right) + \frac{3}{5} \frac{1}{2b_1} \left(1 - \frac{1}{F_1^2} \right) \right], \quad (2.33) \\ m_{\tilde{e}_R}^2 &= m_0^2 + m_W^2 \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} (\tan^2 \theta_W) \\ &+ m_{1/2}^2 \left[\frac{12}{5} \frac{1}{2b_1} \left(1 - \frac{1}{F_1^2} \right) \right], \quad (2.34) \end{aligned}$$

and the $\tilde{\mu}$ masses-squared differ only in $O(m_\mu^2)$, which can be neglected. The corresponding $\tilde{\tau}$ masses-squared are obtained by solving eqs.(2.21-22), and are very close to the results of eqs.(2.33-34) over most of the allowed parameter space. The masses-squared of the squarks are given by formulae analogous to (2.33-34) that we do not present, except for the stop squark $(\tilde{t}_L, \tilde{t}_R)$ mass-squared matrix, in which there can be significant mixing

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2} \left[(m_{LL}^2 + m_{RR}^2) \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4m_{LR}^4} \right], \quad (2.35)$$

¹In the present work, we use the values $\alpha_U \equiv g_U^2/4\pi = 0.04$ and $M_U = 1.335 \times 10^{16}$ GeV; some of the uncertainties connected with the dependence of these values on $\alpha_3(m_W)$, $\alpha_e m(m_W)$ and the supersymmetric particle thresholds have been discussed in ref.[5].

where

$$\begin{aligned} m_{LL}^2 &= m_Q^2 + m_t^2 - \frac{g^2}{4}(v_2^2 - v_1^2) + \frac{g^2}{12}(v_2^2 - v_1^2), \\ m_{RR}^2 &= m_{U^c}^2 + m_t^2 - \frac{g^2}{3}(v_2^2 - v_1^2), \quad (2.36) \\ m_{LR}^2 &= h_t A_t v_2 + h_t \mu v_1, \end{aligned}$$

and m_Q, m_{U^c}, m_t can be obtained from eqs.(2.18), (2.19) and (2.24), respectively. The lighter state $\tilde{t}_1$ is lighter than the other squarks and could in principle be lighter than the t quark itself, a possibility we study in section 3. We conclude this review of basic formulae by reminding the reader of the chargino mass-squared matrix

$$M_C^2 = \begin{pmatrix} M_2^2 + g_2^2 v_1^2 & -M_2 g_2 v_2 + \mu g_2 v_1 \\ -M_2 g_2 v_2 + \mu g_2 v_1 & \mu^2 + g_2^2 v_2^2 \end{pmatrix}, \quad (2.37)$$

and of the neutralino mass matrix

$$M_{\tilde{N}} = \begin{pmatrix} M_2 & 0 & -\frac{g_2 v_2}{\sqrt{2}} & \frac{g_2 v_1}{\sqrt{2}} \\ 0 & M_1 & \frac{g' v_2}{\sqrt{2}} & -\frac{g' v_1}{\sqrt{2}} \\ -\frac{g_2 v_2}{\sqrt{2}} & \frac{g' v_2}{\sqrt{2}} & 0 & \mu \\ \frac{g_2 v_1}{\sqrt{2}} & -\frac{g' v_1}{\sqrt{2}} & \mu & 0 \end{pmatrix}, \quad (2.38)$$

where $M_2 = (g_2^2/g_U^2)m_{1/2}$ and $M_1 = (g_1^2/g_U^2)m_{1/2}$.

2.2 Theoretical constraints

When solving the renormalization group equations for the soft supersymmetry breaking parameters, one must make some assumptions about their values at the scale M_U , where the running gauge coupling constants unify. If the super-Higgs mechanism which describes spontaneous supersymmetry breaking takes place in a ‘hidden’ sector of the theory, which interacts with the ‘observable’ sector only gravitationally, it is natural to assume that

$$M_3(M_U) = M_2(M_U) = M_1(M_U) = m_{1/2}, \quad (2.39)$$

$$m_{H_1}^2(M_U) = m_{H_2}^2(M_U) = m_L^2(M_U) = m_{E_c}^2(M_U) \quad (2.40)$$

$$= m_Q^2(M_U) = m_{U^c}^2(M_U) = m_{D^c}^2(M_U) = m_0^2,$$

$$A_t(M_U) = A_b(M_U) = A_\tau(M_U) = A_0, \quad B(M_U) = B_0. \quad (2.41)$$

More restrictive assumptions would require additional information about the mechanism for supersymmetry breaking in the underlying fundamental theory. Various authors have argued, on different grounds [3], that the dominant source of supersymmetry breaking at the scale M_U could be a universal gaugino mass $m_{1/2}$, but their arguments are model-dependent and do not exclude other possibilities. On the other hand, a general treatment of the soft supersymmetry-breaking terms, including the four independent parameters $m_{1/2}$, m_0 , A_0 and B_0 , would be very complicated, and will not be attempted here. If one wants to assume, for simplicity, that one of the four terms in eqs.(2.39-41) is dominant, the only possibility is $m_{1/2}$, since the remaining ones are immediately ruled out by theoretical and/or phenomenological considerations (lower experimental limit on the gluino mass, absence of charge- or colour-breaking minima, etc.). We therefore assume that

$$m_{1/2}^2 \gg m_0^2, A_0^2, B_0^2 \simeq 0 \quad (2.42)$$

in our subsequent analysis.

Apart from the assumption of eq.(2.42), the main theoretical constraint we are going to impose on the input parameters is the requirement of a correct breaking of the electroweak symmetry, with v.e.v.'s v_1, v_2 corresponding to the measured values of m_W and m_Z via eq.(2.23). The conventional approach to radiative $SU(2)_L \times U(1)_Y$ symmetry breaking makes use of the renormalization group improved tree-level potential of eq.(2.3), with running masses and coupling constants evaluated at a renormalization scale $Q_0 \sim m_W$. However, it has recently been shown [5] that the results obtained in this way are very sensitive to the choice of the renormalization scale Q_0 , and only for a small and model-dependent range of Q_0 does the potential of eq.(2.3) give reliable results, whereas the results obtained from a complete one-loop calculation are remarkably stable for a wide range of Q_0 around

the electroweak scale. Therefore, for each given set of boundary conditions, we shall perform the determination of the v.e.v.'s v_1, v_2 making use of the full one-loop effective potential

$$V_1 = V_0 + \Delta V_1, \quad (2.43)$$

where

$$\Delta V_1 = \frac{1}{64\pi^2} \text{Str} \left[\mathcal{M}^4 \left(\log \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \right] \quad (2.44)$$

depends on v_1, v_2 through the tree-level squared-mass matrix $\mathcal{M}^2$. A detailed discussion of the structure of the one-loop effective potential in the MSSM and of its minimization can be found in ref.[5].

The motivation for supersymmetry based on the hierarchy problem and naturalness suggests that all sparticles weigh less than $O(1)$ TeV. This is supported by specific calculations [7], indicating that severe fine-tuning of the input parameters is needed to accommodate large sparticle masses. Moreover, a superheavy sparticle spectrum would not be of immediate phenomenological relevance. Motivated by these considerations, in our analysis we will focus on the points in the parameter space corresponding to

$$m_{\tilde{g}} < 500 \text{ GeV}, \quad (2.45)$$

although the parameter space could be slightly expanded by relaxing this constraint. Notice, however, that the constraint of eq.(2.45) becomes unnecessary if one imposes the cosmological constraints discussed in the following subsection.

In the previous studies of radiative gauge symmetry breaking, which were using the renormalization-group-improved tree-level potential of eq.(2.3) at some conventional renormalization scale Q_0 , there were two conditions on the running mass parameters of theory that had to be satisfied in order to obtain the desired vacuum with $v_1, v_2 \neq 0$:

1. The tree-level 'breaking condition', requiring a negative eigenvalue for the neutral Higgs mass-squared matrix at the origin,

$$m_1^2 m_2^2 - m_3^4 < 0. \quad (2.46)$$

2. The tree-level ‘stability condition’, requiring the potential of eq.(2.3) to be bounded from below,

$$m_1^2 + m_2^2 + 2m_3^2 \geq 0. \quad (2.47)$$

As explained in detail in ref.[5], when using the full one-loop effective potential of eqs.(2.43-44), we do not need to (more precisely, we *do not have to*) worry about the conditions of eqs.(2.46-47).

Similarly, the conditions that were traditionally imposed [6] to avoid the appearance of unwanted minima (corresponding to the breaking of lepton number, colour or electric charge) have been also recently re-analyzed in ref.[5], taking into account the structure of the full one-loop effective potential, and found to be in general much weaker than previously thought. We have checked explicitly that none of these conditions, when properly applied, puts any further constraints on our parameter space, so we shall omit their detailed discussion here.

2.3 Phenomenological constraints

We now list the constraints that experiment imposes on the MSSM. Some of these are automatically respected once the above theoretical constraints are imposed, whilst others do restrict the allowed parameter space.

- The Tristram e^+e^- ring tells us [8,9] that

$$m_t > 28 \text{ GeV}. \quad (2.48)$$

This constraint is quite model-independent and has to be strictly enforced. There are also more stringent bounds on the top quark mass, coming from the experiments at the CERN and FNAL $p\bar{p}$ colliders, but they are not necessarily valid in the MSSM, so we shall discuss them later. In any case, as we will see, all these bounds are automatically respected in the version of the MSSM presently under discussion².

²Analyses of $B_d^0 - \bar{B}_d^0$ mixing also suggest [19] that $m_t > 50 \text{ GeV}$ in the Standard Model, but this bound is not necessarily valid in a supersymmetric model which has a richer spectrum.

- Analyses of the world’s neutral current data indicate [10] that

$$m_t < 200 \text{ GeV}. \quad (2.49)$$

In supersymmetric models, this bound can become slightly more stringent [20], as a result of the sparticle contributions to the one loop corrections to the effective rho parameter. Even taking this into account, the bound is also automatically respected in the version of the MSSM presently under discussion.

- Published CDF data [11] establish the bounds

$$m_{\tilde{g}} > 73 \text{ GeV} \quad (2.50)$$

if $m_{\tilde{q}} \gg m_{\tilde{g}}$ and

$$m_{\tilde{q}} > 74 \text{ GeV} \quad (2.51)$$

if $m_{\tilde{g}} \gg m_{\tilde{q}}$. These bounds are strengthened to $\sim 105 \text{ GeV}$ if $m_{\tilde{q}} \sim m_{\tilde{g}}$, which is closer to the situation expected in the MSSM if $m_{1/2}$ is the only seed of supersymmetry breaking, as we have assumed. In deriving the above bounds, several assumptions have been made by the CDF experimentalists: a) there are five degenerate left- and right-handed squark flavours; b) the lightest supersymmetric particle is a neutralino $\tilde{N}_1$ with dominant photino component and a negligible mass; c) the dominant decay modes are $\tilde{g} \rightarrow q\bar{q}\tilde{N}_1$ for $m_{\tilde{q}} > m_{\tilde{g}}$ and $\tilde{q} \rightarrow q\tilde{N}_1$ for $m_{\tilde{g}} > m_{\tilde{q}}$. A more complete treatment [21] would somewhat decrease the published CDF bounds for generic values of the MSSM parameters. However, we have simply required eqs.(2.50) and (2.51), since we will find that always $m_{\tilde{g}} > 160 \text{ GeV}$, $m_{\tilde{q}} > 140 \text{ GeV}$, with the possible exception of the lighter stop quark $\tilde{t}_1$, which obeys the weaker limit $m_{\tilde{t}_1} > 100 \text{ GeV}$.

- Experiments at Tristram tell us [9] that

$$m_{\tilde{e}_L}, m_{\tilde{e}_R} > 25 \text{ GeV}, \quad (2.52)$$

for almost all masses of the lightest neutralino $\tilde{N}_1$ which are less than $m_{\tilde{e}}$ and irrespectively of its composition. To investigate the

discovery potential of SLC and LEP, we shall also examine in the following the effects of imposing the more stringent bounds

$$m_{\tilde{e}_L}, m_{\tilde{e}_R} > 45 \text{ GeV}. \quad (2.53)$$

- An unsuccessful search by the UA1 Collaboration [12] for $W \rightarrow \tilde{e}_L \tilde{\nu}$ imposes a constraint in the $(m_{\tilde{e}_L}, m_{\tilde{\nu}})$ plane, that we approximate by

$$\left(\frac{m_{\tilde{e}_L}}{33 \text{ GeV}} \right)^2 + \left(\frac{m_{\tilde{\nu}}}{40 \text{ GeV}} \right)^2 > 1. \quad (2.54)$$

In principle, the agreement of the UA1 and UA2 data for $R \equiv \sigma \cdot B(W \rightarrow e\nu)/\sigma \cdot B(Z \rightarrow e^+e^-)$ with the Standard Model with three neutrino species could be used to constrain $Z \rightarrow$ unobserved neutrals such as $\tilde{\nu}\tilde{\nu}$. However, since $W \rightarrow \tilde{e}\tilde{\nu}$, $Z \rightarrow \tilde{e}\tilde{e}$, $W \rightarrow \tilde{C}\tilde{N}$, $Z \rightarrow \tilde{C}\tilde{C}$ and $\tilde{N}\tilde{N}$ decays could be present and complicate the analysis, and there are unresolved ambiguities in our knowledge of the relevant structure functions, we have not imposed such a constraint.

- Unsuccessful searches at Tristram [9] tell us that

$$m_{\tilde{C}_1} > 26 \text{ GeV}, \quad (2.55)$$

for almost all masses and compositions of a lighter neutralino $\tilde{N}_1$. Searches at the CERN $p\bar{p}$ Collider, for example by UA1 for heavy leptons [22], might impose a tighter constraint on $m_{\tilde{C}_1}$, but this bound can be very model-dependent [23] and requires further analysis. In any case, SLC and LEP should soon be able to discover charginos or to establish the more stringent bound

$$m_{\tilde{C}_1} > 45 \text{ GeV}, \quad (2.56)$$

whose consequences will also be explored in the following.

- Unsuccessful searches for $e^+e^- \rightarrow \gamma +$ missing neutrals, in particular by the ASP experiment [13], impose a constraint in the $(m_{\tilde{e}}, m_{\tilde{N}_1})$ plane. A detailed analysis of this constraint has been

recently given, for a general composition of the lightest neutralino, by Griest [17], whose treatment we have followed. The same experiments impose a constraint in the $(m_{\tilde{\chi}^\pm}, m_{\tilde{\nu}})$ plane that is not interesting for sneutrinos as heavy as we find in the present version of the MSSM (see section 3).

- The lightest supersymmetric particle (LSP) is stable in the MSSM, and hence should be present in the Universe today as a cosmological relic. Motivated by astrophysical observations and by inflation, when $\tilde{N}_1$ is the LSP, as holds in most of our parameter space, we impose

$$\Omega_{\tilde{N}_1} < 1, \quad (2.57)$$

where $\Omega_{\tilde{N}_1} \equiv \rho_{\tilde{N}_1}/\rho_c$; ρ_c is the closure density, which is approximately $\rho_c = 1.8 \times 10^{-29} \text{ g cm}^{-3} \cdot h^2$, where the present Hubble expansion rate H is parametrized by $h \equiv H/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Standard formulae are used to relate $\rho_{\tilde{N}_1}$ to $\sigma(\tilde{N}_1\tilde{N}_1 \rightarrow f\bar{f})$, which we do not reproduce here [16]. A more sophisticated analysis can be found in ref.[17]. Taking the usually quoted observational range $\frac{1}{2} < h < 1$, we get from eq.(2.57) the upper bound

$$\rho_{\tilde{N}_1} < 1.8 \times 10^{-29} \text{ g cm}^{-3}. \quad (2.58)$$

If one believes that the supersymmetric relic is the dominant matter density in the Universe, and that the total density is critical (inflation?): $\Omega_{\tilde{N}_1} = 1$, then the observational lower bound $h > \frac{1}{2}$ leads to

$$\rho_{\tilde{N}_1} > 0.45 \times 10^{-29} \text{ g cm}^{-3}. \quad (2.59)$$

We will not consider this as an inviolable constraint, but will mention its implications in Section 3. In imposing the constraints of eqs.(2.58) and (2.59), we take the most conservative values for the quark-hadron phase transition temperature T_H : $T_H = 150 \text{ MeV}$ when applying eq.(2.58), $T_H = 400 \text{ MeV}$ when applying eq.(2.59).

- In the case that $m_{\tilde{\nu}} < m_{\tilde{N}_1}$, there are arguments based on the absence of high-energy solar neutrinos [14] that

$$m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu} < 3 \text{ GeV}, \quad (2.60)$$

- In the case that $m_{\tilde{\nu}} < m_{\tilde{N}_1}$, there are arguments based on the absence of high-energy solar neutrinos [14] that

$$m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu} < 3 \text{ GeV}, \quad (2.60)$$

if $\tilde{\nu}_e$ or $\tilde{\nu}_\mu$ is the lightest sneutrino, and arguments based on the absence of coherent elastic scattering off ^{76}Ge nuclei [15] that

$$m_{\tilde{\nu}} < 12 \text{ GeV} \quad \text{or} \quad m_{\tilde{\nu}} > 1.4 \text{ TeV}, \quad (2.61)$$

whichever is the lightest sneutrino. As we will see in section 3, these constraints kill an otherwise acceptable corner of parameter space in which $m_{\tilde{\nu}} < m_{\tilde{N}_1}$.

3 Consistent solutions and particle masses

We now look for values of the MSSM parameters that are consistent with all the above constraints. Given our theoretical assumptions, the parameter space to be explored is 3-dimensional $(h_U, m_{1/2}, \mu_U)$, and it is not restrictive to assume positive values for h_U , $m_{1/2}$ and μ_U . It is then convenient to work with two dimensionless parameters $(h_U, \hat{m} \equiv m_{1/2}/\mu_U)$ and $m_{1/2}$. For each point in the $(h_U, \hat{m})$ -space, we look for values of $m_{1/2}$ such that the corresponding minimum (v_1, v_2) of the one-loop effective potential fits the experimental value $m_W \simeq 81 \text{ GeV}$. The Yukawa couplings $h_b(M_U)$ and $h_\tau(M_U)$ are also adjusted, through an iterative procedure, in order to reproduce the correct values of m_b and m_τ . We then impose on any given solution all the remaining theoretical and phenomenological constraints discussed in the previous section. In order to determine the physical masses for the gauge bosons and the third generation fermions, we include threshold effects in the simplified way discussed in ref.[5]. For all the Higgs and supersymmetric particle masses, instead, we just stick to the tree-level expressions, but we evaluate all parameters appearing in those expressions at the scale $\hat{Q}$, where the one-loop field-dependent correction ΔV_1 to the renormalization-group-improved tree-level potential $V_0(Q)$ is minimized. An empirical expression for $\hat{Q}$, valid for our particular set of boundary conditions, is $\hat{Q} \simeq 1.8 m_{1/2} + 20 \text{ GeV}$.

Fig. 1 shows the allowed region in the $(h_U, \hat{m})$ -plane. It is useful to describe the constraints that become violated if one stays outside the allowed region. Points in the excluded region denoted by ‘A’, at the bottom and at the left of fig.1, are such that for all values of $m_{1/2}$ the v.e.v.’s v_1 and v_2 correspond to $m_W \ll 81 \text{ GeV}$. In the conventional approach, choosing appropriately the renormalization scale $Q_0 = \hat{Q}$, one would find that for most of these points eq.(2.46) is not satisfied. For points in the excluded region denoted by ‘B’, at the top right of fig.1, one finds that, for values of $m_{1/2}$ such that the condition of eq.(2.45) is satisfied, the corresponding v.e.v.’s v_1 and v_2 generate too large a value for m_W . In the conventional approach one would find that, stopping the renormalization group evolution at $Q_0 = \hat{Q}$, most of these points are ruled out by the stability constraint of eq.(2.47). Notice that up to now we have not imposed any phenomenological constraint, only theoretical ones. The only phenomenological particle physics constraint which imposes significant restrictions on the parameter space is the experimental limit on the lightest chargino $\tilde{C}_1$, eq.(2.55), which rules out the region denoted by ‘C’, at the top left of fig.1. The other excluded region at the top of fig.1, denoted by ‘D’, is ruled out by the cosmological upper limit on the relic LSP density, eq.(2.58). There is also a small set of points, corresponding to a sneutrino being the LSP, which are ruled out by the constraints of eqs.(2.60) and (2.61). If the LSP is to account for the dark matter density, then we have to impose also the cosmological limit of eq.(2.59): this constraint has the effect of ruling out the region of parameter space below the dashed line in fig.1, corresponding to relatively heavy sparticle spectra. This fact is made clear if one considers the dotted line, corresponding to the limits of eqs.(2.53) and (2.56), to be soon achieved at SLC or LEP I. One can see that only a very small region of parameter space can survive simultaneously SLC/LEP I and the cosmological tests! We stress once again that, once the chargino and the cosmological constraints are satisfied, all the remaining particle physics constraints are automatically satisfied.

When exploring the masses of particles, for parameter choices within the allowed region of fig.1, we start by plotting their allowed ranges

versus the gluino mass $m_{\tilde{g}}$, which sets the scale of supersymmetry breaking in the observable sector. We see in fig.2 that the top quark mass is bounded from below,

$$m_t > 76 \text{ GeV}. \quad (3.1)$$

In the framework of the present model, we regard this as a very sound limit, since for smaller top quark masses the electroweak gauge symmetry cannot be broken in an acceptable way. The dependence of this limit on the values of $\alpha_3(m_W)$ and $\alpha_e m_c(m_W)$, which determine in turn α_U and M_U , has been studied in ref.[5]: the corresponding uncertainty in the m_t determination has been estimated to be ± 3 GeV. We should also stress that, if we had used the conventional approach and, for example, $Q_0 = m_W$, we would have (incorrectly) obtained a significantly lower limit, $m_t > 65$ GeV. From fig.2 we can also see that there is a definite lower limit on the gluino mass,

$$m_{\tilde{g}} > 160 \text{ GeV}. \quad (3.2)$$

The dots inside the allowed region of fig.2 correspond to a set of Monte Carlo points, with uniform probability density in the $(h_U, \hat{m})$ -plane, which survive also the cosmological constraint of eq.(2.59). Imposing further the SLC/LEP I perspective bounds, eqs.(2.53) and (2.56), would leave only a very small subset of allowed points. We observe that, if $\tilde{N}_1$ is to solve the dark matter problem, one obtains effective upper limits for the t-quark and gluino masses,

$$m_t < 90 \text{ GeV} \quad (3.3)$$

and

$$m_{\tilde{g}} < 400 \text{ GeV}. \quad (3.4)$$

We do not regard the upper limits of eq.(3.3,3.4) as absolutely inviolable, for two reasons. First, it could be that $\Omega_{\tilde{N}_1} < 1$, and that the dark matter problem is not solved by the LSP. Second, there is an extremely small set of points in parameter space, squeezed between the dashed line and the upper solid line at the right of fig.1, which can

give $m_t > 90$ GeV and survive the cosmological constraint of eq.(2.59). This set has a negligible measure in the $(h_U, \hat{m})$ -plane, and therefore could not be explored by our Monte Carlo search. Given the above caveats, we can now comment on the restricted range of allowed top masses we find, which is directly related to the small region in h_U spanned by the allowed points in fig.1 which sit above the dashed line. There is a published lower limit $m_t > 44$ GeV (at 95% c.l.) from a UA1 search for semileptonic $t \rightarrow b l \nu$ decay at the CERN $p\bar{p}$ Collider [24]. In principle, there could be exotic t decays such as $t \rightarrow b H^+$ or $t \rightarrow \tilde{t}_1 \tilde{N}_1$ that suppress semileptonic decays and invalidate this bound [25]. However, neither of these occur in our allowed parameter region, and hence the UA1 bound must be respected. Unpublished reports from UA1, UA2 and CDF [26], also looking for $t \rightarrow b l \nu$ decays, indicate that $m_t \geq (60 - 70)$ GeV, which is just what we expect from eq.(3.1). We offer them the prospect of a t-quark discovery 'just around the corner'.

The masses of sleptons and squarks are strongly correlated with that of the gluino. They would be strictly proportional if the D-term contributions to the masses could be neglected as well as the F-terms, and if the threshold renormalization scale $\hat{Q}$ were fixed. In practice, the slepton and squark masses do not depend strongly on these variations. The lightest slepton is the $\tilde{e}_R$ (expected to be almost degenerate with the $\tilde{\mu}_R, \tilde{\tau}_R$), and we see from the lower set of points in fig.3 that

$$m_{\tilde{e}_R} > 40 \text{ GeV}. \quad (3.5)$$

On the other hand, if we are willing to accept the cosmological constraint of eq.(2.59), it must also be that

$$m_{\tilde{e}_R} < 75 \text{ GeV}, \quad (3.6)$$

as we can read off figs.2 and 3. LEP and SLC should soon be able to explore $m_{\tilde{e}_R} < 45$ GeV, and the full range in eqs.(3.5) and (3.6) can be explored at LEP II: a guaranteed discovery within our theoretical framework! We see from the upper set of points in fig.3 that the $\tilde{e}_L$ (and $\tilde{\mu}_L, \tilde{\tau}_L$) is expected to be somewhat heavier

$$55 \text{ GeV} < m_{\tilde{e}_L} < 120 \text{ GeV}, \quad (3.7)$$

where once again the upper limit can be obtained combining figs.2 and 3 and holds if the cosmological constraint of eq.(2.59) has to be satisfied. Therefore, we do not expect the $\tilde{e}_L$ to be discovered at LEP I or the SLC, but it is likely to be seen at LEP II, although there is a possibility that it might require a higher energy e^+e^- collider.

The upper set of points in fig.4 shows the strong correlation between a typical squark mass, namely $m_{\tilde{u}_L}$, and the gluino mass $m_{\tilde{g}}$. We find

$$140 \text{ GeV} < m_{\tilde{u}_L} < 350 \text{ GeV} \quad (3.8)$$

for the range of gluino masses of eqs.(3.2) and (3.4). These results are not encouraging for LEP or the SLC, nor are they particularly encouraging for present hadron colliders, since only the FNAL collider can reach even the lower end of the range in eq.(3.8). Finding squark and gluinos may be a job for higher-energy hadron colliders such as the LHC or the SSC. As discussed in section 2, the lighter stop squark $\tilde{t}_1$ is expected to be lighter than the other squarks, and we see from the lower set of points in fig.4 that

$$100 \text{ GeV} < m_{\tilde{t}_1} < 300 \text{ GeV}. \quad (3.9)$$

However, it is not light enough for $t \rightarrow \tilde{t}_1 \tilde{N}_1$ decay to be kinematically possible, and is still beyond the reach of LEP.

We next turn to neutral Higgs bosons. We see in fig.5a that

$$13 \text{ GeV} < m_H < 72 \text{ GeV}, \quad (3.10)$$

in agreement with the general theorem that $m_H < m_Z$. The range in eq.(3.10) puts the lightest scalar Higgs beyond the reach of $Y \rightarrow H \gamma$ decays, but indicates that it lies in the mass range accessible to LEP ($\leq 50 \text{ GeV}$ at LEP I, $\leq 80 \text{ GeV}$ at LEP II). Another relevant parameter for the Higgs sector is $\tan \beta$, whose allowed range of variation is plotted against the gluino mass $m_{\tilde{g}}$ in fig.5b: the fact that always $\tan \beta > 1.5$ explains the lower limit on m_H in eq.(3.10). Finally, Fig.5c shows that the CP-odd neutral Higgs P might also be light enough to be detected at LEP I via $Z \rightarrow P \gamma$ or $Z \rightarrow P H$. (LEP II does not have much chance of finding it, since $e^+e^- \rightarrow Z^{0*} \nrightarrow Z^0 P$.)

The mass of the lightest neutralino $\tilde{N}_1$ is plotted versus $m_{\tilde{g}}$ in fig.6a, where we see that

$$4 \text{ GeV} < m_{\tilde{N}_1} < 20 \text{ GeV}, \quad (3.11)$$

with the upper limit holding as usual only if the constraint of eq.(2.59) is imposed. The mass of the neutralino should be compared with that of the sneutrino, which is seen in fig.6b. In our search we find that there would be a small region below the diagonal solid line where $m_{\tilde{\nu}} < m_{\tilde{N}_1}$ and

$$15 \text{ GeV} < m_{\tilde{\nu}} < 45 \text{ GeV}, \quad (3.12)$$

but this lies in the range [eq.(2.60)] excluded by the absence of high-energy solar neutrinos from $\tilde{\nu}_e \tilde{\nu}_e$ or $\tilde{\nu}_\mu \tilde{\nu}_\mu$ annihilation in the Sun, and [eq.(2.61)] by the absence of coherent $\tilde{\nu} {}^{76}\text{Ge}$ scattering. The range of the second neutralino mass $m_{\tilde{N}_2}$ is plotted against $m_{\tilde{N}_1}$ in fig.6c. We see that there is quite a range of parameter space in which $m_Z > m_{\tilde{N}_1} + m_{\tilde{N}_2}$, so that $Z \rightarrow \tilde{N}_1 \tilde{N}_2$ decays are kinematically possible, but we have not investigated their rate and possible $Z \rightarrow \gamma + \text{missing neutral energy}$ signature. We study the observability of $Z \rightarrow \tilde{N}_1 \tilde{N}_1$ decay in section 4. Fig.6d shows the allowed range of the lightest chargino mass $M_{\tilde{C}_1}$ as a function of $m_{\tilde{N}_1}$. We see that

$$26 \text{ GeV} < m_{\tilde{C}_1} < 65 \text{ GeV}, \quad (3.13)$$

which would make detection at LEP II certain. The lower end of the range in eq.(3.13) ($m_{\tilde{C}_1} < 40 \text{ GeV}$) could perhaps be explored via $W \rightarrow \tilde{C}_1 \tilde{N}_1$ and $Z \rightarrow \tilde{C}_1 \tilde{C}_1$ decays at present $p\bar{p}$ colliders [23], but will certainly soon be checked in detail by the SLC and LEP I. Also the above upper limits on $m_{\tilde{N}_1}$ and $m_{\tilde{C}_1}$ are a consequence of the plausible cosmological lower bound [eq.(2.59)] on the relic density. Notice that eq.(3.11), giving the range of variation of the LSP mass, places the supersymmetric relic in the mass range where laboratory detection is feasible³.

³We plan to return to this issue in a future publication.

4 Particle couplings and signatures

The observability of the Higgs particles and neutralinos, whose masses we discussed in the previous section, depends crucially on their couplings, which are in turn determined by mixing angles. For generic values of the MSSM parameters, the resulting phenomenology has been discussed in great detail in refs.[27] and [28], respectively. Here we simply outline the effects of our theoretical and phenomenological constraints.

The lightest neutral Higgs H can be produced in Z decay: $Z \rightarrow H + (Z^* \rightarrow l^+ l^-, \bar{\nu} \nu \text{ or } \bar{q} q)$, or in association with the Z : $e^+ e^- \rightarrow Z^* \rightarrow H + Z$. Both processes are controlled by the ZZH coupling

$$g_{ZZH} = (g_{ZZH})_{S.M.} \cdot \sin(\beta - \alpha), \quad (4.1)$$

where $(g_{ZZH})_{S.M.}$ is the corresponding coupling for the Standard Model Higgs, and α ($-\pi/2 \leq \alpha \leq 0$ in the notation of ref.[27]) is the mixing angle in the CP-even neutral Higgs sector, whose mass matrix has been given in eq.(2.29). We therefore have

$$\frac{\text{BR}(Z \rightarrow H + (Z^* \rightarrow l^+ l^-, \bar{\nu} \nu \text{ or } \bar{q} q))}{\text{BR}(Z \rightarrow H + (Z^* \rightarrow l^+ l^-, \bar{\nu} \nu \text{ or } \bar{q} q))_{S.M.}} = \sin^2(\beta - \alpha) \quad (4.2)$$

and

$$\frac{\sigma(e^+ e^- \rightarrow Z^* \rightarrow H + Z)}{\sigma(e^+ e^- \rightarrow Z^* \rightarrow H + Z)_{S.M.}} = \sin^2(\beta - \alpha). \quad (4.3)$$

Then H decays mainly into fermion-antifermion pairs $f\bar{f}$, and the corresponding decay rates are modified, with respect to the Standard Model ones, by a factor $\cos^2 \alpha / \sin^2 \beta$ ($f = u, c, t$) and $\sin^2 \alpha / \cos^2 \beta$ ($f = e, \mu, \tau, d, s, b$), respectively. A Standard Model Higgs boson could be found at LEP I, in $\sim 10^6$ Z decays, if $m_H \leq 50$ GeV, and at LEP II if $m_H < 80$ GeV. However, we see from fig.7a that

$$0.25 < \sin^2(\beta - \alpha) < 0.9, \quad (4.4)$$

so we expect that a smaller range of m_H can be explored by the above processes in our version of the MSSM, since in some region of the allowed parameter space the event rate may be too low to be detectable.

The CP-odd neutral Higgs P cannot be produced by either of the above two processes, because there is no ZZP coupling. However, there is a ZPH coupling that is proportional to a complementary mixing angle factor $\cos(\beta - \alpha)$, giving

$$\frac{\Gamma(Z \rightarrow H + P)}{\Gamma(Z \rightarrow \nu \bar{\nu})} = \frac{1}{2} \cos^2(\beta - \alpha) \phi\left(\frac{m_H^2}{m_Z^2}, \frac{m_P^2}{m_Z^2}\right), \quad (4.5)$$

where $\phi(x, y) = [(1-x-y)^2 - 4xy]^{3/2}$ is the standard phase space factor. Fig.7b shows the range of values of $\sin^2(\alpha - \beta)$ allowed for different values of m_P (notice that $m_P \geq m_H$ in the MSSM, as is apparent from fig.5c). Combining fig.5c and fig.7b, we see that there is a region of parameter space for which $m_H + m_P \ll m_Z$ and $\cos^2(\beta - \alpha) \geq 1/3$, in which the process $Z \rightarrow H + P$ should be observable.

Another new particle whose observability depends sensitively on mixing angles is the lightest neutralino $\tilde{N}_1$. We see from eq.(2.38) that it is in general a non-trivial combination of the neutral wino $\tilde{W}_3$, the $U(1)$ gaugino $\tilde{B}$, and the neutral Higgsinos $\tilde{H}_{1,2}^0$:

$$\tilde{N}_1 = \alpha \tilde{W}_3 + \beta \tilde{B} + \gamma \tilde{H}_2^0 + \delta \tilde{H}_1^0. \quad (4.6)$$

Figs.8a,b,c,d show the fractions $\alpha^2, \beta^2, \gamma^2, \delta^2$ of these sparticle components in the physical mass eigenstate $\tilde{N}_1$ for different values of $m_{\tilde{N}_1}$. We see from fig.8a that the $\tilde{W}_3$ component α^2 is always relatively small, whilst fig.8b shows that the $\tilde{B}$ component β^2 can become large for $m_{\tilde{N}_1} \geq 30$ GeV. As can be seen in figs.8c,d, the $\tilde{N}_1$ is mainly a Higgsino mixture, $\gamma^2 + \delta^2 \geq \frac{1}{2}$, for $m_{\tilde{N}_1} \leq 30$ GeV and retains a substantial Higgsino component even if it is heavier. This opens up the prospect of $Z \rightarrow \tilde{N}_1 \tilde{N}_1$ decay, which has a rate

$$R_{\tilde{N}_1} \equiv \frac{\Gamma(Z \rightarrow \tilde{N}_1 \tilde{N}_1)}{\Gamma(Z \rightarrow \nu \bar{\nu})} = (\gamma^2 + \delta^2) \left(1 - \frac{4m_{\tilde{N}_1}^2}{m_Z^2}\right)^{3/2}. \quad (4.7)$$

The possible range of this ratio as a function of $m_{\tilde{N}_1}$ is shown in fig.9a. We see that, for our most conservative set of constraints,

$$R_{\tilde{N}_1} < 0.7, \quad (4.8)$$

with larger values of $R_{\tilde{N}_1}$ being favoured for small values of $m_{\tilde{N}_1}$. Neutrino-counting experiments at LEP hope to reach the precision of $\Delta N_\nu = \pm 0.1$ in counting the apparent number of neutrino species. Since $Z \rightarrow \tilde{N}_1 \tilde{N}_1$ decays are indistinguishable from $Z \rightarrow \bar{\nu} \nu$ decays, this precision should enable to measure $R_{\tilde{N}_1}$ if it is larger than ~ 0.1 . Looking back at fig.9a, we recall that if the $\tilde{N}_1$ is to close the Universe, then $m_{\tilde{N}_1} < 20$ GeV, as from eq.(3.11). Fig.9a tells us that in this case $R_{\tilde{N}_1} > 0.1$, so that there is a good chance of being able to detect $Z \rightarrow \tilde{N}_1 \tilde{N}_1$ decay at LEP. In certain regions of parameter space, $Z \rightarrow \tilde{\nu} \tilde{\nu}$ decays are kinematically allowed, and we expect the invisible $\tilde{\nu} \rightarrow \nu \tilde{N}_1$ to dominate, so that this process would also contribute to $Z \rightarrow \text{invisible neutrals}$. Therefore in fig.9b we plot⁴ $R_{\text{inv}} \equiv [\Gamma(Z \rightarrow \tilde{N}_1 \tilde{N}_1) + \sum_{i=1}^3 \Gamma(Z \rightarrow \tilde{\nu}_i \tilde{\nu}_i)] / \Gamma(Z \rightarrow \bar{\nu} \nu)$ versus $m_{\tilde{N}_1}$. Certainly no-one 'counting neutrinos' should assume that the result must necessarily be an integer, and should not discard any fractional result, but instead rejoice!

We have no doubt that, in addition to the above processes, LEP I or II can detect the t quark, the $\tilde{e}_{L,R}$, $\tilde{\mu}_{L,R}$, $\tilde{\tau}_{L,R}$ and $\tilde{C}_1$ if they have masses < 90 GeV, which our analysis indicates is quite possible. Moreover, CDF at the FNAL $\bar{p}p$ collider can probably detect the t quark over all our allowed range, eqs.(3.1) and (3.3). We therefore conclude that forthcoming experiments may well find evidence for the 'minimal' version of the MSSM discussed in this paper, or, alternatively, rule out almost completely its presently allowed region of parameter space.

Acknowledgements

One of the authors (F.Z.) would like to thank M. R. Barnett, G. F. Giudice, J. S. Hagelin, and in particular G. Gamberini and G. Ridolfi, for useful discussions and suggestions.

⁴We do not exclude the possibility that, at least for some region of the allowed parameter space, $Z \rightarrow \tilde{\nu} \tilde{\nu}$ decay could be visible [29], but a detailed study of this point would require further analysis and will not be pursued here.

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Figure captions

Figure 1

The region of the $(h_U, \hat{m} \equiv m_{1/2}/\mu_U)$ -plane allowed by the theoretical and phenomenological constraints described in section 2. The excluded areas labelled 'A' and 'B' give too small and too large v.e.v.'s for the Higgs fields, respectively; area 'C' corresponds to $m_{\tilde{C}_1} < 26$ GeV, and area 'D' to $\Omega_{\tilde{N}_1} > 1$ for the conservative assumptions $h = 1$ and $T_H = 150$ MeV. The area below the dashed line corresponds to $\Omega_{\tilde{N}_1} < 1$ for the conservative assumptions $h = \frac{1}{2}$ and $T_H = 400$ MeV. The area above the dotted line would be excluded if SLC or LEP I were able to establish the lower limits of eqs.(2.53) and (2.56).

Figure 2

Allowed region in the plane $(m_{\tilde{g}}, m_t)$. The area below and to the left of the solid line is ruled out by our most conservative set of constraints. The dots inside the allowed region correspond to a set of Monte Carlo points, with uniform probability density over the $(h_U, \hat{m})$ -plane, which survive the cosmological constraint of eq.(2.59).

Figure 3

Values of $m_{\tilde{e}_R}$ and $m_{\tilde{e}_L}$, plotted versus the gluino mass $m_{\tilde{g}}$. They correspond to a set of Monte Carlo points, with uniform probability density over the $(h_U, \hat{m})$ -plane, which survive our most conservative set of constraints.

Figure 4

The same as before but for $m_{\tilde{u}_L}$ and $m_{\tilde{t}_1}$.

Figure 5

Allowed regions in the planes: (a) $(m_{\tilde{g}}, m_H)$; (b) $(m_{\tilde{g}}, \tan \beta)$; (c) (m_H, m_P) . In this and in all the following figures, the area within

the solid lines is permitted by our most conservative set of constraints. The dots are Monte Carlo points which survive also the cosmological constraint of eq.(2.59).

Figure 6

Allowed regions in the planes: (a) $(m_{\tilde{g}}, m_{\tilde{N}_1})$; (b) $(m_{\tilde{N}_1}, m_{\tilde{\nu}})$; (c) $(m_{\tilde{N}_1}, m_{\tilde{N}_2})$; (d) $(m_{\tilde{N}_1}, m_{\tilde{C}_1})$.

Figure 7

Allowed regions for some masses and mixing angles in the neutral Higgs sector: (a) $[m_H, \sin^2(\beta - \alpha)]$; (b) $[m_P, \sin^2(\beta - \alpha)]$.

Figure 8

Composition of the lightest neutralino $\tilde{N}_1$ versus its mass $m_{\tilde{N}_1}$. In the notation of eq.(4.6), (a), (b), (c) and (d) show α^2 , β^2 , γ^2 and δ^2 , respectively.

Figure 9

Allowed regions for the ratios: (a) $R_{\tilde{N}_1} \equiv \Gamma(Z \rightarrow \tilde{N}_1 \tilde{N}_1)/\Gamma(Z \rightarrow \bar{\nu}\nu)$; (b) $R_{inv} \equiv \Gamma(Z \rightarrow \text{invisible neutrals})/\Gamma(Z \rightarrow \bar{\nu}\nu)$; plotted versus the lightest neutralino mass $m_{\tilde{N}_1}$.

Fig.1

Fig.2

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Fig.4

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Fig.5

Fig.6

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Fig.6

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Fig.6

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