CERN-TH-5471/89

On the polarization of gluons in the proton

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ABSTRACT

We analyze the possibility that polarized gluons could make a substantial contribution to the value of $\int_0^1 dx g_1^p(x)$ estimated by the EMC. We show that with a physically motivated kernel for $\gamma^* g \rightarrow \bar{q}q$, where the q are light quarks, a very substantial part of the possible polarized gluon contribution to $g_1^p(x)$ is in the region $x < 0.01$ not measured by the EMC. This means that the polarized gluon distributions proposed so far are too small to explain the EMC data via their effects on light quarks alone. However, they do generate a significant charm contribution to $g_1^p(x)$ for $x > 0.01$ which could help explain the data. We present quantitative predictions for this charm cross-section asymmetry in future polarized lepton-nucleon scattering experiments.

CERN-TH-5471/89
July 1989

The recent EMC measurement of the polarized proton structure function $g_1^p(x)$ has generated much theoretical and experimental interest. When extrapolated to $x=0$ according to theoretical prejudice or to the trend of the data themselves, the EMC measurements at $x > 0.01$ indicate that^[1]

$$\int_0^1 dx g_1^p(x) = 0.126 \pm 0.010 \pm 0.015 \quad (1)$$

This is considerably below the naïve quark model (NQM) expectation^[2] $\int_0^1 dx g_1^p(x) = 0.19$, obtained by assuming that strange quarks and gluons in the proton do not contribute to the sum rule. Two main interpretational trends can be distinguished among theorists who take the result (1) at its face value. According to one possible interpretation,^[3] the discrepancy with the NQM is mainly due to polarized strange quarks in the proton. This interpretation is supported^[4] by the Skyrme model which is believed to describe correctly the physics in the chiral symmetry limit of light quarks, $m_q \rightarrow 0$, and a large number of colors, $N_c \rightarrow \infty$. The alternative interpretation is based on the observation^[5,6,7] that because of the $U(1)$ axial anomaly, there is an $\mathcal{O}(\alpha_s/\pi)$ polarized gluon contribution to the sum rule, which could be responsible^[8] for the discrepancy with the NQM. In this case, the net contribution of polarized gluons to the proton spin must be $\mathcal{O}(6)$ at $\langle Q^2 \rangle \simeq 10 \text{ GeV}^2$, with an almost equal and opposite contribution from orbital angular momentum: $L_z \simeq -\mathcal{O}(6)$.^[9] Of course, the true interpretation of the EMC data is likely to be some combination of these strange quark and gluonic contributions. In this paper we discuss a surprising phenomenological aspect of the purely gluonic interpretation, and a way to test it by measuring an asymmetry in charm production in forthcoming lepton-nucleon scattering experiments.^[10,11]

We argue that the gluonic contribution to $g_1^p(x)$ should not be taken as simply proportional to the polarized gluon distribution $\Delta G(x)$, but should rather be taken as a convolution of $\Delta G(x)$ with $\sigma(\gamma^* g \rightarrow \bar{q}q)$. This has the effect that the gluon contribution to $g_1^p(x)$ via light quarks q is shifted to low x . In particular, if we use parametrizations of $\Delta G(x)$ proposed^[9,12] to explain the EMC result, we find that less than 50% of the gluon contribution to the sum rule comes at $x > 0.01$ where the EMC took data. Moreover, the proposed $\Delta G(x)$ would give a contribution to $g_1^p(x)$ whose behaviour at small x disagreed with most expectations.^[13] The upper bound provided by the unpolarized gluon distribution at low x prevents $\Delta G(x)$ from being greatly increased at $x > 0.01$, and therefore makes it difficult, if not impossible, to explain the EMC result (1) with gluon polarization and light quarks alone.

However, gluon polarization would also give an asymmetry in the charm production cross-section with polarized beam and target. We present calculations of this effect using various proposed forms of $\Delta G(x)$ ^[6,12], in the x and Q^2 ranges accessible to the EMC^[1] and future experiments. We find an effect on $g_1^p(x)$ which could save the polarized gluon interpretation of the present data. The charm cross-section asymmetry could in principle be measured by the SMC^[10] and HERA^[11] experiments. We therefore urge that these experiments make every effort to identify charmed particles in the final state, so as to distinguish experimentally between the strange quark^[3] and gluon^[6,8] interpretations of the EMC result.

In the naive parton model, the polarized structure function $g_1^p(x)$ is given by

$$g_1^p(x) = \frac{1}{2} \sum_q Q_q^2 \Delta q(x) \quad (2)$$

where Q_q is the quark charge, $\Delta q(x) = q_1(x) + \bar{q}_1(x) - q_1(x) - \bar{q}_1(x)$ is the polarized quark distribution, and the sum is over the light quarks, $q = u, d, s$. When integrated over x , equation (2) becomes

$$\int_0^1 dx g_1^p(x) = \frac{1}{2} \left(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right) \quad (3)$$

Before the EMC result^[1], it was generally expected^[2] that $\Delta s \simeq 0$, in which case $\int_0^1 dx g_1^p(x) \simeq 0.19$, whereas the EMC found the result in eq. (1). This discrepancy could not be removed by the perturbative QCD corrections to the non-singlet part of $g_1^p(x)$,^[14] which are small and have in fact been included in the above estimate of 0.19 if $\Delta s = 0$. It was therefore suggested^[3,4] that $\Delta s \neq 0$ and a simple fit to the EMC value of $\int_0^1 dx g_1^p(x)$ and nucleon axial current matrix elements gives

$$\Delta u = -0.75 \pm 0.06, \quad \Delta d = -0.50 \pm 0.06, \quad \Delta s = -0.22 \pm 0.06 \quad (4)$$

However, it has recently been realized^[5,6,7] that a new type of perturbative QCD correction to the singlet part of $g_1^p(x)$ might be important. This can be expressed by replacing the Δq by $\widetilde{\Delta q} \equiv \Delta q - \frac{\alpha_s}{2\pi} \Delta G$ in equations (3) and (4), where $\Delta G \equiv \int_0^1 \Delta G(x)$ and $\Delta G(x) = G_1(x) - G_1(x)$ is the polarized gluon distribution. One extreme possibility^[6,8] is therefore that $\Delta s = 0$ so that $\widetilde{\Delta s} = -\alpha_s/2\pi \Delta G = -0.22 \pm 0.06$ and the EMC spin

effect is due to polarized gluons. However, this would require a surprisingly large value of ΔG . At $\langle Q^2 \rangle \simeq 10 \text{ GeV}^2$ as in the EMC experiment, it is believed that $\alpha_s \simeq 0.23$, so that $\Delta G \simeq 6$ would be needed^[9]. This would imply a rather counter-intuitive decomposition of the proton spin

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \Delta G + L_z \quad (5)$$

in which the orbital angular momentum $L_z \sim -6$, in order to cancel the large value of ΔG . Of course, the truth probably lies somewhere between the extreme $\Delta s \neq 0$, $\Delta G = 0$ and $\Delta s = 0$, $\Delta G \neq 0$ possibilities. Additional experimental information is required to determine unambiguously Δu , Δd and ΔG , and we will propose one such probe later in the paper. However, first we note a problem for the ΔG interpretation, at least in the forms proposed so far.

The axial anomaly tells us^[5,8,7] unambiguously that the first moments of $\Delta q(x)$, namely the Δq , are modified to $\widetilde{\Delta q} = \Delta q - \alpha_s/2\pi \Delta G$, but does not fix the modification of $\Delta q(x)$ as a function of x . The natural prescription, which would certainly be well motivated for massive quarks, is to take

$$\widetilde{\Delta q}(x) = \Delta q(x) - \int_{x/z_{\max}}^1 \frac{dx'}{x'} \Delta G(x') \sigma(x/x') \quad (6)$$

where

$$x_{\max} = \frac{1}{1 + 4m_q^2/Q^2} \quad (7)$$

with $Q^2 = -q^2$ where q is the 4-momentum of the virtual photon, and $\sigma(z)$ is the cross-section for $\gamma^* g \rightarrow \bar{q}q$. Its form is well-known for $m_q = 0$

$$\sigma(z) = \hat{\sigma}(z) + f(z) : \quad \hat{\sigma}(z) = \frac{\alpha_s}{2\pi} \left\{ \frac{1}{3} [(1-z^2) - z^2] \ln \left(\frac{1-z}{z} \right) \right\} \quad (8)$$

where $f(z)$ has the property that $\int_0^1 dz f(z) = 0$. It is easy to recover the anomaly result by integrating equation (6) over x . This is true whether or not some $f(z)$ piece in equation (8) is included. These different definitions of $\sigma(z)$ correspond to different prescriptions for defining $\Delta G(x)$. Almost independently of the precise form of $\sigma(z)$, the effect of the convolution (6) is that $\Delta G(x)$ at some $x=x_0$ contributes to $\widetilde{\Delta q}(x)$ at some $x < x_0$. This is

because after gluon splitting, the momentum fraction of the quark struck by the γ^* is necessarily smaller than that of its parent gluon. Although the $\Delta G(x)$ contribution to $\widetilde{\Delta q}(x)$ is prescription-dependent, we believe the choice $\sigma(z) = \hat{\sigma}(z)$ to be physically motivated: it is plotted as the solid line in fig. 1. The formula $\widetilde{\Delta q}(x) = \Delta q(x) - (\alpha_s/2\pi) \Delta G(x)$ proposed in ref. 8 implies a kernel $\delta(x/x' - 1)$ in equation (6), requiring an extreme choice of $f(z)$.

We have investigated the softening of the gluon contribution to $\widetilde{\Delta q}(x)$ using $\sigma(z) = \hat{\sigma}(z)$ and two recently-proposed polarized gluon distributions^[8,12]. One is of the form

$$\Delta G(x) = Ax^{-0.3}(1-x)^7 \quad (9)$$

with $A = 16.3$ chosen such that $\Delta G = 5$. The percentages of the gluonic contribution to $\widetilde{\Delta q}(x)$ that appear in different x bins are tabulated in Table I, and the shape of the gluonic contribution to $g_1^p(x)$ is shown as a solid line in fig. 2. We see that about 50% of the gluonic contribution comes from $x < 10^{-2}$, which one might have expected on the basis of solid line in fig. 1. In contrast, the EMC result (1) was based on data with $x > 10^{-2}$. In order to estimate $\int_0^1 dx g_1^p(x)$, the EMC had to extrapolate to $x=0$, which they did by assuming an essentially constant structure function at small x . This was in agreement^[9] with the trend of their data at $x < 0.1$ and with expectations based on Regge theory^[13]. According to this assumption, the EMC measured almost all of $\int_0^1 dx g_1^p(x)$, and the contribution of the unmeasured region was negligible. This would not be true in the polarized glue interpretation of their result, which would imply a very different behaviour of $g_1^p(x)$ at small x , as seen in fig. 2, unless the quark contribution mysteriously cancelled the gluon contribution.

However, the main problem is that any interpretation of EMC data should fit the data where they are measured, namely at $x > 10^{-2}$. This means that one must require

$$\int_{0.01}^1 dx (\Delta q(x) - \widetilde{\Delta q}(x)) \simeq 0.22 \quad (10)$$

whereas Table I indicates that the polarized gluon distribution (9) gives less than half this value. A similar statement is also true of the polarized gluon distributions proposed in Ref. 12, as also seen in Table I. The normalization of these distributions would apparently need to be more than doubled in order to fit the data. However, this is not possible without violating the upper bound on $\Delta G(x)$ given by the unpolarized gluon distribution $G(x)$.

As an exercise, we have taken $\Delta G(x) = G(x)$ for $x \geq 0.01$, with the unpolarized gluon distribution taken from ref. 15, with $Q^2 = 5 \text{ GeV}^2$ and $\Lambda_{QCD} = 200 \text{ MeV}$. We then find

$$\int_{0.01}^1 dx \left(\Delta q(x) - \widetilde{\Delta q}(x) \right) = 0.14 \quad (11)$$

which is still somewhat too small.

However, it may be possible to save the polarized gluon interpretation of the EMC result (1) by including charm production. It is in any case important to measure $\Delta G(x)$ experimentally, and we now discuss how to do this by looking for a charm production asymmetry in polarized lepton-nucleon scattering.* We have computed exactly the polarization-dependent part of the Born cross-section for $\gamma^* g \rightarrow \bar{q}q$, assuming $p_g^2 \ll m_q^2, Q^2$, and keeping all orders in m_q^2/Q^2 :

$$\begin{aligned} \sigma_m = \frac{4\alpha_s}{9} \left\{ -\frac{1}{2\pi P^2 \sqrt{s}} [q^2 P + P_s + 2m^2 \sqrt{s}] \log \frac{p^0 + l}{p^0 - l} \right. \\ \left. + \frac{l}{\pi \sqrt{s} P^2} [2P^2 + 5Pq + q^2 + 2m^2] \right. \\ \left. + \frac{2q^2 l^3}{\pi P^2 z s^{3/2}} + \frac{m^2(-q^2)}{2\pi P^2 z s} \log \frac{p^0 + l}{p^0 - l} \right. \\ \left. + \frac{m^2}{\pi \sqrt{s} P^2} (\sqrt{s} - P) \log \frac{p^0 + l}{p^0 - l} - \frac{l}{\pi \sqrt{s} P^2} (s - 2P\sqrt{s}) - \frac{2lm^2}{\pi \sqrt{s} P^2} \right\} \end{aligned} \quad (12)$$

where non-covariant quantities are evaluated in the $\gamma^* - g$ center-of-mass system: $p = (P, 0, 0, P)$ and $q = (\sqrt{P^2 + q^2}, 0, 0, -P)$ are the gluon and γ^* 4-momenta, $s = (p + q)^2$, l^0 and l are the energy and momentum of the struck quark, and $z \equiv -q^2/2p \cdot q$.

When inserted into equation (6), we expect this to give a representation of the charm contribution to $g_1^p(x)$ which is accurate to $\mathcal{O}(1) \times (\alpha_s/\pi)$ when $Q^2 \lesssim 10 \text{ GeV}^2$. Certainly, virtual photon-gluon fusion is known to give an adequate representation of the charm production with unpolarized beam and target^[20] and we expect the same to be true in the polarized case. Confirmation of this expectation would require a complete calculation of the

* Polarized photoproduction has been considered by M. Peskin^[16] and by ref. 17, but there is no planned experiment at $Q^2 = 0$. Inelastic photoproduction of the J/ψ off polarized protons have been discussed in ref. 18 as a probe of gluon polarization, and the hadro- and photoproduction of jet pairs in ref. 12.

one-loop radiative corrections to the cross-section, which has been done for the unpolarized photoproduction case.^[19] There it was found that the Born term was an accurate estimate for $s/m_c^2 \lesssim 10$. In the case of a virtual photon, we expect on general grounds that the effective $p_g^2 = \mathcal{O}(\alpha_s/\pi) \times Q^2$, and hence that the $p_g^2 \ll m_c^2$ approximation used above may be useful for $Q^2 \lesssim 10 \text{ GeV}^2$, which is the EMC range and that to be explored in the forthcoming SMC^[10] and HERA^[11] experiments on polarized lepton-nucleon scattering.

Numerical values of the charm contribution to $g_1^p(x)$ are presented in Table 2. They have been calculated using the full massive-quark Born cross-section (12) in formula (6), and either the Altarelli-Stirling^[8] or Kunszt^[12] models for the polarized gluon distribution. The charm contribution to $g_1^p(x)$ is also shown as the dashed line in fig. 2.

Since we consider $\sigma_m(z, p_g^2, q^2)$ in the limit $p_g^2 \ll m^2 \lesssim Q^2$, $\int_0^1 dz \sigma_m$ formally vanishes as $Q^2 \rightarrow \infty$.^[6,7] At large Q^2 : $p_g^2 = \mathcal{O}(\alpha_s/\pi) \times Q^2 \gg m^2$, the charm contribution would be given by the anomaly, as for the light quarks. But our formula for σ_m should be an adequate representation at $Q^2 \lesssim 10 \text{ GeV}^2$, and its overall contribution to $\int_0^1 dz g_1^p(x)$ is small. However, even this charm contribution to $g_1^p(x)$ does not vanish locally in x , reflecting the fact that $\sigma_m(z) > 0$ for large z and < 0 for small z , changing sign at intermediate z as seen in fig. 1. The effect of adding the charm contribution to the light quark contribution to $\int_{0.01}^1 g_1^p(x)$ is to compensate for much of what was lost from Δq by using the kernel $\hat{\sigma}(z)$ for light quarks (see the first two rows of table I). Indeed, adding the light and massive quark contributions to $\int_{0.01}^1 g_1^p(x)$ we obtain:

$$\delta \int_{0.01}^1 g_1^p(x) = \begin{array}{ccc} \text{ref. 8} & Q^2 = 4\text{GeV}^2 & Q^2 = 10\text{GeV}^2 \\ -5.5 \cdot 10^{-2} & & -5.7 \cdot 10^{-2} \\ \text{ref. 12} & -4.1 \cdot 10^{-2} & -5.2 \cdot 10^{-2} \end{array} \quad (13)$$

The numerical estimates of Table 2 suggest that the charm contribution may violate scaling in $g_1^p(x)$ to an extent which is on the verge of being visible in the present data.^[1] Fig. 3 shows the charm contribution to $g_1^p(x)$ at $x = 0.01$ and $x = 0.06$ for $1 < Q^2 < 10 \text{ GeV}^2$, compared with the average of the EMC data in this region. A precise investigation of the observability of the charm scaling violation in $g_1^p(x)$ would require incorporating the Q^2 -dependence of $\Delta G(x)$ and α_s , understanding of the $\mathcal{O}(\alpha_s/\pi)$ corrections to the charm

production cross-section, and control over the scaling violations in other contributions to $g_1^p(x)$, which lies beyond the scope of this paper.

It is clear that the gluonic interpretation^[6,8] of the EMC spin effect requires a large charm contribution to $g_1^p(x)$. We therefore urge that the forthcoming SMC^[10] and HERA^[11] experiments make every effort to detect charmed final states, as was done previously by the EMC^[20]. Such measurements could play a key rôle in discriminating between the proposed strange quark^[3,4] and gluon^[6,8] interpretations of the EMC spin effect.

ACKNOWLEDGEMENTS

We thank G. Altarelli, Y. Frishman, G. Ross and W. J. Stirling for useful discussions. We also particularly thank M. Peskin for his early and continued interest and comments.

Table I: Percentage of polarized gluon contributions to $\widetilde{\Delta q(x)}$ via light quarks

x range	Proposed polarized gluon distributions	
	ref. 8	ref. 12
$< 10^{-3}$	18	21
10^{-3} to 10^{-2}	30	34
10^{-2} to 10^{-1}	39	39
0.1 to 0.2	9.2	4.8
0.2 to 0.3	2.8	0.5
> 0.3	1.1	0

To obtain the corresponding contributions to $\int_0^1 dx g_1^p(x)$, multiply these numbers by $-6.1 \cdot 10^{-4}$ (ref. 8) and $-5.7 \cdot 10^{-4}$ (ref. 12, where we chose $Q^2 = 10 \text{ GeV}^2$).

Table II: Charm contributions to $\int dx g_1^p(x)$

x range		Proposed polarized gluon distributions	
		ref. 8	ref. 12
$Q^2 = 4 \text{ GeV}^2$	0.01 to 0.1	$-1.1 \cdot 10^{-2}$	$-7.0 \cdot 10^{-3}$
	0.1 to 0.2	$-1.0 \cdot 10^{-4}$	$-2.3 \cdot 10^{-6}$
$Q^2 = 6 \text{ GeV}^2$	0.01 to 0.1	$-1.5 \cdot 10^{-2}$	$-1.1 \cdot 10^{-2}$
	0.1 to 0.2	$-4.6 \cdot 10^{-4}$	$-2.6 \cdot 10^{-5}$
	0.2 to 0.3	$-5.3 \cdot 10^{-6}$	$-9.1 \cdot 10^{-9}$
$Q^2 = 8 \text{ GeV}^2$	0.01 to 0.1	$-1.9 \cdot 10^{-2}$	$-1.5 \cdot 10^{-2}$
	0.1 to 0.2	$-9.8 \cdot 10^{-4}$	$-8.7 \cdot 10^{-5}$
	0.2 to 0.3	$-2.8 \cdot 10^{-5}$	$-1.6 \cdot 10^{-7}$
	0.3 to 0.4	$-2.3 \cdot 10^{-7}$	$-2.7 \cdot 10^{-11}$
$Q^2 = 10 \text{ GeV}^2$	0.01 to 0.1	$-2.1 \cdot 10^{-2}$	$-1.8 \cdot 10^{-2}$
	0.1 to 0.2	$-1.5 \cdot 10^{-3}$	$-1.8 \cdot 10^{-4}$
	0.2 to 0.3	$-7.1 \cdot 10^{-5}$	$-7.8 \cdot 10^{-7}$
	0.3 to 0.4	$-1.5 \cdot 10^{-6}$	$-7.3 \cdot 10^{-10}$

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FIGURE CAPTIONS

1. The form of the polarized kernel $\hat{\sigma}(z)$ used for light quarks in equation (6) (solid line). The prescription of ref. 8 corresponds to choosing a kernel $\propto \delta(z-1)$. Also shown is the corresponding polarized kernel $\sigma_m(z)$ (12) for massive quarks: $m = 1.5$ GeV, $Q^2 = 10$ GeV² (dashed line).
2. The shape of the polarized gluon contribution to $g_1^p(x)$ obtained using the distribution (9) in the formula (6) using the Born cross-section kernel $\hat{\sigma}(z)$ shown in fig. 1 for light quarks (solid line), and equation (12) for the charmed quark: $m = 1.5$ GeV, $Q^2 = 10$ GeV² (dashed line). Also shown are the EMC data for $g_1^p(x)$.
3. The charm contributions to $g_1^p(x)$ obtained using the polarized gluon distribution (9) in equation (6), for $x = 0.01, 0.06$ and $Q^2 < 10$ GeV². Also shown is the EMC data point for $\langle x \rangle = 0.05$, $\langle Q^2 \rangle = 8$ GeV².

Fig. 1

Fig. 2

Fig. 3