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THE PHYSICS OF DOUBLE LAYERS
AND THEIR ROLE IN ASTROPHYSICS

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THE PHYSICS OF DOUBLE LAYERS AND THEIR ROLE IN ASTROPHYSICS

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Abstract. Research on electrostatic double layers (DL's) is surveyed and astrophysical applications are reviewed. Strong highly relativistic DL's directly accelerate equal numbers of ions and electrons, whereas in the non-relativistic limit electron acceleration predominates. The existence criteria for DL's are discussed. There is a close relationship between the Bohm criteria, the plasma dispersion relation and the Penrose stability condition. Current driven instabilities can lead to DL formation, and account for the fluctuations associated with strong DL's. A class of weak DL's may be described using the modified Korteweg-de Vries equation, and the conditions for the emergence of a DL from arbitrary initial conditions can be specified. Simulation studies of strong DL formation reveal important dynamical features. Weak DL's are found in simulations of ion-acoustic instabilities. Laboratory DL's have been studied in great detail. There is a strong coupling between a DL and the global current system. The results of theoretical, simulation and observational investigations of magnetospheric DL's are presented. High energy observations of solar flares agree well with a DL model. DL's may be a feature of many high energy astrophysical phenomena.

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1. Introduction

A double layer (DL) is an electrostatic structure which can appear within a current carrying plasma and sustain a significant net potential difference. The thickness, which depends on the local plasma parameters, can be very small compared to the dimensions of the region in which the DL is embedded. Internally the DL is not charge neutral and must contain at least two layers of opposite net charge (hence the nomenclature “double layer”) associated with the internal electric field. This field has a component parallel to the magnetic field so that the DL is a region where ideal magnetohydrodynamics breaks down. As a whole the DL has no net charge and the surrounding plasma has no significant electric field and is approximately charge neutral. However the plasma may be perturbed by extended presheaths matching the conditions at the edges of the DL to those in the undisturbed plasma.

A basic property, which is of particular interest for applications to astrophysics, is that particles traversing the DL are directly accelerated by the net potential difference, ϕ_{DL} . The DL acts as an electrical load dissipating energy at a rate $I\phi_{DL}$ (where I is the total current through the DL), which is transformed to the directed kinetic energy of accelerated particles. In other words the DL exhibits inertial resistance. This should be contrasted with the effect of ohmic or anomalous resistance which is to transform electrical energy to thermal or random particle motions. Since the DL acts as a load there has to be an external source maintaining the potential difference, ϕ_{DL} , and driving the current, I . In the laboratory this source is usually an electrical power supply, whereas in space it may be the magnetic energy stored in an extended current system which responds to a change in current with an inductive voltage.

The formation of a DL is one process by which a current carrying plasma may produce a local region with electric field parallel to an ambient magnetic field leading to energy dissipation. This property is shared by regions of anomalous resistivity produced by current driven instabilities. In the case of ion-acoustic instabilities computer simulations have revealed that the anomalous resistivity may in reality be the large scale result of the formation of many weak ion-acoustic

DL's (Section 6.2). Steady state one-dimensional DL's may be described as solutions of the Vlasov-Poisson equations (Section 2.) and in this way closely resemble laminar shocks. The method used to find solutions for such shocks may be extended to DL's. DL solutions may also be obtained by the Bernstein- Greene-Kruskal- (BGK) method for finding nonlinear electrostatic waves, and as such DL's may be regarded as BGK-states. They share the common problem of the accessibility of these states from an initially undisturbed plasma. The dynamics of very weak DL's can in certain cases be described using the modified Korteweg-de Vries equation (Section 5.2) and the DL appears as a particular form of soliton. The initial value problem can then be completely solved by inverse scattering methods, and conditions for DL formation explicitly expressed.

Computer simulations have lead to a deeper understanding of the formation mechanisms and non-linear dynamics of DL's (Section 6). They also provide an important tool for understanding experimentally observed DL's. The results of laboratory experiments in many different plasma configurations (Section 7) have established the importance of DL formation in the physics of current carrying plasmas, and indicate the type of behaviour that can be significant in astrophysics. The magnetosphere is an important testing ground for the application of DL's in naturally occuring plasmas (Section 9). The accessibility of the Earth's magnetosphere to detailed *in situ* plasma measurements has stimulated the development of detailed DL models and many of the computer simulation studies.

Since electrostatic DL's are by their nature limited to small thin regions within a plasma, their direct detection in more distant astronomical plasmas is difficult. Their presence may however be indicated indirectly by their global effects (Section 8), as for example the magnetohyromagnetic relaxation that should accompany the release of stored magnetic energy. Such considerations provide important tests for the viability of DL's in particular astrophysical phenomena, and together with the constraints set by the observational parameters have been applied in several proposed models for energy release in astrophysics. One of the earliest suggested applications is to solar flares (Section 10). Recent observations indicate prompt acceleration to very high energies, setting a strong restriction on

possible mechanisms, which is met by the direct acceleration of equal numbers of both ions and electrons in strongly relativistic DL's. Applications to more distant objects must rely on the large body of experimental and theoretical knowledge of DL's that has been built up, as well as the experience gained from the study of DL's in nearby plasmas.

The reader is referred to conference proceedings for a broader view of DL research than can be given by any individual reviewer (Michelsen and Rasmussen 1982 [1], Schrittwieser and Eder 1984 [2], Williams and Moorehead 1987 [3]). This particular review is not intended to be comprehensive, but rather to develop a general view of the physical properties of DL's, in particular those which may help to elucidate their possible rôle in astrophysics.

2. Steady state double layers

The structure of a DL is determined self consistently by the particles moving in the electric field set up by their net charge distribution. The essential features of the DL structure are shown in Figure 1. In the most general case both free and reflected ion and electron components are required. The current is carried by the free particles and those accelerated by the DL electric field emerge as energetic beams. The regimes of ion and electron acceleration are shown as hatched areas in Figure 1. The DL potential acts as a barrier for the reflected particles i.e. the plasma electrons entering on the high potential side and ions on the low potential side. If the potential is very large compared to the equivalent thermal potentials of these particles ($kT_{i,e}/e$, where in general $kT_{i,e}$ is taken to be a measure of the mean random kinetic energy) they will barely penetrate the DL and the internal structure will be determined by the net charge and dynamics of the free particles. An example of such a "strong" DL is the model derived by Langmuir (Langmuir 1929 [4]) with only cold ions and electrons. For a potential difference ϕ_{DL} , and ion and electron currents $i_{i,e}$ Poisson's equation becomes (for singly charged ions),

$$-\epsilon_0 \frac{d^2 \phi}{dx^2} = \frac{i_i}{\sqrt{2e(\phi_{DL} - \phi)/m_i}} - \frac{i_e}{\sqrt{2e\phi/m_e}} \quad (2.1)$$

where $\phi(0) = \phi_{DL}$ and $\phi(d) = 0$ and d is the thickness of the DL. There are weak singularities at the boundaries, since the injection velocities are approximated

to zero, but these disappear on integration. Finite injection velocities lead to modifications (Levine and Crawford 1980 [5]) which become small if the velocities tend to zero. Integration of Equation (2.1) leads to Langmuir's DL relation,

$$i d^2 \equiv (i_e + i_i) d^2 = (1 + \sqrt{m_e/m_i})(4C_0 \epsilon_0/9) \sqrt{2e/m_e} \phi_{DL}^{3/2} \quad (2.2)$$

where, in terms of the elliptic integrals E and K , C_0 may be expressed as (Raadu 1982 [6]),

$$C_0 = 2^{-3/2} (4\sqrt{2}E(\sin \pi/8) - (1 + 2\sqrt{2})K(\sin \pi/8))^2 \approx 1.86518 \quad (2.3)$$

It is also assumed that the electric field E vanishes at the edges of the DL, which leads to the Langmuir condition for the current ratio,

$$i_e/i_i = \sqrt{m_i/m_e} \quad (2.4)$$

which shows that the electron current dominates in such a DL, which has implicitly been assumed to be non-relativistic.

For astrophysical applications it is often necessary to consider relativistic effects, for example in the DL model for energy release in solar flares potentials $\simeq 10^9$ V are required (Alfvén and Carlqvist 1967 [7]). The internal structure of a relativistic DL differs from that of a non-relativistic DL, and the relations between the currents $i_{i,e}$ and the potential ϕ_{DL} are quite different (Carlqvist 1969 [8], 1982 [9]). Apart from the relativistic effects the assumptions are the same as for the Langmuir DL (Langmuir 1929 [4]). Following the original derivation of the relativistic DL model (Carlqvist 1969 [8]) the Poisson equation corresponding to Equation (2.1) is,

$$-\epsilon_0 \frac{d^2 \phi}{dx^2} = \frac{i_i/c (m_i c^2/e + \phi_{DL} - \phi)}{\sqrt{\phi_{DL} - \phi} \sqrt{2m_i c^2/e + \phi_{DL} - \phi}} - \frac{i_e/c (m_e c^2/e + \phi)}{\sqrt{\phi} \sqrt{2m_e c^2/e + \phi}} \quad (2.5)$$

where the ion and electron charges have been found from $i_{i,e}/v_{i,e}$ and their velocities $v_{i,e}$ follow from relativistic energy conservation (N.B. here $i_{i,e} > 0$, $v_i > 0$ and $v_e < 0$). A first integration of Equation (2.5) gives,

$$-\frac{1}{2}\epsilon_0 \left(\frac{d\phi}{dx} \right)^2 + (i_i/c) \sqrt{\phi_{DL} - \phi} \sqrt{2m_i c^2/e + \phi_{DL} - \phi} +$$

$$+(i_e/c)\sqrt{\phi}\sqrt{2m_e c^2/e + \phi} = -\Pi \quad (2.6)$$

The constant of integration, Π , is determined by setting $d\phi/dx = 0$ at the DL edges $x = 0$ and $x = d$ and for consistency this requires that the current ratio (cf Equation (2.4)) is given by a generalised Langmuir condition,

$$\frac{i_e}{i_i} = \frac{\sqrt{2m_i c^2/e + \phi_{DL}}}{\sqrt{2m_e c^2/e + \phi_{DL}}} \quad (2.7)$$

which reduces to the non-relativistic form, Equation (2.4), if $\phi_{DL} \ll m_e c^2/e \approx 5 \cdot 10^5 V$. On the other hand for a strongly relativistic DL in a hydrogen plasma i.e. if $\phi_{DL} > m_i c^2/e \approx 9.4 \cdot 10^8 V$, $i_i \approx i_e$. A strongly relativistic DL accelerates equal numbers of ions and electrons (Figure 2). Also in the strongly relativistic case the generalised form of Langmuir's DL relation becomes,

$$i d^2 = (i_e + i_i) d^2 \approx \frac{4\epsilon_0 c}{m_i c^2/e} \phi_{DL}^2 \quad (2.8)$$

In the intermediate case $m_e c^2 < e\phi_{DL} < m_i c^2$ this relation is given by (Raadu 1982 [6]),

$$i d^2 = (i_e + i_i) d^2 \approx \left(1 + \sqrt{e\phi_{DL}/2m_i c^2}\right) \frac{\pi^2 \epsilon_0 c}{2} \phi_{DL} \quad (2.9)$$

where the Langmuir condition is $i_e/i_i \approx \sqrt{2m_i c^2/e\phi_{DL}}$. In the general case a numerical integration is necessary to find the DL relation (Carlqvist 1969 [8]) and the approximate dependences of the current on ϕ_{DL} may be confirmed within the three regimes: non-relativistic, intermediate and highly relativistic (Figure 2).

2.1 Kinetic description

The structure of a steady state plane parallel (one-dimensional) DL can be found by a direct integration of Poisson's equation and the time-independent Vlasov equations for the separate particle components (Andrews and Allen 1971 [10], Swift 1975 [11], Schamel and Bujarbarua 1983 [12]). The principles are the same as for laminar shocks (Montgomery and Joyce 1969 [13]). Solutions of the Vlasov equations,

$$\frac{Df_{i,e}}{Dt} \equiv \frac{\partial f_{i,e}}{\partial t} + v \frac{\partial f_{i,e}}{\partial x} + \frac{q_{i,e} E}{m_{i,e}} \frac{\partial f_{i,e}}{\partial v} = 0 \quad (2.1.1)$$

are then given by defining the ion and electron distribution functions, $f_{i,e}(x, v)$, to be functions of the energies, $w_{i,e} = \frac{1}{2}m_{i,e}v^2 + q_{i,e}\phi(x)$, which are integrals of motion. The particle densities, $n_{i,e}$, are found by integrating the distribution functions over velocity, and are therefore only functions of the electric potential ϕ ($E = -d\phi/dx$). Poisson's equation,

$$\epsilon_0 \frac{d^2\phi}{dx^2} = -(q_i n_i(\phi) + q_e n_e(\phi)) \quad (2.1.2)$$

where $q_i = Ze$ and $q_e = -e$ are the ion and electron charges, may now be integrated once to give,

$$\frac{1}{2}\epsilon_0 \left(\frac{d\phi}{dx} \right)^2 + V(\phi) = \Pi \quad (2.1.3)$$

where Π is a constant. The term $V(\phi)$ is commonly called the Sagdeev or classical potential, and formally the integral may be regarded as the 'energy' of a fictitious particle located at a 'position' ϕ where the 'time' is given by x . To within an additive constant depending on the choice of Π , $V(\phi)$ is equal to minus the total particle pressure (i.e. the sum of the dynamical and thermal pressure) given by the second moments of the distribution functions (Andrews and Allen 1971 [10], Torvén 1980 [14]). Equation (2.1.3) is therefore exactly equivalent to the condition of stress balance,

$$-\frac{1}{2}\epsilon_0 E^2 + \sum_{i,e} \int_{-\infty}^{+\infty} m_{i,e} v^2 f_{i,e}(x, v) dv = -\Pi \quad (2.1.4)$$

where, since a constant magnetic field is assumed, only the electric field part of the Maxwell stress tensor appears. Comparison of Equations (2.1.4) and (2.1.3) gives an explicit expression for the classical potential $V(\phi)$ in terms of the particle distributions. (Here Π is chosen so that $-V(\phi)$ is the total pressure. However the choice $\Pi = 0$ is often made so that $V(\phi) = 0$ for $E = 0$.) A second integration of Equation (2.1.3) implicitly gives the spatial structure of the DL, i.e.

$$x - x_0 = \pm \sqrt{\frac{\epsilon_0}{2}} \int_{\phi_0}^{\phi} \frac{d\phi'}{\sqrt{\Pi - V(\phi')}} \quad (2.1.5)$$

It may be argued that this is physically the most reasonable way to determine the structure of a DL (Schamel 1982 [15]). Some care is needed in specifying the distribution functions since they should be determined at the appropriate sources

at $x = \pm\infty$ which are at potentials differing by the DL potential ϕ_{DL} . Thus the value of the distribution function at a particular particle velocity should be related to a specified value at its source of origin. Also the distribution of locally trapped particles in the case of a non-monotonic DL potential profile must in reality depend on the unspecified time history of the formation of the DL i.e the dynamical trapping process.

An alternative procedure is to predetermine the potential profile and then to solve the time independent Vlasov–Poisson equations by adjusting the distribution function for one of the particle populations e.g. the reflected electrons (Knorr and Goertz, 1974 [16]). This corresponds to the BGK method for non-linear electrostatic waves (Bernstein *et al* 1957 [17]). Suppose then that the distribution functions for free and reflected ions and for free electrons have been specified and written as functions of the energy integrals. The corresponding densities as functions of the potential ϕ follow from an integration over velocity. Now if the (monotonic) potential profile $\phi(x)$ is given as some function of x ($0 \leq \phi(x) \leq \phi_{DL}$) the reflected electron density $n_{e,r}(\phi)$ is required to be a specific function of ϕ in order to satisfy Poisson's equation,

$$n_{e,r}(\phi) = -n_{e,f}(\phi) - \frac{q_i}{q_e} (n_{i,r}(\phi) + n_{i,f}(\phi)) - \frac{\epsilon_0}{q_e} \frac{d^2 \phi}{dx^2} \quad (2.1.6)$$

where the inverse function $x(\phi)$ must be used to write the last term as a function of ϕ . Now a distribution function for the reflected electrons which satisfies the Vlasov equation may be defined by $f_{e,r}(x, v) = G(\psi)$ where $\psi = -w_e/e = \phi - m_e v^2/2e$ is a constant of motion. The function $G(\psi)$ is now determined by the integral equation,

$$n_{e,r}(\phi) = 2 \int_0^{\sqrt{2e\phi/m_e}} G(\phi - m_e v^2/2e) dv \quad (2.1.7)$$

using the fact that the reflected distribution function is symmetric about $v = 0$ and by definition zero for $|v| > \sqrt{2e\phi/m_e}$ i.e. for non-trapped electrons. Changing to the variable ψ gives an Abel equation,

$$n_{e,r}(\phi) = \sqrt{\frac{2e}{m_e}} \int_0^\phi \frac{G(\psi) d\psi}{\sqrt{\phi - \psi}} \quad (2.1.8)$$

with the solution,

$$G(\psi) = \frac{1}{\pi} \sqrt{\frac{m_e}{2e}} \frac{d}{d\psi} \int_0^\psi \frac{n_{e,r}(\phi) d\phi}{\sqrt{\psi - \phi}} \quad (2.1.9)$$

Since by definition there are no reflected electrons at $\phi = 0$, $n_{e,r}(0) = 0$ and the solution may be expressed in the form,

$$G(\psi) = \frac{1}{\pi} \sqrt{\frac{m_e}{2e}} \int_0^\psi \frac{dn_{e,r}(\phi)/d\phi d\phi}{\sqrt{\psi - \phi}} \quad (2.1.10)$$

where the lower limit to the integration $\phi = 0$ depends on the assumption $\phi(x) \geq 0$ and in general may be replaced by the minimum of $\phi(x)$ (Knorr and Goertz 1974 [16]). Here it can be seen that in principle spatially extended quasi-neutral DL's may be found by choosing $\phi(x)$ with a length scale much greater than a Debye length. However in each particular case the resulting distribution for reflected electrons $f_{e,r}(x, v) = G(\phi - m_e v^2/2e)$ must be physically reasonable i.e. everywhere positive and without singularities. Extended potential drops (EPD's) have been seen in some DL experiments (Carpenter and Torvén 1987 [18]) and may thus be sustained by strongly modified reflected electron distributions.

3. Existence criteria

In order that DL solutions may be obtained by the method of direct integration for given velocity distributions (Equations (2.1.3) – (2.1.5)) the Sagdeev potential $V(\phi)$ must satisfy certain conditions. These incorporate matching conditions between the DL and the ambient plasmas on both sides: It is normally required that the net charges and the electric field E tend to zero at the edges of the DL. These conditions are to ensure that the plasma in contact with the DL is effectively shielded from the internal structure of the DL and hence undisturbed.

To implement the charge neutrality conditions in terms of the Sagdeev potential, it may be found from Equations (2.1.2) and (2.1.3) that,

$$q_i n_i(\phi) + q_e n_e(\phi) = -\epsilon_0 \frac{d^2 \phi}{dx^2} = V'(\phi) \quad (3.1)$$

Therefore for charge neutrality at the edges of the DL,

$$V'(0) = V'(\phi_{DL}) = 0 \quad (3.2)$$

where without loss of generality it is assumed that $\phi = 0$ and $\phi = \phi_{DL}$ there. Next from the zero electric field condition and Equation (2.1.3),

$$V(0) = V(\phi_{DL}) = \Pi \quad (3.3)$$

This also ensures that the DL as a whole has no net charge ($\epsilon_0(E(\phi_{DL}) - E(0)) = 0$). However the most physically illuminating interpretation follows from Equation (2.1.4) which gives,

$$\sum_{i,e} \int_{-\infty}^{+\infty} m_{i,e} v^2 f_{i,e}(x_0, v) dv = \sum_{i,e} \int_{-\infty}^{+\infty} m_{i,e} v^2 f_{i,e}(x_1, v) dv \quad (3.4)$$

where $x_{0,1}$ are the space coordinates at the DL edges. Hence the condition is equivalent to requiring that the total particle pressures (dynamical plus “thermal”) are in balance across the DL. This is the generalised Langmuir condition corresponding to that for strong DL’s. In the case of strong DL’s Equation (2.4) follows from Equation (3.4) if it is required that the dynamical pressures of the emerging particle beams dominate. Stated in terms of pressure balance the generalised Langmuir condition should apply quite generally to steady state DL’s.

Given that the generalised Langmuir condition is satisfied, it is also required that $E^2 \geq 0$ so that (Equation (2.1.3)),

$$V(\phi) < \Pi \quad (3.5)$$

everywhere inside the DL. The DL potential profile is then given implicitly by Equation (2.1.5). In brief the set of existence conditions, Equations (3.2) (3.3) (3.5), require that the Sagdeev potential has two equal absolute maximum values at $\phi = 0$ and $\phi = \phi_{DL}$ as for example in Figure 4. In terms of the fictitious ‘particle’ interpretation, where Equation (2.1.3) is regarded as an ‘energy’ equation, a DL solution may then be given by a ‘particle’ starting at rest ($d\phi/dx = 0$) at one of the maxima, rolling down the potential $V(\phi)$ and then upwards coming to rest at the other maximum. The ‘velocity’ corresponds to the field E and is a maximum at the minimum of $V(\phi)$ (see Figure 4).

The DL existence condition, Equation (3.5), is most sensitive near the edges of the DL where $V = \Pi$. Introducing $\delta\phi$ such that $0 < \delta\phi \ll \phi_{DL}$ and since $V(\phi)$ is differentiable it follows that as $\delta\phi \rightarrow 0$,

$$V(0 + \delta\phi) \rightarrow \Pi + V'(0)\delta\phi ; \text{ and } V(\phi_{DL} - \delta\phi) \rightarrow \Pi - V'(\phi_{DL})\delta\phi \quad (3.6)$$

Therefore from Equation (3.2) $V(0 + \delta\phi) = V(\phi_{DL} - \delta\phi) = \Pi$ to $O(\delta\phi)$ and it is not possible to test the existence condition. Instead using Equation (3.1) the existence condition near the edges of the DL may be written,

$$V(\delta\phi) - \Pi = \int_0^{\delta\phi} q_i n_i(\phi) + q_e n_e(\phi) d\phi < 0$$

and

$$V(\phi_{DL} - \delta\phi) - \Pi = - \int_{\phi_{DL} - \delta\phi}^{\phi_{DL}} q_i n_i(\phi) + q_e n_e(\phi) d\phi < 0 \quad (3.7)$$

and as $\delta\phi \rightarrow 0$ these reduce to requirements that the net charges have an appropriate sign. Quite generally, assuming that the electric field is zero at the edges of the DL, it can be argued directly that near the boundaries within the DL self consistency between the electric field and charges requires that the net charge on the low and high potential sides are negative and positive respectively (for a monotonic potential). Similar requirements for wall sheaths are known as Bohm criteria (Bohm 1949 [19]) and the same term is used for the more general case of DL's, which are in contact with an ambient plasma on both sides. In the case of strong DL's, for which the reflected particle populations hardly penetrate the structure, the generalised Bohm criteria on the low and high potential side approximate those for positive and negatively biased wall sheaths. The incoming free particles are required to have velocities exceeding their acoustic or thermal velocities (Block 1972 [20](a)).

In cases where $V(\phi)$ has a second derivative at the DL edges, i.e. if the net charge is differentiable there with respect to ϕ , the existence condition, Equation (3.5), may be expanded to $O(\delta\phi^2)$ giving the generalised Bohm criteria,

$$V''(0) < 0 ; \text{ and } V''(\phi_{DL}) < 0 \quad (3.8)$$

The maximum values of $V(\phi)$ at $\phi = 0, \phi_{DL}$ are then true maxima. The Bohm criteria in this form are noted in a derivation of fluid models of double layers with four particle components including thermal effects (Levine and Crawford 1980 [5]). Next from Equation (3.1),

$$V''(\phi) = \sum_{i,e} q_{i,e} n'_{i,e}(\phi) \quad (3.9)$$

where using the Vlasov equations (2.1.1) in the time independent form ($\partial f_{i,e}/\partial t = 0$),

$$n'_{i,e}(\phi) = \int_{-\infty}^{+\infty} -\frac{\partial f_{i,e}(x,v)}{\partial x} \frac{dx}{d\phi} dv = \int_{-\infty}^{+\infty} \frac{q_{i,e}}{m_{i,e}v} \frac{\partial f_{i,e}(x,v)}{\partial v} dv \quad (3.10)$$

Combining these two equations gives the generalised Bohm criteria (3.8) in terms of the distribution functions,

$$V''(\phi) = \sum_{i,e} \frac{q_{i,e}^2}{m_{i,e}} \int_{-\infty}^{+\infty} \frac{1}{v} \frac{\partial f_{i,e}(x,v)}{\partial v} dv < 0 \text{ as } \phi \rightarrow 0, \phi_{DL} \quad (3.11)$$

Practically equivalent forms of this condition have been discussed in connection with wall sheaths (Harrison and Thompson 1959 [21], Allen 1976 [22]). These forms of the Bohm criteria bear a close relationship to the dispersion relation for electrostatic waves in a uniform plasma,

$$k^2 = \sum_{i,e} \frac{q_{i,e}^2}{\epsilon_0 m_{i,e}} \left(P \int_{-\infty}^{+\infty} \frac{1}{v - \omega/k} \frac{df_{i,e}(v)}{dv} dv + i\pi \frac{df'_{i,e}(\omega/k)}{dv} \right) \quad (3.12)$$

where ω and k are the frequency and wave number of a small perturbation. Inside the DL all the distribution functions are symmetric about $v = 0$ over a velocity range which is sufficiently small so that all particles are reflected towards the low or high potential sides i.e. for $-v_s < v < v_s$, where $v_s = \min(\sqrt{2e\phi/m_e}, \sqrt{2q_i(\phi_{DL} - \phi)/m_i})$. Hence $\partial f_{i,e}(x, 0)/\partial v = 0$ inside the DL, so that the Bohm criteria become equivalent to the condition that $\epsilon_0 k^2 < 0$ for zero phase velocity ($\omega/k = 0$) (Raadu and Rasmussen 1988 [23]). Therefore the Bohm criteria require that the electrostatic wave with zero phase velocity is spatially evanescent (i.e. the wave number k is imaginary). For a Maxwellian plasma this reduces to the Debye shielding condition, $k^2 = -\lambda_D^{-2}$. The Bohm criteria require that for general plasma conditions the ambient plasma is shielded from the

DL. In the case of a wall sheath it has been argued that the inequality in the Bohm condition should be replaced by an equality, implying that for a particular example an ion acoustic wave has zero phase velocity so that it is unable to travel back into the plasma (Allen 1976 [22]). This need not apply to the case of DL's since the formation process should be different to that for a wall sheath.

A complicating factor is that there can be near discontinuities in the ion and electron distribution functions at the separatrices between free and reflected particles, located at $v = \pm v_{s,e} = \pm \sqrt{2e\phi/m_e}$ and $v = \pm v_{s,i} = \pm \sqrt{2q_i(\phi_{DL} - \phi)/m_i}$ respectively (Raadu and Rasmussen 1988 [23]). For example suppose that the reflected electrons within a DL have a waterbag distribution,

$$\begin{aligned} f_e(x, v) &= f_0 \quad \text{for } -\sqrt{2e\phi/m_e} < v < +\sqrt{2e\phi/m_e} \\ f_e(x, v) &= 0 \quad \text{for } |v| > \sqrt{2e\phi/m_e} \end{aligned} \quad (3.13)$$

The corresponding charge density is then given by (cf Equation (3.1)),

$$V_e'(\phi) = -en_e(\phi) = -2ef_0\sqrt{2e\phi/m_e} \quad (3.14)$$

where V_e is the corresponding contribution to the Sagdeev potential. The contribution to the generalised Bohm criterion (3.8) at $\phi = 0$ is then, taking the derivative of Equation (3.14),

$$V_e''(\phi) = -\frac{2e^2}{m_e} \frac{f_0}{\sqrt{2e\phi/m_e}} \quad \text{where } \phi \rightarrow 0 \quad (3.15)$$

which also follows from Equation (3.11) summing over the two δ -function contributions at the edges of the waterbag distribution. Here $V''(\phi)$ diverges to $-\infty$ at $\phi = 0$ but from Equation (3.14) the charge density is finite and negative as required by the Bohm criterion. Thus if the generalised Bohm criterion (3.8) is derived by a limiting process the value $V'' = -\infty$ is correctly interpreted as satisfying the criterion. In general $V''(\phi)$ should be found first by summing the separate contributions from the ion and electron distribution functions with possible discontinuities at $v = v_{s,i}$ and $v = v_{s,e}$ respectively. Then the Bohm criteria follow in the limits $\phi \rightarrow 0+$ and $\phi \rightarrow \phi_{DL}-$.

For a general distribution $f(v)$ with discontinuities $\Delta f_{\pm}$ at the separatrices $\pm v_s$, the δ -function contributions in Equation (3.11) lead to a contribution V_{Δ}'' to the Bohm criterion given by,

$$\begin{aligned} V_{\Delta}'' &= \frac{q^2}{m} \left(\frac{\Delta f_-}{-v_s} + \frac{\Delta f_+}{+v_s} \right) = \\ &= (q^2/mv_s)(f(-v_s-) - f(-v_s+) - f(+v_s-) + f(+v_s+)) \end{aligned} \quad (3.16)$$

and $f(-v_s+) = f(+v_s-) \equiv f_r$ since they are in effect values on the same reflected particle orbit in phase space just inside the separatrix (Raadu and Rasmussen 1988 [23]). Since $1/v_{s,i} \rightarrow \infty$ as $\phi \rightarrow \phi_{DL}$, and $1/v_{s,e} \rightarrow \infty$ as $\phi \rightarrow 0$, V_{Δ}'' dominates the Bohm criterion unless $f(-v_s-) + f(+v_s+) = 2f_r$. Therefore there is a critical value of f_r to satisfy the Bohm criterion. This leads to the critical trapped particle density found in DL models with discontinuous distributions (Kan and Lee 1980 [24]).

It should be noted that the Bohm criteria (3.8) are in principle only sufficient to ensure that the general existence condition (3.3) is met at the edges of the DL, although in practice once they are satisfied the general condition is often also fulfilled. As remarked above the general condition is most sensitive just at the DL edges.

If DL solutions are derived by the BGK method (Knorr and Goertz 1974 [16]) the only requirement is that the deduced reflected electron distribution is physically acceptable, in particular that it should be positive. The potential profile $\phi(x)$ is prescribed and the Sagdeev potential given by Equation (2.1.3) automatically meets all the existence criteria; the solution $\phi(x)$ is already known! The interesting physical conclusion is that the existence criteria, i.e the generalised Langmuir and Bohm criteria, may be met if the trapped electron distribution may be freely adjusted, provided it is positive.

In reality the Bohm criteria may be an over restriction, since the edges of a DL, particular on the low potential side, nearly always show strong fluctuations. Moreover the DL need not match directly onto the asymptotic plasma conditions. Analogously with wall sheaths, there may be extended, dynamical or collisionally dominated, presheaths allowing a gradual transition from the conditions at

the edges of the DL to the distant plasma (Raadu and Rasmussen 1988 [23]). Numerical studies have shown that a wall sheath may for example be matched to a collisionless plasma by a presheath expanding at the ion-acoustic velocity which accelerates ions towards the sheath so as to fulfill the Bohm criterion there (Cipolla and Silevitch 1981 [25]). Also, a potential minimum often appears on the low potential edge of strong DL's. This reflects electrons, hence controlling the current through the DL, and the electron velocity distribution at the minimum must satisfy the Bohm criterion, rather than that in the adjacent plasma region. Such a minimum is only possible in a dynamical state for strong DL's (Raadu and Rasmussen 1988 [23]), or possibly as a consequence of the three-dimensional structure of the DL.

4. Double Layer Models

A DL is said to be "strong" if the energies acquired by the ions and electrons accelerated by the potential difference ϕ_{DL} are very large compared to the energies of the incoming and reflected particles (Figure 3). The reflected populations then hardly penetrate the DL so that the internal structure is determined by the accelerated particles, which may be treated as approximately cold beams. Poisson's equation may then be approximated by Equation (2.1), the form taken for a Langmuir DL. Also, the generalised Langmuir condition, Equation (3.4), is dominated by the dynamical pressures of the accelerated particles and may be approximated by Equation (2.4) the condition for the Langmuir DL. However the generalised Bohm criteria, Equation (3.8) depend on the low energy particle populations and should be verified in a complete model, giving the matching of the interior of the DL to its outer edges and the ambient plasma. To see how this is done it is useful to consider DL models which may be both weak and strong.

As a particular example the structure of a DL with both monoenergetic beams of ions and electrons and partially reflected Maxwellian distributions of ions and electrons may be considered (Swift 1975 [11]). The beams enter the DL so as to be accelerated by the potential. Such a model is in principle the same as a fluid model with isothermal reflected particles and cold fluid beams. For a fluid model with

finite temperature beams, there is also a kinetic equivalent since in one-dimension a collisionless non-viscous fluid component with ratio of specific heats $\gamma = 3$ is exactly equivalent to a water-bag distribution (Bertrand and Feix 1968 [26]). There is therefore a close relationship between the kinetic model under discussion (Swift 1975 [11]) and earlier fluid models of DL's (Block 1972[a] [20], Block 1972[b] [27]), and in particular the existence criteria may be compared directly. For the kinetic model, The Maxwellian distribution function of, for example, the electrons is built up of a central range of reflected particles and a high velocity tail which can penetrate the DL, both originating on the high potential side, and a matching tail from the low potential side, which is accelerated by the DL and must be assumed by some means to be adjusted to produce the complete Maxwellian distribution. (Since the Maxwellian has no mean drift velocity the distribution is symmetric, and the matching source on the low potential side can therefore be replaced by assuming a perfect reflection of all incoming electrons.) The same argument applies for the ions. It would be more reasonable to use truncated distributions from single sources, but if the DL potential is large the reflected part of the Maxwellian distribution can be expected to dominate, so that the problem of providing full Maxwellian distributions is not essential. The Sagdeev potential $V(\phi)$ for the model is given by,

$$\begin{aligned}
 -V(\phi) = & P_{0e} e^{-e(\phi_{DL}-\phi)/kT_e} + \frac{m_e i_e}{e} \sqrt{v_{0e}^2 + \frac{2e\phi}{m_e}} + \\
 & + P_{0i} e^{-e\phi/kT_i} + \frac{m_i i_i}{e} \sqrt{v_{0i}^2 + \frac{2e(\phi_{DL}-\phi)}{m_i}} \quad (4.1)
 \end{aligned}$$

The generalised Langmuir condition is then given by $V(0) = V(\phi_{DL})$ using this particular form of $V(\phi)$. There are in principle many free parameters, but here it will now be assumed that the DL has a very high degree of symmetry in order to find a restricted family of solutions. First it will be assumed that the thermal populations have the same temperatures, $T_i = T_e = T$, and a parameter $\alpha = kT/e$ introduced. Next suppose that the dynamical pressures of the incoming particle beams are equal and that this also applies to the emerging accelerated beams. This condition is rather artificial, and also implies that the thermal populations are

independently in pressure balance across the DL in order to satisfy the generalised Langmuir criterion. It also requires that the ion and electron currents, $i_{i,e}$, satisfy Equation (2.4), the Langmuir condition for a strong DL, even if the particular solutions are “weak”. The conditions on the incoming and accelerated beams also imply that,

$$2e\phi_{DL}/m_e v_{0e}^2 = 2e\phi_{DL}/m_i v_{0i}^2 \equiv \beta \quad (4.2)$$

and that the incoming beam densities are equal $n_{0eb} = n_{0ib} = n_{0b}$ as are even the accelerated densities. The condition that the thermal populations are in pressure balance, given that their temperatures are equal, reduces to $P_{0e} = P_{0i} = P_0$, also implying that their densities on either side of the DL are equal ($n_{0t} = P_0/kT$). The charge neutrality conditions at the edges of the DL, Equation (3.2) are now both satisfied if,

$$n_{0t} (1 - e^{-\alpha}) = n_{0b} (1 - [1 + \beta]^{-1/2}) \quad (4.3)$$

The remaining conditions, the Bohm criteria Equation (3.8), reduce to,

$$\frac{m_i v_{0i}^2}{kT} = \frac{m_e v_{0e}^2}{kT} > \left(\frac{n_{0b}}{n_{0t}} \right) \frac{1 + [1 + \beta]^{-3/2}}{1 + e^{-\alpha}} \quad (4.4)$$

In the limit of a strong DL Equations (4.3) and (4.4) reduce to $n_{0t} = n_{0b}$, i.e. on either side of the DL the densities of the incoming beam particles and the reflected particles dominate, and

$$v_{0i} > \sqrt{kT/m_i} \quad \text{and} \quad v_{0e} > \sqrt{kT/m_e} \quad (4.5)$$

These correspond to the Bohm criteria found for the equivalent fluid analysis (Block 1972[a] [20]), given that here $T_e = T_i$.

An extensive class of monotonic “analytical” DL’s has been presented, where the particle distributions correspond to Maxwellians for trapped (reflected) and shifted Maxwellians for free particles (Schamel and Bujarbarua 1983 [12]). The steady state Vlasov equations are satisfied by giving the distributions as functions of the particle energies $w_{i,e}$. For example the electron distribution is divided into free and trapped parts, $f_{ef}(x, v)$ and $f_{et}(x, v)$, which are only non-zero for free

($m_e v^2/2 > e\phi$) and trapped ($m_e v^2/2 \leq e\phi$) particles respectively. Without normalisation $f_{ef,t}$ are defined by,

$$\begin{aligned} f_{ef} &= c_e \exp \left[-m_e \left(-\sqrt{v^2 - 2e\phi/m_e} + v_{e0} \right)^2 / 2kT_e \right] \quad \text{for } v < -\sqrt{2e\phi/m_e} \\ f_{et} &= c_e e^{-m_e v_{e0}^2 / 2kT_e} \exp \left[-\left(\frac{1}{2} m_e v^2 - e\phi \right) / kT_{et} \right] \quad \text{for } \frac{1}{2} m_e v^2 \leq e\phi \\ f_{ef} &= c_e \exp \left[-m_e \left(+\sqrt{v^2 - 2e\phi/m_e} + v_{e0} \right)^2 / 2kT_e \right] \quad \text{for } v > +\sqrt{2e\phi/m_e} \end{aligned} \quad (4.6)$$

where c_e is a constant determined by the total electron density. T_e and T_{et} are the free and trapped electron temperatures. The free electrons are assumed to form a full Maxwellian distribution with a drift velocity $-v_{e0}$ to the right of the DL (at $\phi = 0$). An important feature is that the amplitudes of the distributions are adjusted so that they are continuous at $v = \pm\sqrt{2e\phi/m_e}$. It should be remarked that the free electrons travelling to the right must have a different source to the others. Thus the assumption that they form a complete Maxwellian with the other electrons at $\phi = 0$ is arbitrary. Moreover, since the sources are distinct, there may in a real situation be a near discontinuity in the distribution of electrons at $v = -\sqrt{2e\phi/m_e}$. However the choice defined by Equation (4.6) and a corresponding one for the ions permits a diversity of physically interesting DL solutions.

A great simplification resulting in a high degree of symmetry of the solutions may be achieved if it is assumed that $T_e = T_i = T$ and that the current ratio $i_e/i_i = \sqrt{m_i/m_e}$ (Schamel and Bujarbarua 1983 [12]). It then follows from the existence conditions that $T_{et} = T_{it}$, and that there are three dimensionless free parameters: $\alpha = T_i/T_{it} = T_e/T_{et}$, $u_0 = v_{i0}/\sqrt{kT_i/m_i} = v_{e0}/\sqrt{kT_e/m_e}$ and $\psi = e\phi_{DL}/kT$. For a given DL strength ψ there is a lower limit on the drift velocities, $u_0 \geq u_{0c}(\psi)$, and an upper limit on the trapped particle temperatures $\alpha^{-1} \leq 1/\alpha_c(\psi)$. Here it should be noted that by assumption (see Equation (4.6)) there are always trapped particles since for all trapped particle temperatures $f_{et} \geq \exp(-m_e v^2 / 2kT_e)$, equality being approached as $T_{ef} \rightarrow \infty$, and similarly for ions. However the limit on $T_{et} = T_{it}$ implies that there is a minimum density above that formally allowed by varying the parameters in Equation (4.6). Next, in all

cases $u_{0c} > 1$ so that the classical Bohm criteria are required (cf Equation (4.5)). From the analysis there is a parameter $G \equiv 2\alpha\psi/u_0^2$ which remains nearly constant ($= O(1)$). A consequence is that for weak DL's ($\psi \rightarrow 0$) $\alpha \rightarrow \infty$ so that the trapped particles become cold. It is argued that this may cause stability problems. The high drift velocities u_0 found for the models imply several types of two-stream instabilities which may be related to the fluctuations in real DL's. Such instabilities should produce some diffusion in phase space but need not destroy the structure. Here it may be noted that instabilities are relatively harmless if they propagate away from the DL (Raadu and Rasmussen 1988 [23]). The thickness Δ of the DL decreases in the range $0.5 \leq \psi \leq 5$ but for $\psi > 5$ scales as $\Delta \approx 5\sqrt{\psi}$ (in Debye lengths, λ_D). Solutions are also found when the condition $i_e/i_i = \sqrt{m_i/m_e}$ is relaxed. Then if $G \equiv 2\alpha\psi/u_0^2$ is defined for the ions it is still nearly constant. Typical drift velocities are higher implying stronger fluctuations for these more general solutions.

Two types of monotonic DL's related respectively to electron and to ion holes have been analysed and shown to have a small amplitude limit (Hudson *et al* 1983 [28], Kim 1983 [29], Schamel 1983 [30]). Since they may be related to linear electrostatic plasma modes they are termed slow electron acoustic DL's (SEADL's) and slow ion acoustic DL's (SIADL's). The distributions closely resemble those prescribed for the "analytical" DL's (cf Equation (4.6) and analogously for the ions), the only difference being that the trapped particle distributions may have the character of a hole i.e. have a minimum for zero velocity (Figure 3). Thus for the trapped electrons,

$$f_{et} = c_e e^{-m_e v_{e0}^2 / 2kT_e} \exp \left[-\beta \left(\frac{1}{2} m_e v^2 - e\phi \right) / kT_e \right] \quad \text{for} \quad \frac{1}{2} m_e v^2 \leq e\phi \quad (4.7)$$

where β may be both positive, reducing to the previous case with $T_{et} = T_e/\beta$, or negative giving a hole distribution. Similarly for the trapped ions,

$$f_{it} = c_i e^{-m_i v_{i0}^2 / 2kT_i} \exp \left[-\alpha \left(\frac{1}{2} m_i v^2 - e(\phi_{DL} - \phi) \right) / kT_i \right] \\ \text{for} \quad \frac{1}{2} m_i v^2 \leq e(\phi_{DL} - \phi) \quad (4.8)$$

where α may be both positive or negative. For weak SEADL's a series expansion leads to a small amplitude approximation for the Sagdeev or "classical" potential, in dimensionless form (Schamel 1983 [30]),

$$-V(\Phi) = B_2 \left[\Phi_{DL}^{1/2} \Phi (\Phi_{DL} - \Phi) - \frac{2}{5} \left(\Phi_{DL}^{5/2} - \Phi^{5/2} - [\Phi_{DL} - \Phi]^{5/2} \right) \right] + \frac{C_1}{3} \Phi^2 \left(\Phi_{DL}^{1/2} - \Phi^{1/2} \right)^2 \quad (4.9)$$

where the coefficients B_2 and C_1 depend on the drift velocities v_{i0} and v_{e0} of the free distributions, on the temperature ratio $\theta = T_e/T_i$, and on α . The generalised Langmuir condition $V(\phi_{DL}) = V(0)$ (also termed "nonlinear dispersion relation") has already been imposed to reduce $V(\Phi)$ to this form. If the problem is restricted to the case of no ion current, $v_{i0} = 0$, the temperature ratio must satisfy $T_i/T_e > 3.5$ and the electrons must carry a finite current, $v_{e0} \geq 0.924\sqrt{2}\sqrt{kT_e/m_e}$. In the special case where $B_2 \ll |C_1| \sqrt{\Phi_{DL}}$, which implies $\alpha \approx 1$ and $v_{i0} \approx 0$ i.e. that the ion distributions add up to a full Maxwellian with no drift velocity in the reference frame of the DL, Equation (4.9) can be integrated to give the DL profile (Kim 1983 [29]),

$$\Phi(x) = \frac{\Phi_{DL}}{4} \left[1 + \tanh \left(\sqrt{C_1 \Phi_{DL}} / 24x \right) \right]^2 \quad (4.10)$$

It is necessary that $\beta < -0.71$ i.e that the hole in the trapped electron distribution is sufficiently deep. Corresponding results may be obtained for SIADL's interchanging the conditions on the ions and electrons. For example if $v_{e0} = 0$ then $T_e/T_i > 3.5$ and $\alpha < -0.71$.

The physical significance of the SEADL's and the SIADL's depends on whether the asymptotic injected distributions can be imposed in a natural way. For example in the case of a SIADL (Figure 3) the ions have a "tuning fork" phase space distribution. On the low potential side the distribution reduces to a full Maxwellian, i.e. the "handle". The ions with positive and negative velocities originate from separate sources at $x = \pm\infty$ which have to be matched if a full Maxwellian is to be achieved. On the low potential side the ion population consists of free accelerated and injected ions, and of mainly high velocity ions reflected by the DL (i.e. ions with $|v| \approx$ and $< \sqrt{2e\phi_{DL}/m_i}$) forming the "prongs".

The reflected ions must be supplied by injecting ions with an energy close to but under $e\phi_{DL}$. Thus to produce a SIADL accelerated ions must be produced from the low potential source. Corresponding arguments apply to the SEADL's. Appropriate conditions have been argued to exist for both types of slow acoustic DL's in particular experiments on DL's (Schamel 1983 [30]).

Non-monotonic DL's in the form of asymmetric electron or ion holes are also possible (Schamel 1982[a] [15], 1982[b] [31] and 1986 [32]). In the case of electron holes these have a potential maximum on the high potential side corresponding to trapped electron orbits with low phase space density. The asymmetric ion holes (Figure 3), having a potential minimum on the low potential side, may be identified with ion acoustic DL's.

The nature of the negative potential dip associated with the ion-acoustic DL's formed in a plasma simulation (Sato and Okuda 1980 [33]) was investigated using analytical models (Hasegawa and Sato 1982 [34]). A high ion to electron temperature ratio ($T_e \gg T_i$) allows the formation of positive potential solitons. For the same conditions it is argued that a local adiabatically generated negative potential will tend to form a hole in the ion phase space distribution. An equilibrium is possible where fluctuations lead to a partial trapping of ions. These ions provide local net positive charge regions on the edges of the potential dip compensating the negative charge, due to the paucity of ions in the centre. Symmetric ion hole solutions are found assuming a Maxwellian electron distribution. The ions were also assumed to have a Maxwellian distribution on the untrapped orbits with $w_i \geq 0$ (where $w_i = m_i v^2/2 + e\phi$), as well as for a range of weakly trapped orbits with $-\phi_0 < w_i < 0$ determined by the level of the potential fluctuations ϕ_0 responsible for the trapping. As for the DL models a first integral of the stationary Vlasov-Poisson equations is obtained in the same form as Equation (2.1.4) and a Sagdeev potential determined (cf Equation (2.1.3)). There is now however only one Bohm condition, $V''(0) < 0$, at the edges of the symmetric potential dip. At the potential minimum $\phi = -\phi_m$ there is a negative charge ($V'(-\phi_m) < 0$), and since $E = 0$ there it is required that $V(-\phi_m) = V(0)$. This requirement leads to an upper limit to T_i/T_e for a given degree of ion trapping, $e\phi_0/kT_i$ e.g.

$T_i/T_e < 0.089$ if $e\phi_0/kT_i = 2$. Also $\phi_m = O(kT_e/e)$ for $\phi_0 = O(kT_i/e)$. A further integration of Equation (2.1.3) with $V(\phi)$ satisfying the present conditions leads to a symmetric potential dip.

Asymmetric ion holes require an asymmetry between the trapped electrons distributions on either side of the potential minimum. It was assumed that the electrons passing the potential minimum form a beam on the high potential side and that the density of the Maxwellian electrons is different there. The ions are assumed to have a constant density for positive potentials, on an electron inertial time scale. Asymmetric ion hole solutions are found which are non-stationary beyond an ion inertial time scale, as observed in computer simulations.

There is a class of DL models, for which all the ions form a simple cold fluid or beam distribution and the electrons have a non-Maxwellian distribution. Here they are denoted "cold-ion" DL's (see Figure 3). As a specific example the case of a double Maxwellian distribution may be chosen as a suitable non-Maxwellian distribution of electrons. The Sagdeev potential (Equations (2.1.3) and (2.1.4)) is then given by,

$$-V(\phi) = n_{h0}kT_h e^{+e\phi/kT_h} + n_{c0}kT_c e^{+e\phi/kT_c} + m_i v_{i0} / \sqrt{v_{i0}^2 - 2e\phi/m_i} \quad (4.11)$$

where $T_{h,c}$ and $n_{h,c}$ are the temperatures and densities of the hot and cold Maxwellian components of the electron distribution, and v_i the ion velocity. The subscript 0 indicates the values at $\phi = 0$. Both large amplitude and weak solitary waves may be found for this case, and depending on the relative proportions of the hot and cold electron components even negative potential solutions, i.e density dips, may be found (Buti 1980 [35], Nishihara and Tajiri 1981 [36]). At the transition between negative and positive solitary wave solutions the Sagdeev potential takes the correct form for a DL solution. Thus for $T_h/T_c = 20$ and $n_{h0}/n_{c0} = 3.5$ there is a DL with potential $\phi_{DL} = -0.55kT_h/e$ if the ion beam velocity $v_{i0} = 1.2c_s$ (Nishihara and Tajiri 1981 [36]) (c_s is the effective ion acoustic velocity defined in Section 5.2), or equivalently if the DL moves with this velocity in the frame of reference of the ions on the high potential side (N.B. $\phi_{DL} < 0$). The properties of these large amplitude DL solutions are only explicitly

treated elsewhere (Bharuthram and Shukla 1986 [37]). Small amplitude solutions have been surveyed (Goswami and Bujarbarua 1985 [38]). In the limit of weak DL's the case of a general non-Maxwellian electron distribution has been analysed and may be treated dynamically using the modified Korteweg-de Vries equation (see references in Section 5.2). For the two temperature case the treatment has been extended to find a multi-dimensional KdV-equation for a magnetised plasma (Bharuthram and Shukla 1986[b] [39]). Large amplitude solutions are found for a more general one-dimensional case with finite ion temperature, and an mKdV-equation is again found in the weak perturbation limit (Baboolal *et al* 1988 [40]).

In general steady state weak DL models are found by expanding the ion and electron contributions to the Sagdeev potential, $V(\phi)$ as power series and solving Equation (2.1.3) within this approximation. It is important to note that the particle distribution functions may well have discontinuities of varying degree at the separatrix between particles originating from distinct sources at $x = \pm\infty$. For example, for a DL with a potential in the range $0 \leq \phi \leq \phi_{DL}$, the separatrix for the electrons at $\phi = 0$ is at $v = 0$. Since the electrons travelling to the left and to the right have different sources the distribution can have a discontinuity there and as a consequence it is found that $V_e(\phi)$, The electron contribution to $V(\phi)$ must be expanded as a power series in $\phi^{1/2}$,

$$V_e(\phi) = a_{e1}\phi^{3/2} + a_{e2}\phi^2 + a_{e3}\phi^{5/2} + a_{e4}\phi^3 + a_{e5}\phi^{7/2} + a_{e6}\phi^4 + \text{etc} \quad (4.12)$$

The case where the sources contribute two half Maxwellian's has been treated in this way (Torvén 1986 [41]), and it is noted that even if the temperatures and densities at the sources are the same there will still be a discontinuity at the separatrix since the sources are assumed to be at different potentials ($\phi = 0, \phi_{DL}$). Only powers up to $\phi^{5/2}$ are retained, and assuming cold ions these only contribute to the ϕ^2 term i.e. only the linear response of the ions is needed. In a suitable moving frame of reference and applying the standard boundary conditions the Sagdeev potential takes the form,

$$V(\phi)/V_0 = \frac{2}{5}\phi^{3/2} \left(\phi_{DL}^{1/2} - \phi^{1/2} \right)^2 \quad (4.13)$$

This is shown in Figure 4 where the minimum corresponding to the maximum electric field (cf Equation (2.1.3)) in the DL is indicated, as well as the resulting DL profile. The DL solution is,

$$\phi(x) = \phi_{DL} \tanh^4 [(x - v_\phi t)/d] \quad \text{for } x \leq v_\phi t$$

$$\phi(x) = 0 \quad \text{for } x > v_\phi t \quad (4.14)$$

where the relationship between v_ϕ (the phase velocity in the stationary reference frame), d and ϕ_{DL} may be found in the original reference (Torvén 1986 [41]). A mirror reflection of Equation (4.14) ($x \rightarrow -x$) gives a second solution propagating in the opposite direction, with a potential profile as shown in Figure 4. The amplitude of the DL is proportional to the discontinuity in the distribution function. With the same degree of approximation an evolution equation of KdV-type is also found,

$$2\psi_\tau - (\frac{3}{5}\psi^{1/2} + \psi^{3/2})_\xi - \psi\xi\xi\xi = 0 \quad (4.15)$$

where ψ is a dimensionless potential and where ξ and τ are stretched space and time coordinates.

If there is no jump discontinuity in the electron distribution function at the separatrix $a_{e1} = 0$ in Equation (4.12), but if there is a discontinuity in the derivative of the distribution $a_{e3} \neq 0$. Similar arguments apply for the ions leading to an expansion of the ion contribution $V_i(\phi)$ as a power series in $(\phi_{DL} - \phi)^{1/2}$. This explains the forms of the Sagdeev potentials found for weak SEADL's and SIADL's (Schamel 1983 [30]) (e.g. Equation (4.9)), which are therefore dependent on the existence of discontinuities in the derivatives of the distribution functions at the separatrices.

If there are no discontinuities in the distributions or their derivatives up to second order at the separatrices, the Sagdeev potential may be expanded up to ϕ^4 in powers of ϕ giving,

$$-V(\phi) = A_1\phi^2 + A_2\phi^3 + A_3\phi^4 \quad (4.16)$$

in suitably normalised form. This also applies if there are any fluid components. The coefficients A_j contain parameters relating to the component particle species and the chosen moving frame of reference. If there is a DL solution with potential ϕ_{DL} , charge neutrality at $\phi = \phi_{DL}$ requires $V'(\phi_{DL}) = 0$ (Equation (3.2)) and the generalised Langmuir condition (Equation (3.3)) requires $V(\phi_{DL}) = 0$. Therefore for a DL solution Equation (4.16) must have a double root for some $\phi \neq 0$, i.e. $A_2 = 4A_1A_3$. The Bohm criteria or the condition that $V(\phi) < 0$ are then all satisfied if $A_1 > 0$. The DL solution is then of the form,

$$\phi(x) = -\frac{1}{2} \left(\frac{A_2}{2A_3} \right) (1 - \tanh[(x - v_\phi t)/d]) \quad (4.17)$$

An example is given by the case of weak cold-ion DL's (Torvén 1981 [42], Goswami and Bujarbarua 1985 [38]). An essential requirement on the parameters (Raadu and Rasmussen 1988 [23]) is given by the condition that the DL potential is in fact small as assumed in using the truncated power series expansion,

$$|A_2| \ll |A_3| \quad (4.18)$$

In the case of the weak cold-ion DL's this implies a relation between the ratio of the temperatures and the ratio of the densities of the hot and cold electron components (see Section 5.2).

Several DL models motivated by real physical situations have appeared in the literature. For example, an auroral DL model has been derived where the particle populations originate directly from the magnetosphere and the ionosphere as well as indirectly through implicitly assumed scattering processes leading to a trapped secondary population of energetic electrons between the DL and the ionosphere (Kan and Lee 1980 [24]). Secondary ions produced by ionisation on the high potential side of a DL are included in a model applying to a partially ionised laboratory plasma with only one source on the low potential side (Andersson and Sörensen 1983 [43]).

5. Double layer dynamics

Experiments on DL's (Section 7) clearly demonstrate that they are in reality inherently dynamical structures. This view is supported by the results of computer

simulations (Section 6) which allow a treatment of the time-dependent non-linear behaviour. Weak DL's are most nearly steady structures, and can in certain circumstances have the character of solitons (Section 5.2). In the case of weak DL's it has also been possible to find various evolution equations of mKdV-type (Torvén 1981 [42], Nishihara *et al* 1982 [44], Chanteur *et al* 1983 [45], Torvén 1986 [41]). Strong DL's are less amenable to analytical treatment. Since they produce beams of accelerated particles instabilities are expected in the ambient plasma which may react back on the DL itself leading to fluctuations as observed in the laboratory. The Bohm criteria in general require drift motions for incoming particles which can drive electrostatic instabilities, and this may also account for the observed dynamical behaviour. The existence of such current driven instabilities was once believed to prevent the formation of DL's, the unstable growth leading instead to a state of plasma turbulence (Smith and Priest 1972 [46]). However there is now abundant evidence that DL's do form under such conditions, although they are often embedded in a strongly fluctuating plasma. Current driven instabilities provide a mechanism for the formation of DL's. This has been shown in computer simulations (Section 6). The dynamics of DL formation is also discussed in the following subsections.

The strictly local stability of a one dimensional DL model has been treated (Knorr and Goertz 1974 [16]) by applying the Penrose criterion (Penrose 1960 [47]) for stability of an infinite uniform plasma. The method is only an approximation when applied to the interior of the DL and may give some indication of instabilities, particularly if the the wavelengths of the instabilities can be shown by an additional analysis to be small compared to the spatial variation of the structure. Applied asymptotically to the ambient plasmas on either side of the DL, the criterion indicates whether fluctuations may develop possibly perturbing the DL. The particular models considered were found for a given potential profile, $\phi(x) = \phi_0 \tanh(x/\xi)$ (i.e. $\phi_{DL} = 2\phi_0$ and $-\phi_0 < \phi < \phi_0$), using an Abel inversion to determine the distribution of reflected electrons (Equation (2.1.10)) when the

other distributions are specified, as described in Section 2.1. The corresponding Sagdeev potential (see Equation (2.1.3)) is also predetermined,

$$V(\phi) = -\frac{1}{2}\epsilon_0 \frac{\phi_0^2}{\xi^2} \left[1 - \left(\frac{\phi}{\phi_0} \right)^2 \right]^2 \quad (5.1)$$

The Penrose analysis is greatly simplified if it is related to the form of $V(\phi)$, as has been done in an earlier analysis of the asymptotic stability of collisionless shock waves (Biskamp 1969 [48]). The stability test is made for a minimum in the combined distribution, $f_c(v) = f_e(v) + (m_e/m_i)f_i(v)$, at $v = 0$. The Penrose condition for stability at $v = 0$ is,

$$\int_{-\infty}^{+\infty} \frac{f_c(v) - f_c(0)}{v^2} dv \equiv \int_{-\infty}^{+\infty} \frac{1}{v} \frac{\partial f_c(v)}{\partial v} dv < 0 \quad (5.2)$$

Comparing with Equation (3.11) and using Equation (5.1) the condition for stability is simply,

$$V''(\phi) = 2\epsilon_0/\xi^2 [1 - 3(\phi/\phi_0)^2] < 0 \quad (5.3)$$

At the edges of the DL where, $\phi = \pm\phi_0$, this is identical to the Bohm criteria and is satisfied since there $V''(\pm\phi_0) = -4\epsilon_0/\xi^2 < 0$. Thus, if there is a minimum at $v = 0$ in the combined distribution function at the edges of a DL, the Bohm criteria imply asymptotic Penrose stability. (This is a somewhat paradoxical result in view of earlier remarks, but it should be recalled that the unstable distributions of injected particles often required by the Bohm criteria are associated with further minima in the combined distribution for $v \neq 0$.) There are in general other minima for $v \neq 0$ associated for example with the existence of particle beams from strong DL's which can give asymptotic instability. Returning to the present case it may also be seen for example that at the centre of the DL ($\phi = 0$), where $V(\phi)$ has a minimum, the Penrose criterion implies local instability. This is a general property of DL's since the Sagdeev potential must always have a local minimum where $V'' > 0$ somewhere inside the DL.

The full non-local stability analysis has been made for a particular case of a nonlinear electrostatic wave (Infeld 1972 [49]), and a general study has been made for stationary solutions of the Vlasov-Poisson equations (Lewis and Symon

1979 [50]). In principle it should be possible to extend such methods to DL models. An instability has been shown to exist for trapped electron plasma waves within a DL (Wahlberg 1979 [51]). The trapping is due to the density depression within the DL. An electron distribution consisting of a water bag with a cold beam at one edge, which is locally Penrose stable, is considered. The ions are held fixed so that only the electron response is involved. The trapping involves wave coupling due to the spatial inhomogeneities. The plasma mode travelling towards the low potential side couples to the fast beam mode, whereas both beam modes couple to the oppositely directed plasma mode in the interior. At the high potential edge the modes considered become evanescent, so that the fast beam mode is reflected as a slow beam mode. The slow beam mode is in turn reflected at the low potential edge as a fast mode, completing the coupling cycle. The beam provides free energy and there are negative energy modes capable of feeding oscillations to produce an instability. The analysis is further developed for the case of very strong DL's. There is a critical thickness above which the instability appears, with a high growth rate approaching the plasma frequency. Since the waves are trapped within the DL the instability is absolute rather than convective. It is suggested that the instability may in fact be important for the formation of the DL, and it is argued that there should be a drift velocity threshold at $v_d \approx \sqrt{kT_e/m_e}$. Related work has recently been reported (Teichmann 1987 [52]).

A stability analysis for weak slow ion acoustic and slow electron acoustic DL's (SIADL's and SEADL's) has established linear stability with respect to one-dimensional perturbations and instability for two-dimensional electrostatic perturbations in both cases (Schamel 1983 [30]). The two-dimensional analysis leads to an inhomogeneous eigenvalue problem, where the frequency of the perturbation is the eigenvalue and the perpendicular wavenumber may be given. For the unstable modes the typical transverse wavelength is comparable to the length scale of the inhomogeneity. The growth rate normalised to ω_{pe} scales as $(e\phi_{DL}/kT_e)^{1/4}$.

Simulated and laboratory DL's normally have significant motions with respect to the surrounding plasma. The movement can be interpreted by applying a steady state analysis in the moving frame of reference (Singh 1979 [53], Singh

and Schunk 1982 [54], Raadu and Rasmussen 1984 [55]). Thus for a strong DL it may be argued that if the Langmuir condition $|i_e/i_i| = |\sqrt{m_i/m_e}|$ (Equation (2.4)) is not satisfied in the rest frame, the DL will begin to move so as to adjust the current balance to the correct value in the moving frame of reference. The typically observed motion of a DL towards the high potential side would then be caused by the need to increase the ion current to meet the Langmuir condition (Raadu and Rasmussen 1984 [55], Raadu and Rasmussen 1988 [23]). Initially the ions on the low potential side will lag behind, and the electrons will set up an ambipolar electric field to accelerate them. This results in a potential dip on the low potential side of the DL, which reflects electrons reducing the electron current. Consequently from the Langmuir condition the DL slows down and the dip begins to disappear. For a strong DL there are no one-dimensional solutions with a steady state potential dip. This is due to the reduced particle pressure on the high potential side of the dip as a result of the electron reflection and the non-zero electric field. Both these effects make it impossible to satisfy Equation (2.1.4), the stress balance condition, both at the low potential edge of the DL and at the same potential inside the DL. However a potential minimum may be set up self consistently on an intermediate time scale when the ions may be regarded as fixed (Raadu and Rasmussen 1988 [23]).

As well as propagating a DL may also evolve dynamically, expanding or contracting by a variety of mechanisms (Carlqvist 1984 [56]). It can for example be argued that the Langmuir condition leads to a movement of only the edge of the DL into the high potential plasma, while the low potential edge is at rest, resulting in an expansion of the DL. The interaction of the accelerated particle beams with the ambient plasma can lead to a dynamical evolution of the DL itself due to the resulting changes in the boundary conditions. Beam-plasma collisions result in a transfer of momentum to the plasma, tending to push plasma away from the DL (Carlqvist 1984 [56]). For a strong DL the dynamical pressures of the ion and electron beams, p_{ib} and p_{eb} , are

$$p_{ib} = p_{eb} \equiv p_b = m_e i_{eb} / e \sqrt{2e\phi_{DL}/m_e} \quad (5.4)$$

where the electron beam current density, i_{eb} , is close to the total current density in the ambient plasma. Thus if the current drift velocity in the plasma is v_d and the plasma pressure, p_0 , is $n_0 kT_0$ it follows that,

$$\frac{p_b}{p_0} = \frac{v_d}{\sqrt{kT_0/m_e}} \sqrt{\frac{2e\phi_{DL}}{kT_0}} \quad (5.5)$$

If, for example, the drift velocity is comparable to the ion-acoustic velocity, $\sqrt{kT_0/m_i}$, the beam pressure will dominate the plasma pressure for strong double layers such that, $e\phi_{DL} \gg (m_i/m_e)kT_0$. If v_d is close to the electron thermal velocity as in some laboratory experiments the large factor (m_i/m_e) should be replaced by unity and the condition becomes identical to that for a strong DL. It can be argued that in such circumstances the plasma may be swept away by the particle beams leading to an expansion of the DL (Carlqvist 1984 [56]), although the process must in reality be complex due to the large number of free parameters and complicating factors such as the geometry. Alternatively it may be argued that the DL does not itself expand but that the adjacent regions where the plasma density is being reduced by the action of the beams form dynamical pre-sheaths onto which the DL must be matched. These strongly modified matching conditions must require an altered internal structure for the double layer. In particular the reduced densities imply larger current drift velocities making it easier to fulfill the generalised Bohm criteria.

5.1 Formation via instabilities

The Bohm criteria for strong DL's, see for example Equation (4.5), often imply that there should be current driven instabilities in the ambient plasmas. Arguments have been presented that such instabilities are not damaging for the DL but rather account for the fluctuations seen in real DL's, particularly at the low potential edge (Schamel and Bujarbarua 1983 [12], Raadu and Rasmussen 1988 [23]). Instead the instability of the current carrying plasma can lead to the formation of DL's. The initial stage involves an "evacuation" mechanism as first proposed for a two-stream situation (Alfvén and Carlqvist 1967 [7]). In that case it was argued that the electrons because of their low mass should show a quasi-steady response to changes in the ion density. If there is a region with reduced density,

n , the electrons must be accelerated there so as to reduce their density, and an electric field E is set up such that,

$$E \equiv -\frac{\partial \phi}{\partial x} = -\frac{m_e v_e}{e} \frac{\partial v_e}{\partial x} = -\frac{\partial}{\partial x} \left(\frac{m_e i_e^2}{2e^3} \frac{1}{n^2} \right) \quad (5.1.1)$$

where i_e is the electron current, and assuming that the region is sufficiently extended so that the plasma is quasi-neutral ($n_e \approx n_i \equiv n$). This electric field is directed out of the density depression, and accelerates ions outwards resulting in a further reduction of the density. Thus there is a tendency to produce local regions with extremely low densities, which should favour the formation of a DL. The mechanism is identical to that of the cold two-stream instability in the long wavelength regime, non-dispersive regime (Carlqvist 1973 [57]). From Equation (5.1.1) it may be seen that the potential ϕ has a maximum at the centre of the density depression. The original analysis can be extended to include the effects of finite particle temperatures (Carlqvist 1972 [58]). The evacuations are produced by electrostatic current driven instabilities in a finite temperature plasma, provided that the critical current is exceeded (Raadu and Carlqvist 1981 [59]). The linear development can be followed by representing the initial density depression by a superposition of unstable waves. In the long wavelength these all have the same phase velocity and the growth rate is linearly proportional to the wavenumber. An initial depression deepens, becomes narrower and the evacuated plasma produces density maxima on either side. For the ion-acoustic case the electric field is,

$$E \equiv -\frac{\partial \phi}{\partial x} \approx -\frac{\partial}{\partial x} \left(\frac{kT_e}{e} \log n \right) \quad (5.1.2)$$

and there is a potential minimum at the centre of the density depression. The driving mechanism for the instability, which occurs for a relative drift velocity $v_d > \sqrt{kT_e/m_i}$, is then inverse Landau damping and depends on kinetic effects. The non-linear development in the case of cold electrons and warm ions has been treated numerically solving the Vlasov equation for the ions and assuming quasi-steady electron motion (Raadu and Carlqvist 1981 [59]). The evacuations continued the linear trend and were followed to a central density of less than 30% of the ambient plasma.

An exact non-linear analysis of the mechanism has been made for a fluid description (Galeev *et al* 1981 [60]). The basic equations, assuming isothermal perturbations, are,

$$m_i \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right) = -e \frac{\partial \phi}{\partial x} - \frac{kT_i}{n} \frac{\partial n}{\partial x}$$

$$\frac{\partial n}{\partial t} + \frac{\partial nv}{\partial x} = 0$$

$$e\phi = \frac{m_e i_e^2}{2e^2} \left(\frac{1}{n^2} - \frac{1}{n_0^2} \right) + kT_e \log \frac{n}{n_0} \quad (5.1.3)$$

assuming quasi-neutrality and where n_0 is the unperturbed density and i_e the electron current. The solution is found by a Legendre transformation, introducing the velocity potential ψ by setting $v = \partial\psi/\partial x$. The equation of motion for the ions may be integrated to give,

$$\frac{\partial \psi}{\partial t} + \frac{v^2}{2} + \left[\frac{e\phi}{m_i} + \frac{kT_i}{2m_i} \log \frac{n^2}{n_0^2} \right] = 0 \quad (5.1.4)$$

where the term in square brackets is defined to be w , the effective "enthalpy", which may be used together with v to give two new independent variables. The generating function for the transformation is

$$\chi = \psi - xv + t(v^2 + w) \quad (5.1.5)$$

and the transformation is defined using,

$$d\chi = \frac{\partial \chi}{\partial v} dv + \frac{\partial \chi}{\partial w} dw = -(x - vt)dv + t dw \quad (5.1.6)$$

Finally an equation for the generating function χ is found from the ion continuity equation,

$$\frac{\partial \chi}{\partial w} + \frac{n}{dn/dw} \frac{\partial^2 \chi}{\partial w^2} - \frac{\partial^2 \chi}{\partial v^2} = 0 \quad (5.1.7)$$

Since this is a linear equation the original non-linear equations (5.1.3) are now reduced to a linear form. For simplicity the Buneman instability regime ($m_e i_e^2 / 2n^2 e^2 \gg kT_e + kT_i$) was then assumed and the general solution found.

As specific examples an initial harmonic perturbation and an initial localised perturbation were used. In the latter case an initial density distribution of the form,

$$n(x, 0) = n_0 (1 - \frac{1}{2}\epsilon \operatorname{sech}(x/L)) \quad (5.1.8)$$

and an initial zero velocity were assumed. It was assumed that $\epsilon \ll 1$, but the perturbation grows to large amplitude so that the analysis is fully non-linear. The final solution may here be written in a compact form,

$$\frac{n_0}{n} - 1 = \frac{1}{2}\epsilon \operatorname{Re} \operatorname{sech} \left(\frac{x'}{L} + \frac{i\pi t}{2t_0} \right)$$

$$v = \frac{1}{2}\epsilon \sqrt{\frac{m_e}{m_i}} \left(\frac{i_e}{n_0 e} \right) \operatorname{Im} \operatorname{sech} \left(\frac{x'}{L} + \frac{i\pi t}{2t_0} \right)$$

$$\text{where } x' = x - \arctan \left[\tanh \left(\frac{x}{L} \right) \tan \left(\frac{\pi t}{2t_0} \right) \right] \quad (5.1.9)$$

and there is an explosive singularity at $x' = 0$ when $t = t_0 = \pi L / [2\sqrt{m_e i_e^2 / m_i n_0^2 e^2}]$, where $v \rightarrow \infty$ and $n \rightarrow 0$. In particular from Equation (5.1.9) the minimum density at the centre of the dip is given by

$$n_{min} = n_0 / (1 + \frac{1}{2}\epsilon \sec(\pi t / 2t_0)) \quad (5.1.10)$$

These results have been compared with previously obtained self similar solutions and a computer simulation (Belova *et al* 1980 [61]). The simulation is performed for a system length of $100 \lambda_D$ for an isothermal plasma, with continuous injection of particles with shifted Maxwellian distributions such that the Buneman instability threshold is exceeded ($u_e = 1.8\sqrt{T_e/m_e}$). DL's form with an explosive rise of the potential difference ($\Delta\phi \sim (t_0 - t)^{-2}$), and are short lived. The explosive growth of the potential agrees with that obtained in the nonlinear Buneman analysis (Galeev *et al* 1981 [60]) although in the latter case no net potential difference was developed. An external current generator or applied electric field would be needed. Also the Buneman analysis was strictly quasineutral, and this should break down during the explosive development of the density depression. A generalisation of the two fluid analysis has been carried out dropping the quasineutrality condition (Davey 1983 [62]), but the stage of DL formation was not reached.

Particle simulation studies have confirmed the formation of weak DL's as the result of the growth of ion-acoustic instabilities (see Section 6.2). The formation of such ion-acoustic DL's is associated with a growing potential minimum which reflects electrons. As this minimum grows the density must also decrease, and there is therefore an evacuation process accompanying the DL formation. The development can be described by a modified form of the Korteweg-de Vries equation with a term including the kinetic effects of electron reflection by a potential minimum (Chanteur *et al* 1983 [45]),

$$\begin{aligned} \frac{\partial \phi}{\partial t} + \frac{v_b}{2} \frac{\partial}{\partial x} \left[\left(1 - \frac{3}{2} v_d^2 \right) \phi^2 + (1 + v_d^2) \frac{\partial^2 \phi}{\partial x^2} + \right. \\ \left. + \operatorname{sgn}(x - x_m) \sqrt{2/\pi} v_d \phi_m Y(\phi/\phi_m) \right] = 0 \end{aligned}$$

with $Y(\theta) = \sqrt{1 + \theta} - \theta \sinh^{-1} \sqrt{-1 - \theta^{-1}}$ (5.1.11)

where v_d is the drift velocity of the electrons, which have a Maxwellian distribution. A frame of reference moving with velocity $v_b = \sqrt{kT_e/m_i}(1 - v_d^2)^{-1/2}$ is taken, in which the potential varies slowly. It is assumed that the potential ϕ has a unique minimum $-\phi_m$ at x_m . Similar equations have also been derived elsewhere in the literature (Karpman *et al* 1979 [63], Nishihara *et al* 1982 [44]). The asymmetry around $x = x_m$ given by the factor $\operatorname{sgn}(x - x_m)$ comes from those electrons in the shifted Maxwellian distribution that have sufficiently low energy to be reflected by the potential minimum $-\phi_m$. There is an overpopulation of such electrons travelling in the direction of the mean drift velocity v_d , and it is this which leads to the growth of the potential according to the energy equation,

$$\frac{\partial}{\partial t} \int \phi^2 dx = \sqrt{32/\pi} v_d \phi_m^2 \int_{-1}^0 Y(\theta) d\theta \quad (5.1.12)$$

An estimate of the evolution of an initial perturbation gives a growth rate,

$$\gamma = \frac{1}{\phi_m} \frac{d\phi_m}{dt} = \sqrt{\frac{m_e}{m_i}} \frac{v_d}{a(t)}$$

$$\text{where } a(t) = \sqrt{2\phi(x_m, t)/\phi''(x_m, t)} \quad (5.1.13)$$

assuming a quadratic fit to the potential minimum. There is also a rapid localisation of the minimum. Effects due to a finite time of flight for very slow electrons and those just reflected at the potential minimum are neglected. Comparison with a numerical simulation of the formation of ion-acoustic DL's shows good agreement for the energy and momentum losses of the electrons.

5.2 mKdV description

The fully non-linear time dependent dynamics of weak DL's is intrinsically difficult to deal with. In general the electrostatic behaviour of a plasma may be described by solving Poisson's equation together with the time dependent Vlasov equations for each species. This system of equations is strongly nonlinear for large disturbances of the plasma. However for small but finite perturbations simplifications can be made. Thus weakly nonlinear ion acoustic waves may be described by the Korteweg de Vries (KdV) equation (Washimi and Taniuti 1966 [64]). In standard form this may be written,

$$v_t \pm 6v v_x + v_{xxx} = 0 \quad (5.2.1)$$

Although the derivation involves losing certain aspects of the physics, standard methods may then be used to follow the time dependent behaviour. The modified Korteweg-de Vries (mKdV) equation has also been shown to apply to various plasma physical problems. There are two distinct forms of the equation

$$u_t \pm 6u^2 u_x + u_{xxx} = 0 \quad (5.2.2)$$

which are denoted by mKdV[±] according to whether the nonlinear term has a positive or negative sign. The mKdV⁺ equation, which can support both positive and negative soliton solutions, may be applied to the case of a plasma with a negative ion component (Watanabe 1984 [65]). The mKdV⁻ equation may be used to describe weak perturbations of a plasma with cold ions and a non-Maxwellian electron distribution (Torvén 1981 [42], Tajiri and Nishihara 1985 [66]) and supports both solitons and DL solutions. The DL solutions behave as true solitons, as has been demonstrated in connection with a different physical model. (Perelman

et al 1974[a] [67], Perelman *et al* 1974[b] [68]). The existence of (shock like) DL solitons is a special feature of the mKdV⁻ equation; DL solitons do not occur for the mKdV⁺ equation. The mKdV⁻ solitons have the general form,

$$u = (a/v) - a [\tanh(\eta + \Delta) - \tanh(\eta - \Delta)] \quad (5.2.3)$$

where $\eta = ax + (6v^2 - 4)a^3t + \eta_0$ and $\Delta = \frac{1}{4}\log[(1+v)/(1-v)]$. The DL solitons appear as limiting forms for $v \rightarrow \pm 1$,

$$u = \pm a \tanh(ax + 2a^3t + \eta_1) \quad (5.2.4)$$

The existence of a regime of weak DL's governed by the mKdV⁻ equation was first established for a general non-Maxwellian distribution of electrons fulfilling certain conditions (Torvén 1981 [42]). As a specific example it is of interest to consider a double Maxwellian distribution consisting of a hot component with temperature T_h and density n_h , and a cold component with temperature T_c and density n_c (Raadu and Chanteur 1986 [69], Chanteur and Raadu 1987 [70]). Such a distribution may be expected when plasmas of separate origin, e.g. cold ionospheric plasma and magnetospheric plasma in the Earth's magnetosphere, interpenetrate; or for example in experiments where energetic primary electrons ionise a neutral gas to produce a plasma with secondary lower energy electrons. An effective temperature T_{eff} may be introduced such that $T_{eff}^{-1} = \alpha_c T_c^{-1} + \alpha_h T_h^{-1}$, where $\alpha_{c,h}$ are the fractional densities in the undisturbed state i.e. $\alpha_{c,h} = n_{0c,h}/n_0$. For wavelengths greater than the effective Debye length $\sqrt{\epsilon_0 T_{eff}/n_0 e^2}$, ion acoustic waves then propagate with an effective sound velocity $c_s \equiv \sqrt{k T_{eff}/m_i}$. The space and time coordinates and the plasma physical quantities may be non-dimensionalised, using n_0 , λ_D , c_s and the "thermal" potential kT_{eff}/e . The ion continuity and momentum equations take the dimensionless form

$$n_t + (nu)_x = 0$$

$$u_t + uu_x = -\phi_x \quad (5.2.5)$$

where $n = (n_i - n_0)/n_0$ is the normalised density perturbation. Also, assuming that the electron densities are separately in thermal equilibrium with the potential, the dimensionless Poisson's equation becomes,

$$\phi_{xx} = \alpha_c \exp(\theta_c \phi) + \alpha_h \exp(\theta_h \phi) - (1 + n) \quad (5.2.6)$$

where $\theta_{c,h} = T_{eff}/T_{c,h}$. A smallness parameter ϵ is introduced by setting $\phi = \epsilon \Phi$ where $\Phi = O(1)$ and the Poisson equation expanded to third order in ϵ ,

$$n = -\epsilon \Phi_{xx} + \epsilon \Phi + \epsilon^2 (A/2) \Phi^2 + \epsilon^3 (B/6) \Phi^3 + O(\epsilon^4) \quad (5.2.7)$$

where $A = \alpha_c \theta_c^2 + \alpha_h \theta_h^2$ and $B = \alpha_c \theta_c^3 + \alpha_h \theta_h^3$. In general dispersive and quadratic nonlinear terms are equally important if $\epsilon \Phi_{xx} = O(\epsilon^2 A)$ i.e. for spatial variations on a length scale $l = \epsilon^{-1/2}$. In this case a standard analysis leads to an ordinary KdV equation for ϕ in a frame moving with the sound velocity c_s ,

$$\phi_t + \frac{1}{2}(3 - A)\phi\phi_x + \frac{1}{2}\phi_{xxx} = 0 \quad (5.2.8)$$

For $A < 3$ ($A > 3$) there are positive (negative) soliton solutions. However close to the transition if $1/2(A - 3) = O(\epsilon)$ higher order terms become equally important in the analysis. Then in Poisson's equation (5.2.7) $\epsilon \Phi_{xx} = O(\epsilon^3 B)$ for a length scale $l = \epsilon$. The linear terms in the KdV equation, which are independent of the amplitude of ϕ , satisfy $\phi_t = O(\phi_{xxx})$ for a time scale $\tau = \epsilon^{-3}$. Now assuming variations on these length and time scales the procedure is to expand the ion velocity to third order $u = u_1 + u_2 + u_3 + \dots$. As before a frame of reference moving at the sound velocity c_s is assumed. Equating terms of equal order in Equations (5.2.5) and (5.2.7) leads to the mKdV equation,

$$\phi_t + \frac{1}{2}(3 - A)\phi\phi_x + \frac{1}{4}(15 - B)\phi^2\phi_x + \frac{1}{2}\phi_{xxx} = 0 \quad (5.2.9)$$

which is the same as Equation (5.2.8) except for the extra higher order non-linear term (note that $\phi = O(\epsilon)$). After a linear transformation of the potential given by,

$$\phi = \left[\frac{12}{|15 - B|} \right]^{1/2} \psi - \frac{3 - A}{15 - B} \quad (5.2.10)$$

and of the space and time coordinates Equation (5.2.9) takes the standard form,

$$\psi_t + 6 \operatorname{sgn}(15 - B)\psi^2\psi_x + \psi_{xxx} = 0 \quad (5.2.11)$$

Shock like DL solitons exist if this is the mKdV⁻ equation i.e. if $B > 15$. Also to be in the appropriate regime $A - 3 = O(\phi)$ so that $A \approx 3$. These conditions then require that $T_h/T_c > q$ and $n_h/n_c > q$ where $q = 5 + 2\sqrt{6} \approx 9.90$.

To determine the conditions under which a DL will form for given initial conditions an inverse scattering method has been developed (Raadu and Chanteur 1986 [69], Chanteur and Raadu 1987 [70]) for the mKdV⁻ equation,

$$\psi_t - 6\psi^2\psi_x + \psi_{xxx} = 0 \quad (5.2.12)$$

The restricted case where $|\psi(x \sim -\infty)| = |\psi(x \sim +\infty)|$ has been discussed elsewhere (Perelman *et al* 1974[b] [68], Ono 1976 [71], Romanova 1979 [72], Torvén 1981 [42]) and the formalism resembles that used for the mKdV⁺ equation (Wadati 1975 [73]). A scattering problem $M\eta = k\eta$ where,

$$M = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \partial_x + \begin{pmatrix} 0 & -i\psi \\ i\psi & 0 \end{pmatrix} \quad (5.2.13)$$

with an evolution equation $M_t = BM - MB$, where the matrix B is chosen so that ψ satisfies the mKdV⁻ equation, is considered. A diagonalisation procedure leads to a pair of Schrödinger equations corresponding to the eigenvalue problem $M^2\eta = k^2\eta$,

$$\begin{aligned} g_{xx} + [\lambda - (\psi^2 - \psi_x)]g &= 0 \\ h_{xx} + [\lambda - (\psi^2 + \psi_x)]h &= 0 \end{aligned} \quad (5.2.14)$$

where $\lambda = k^2$. The functions $v \equiv \psi^2 \pm \psi_x$ are as given by the Miura transformation (Miura 1968 [74]) and satisfy the KdV equation (5.2.1). The Equations (5.2.14) are related by a Darboux transformation and have identical non-zero eigenvalues. For $\lambda = 0$ they have solutions,

$$g_0(x) = \exp\left(-\int \psi dx\right) \quad \text{and} \quad h_0(x) = \exp\left(+\int \psi dx\right) \quad (5.2.15)$$

Since $g_0(x) = h_0(x)^{-1}$ at most only one of these can be an eigenfunction ($\rightarrow 0$ as $x \rightarrow \pm\infty$). The inverse scattering method (ISM) for the KdV equation (Gardner

et al 1974 [75]) may now be used in the generalised form (Khruslov 1975 [76]) to allow $|\psi(x \sim -\infty)| \neq |\psi(x \sim +\infty)|$.

It is now possible to predict the outcome of a particular initial perturbation $\psi(x, 0)$ for the mKdV⁻ equation (5.2.12). For each non-zero eigenvalue of Equations (5.2.14) a soliton will appear. A soliton wave train will appear if $|\psi(+\infty, 0)| > |\psi(-\infty, 0)|$, according to the generalised ISM. A DL corresponds to a soliton with eigenvalue $\lambda = 0$. Therefore a DL will occur if either $h_0(x)$ or $g_0(x)$ (Equation (5.2.15)) is localised. This means that a DL will appear if and only if the initial potential $\psi(x, 0)$ changes sign between $-\infty$ and $+\infty$ i.e. if $\psi(-\infty, 0)\psi(+\infty, 0) < 0$. The shape of the DL is then given by,

$$\psi = \psi_{\infty} \tanh(\psi_{\infty} x) \quad (5.2.16)$$

where $\psi_{\infty} = \psi(+\infty, 0)$. It should be recalled that the potential ψ is related to the original potential ϕ in the physical model by a simple linear transformation, Equation (5.2.10). The results were also confirmed for a representative choice of initial perturbations by solving the mKdV⁻ equation numerically using a pseudospectral scheme (Raadu and Chanteur 1986 [69], Chanteur and Raadu 1987 [70]).

6. Computer simulations

In one of the first plasma simulation studies of current driven instabilities (Boris *et al* 1970 [77]) anomalous resistivity effects were investigated in one- and two-dimensional systems. Periodic boundary conditions were used with an externally imposed electric field. In one case conditions were chosen so that ion-acoustic instabilities should be excited. A flattening of the electron distribution function was found in the ion-acoustic velocity region, but the bulk of the electrons were accelerated until their drift velocity v_d was comparable to their thermal velocity $v_{th,e}$ driving the two-stream instability. As a general result it was found that electron heating by the two-stream instability tended to maintain $v_{th,e} \approx v_d$ as the drift velocity increased. This was also found in a corresponding case with constant applied electric field in one-dimensional studies with a realistic hydrogen-electron mass ratio (Morse and Nielson 1971 [78]). Very little ion heating was found,

although in a case with no applied field high energy tails formed in the space averaged ion distribution function. These came from localised jets with high velocities, in both directions, due to non-linear wave breaking. Later studies (Biskamp and Chodura 1971 [79]) in two-dimensions with a constant current condition indicated that saturation of the instability could occur due to ion trapping by waves. A large electron to ion temperature ratio was used ($T_e/T_i = 50$) with mass ratios up to 1600 and an initial drift velocity equal to the electron thermal velocity. The drift velocity of the bulk of the electrons was much reduced and the current was carried by an extended tail in the distribution. By varying the mass ratio it was established that this relaxation of the electron distribution is not connected to the saturation of the instability. The scaling of the effective collision frequency with the mass ratio was weaker than that implied by a simple proportionality to the linear growth rate. This indicated that the effective collision frequency had apparently nothing to do with the growth rate.

In a comparison between possible theoretical results and one-dimensional simulations (Biskamp and Chodura 1973 [80]) it was concluded that the asymptotic behaviour of the two-stream instability cannot be understood from the linear stability properties of the average distribution functions. Instead it was proposed that the instability produces a Bernstein-Greene-Kruskal wave (Bernstein *et al* 1957 [17]). This in turn is subject to a coalescence instability which occurs as the ratio of the drift to thermal electron velocity continues to grow under the influence of the applied electric field. The merging of two neighbouring potential wells doubles the wavelength and is accompanied by a burst of fluctuations. This idea was tested using a particle simulation, doubling the length at a time when the initial strong instability had died down. An abrupt decrease in the drift velocity and a corresponding rise in the electron temperature resulted. Also further tests were made switching off the applied field at the same time as doubling the length. Strong instability was found rapidly breaking the drift velocity. No instability at all was found, however, if the electron distribution was made homogeneous with the same form as the spatial average. This indicated that the instability of a

non-linear spatial structure was necessary to explain the behaviour as in the theoretical model. The importance of such a structure was a first indication that even DL's could play a rôle in what had previously been assumed to be a spatially uniform development of current driven instabilities. Comparisons have been made between the results of simulation studies and turbulence theory in a recent review of ion-acoustic turbulence and anomalous transport (Bychenkov *et al* 1988 [81]).

6.1 Simulations of strong double layers

The first particle simulations of DL's were made assuming an initial drift velocity in a nonperiodic system (Goertz and Joyce 1975 [82]). This work was extended to a case with a fixed externally applied potential (Joyce and Hubbard 1978 [83], Hubbard and Joyce 1979 [84]). For a typical case the applied voltage was $200 kT_e/e$ for a system of length $360 \lambda_D$ and an isothermal plasma. Initially a large potential peak appears near the anode source, with a corresponding vortex structure in the electron phase space. It is argued that here this plays the essential rôle in the DL formation, rather than the Buneman instability which produces initial wavelike oscillations. The potential pulse travels at around the ion acoustic velocity and a DL appears after it reaches the anode source. Eventually a strong steady state DL is established. The emerging particle beams drive instabilities, and it is argued that they may reach unrealistic amplitudes since the unstable waves are trapped within the simulated region by the boundary conditions. For short system lengths the electron beam instability can be suppressed if the anode plasma is shorter than the unstable wavelength. The ion beam driven instabilities produce electron phase space vortex structures with lengths given by the most unstable wavelength. The DL is eventually partly disrupted by the growth of the instabilities, but subsequently recovers its laminar structure. It is proposed that DL's will cyclically pass through a series of turbulent and laminar phases, on a time scale of several ion plasma periods. If the turbulence is too large the DL will disrupt, but will eventually re-form.

A one-dimensional Vlasov simulation of strong DL's has been made by numerically solving the time dependent Vlasov-Poisson equations with boundary conditions coupled by an external circuit (Smith 1982 [85]). The circuit consisted of

an inductance L , a resistance R , and a potential source, Φ , in series connecting the boundary "grids" at the two ends of the system. The net potential over this circuit was then equal to the net potential ϕ_{DL} over the plasma, which was mainly supported by a DL which eventually formed. In principle the current I through the circuit need not equal the current through the plasma if the grids were allowed to carry a variable surface charge. However the magnitudes of the injected drifting Maxwellian distributions were varied in such a way that the net plasma current densities at both ends were equal ($\equiv J_0(t)$) and matched the circuit current I . This implied that there were no surface charges on the grids, and hence that the electric field was always zero at both ends of the system, implying that the plasma as a whole contained zero total net charge. One possible physical motivation for these conditions is that plasmas are usually quasi-neutral except locally at nonlinear structures such as DL's which have no net charge. It is also possible to argue that in real experiments plasma sources may under suitable conditions provide such boundary conditions.

Isothermal conditions were assumed ($T_i = T_e$) and the drift velocities were $-2\sqrt{kT_i/m_i}$ and $2\sqrt{kT_e/m_e}$ respectively for ions and electrons, with mass ratio $m_i/m_e = 16$. An initial applied potential $\Phi = 20 kT_e/e$ and plasma current at the boundaries $J_0(0) = -0.5$ were assumed. Two cases with different induction times L/R were analysed, one very large implying constant current and the other equal to the ion transit time. Initially two-stream instabilities developed producing waves with an envelope increasing in the direction of the electron drift. Subsequently the potential maxima began to trap electrons and to coalesce. A DL formed apparently due to local current interruption by the electron trapping. Finally ions became trapped in the potential minima. The potential over the system ϕ_{DL} rose monotonically in the constant current case. In the second case ϕ_{DL} rose to the level of the applied potential with inductive overshoots on an ion plasma time scale (ω_{pi}^{-1}). After a phase dominated by ion dynamics the DL approached a laminar state, with a scaling law $\phi_{DL} = 0.165 l_*^2 / \lambda_{e*}^2$ ($l_* \equiv \phi_{DL} / \max |E|$ measures the DL thickness; λ_{e*} is the Debye length on the low potential side). This scaling law

applied to both cases and is similar to that found in other studies (Goertz and Joyce 1975 [82], Singh 1980 [86]).

The recurrent formation of DL's has been investigated in a series of one-dimensional Vlasov-Poisson simulation studies (Singh 1982 [87], Singh and Schunk 1982[a] [54], 1982[b] [88], 1983 [89]). These and other studies are summarised in a recent review (Singh *et al* 1986 [90]). An external potential is applied, which initially accelerates the electrons leading to a net positive space charge near the cathode. In this way a positive potential pulse is created with an electron hole (vortex) structure. As in earlier particle simulations (Joyce and Hubbard 1978 [83], Hubbard and Joyce 1979 [84]) this pulse propagates and is destroyed on reaching the anode source. The next phase is characterised by strong plasma turbulence, and the expulsion of plasma to form a cavity, provided the system is sufficiently long. A potential step near the cathode source evolves into a DL with a potential dip on the cathode edge. The DL moves at about the ion acoustic velocity towards the anode closely followed by an expanding plasma. The changes in electron and ion current are consistent with the Langmuir condition (Singh and Schunk 1982[b] [88]). The existence of an ion beam and an electron current on the cathode side allows the formation of a new DL triggered by a potential pulse. In this way a DL is formed as part of a cyclic process on an ion transit time scale. It is argued that the primary cause of this behaviour is the different time scales associated with ions and electrons. The DL formation interrupts the ion flux causing the DL to move to fulfill the Langmuir condition. The electron current is restored in step with the recovery of the ion flux. The fluctuations associated with the restored current lead to the formation of another DL.

The evolution of one-dimensional DL's has been followed in a numerical simulation where particular attention was paid to the electron dynamics (Yamamoto 1985 [91]). Three major phases in the development are distinguished. Initially a positive potential surge appears moving towards the high potential (anode) boundary at about the ion acoustic velocity. This is accompanied by the acceleration of ions on the low potential (cathode) side by an ambipolar electric field. This is followed by the formation of a quasi-steady DL, also moving towards the

anode. The final phase is marked by the collapse of the DL due to an insufficient particle supply for the given imposed potential. These phases repeat periodically, leading to a recurrent formation of the DL as found in other simulations (Singh and Schunk 1982 [54]).

The structure of the quasi-steady DL after $600/\omega_{pe}$ for one particular case is displayed in Figure 5. The imposed potential ($50 kT_{1e}/e$) is taken up by the DL. In this case the cathode plasma source has a temperature, T_{1e} , 10 times that of the anode source. The ions have the same temperatures as the electrons. As for the precursor potential surge, the DL is followed by an ambipolar plasma expansion resulting in the small but persistent potential dip visible at the cathode edge of the DL profile (see Figure 5.). A steepening of the DL is attributed to a cooling of the reflected electrons and to an increase in the density of the preaccelerated beam entering the DL. There is evidence for electron beam driven instabilities in the anode plasma, and for thermal waves in the cathode plasma. A sizeable fraction of the electrons from the cathode source are found by particle tracing to be reflected back to the source before passing through the DL. It is shown that a significant contribution to this reflection can be made by resonant diffusion by the thermal waves. The underlying mechanism is argued to be the relaxation of the asymmetric electron velocity distribution, resulting from the absence of electrons from the anode plasma due to reflection by the strong DL. Such a relaxation naturally leads to a reversal of some electron velocities. The ambipolar potential minimum also contributes to the process since it has a depth of order kT_{1e}/e .

Extended DL's with thickness comparable to the whole simulation region ($256 \lambda_D$) are found for a case with applied potential $10 kT_{1e}/e$ and hot ions ($T_{1i}/T_{1e} = 10$). The electron beam instability is present throughout the system and the unstable wavelength increases towards the anode source due to acceleration of the beam. The electron flow is appreciably inhibited by trapping in the non-linear waves. These results may be compared with recent pulsed DL experiments where an initial phase is found with an EPD (Extended Potential Drop) on an ion transit time scale (Carpenter and Torvén 1987[a] [18] and 1987[b] [92], Bohm and Torvén 1986 [93]).

The formation of plane parallel and two-dimensional strong DL's has been studied using a $2\frac{1}{2}$ -dimensional particle-in-cell plasma simulation method (Borovsky and Joyce 1983 [94]). A two-dimensional grid was used applying periodic boundary conditions in one direction and with Maxwellian particle sources at given potentials on the boundaries in the other direction. Particles leaving the system on one of the non-periodic boundaries were reinjected at the opposite boundary with a velocity corresponding to the source there. Hence the net charge in the system was always zero. Typical dimensions were $60\lambda_D$. Around 20 000 particles of each species were used with a mass ratio $m_i/m_e = 16$ in most of the cases presented. A constant magnetic field at various angles to the direction of the applied potential difference was used ($\theta = 0^\circ, 45^\circ, 60^\circ$).

Plane parallel DL's were produced by applying a constant potential, $\Delta\phi \approx 25 kT_e/e$, over one source ("upper") and keeping the other source ("lower") at zero. The initial response was characterised by potential ridges moving upwards at several times the ion-acoustic velocity, followed by large plasma oscillations settling down to form a DL near the upper boundary. The DL thickness only increased very weakly with an increasing obliqueness of the applied magnetic field (chosen so that $\omega_{ce} = \omega_{pe}$). The beam of accelerated electrons were aligned with the magnetic field, whereas the velocity of the weakly magnetised ion beam was close to the direction of the DL electric field. Detailed comparisons were made with steady state one-dimensional models derived using the Vlasov-Poisson equations. Electrostatic waves produced by the electron beam on the high potential side, were identified as Langmuir waves for a vertical magnetic field and low-frequency electrostatic electron-cyclotron waves for the strongly oblique case ($\theta = 60^\circ$). Also in the latter case upward moving positive pulses, presumably produced by the interaction of the ion beam with electrons on the low potential side, were associated with a periodical instability. The pulses widened eventually forming a ridge dividing the low potential region in two. The potential of the region adjacent to the DL, which was simultaneously drifting slowly upwards, rose to that of the applied potential replacing the original DL with a new one at the dividing ridge.

To produce two-dimensional DL's, the full potential difference $\Delta\phi$ was only applied over the central part of the upper source. A matching potential linearly decreasing to zero was applied on the outer parts. A constant vertical magnetic field was used. The results showed many features in common with the plane parallel cases. Some details of the behaviour depended on the sign of the applied potential difference. For a negative potential ($\Delta\phi < 0$) a strong shielding was found, and to produce a DL the density at the boundary had to be enhanced by modifying the particle injection. The DL formed for a positive applied potential tended to merge irreversibly with the adjacent DL's implicit in the horizontally periodic system. The merging process was apparently due to the same instability involved in the periodic re-formation of DL's, together with the effect of weakly magnetised ions. Four cases with a negative applied potential and different field strengths ($0, \infty$, and such that $\omega_{ce} = \omega_{pe}$ and $\omega_{ci} = \omega_{pe}$) were investigated in detail. DL's formed rapidly across the central region, on less than a thermal ion transit time, being produced directly from an initial positive potential pulse. The ion and electron phase space distributions on a cut along the central magnetic field line closely resemble those for a strong one-dimensional DL (cf Joyce and Hubbard 1978 [83]). The low potential region was flanked by a narrow potential drop nearly perpendicular to the magnetic field, and the penetration of this region increased with magnetic field strength. The oblique regions of the two-dimensional DL had the same thickness as the horizontal region, scaling with the Debye length and insensitive to changes in the ion to electron mass ratio. A periodic instability similar to that for strongly oblique plane parallel DL's was found for both positive and negative applied potentials, although in the latter case re-formation requires that a sufficiently extended negative potential region is formed to allow positive pulses to grow. Characteristic variations of the shape of the negative potential region were seen during the instability cycle. For example, when $\omega_{ci} = \omega_{pe}$, the downward moving rounded DL became v-shaped at the instability onset and after re-formation tended to be squarish. It was noted that the sizes and shapes of the DL's strongly depended on the applied boundary conditions, so that in real situations the plasma sources and sinks and their potentials are crucial.

6.2 Simulations of ion-acoustic double layers

One- and two-dimensional cloud-in-cell particle simulation codes were used to investigate the formation of DL's and the effective resistivity resulting from the thermalisation of the accelerated electrons (DeGroot *et al* 1977 [95]). The simulations were periodic with an externally imposed electric field to maintain a constant current. The system dimensions were $256 \lambda_D$ and $141 \lambda_D$ for the one- and two-dimensional simulations respectively. Ion-electron mass ratios of 100 and 1000 were used. DL's were only found when the electron temperature greatly exceeds the ion temperature. The current drift velocity had to be comparable to $v_{th,e}$, the electron thermal velocity ($\geq 0.8 v_{th,e}$). For large amplitude initial perturbations the DL's formed near the density minima. In 1-dimension they could grow in an initially uniform plasma; whereas in 2-dimensions ion acoustic resistivity strongly heated the electrons so that they had to be cooled to avoid quenching of the instability for the given constant drift velocity. In a real plasma cooling may be expected from convection and collisions with neutrals. The separation of the DL's (typically $60 \lambda_{De}$) was sufficient for the two-stream instability to thermalise the beam of electrons formed by acceleration over the net potential difference. An average effective collision frequency could be found which was a few percent of the electron plasma frequency and increased for an increase in the ion-electron mass ratio. The potential profile was not monotonic and there was a potential minimum on the low potential side of the DL's. The net potential differences were a few times the electron thermal potential. In the non-magnetised two-dimensional simulations the randomly formed DL's moved to form a plane-parallel structure perpendicular to the current. There were sharp peaks in the ion density at the low potential edges of the DL's followed by a reduction inside. The DL's typically moved with the ion-acoustic velocity towards the high potential side.

The influence on DL formation of varying the length of a long system (up to $1024 \lambda_D$) has been studied for periodic boundary conditions with no externally applied electric field (Sato and Okuda 1980 [33]). The current drift velocity was small ($0.6 v_{th,e}$) and since the ion-electron mass ratio was 100 the instabilities were in the ion-acoustic regime for the chosen electron to ion temperature ratio

($T_e/T_i = 20$). A one-dimensional particle simulation was used with initially uniform conditions. Three phases in the development were noted. Initially the field energy increased linearly and the current relaxed indicating an ion acoustically driven anomalous resistivity. Next during an explosive increase in field energy and an abrupt fall of the current a DL was formed. Finally the current was only a tiny fraction of its original value and the resistivity was negligible. The DL potential jump was comparable to the electron thermal potential. On the low potential edge there was a sharp potential minimum and peaks in the ion and electron densities with an extended region of enhanced plasma density. On the high potential side there was a corresponding region of decreased density. By successively halving the length it was found that a system length of $256 \lambda_D$ was marginal for DL formation.

In a continuation of the previous work (Sato and Okuda 1981 [96]) a much longer system was investigated (length $4096 \lambda_D$) to see if the DL potential continued to show a tendency to increase with the system length. Four separate DL's were in fact formed, each closely resembling the DL previously found in the system of length $1024 \lambda_D$. It was concluded that for the given plasma conditions there is a fundamental length of about $1000 \lambda_D$ required to form each ion acoustic DL. The individual DL's decayed after a few hundred plasma periods emitting spikey wave trains on the high potential side moving at the ion acoustic velocity and interpreted as ion acoustic solitons. The effects of an external circuit with a voltage source and a resistance in series with the simulated plasma region (to model the ionospheric coupling to the magnetospheric plasma) were also studied. There was then a net potential over the system but strict periodicity was retained for the particle motions. DL's appeared with similar characteristics to those in the fully periodic simulations. The current decreases suddenly when the DL is formed but recovers its initial value afterwards indicating a very low plasma resistivity. It was noted that the sharp potential dip associated with the DL persists even after the DL itself has been destroyed.

A combination of one- and two-dimensional particle simulations as well as analytical methods were used to further clarify the dynamics of the formation of ion-acoustic DL's (Nishihara *et al* 1982 [44]). Strict periodicity was assumed so

that the potential of the DL formed was compensated by an extended opposite potential opposing the plasma current. In such simulations the energy required to power the DL formation is supplied from the initial kinetic energy in the relative motion between the ions and electrons. This energy is extracted in regions where the current and electric field are oppositely directed as in the studies under discussion. In both the one- and two-dimensional simulations a growing negative potential dip formed with the DL. A sharp current drop signaled the appearance of the dip and was attributed to electron reflection. A Korteweg-de Vries equation modified by an extra term giving the kinetic effects of electron reflection was derived and solved numerically. The solutions showed the growth of a potential dip accompanied by a deceleration as in the simulations. Without the modifying term the equation would only support positive solitons and numerical solutions confirmed the decay of a negative dip in that case. In the simulations the deceleration of the dip eventually allowed ion trapping.

The rôle of a potential minimum in the formation of an ion acoustic DL was elucidated by numerically solving the Vlasov-Poisson equations (Chanteur *et al* 1983 [45]). This method was chosen to avoid the noise intrinsic to particle simulations with moderate numbers of particles per Debye length, λ_D . Periodic boundary conditions were used with no applied electric field for a one-dimensional system of length $512 \lambda_D$. The initial drift velocity, v_d , was chosen to drive ion-acoustic instabilities ($T_e/T_i = 20, m_i/m_e = 100$). An initial localised density dip transiently interrupted the current resulting in a potential minimum moving with the density dip at the drift velocity. A DL formed with a potential difference comparable in size to the potential minimum. Quasi-neutrality requires such a potential difference to compensate for the reduction of the electron density on the down-stream side of the minimum due to electron reflection and a corresponding increase upstream. The growth of the DL was related to the energy lost by electrons reflected by the potential minimum. An adiabatic theory assuming that the electrons were in equilibrium with the moving potential structure gave good agreement between the numerical and theoretical values of the momentum and energy loss rates during the growth phase. This broke down when the DL abruptly began

to decelerate at the same time as ion trapping became evident in their phase space diagrams. An equation of the modified Korteweg–de Vries type (cf Karpman *et al* 1979 [63]), closely related to that used to interpret particle simulation results (Nishihara *et al* 1982 [44]) was derived using the adiabatic theory for the electrons and explained both growth and narrowing of the potential profile.

The formation of DL's has been studied using the Vlasov–Poisson approach even for an initial white noise perturbation of the ion density (Chanteur 1984 [97]). There was then an initial self-organisation phase during which velocity perturbations built up and the electrostatic energy decreased. The initial perturbation had to be very weak to reduce the accompanying velocity diffusion which could quench the instability. Particle codes do not have such a strong restriction. Typically two or three weak DL's developed over a distance of $1024 \lambda_d$. Negative potential structures developed similar to those in particle simulations and Vlasov–Poisson treatments with a strong localised initial perturbation. The growth was however much slower and the net potential difference smaller. An oscillating potential tail developed behind the potential minimum and limited its growth by electron trapping. The separation of the weak DL's tended to decrease with increasing drift velocity. Strictly periodic boundary conditions were used, with no applied electric field and no constant current condition, so that the system contained a fixed finite amount of energy available for redistribution during the plasma relaxation. The long time behaviour was characterised by the concentration of the field into a few coherent wave packets containing trapped ions and electrons.

Dynamical (Nishihara *et al* 1982 [44], Chanteur *et al* 1983 [45]) analyses have been used to describe the ion–acoustic DL's found in simulation studies. It has been seen that such DL's move towards the high potential downstream electron side at approximately the ion–acoustic velocity with respect to the stationary ions. In the reference frame of the DL effectively–cold ions stream in from the high potential side, are accelerated to a maximum velocity at the potential minimum, and emerge with a net increase in velocity corresponding to the DL potential. These ions are not easily trapped in the time dependent DL's studied numerically since a fluctuation comparable to the net potential over the DL (or an equivalent

change during the crossing time) would be necessary. However when the DL velocity begins to drop and eventually reverse direction low velocity ions enter on the low potential side and are reflected inside the DL due to the existence of a net potential over the DL. Such ions are susceptible to trapping as a result of small fluctuations and slow time developments. The ion trapping is thus associated with the decay phase of the DL. Further studies are necessary to clarify the relation between analytical models of asymmetric ion holes (Hasegawa and Sato 1982 [34]) and ion acoustic DL's.

DL formation triggered by an initial localised density depression (Wagner *et al* 1985 [98]) has been further studied using a one-dimensional particle simulation with periodic boundary conditions. The plasma parameters and drift velocities are similar to those used in previous studies of the formation of ion-acoustic DL's (Sato and Okuda 1980 [33], 1981 [96]; Chanteur 1984 [97]), e.g in the two "canonical" cases presented in detail, $m_i/m_e = 64$, $T_e/T_i = 5$, $v_d = 0.75v_{th,e}$. In another case the program was modified to include the effects of an applied electric field due to induction, giving essentially the same results for the evolution of the density depression. In the two canonical cases the width of the initial perturbation, $L_{dep} = 64\lambda_{De}$, is comparable to that used in previous Vlasov-Poisson and particle simulation studies (Chanteur *et al* 1983 [45], Chanteur 1984 [97]). However in the previous studies only a 10% initial depression was treated. In the first canonical case a 40% depression was used, implying a drift velocity at the centre, $v'_d = 1.25v_{th,e}$, sufficient to drive Buneman instabilities. A 15% depression was used in the second case. A comparison with previous work shows many similarities, although the interpretations differ in detail. Interesting ion dynamical effects are seen on the low potential side of the developing DL in both canonical cases. In the first case a large pulse of ions is ejected from the depression during the initial Buneman phase (the mean drift velocity falls steadily since the current is not maintained). In the second case, where only ion acoustic instabilities are possible, a series of solitary ion pulses, in the form of ion phase space holes, are emitted during the decay of the ion-acoustic DL. After the DL has decayed, a new DL forms and the process is repeated.

The results of a particle simulation with a constant current condition, to simulate the effect of a large external inductance are presented in Figure 6. They exhibit some of the basic features of ion-acoustic DL's. The program used is a version of ES1 (Birdsall and Langdon 1985 [99]) modified by the introduction of a constant current algorithm. The particles and the electric field are treated periodically, but there is a non-zero mean electric field and a net potential difference. A mass ratio $m_i/m_e = 100$ is used and 8192 particles of each species are followed. The initial drift velocity is large, $v_d = 2 \sqrt{kT_e/m_e}$, and the temperature ratio is $T_e/T_i = 20$. The DL formation is triggered by an initial density dip with a centrally placed (at $x = 64 \lambda_D$) Gaussian profile of width $16 \lambda_D$ giving a 20% maximum density depression. After a time $t = 80\omega_{pe}^{-1}$ (Figure 6(a)) a DL, amplitude $\approx 5 kT_e/m_e$, has formed close to the centre of the simulated region. There is a very large potential pulse to the right of the DL with an associated vortex structure in the electron phase space. This pulse propagates away from the DL and is well separated from it at $t = 120\omega_{pe}^{-1}$ (Figure 6(b)). The DL accelerates electrons to form a beam on the high potential side, where there is also a local reflected population, as for a typical strong DL. However the ion distribution resembles that seen in simulations of ion-acoustic DL's. The ions are accelerated by the DL but show clear signs of trapping to form a phase space hole structure at later times (see Figure 6(b) for $t = 120/\omega_{pe}$). There is also a persistent potential minimum on the low potential edge of the DL which can be traced back to the initial perturbation.

Particle simulation studies have been made of a nonlinear instability of ion holes in current carrying plasma (Berman *et al* 1985 [100], Berman *et al* 1986 [101]). The associated potential minima become asymmetric giving a DL potential profile similar to the ion-acoustic DL's. However the one-dimensional simulation studies show that growth is possible for current drift velocities *below* the limit for linear growth. The results are in good agreement with the theory of hole instability (Dupree 1982 [102], Tetreault 1983 [103]). The driving mechanism is the exchange of momentum with electrons reflected by the potential minimum, depth

ϕ_0 , associated with the hole. They change momentum at a rate $\dot{p}_e$ given by,

$$\dot{p}_e = -\omega_{pe}^2 \phi_0^2 f'_{0e}(u)/\pi \quad (6.2.1)$$

where $f_{0e}(u)$ is the electron distribution function. The ion hole moves with a velocity u and consists of a localised depletion in ion phase space of depth $\tilde{f}$, length Δx and velocity trapping width $2\Delta v_i$. In equilibrium these parameters are related by the trapping condition and Poisson's equation. The rate of change of ion hole momentum is then,

$$\dot{p}_i = n_0 m_i \Delta x \Delta v_i \tilde{f} \dot{u} \quad (6.2.2)$$

and from consevation of momentum $\dot{p}_i = -\dot{p}_e$. The ion hole decelerates as a result of the momentum exchange ($\dot{u} < 0$) and since the phase space density inside is constant by Liouville's theorem the depth increases according to,

$$\frac{\partial \tilde{f}}{\partial t} \equiv \gamma_H^i \tilde{f} = -\dot{u} \frac{\partial f_{0i}}{\partial v} \quad (6.2.3)$$

Using the equilibrium condition for the hole,

$$\tilde{f} = -\Delta v_i / 2\lambda^2 \omega_{pi}^2 \quad (6.2.4)$$

where λ is the shielding length ($\approx \lambda_D$ for $u \leq \sqrt{kT_i/m_i}$) the growth rate may be written,

$$\gamma_H^i = -\frac{\Delta v_i}{\Delta x} \frac{\lambda^4}{\lambda_{De}^2 \lambda_{Di}^2} v_e^2 f'_{0e} v_i^2 f'_{0i} \quad (6.2.5)$$

The DL potential over the hole, which decelerates it, is

$$\delta\phi = -\phi_0 \frac{\lambda^2}{\lambda_{De}^2} v_e^2 f'_{0e}(u) \quad (6.2.6)$$

This form of the theory which applies when $\epsilon\phi/T_i \ll 1$ shows close agreement with the simulations for equal temperatures, $T_i = T_e$, in the linear stable regime.

The more recent of the two simulation studies of the nonlinear ion hole instability (Berman *et al* 1986 [101]) were made using a one-dimensional electrostatic code with periodic length, $L = 512\lambda_D$, and with 200 particles per Debye length ($= n_0\lambda_D$). The mass ratio, m_i/m_e , was 100. In one case with equal temperatures,

$T_i = T_e$, and an initial relative drift velocity, v_D , below the threshold for linear instability ($v_D/v_e = 1 < 1.44$ where $v_e = \sqrt{kT_e/m_e}$) the particles were loaded to minimise the electrostatic energy (“quiet start”), with an initial hole in the ion distribution. The initial hole was rectangular with dimensions $\Delta x = 10\lambda_D$, $\Delta v = 0.5v_i$, ($v_i = \sqrt{kT_i/m_i}$) and depth, $\tilde{f}$, chosen to approximate a BGK equilibrium (using the ion trapping condition $m_i\Delta v^2 = 2e\phi$ and Poisson’s equation). From an initial velocity, $u_0 = 1.5v_i$, the ion hole was braked on a time scale of a few $100\omega_{pe}^{-1}$. The velocity width increased and the phase space depth increased with the deceleration and the potential developed a DL profile with a deep minimum on the low potential side. Periodic peaks developed on the high potential side due to electron hole formation. Comparisons with the theory for $e\phi/T_e \ll 1$ (Dupree 1982 [102], Dupree 1983 [104]) even showed reasonably good quantitative agreement as well as following the qualitative predictions. Thus at a later time the approximate conditions for BGK equilibrium still held. The average numerical value of the hole acceleration from the phase space diagrams, $\dot{u} = -4.0 \cdot 10^{-4} v_e \omega_{pe}$, was comparable to the theoretical value, $-1.3 \cdot 10^{-4} v_e \omega_{pe}$, as was the value found from following the potential minimum. Also the instability growth rates from the time development of Δv and the potential depth, $2.5 \cdot 10^{-3} \omega_{pe}$ and $1.6 \cdot 10^{-3} \omega_{pe}$, were also close to $\gamma_H^i = 1.3 \cdot 10^{-3} \omega_{pe}$, the theoretical value. The system length, L , should be an important factor for the instability since the increase in available electron momentum due to the growth of the hole width has to be sufficient to sustain the growth (Dupree 1986 [105]) so that approximately, $\gamma_H^i L / \Delta v > 1$. This was tested in two separate simulation runs above and below the critical length, the hole deceleration rate being significantly less in the second case.

Simulations were also made for the unstable regime, $v_D = 0.6v_e$, for hot electrons ($T_e/T_i = 20$) and initially uniform densities. Both “quiet” and “random” (i.e. with a random distribution of particles in space, giving statistical noise) initial conditions were used, ion holes taking longer to form in the first case. The ion phase space developed growing fluctuations “intermittently” along the length of the plasma i.e. over restricted regions separated in space. Intermittency can prevent stabilisation by the formation of a plateau in the distribution function,

provided the transit time between isolated fluctuations is less than the growth time. As a consequence comparisons with theories for uniform plasma turbulence were found to give poor agreement. Although the localised wave-like fluctuations (LWF) grew at approximately the linear growth rate, linear dispersion did not destroy the wave packet, and they reached large amplitudes ($e\phi/T_e \approx 1$). Ion holes formed when the potential became large enough to trap ions significantly. The fully developed holes were decelerated by momentum exchange with electrons and exhibited a DL like potential profile. The simulation results were consistent with an extension of the theory of ion hole growth to the case $e\phi/T_i \gg 1$ (Dupree 1986 [105]).

7. Experiments on double layers

Plasma DL's have been studied in discharge experiments over many years. If the confining tube has sections with different diameters, the plasmas within each section have different properties. DL's may then appear at the constrictions to match the plasmas and can be observed as potential discontinuities. Early experiments (Langmuir and Mott-Smith Jr 1924 [106]) have been followed by a series of studies (Schönhuber 1963 [107], Crawford and Freestone 1963 [108], Andrews and Allen 1971 [10], Anderson 1977 [109], Torvén 1978 [110]). The constriction DL, which bulges out into the wider tube section, acts to increase the effective matching area and accelerates electrons from the cathode plasma into the narrower tube section; numerical solutions have been found (Andrews and Allen 1971 [10]). The constriction DL exists for any discharge current, and is the result of the particular boundary conditions imposed by the constriction.

DL's are also associated with current limitation phenomena in low pressure arc discharges (Torvén 1969 [111]). It was found that DL's can form in an initially axially homogeneous plasma column in a tube with uniform cross-section (Babić *et al* 1971 [112], Babić and Torvén 1974 [113]). The DL appeared when the current was increased sufficiently so that the drift velocity was comparable to the electron thermal velocity, $\sqrt{kT_e/m_e}$, and DL formation was found to be the essential current limitation mechanism. A criterion for two-stream instability, worked

out for the actual discharge conditions (Sreenivasan and Schroeder 1983 [114]), agrees well with experimentally observed current densities when the DL forms. The significance of the DL for current limitation has been confirmed in several experiments (Torvén and Babić 1975 [115], Levine *et al* 1977 [116], Maciel and Allen 1984 [117], Allen 1985 [118]). Wall effects may be reduced by carrying out the experiments with a strong externally imposed magnetic field, and DL's have been also found in this case (Lutsenko *et al* 1975 [119], Torvén and Andersson 1979 [120]).

Although much may be learned from discharge experiments, ionisation effects are of necessity always present and it is not always clear to what extent particular phenomena can occur in a fully ionised plasma. Extrapolation to the case with complete ionisation requires detailed experimental studies to determine the significant parameters. The effects of ionisation can be significantly reduced if the regions of plasma production can be separated from the region where the DL is studied. This is most effectively accomplished in Q-machines where the plasma is produced by surface ionisation on externally heated metal plates at the ends of the plasma column. A detailed review of DL experiments including Q machine experiments may be referred to here (Sato 1982 [121])

An alternative approach is to decouple the plasma production from the plasma current. This may be done by using electrons produced at filaments on the walls of an evacuated chamber to ionise the neutral gas (Taylor *et al* 1972 [122]). The chamber may be divided into two or three separate regions separated by grids, and ionisation effects may be further limited if the experimental region has no wall filaments. A similar approach is also possible for discharge plasma devices, where the experimental region has a reduced neutral gas density produced by pumping, and hence a reduced ionisation rate. Plasma then enters from an adjacent region maintained by a discharge current (Torvén and Andersson 1979 [120]). All such devices may be termed multiple plasma machines. A relatively recent review gives a careful assessment of DL experiments, particularly in double and triple plasma devices (Hershkovitz 1985 [123]). An earlier review (Torvén 1979 [124]) provides a valuable synthesis of the results of a variety of laboratory experiments.

7.1 Multiple plasma machine experiments

The first studies of DL's in a double plasma machine were made with plasma production by heated filaments on the walls of both chambers (Quon and Wong 1976 [125]). The stable steady state DL's, which were formed for suitably adjusted plasma densities and relative potential difference between the two chambers, had a thickness great enough to be resolved by probe measurements. The presence of reflected and accelerated ions and electrons was demonstrated as required for theoretical models of strong DL's. Since ionisation effects were present throughout the device the ambient plasma properties adjacent to the DL were determined locally. The DL's were relatively weak with $1 \leq e\phi_{DL}/kT_e \leq 4.5$. and were produced when the electron drift velocity exceeded their thermal velocity $\sqrt{kT_e/m_e}$.

Strong ($e\phi_{DL}/kT_e \approx 10$) DL's have been produced in a magnetically confined plasma column in a double plasma device (Torvén and Andersson 1979 [120]). The plasma is produced by a mercury arc discharge and enters the main chamber through a circular aperture. An end plate acts as an anode. As the applied potential is increased the anode sheath evolves into a DL as a result of ionisation of the background neutral gas. The DL separates two plasma regions, one originating from the discharge source and the other on the high potential side resulting from volume ionisation by accelerated electrons. Only a few percent of the beam electrons undergo ionising collisions, but this is sufficient to maintain the plasma.

Wave phenomena produced by the strong DL have also been investigated in the same apparatus (Torvén and Lindberg 1980 [126]). Fluctuations at high frequencies close to the electron plasma frequency (greater than the gyro frequency) and at low frequencies below the ion plasma frequency dominate. The high frequency has a localised maximum intensity some 500 – 1000 Debye lengths from the DL in the anode plasma (Figure 7). Correlation measurements indicate axially propagating waves with nearly constant phase velocity, somewhat below the velocity of the electron beam produced by the DL. An electron energy analyser has been used to show that the high frequency field scatters most beam electrons on a single passage through the anode plasma (Lindberg 1982 [127]) (Figure 7).

The collisional scattering of the electrons is relatively unimportant. The scattered electrons form a trapped energetic population.

Three-dimensional time dependent DL's with a potential jump higher than the ionisation potential have been created in argon, helium and xenon plasmas starting from unstable anode sheaths (Andersson 1981 [128]). The apparatus is a form of double plasma machine with a discharge plasma source on the low potential (cathode) side. As the applied voltage is increased an instability appears at the anode sheath, leading to the formation of a DL separated from the anode by a plasma created by volume ionisation. The critical voltage decreases as the background density is decreased. The DL has a three-dimensional structure and there are strong radial fields which are seen to accelerate ions out of the anode plasma. Electrons are accelerated into the anode plasma and retain their identity as a beam there. High frequency fluctuations are found to be confined to the anode plasma. Simple time independent sheath models are insufficient to explain the transition to the DL which requires ionisation for equilibrium but may be the result of plasma instabilities. Plane DL models with ionisation are thicker agreeing better with the observations.

The rôle of a potential dip accompanying stationary and transient DL's in a double plasma machine has been examined in some detail (Fujita *et al* 1983 [129], 1984[a] [130], 1984[b] [131]). A cathode source plasma is produced by an argon discharge between an oxide cathode and a fine meshed grid. This grid acts as a cathode for the main experimental chamber which is terminated by a positively biased metal anode plate. Plasma drifts along the uniform applied magnetic field and enters the main chamber through the grid. As the plate voltage is increased a local discharge produces a secondary plasma on the anode side, separated from the primary plasma from the source by a DL. The secondary plasma is sustained by the ionisation produced by a few percent of the beam electrons accelerated by the DL. The voltage of the DL is not limited to the ionisation potential of the argon gas. The situation resembles that for other single source DL experiments (Torvén and Andersson 1979 [120], Torvén and Lindberg 1980 [126], Andersson

1981 [128]). The potential surfaces are found to wrap around the new anode plasma region. A small potential dip is found at the cathode edge of the DL.

The formation of the DL is studied by applying a pulsed positive potential to the anode plate. A DL appears with a large potential dip on the low potential edge. The amplitude of this dip is $3 - 5 kT_e/e$ and strongly reduces the current to the anode plate. The dip rapidly damps out and propagates towards the cathode source at a few times the ion acoustic velocity, always attached to the DL. Potential contour plots reveal a deep localised potential minimum at the dip. Phase space plots, constructed from measured energy distributions, show an ion hole (vortex) structure at this minimum (Fujita *et al* 1984[b] [131]). The electron phase space shows that most electrons are reflected by the transient potential dip, so that the populations on either side are kept separate. The DL finally reaches a steady state with a small dip.

The steady state is further investigate by constructing one dimensional models (Fujita *et al* 1984[a] [130]). Standard methods are used, except for the ions on the low potential side of the DL. These are treated approximately as a cold beam on the cathode side of the potential minimum, but on the anode side an attenuation factor $\exp(-e\Delta\phi/kT_i)$ is introduced, where $\Delta\phi$ is the potential measured with respect to the minimum value. This factor is to introduce the effects of radial ion loss which are present in the experimental situation due to the outwardly directed fields in the anode plasma region. This allows strong DL solutions to be found with a potential minimum. There is good general agreement although the experimental DL's are much thicker.

In triple plasma machine experiments an extra source provided electrons on the high potential side of the DL's (Coakley *et al* 1978 [132]). Stable strong DL's with $e\phi_{DL}/kT_e > 10$ were found and the accelerated ion and electron beams detected. The electron beams penetrated a distance of more than $200\lambda_{De}$ without dissipation. Potential dips sufficiently great to reflect thermal electrons were seen on the low potential side of the measured axial potential profiles.

The introduction of separate plasma sources on the high and low potential sides of the main chamber with separating grids in a triple plasma machine, creates

the possibility of reducing the ionisation effects in the main chamber in DL experiments (Hershkowitz *et al* 1980 [133]). The particle distributions may then be directly controlled by manipulating the sources and it becomes meaningful to model the plasma state including the DL in the main chamber by solutions of the stationary Vlasov–Poisson equations for an actual experiment (Hershkowitz *et al* 1981 [134]). For the experiments concerned the measured two-dimensional potential contours showed an approximately plane-parallel configuration, suitable for one-dimensional modelling. However it was noted that ions may in fact be lost radially, and that the trapped electrons on the low potential side may be populated by inelastic scattering of energetic electrons from the high potential side (Ionisation degrees are typically small, $\leq 1\%$, in multiple plasma machines).

Multiple DL's have also been reported in a triple plasma experiment (Chan and Hershkowitz 1982 [135]). For conditions supporting a single DL a second DL was formed if either the density was increased or the length of the system changed. The critical parameter appeared to be the length of the chamber measured in Debye lengths, L/λ_{De} . A self consistent mechanism was suggested whereby the density was locally enhanced as a result of scattering by ion-acoustic fluctuations (also seen in other experiments with multiple DL's, Guyot and Hollenstein 1983 [136]) associated with the DL's.

A significant three-dimensional potential structure has been found for DL's in triple plasma machines (Coakley and Hershkowitz 1979 [137], Baker *et al* 1981 [138]). U-shaped contours were found enclosing the plasma on the high potential side of the DL (Baker *et al* 1981 [138]). The associated electric fields were therefore outwardly directed and hence in the opposite sense to those reported in the magnetosphere (Temerin *et al* 1984 [139]) and taken to indicate the presence of DL's.

The formation and properties of multiple DL's are the subject of experiments made in a long triple-plasma device (Hollenstein *et al* 1980 [140], Guyot and Hollenstein 1983 [136], Hollenstein and Guyot 1983 [141]). The cathode and anode plasma sources are maintained by electrons emitted from negatively biased heated tungsten filaments, with a background neutral argon gas pressure of $\approx 50\text{mPa}$.

The electrically insulated sources are separated from the main chamber by grids, one of which is grounded and the other at an adjustable positive potential, V_g . The resulting argon plasma has typical parameters, $n_e = 2 \cdot 10^{15} \text{ m}^{-3}$, $T_e = 1.2 \text{ eV}$ and $T_i = 0.2 \text{ eV}$. Measurements are made with Langmuir and emitting probes.

A DL is observed even for small applied voltages, $V_g = 5 \text{ V}$, and becomes more pronounced as V_g is increased. Eventually a second DL appears at a distance of $\approx 5000 \lambda_{De}$, with a third appearing for even higher voltages $V_g = 50 \text{ V}$. The potential jumps are strong, $e\phi_{DL}/kT_e = 12$, and the thickness $\Delta L = 300 - 600 \lambda_{De}$ for $V_g = 20 \text{ V}$. The maximum field in this case is $\approx 100 \text{ V m}^{-1}$ corresponding to a fractional departure from charge neutrality of only 10^{-4} . The DL's are accompanied by density dips of $\approx 40\%$ and sharp peaks in the low frequency fluctuations. There are adjacent peaks in the ion and electron temperatures on either side of the DL corresponding to the presence of accelerated ion and electron beams. Probe measurements show the existence of an electron beam with a velocity corresponding to direct acceleration from the low potential plasma. This is dissipated within the region of enhanced electron temperature. The two-dimensional potential contours (axially symmetric) exhibit a penetration of the equipotential surfaces towards the cathode at midradial distance from the axis. The electron drift velocities, measured by the method of rotating Langmuir probes (Hirose *et al* 1970 [142]), are always $< \sqrt{kT_e/m_e}$. For example for the case with $V_g = 20 \text{ V}$ the drift velocity $< 0.3 \sqrt{kT_e/m_e}$. This together with the presence of low frequency fluctuations, identified as ion-acoustic turbulence, suggests a similarity to the simulation studies of ion-acoustic DL's. High frequency turbulence at the plasma frequency consists of unstable waves with phase velocities close to the electron beam velocity on the high potential side of the DL.

Observations of DL's with potential drops up to 1 kV in an argon plasma with electron temperature $T_e = 2 - 4 \text{ eV}$ have been reported (Torvén 1982 [143]). The experiments are made in a triple plasma machine. By strong pumping of the main chamber a low neutral gas pressure is maintained ($\leq 5 \text{ mPa}$) essentially eliminating ionisation effects so that the plasma properties can be determined by the discharge sources at either end. The sources produce argon plasma by a DC

discharge between the walls and heated tungsten filaments and the density may be controlled by changing the discharge current. The plasmas enter the main chamber through circular apertures, which determine the diameter of the plasma column given the confining magnetic field of 200 – 400 G. The plasma sets up ambipolar potential drops of ≈ 10 V between the sources and the main chamber in order to retard the electron flow and accelerate ions into the chamber. This resembles the behaviour of the sources in Q machines, when they are operated in the electron rich regime. Charge exchange collisions can build up a low energy ion population. The plasma density is typically $5 \cdot 10^{15} \text{ m}^{-3}$ so that the electron plasma frequency, $\omega_{pe} \approx 4 \cdot 10^9 \text{ s}^{-1}$, is comparable to the gyrofrequency $\omega_{ce} \approx 3.5 - 7.0 \cdot 10^9 \text{ s}^{-1}$.

DL's are produced by drawing a sufficiently strong current between the two sources, and their potential is determined by the applied potential difference. Measurements with emissive probes show that the high potential applied to the "anode" source propagates into the central chamber at about the ion acoustic speed ($2 - 3 \cdot 10^3 \text{ ms}^{-1}$). A DL with a stationary position is formed within the chamber after $50 - 100 \mu\text{s}$. The potential of the whole plasma column rises to increase the electron flux from the "cathode" source to compensate for the inhibition of electron flux by the high potential on the anode side. Along the axis the potential in the cathode plasma decreases gradually towards the DL so that there is a minimum just before the sharp rise to the anode plasma potential which increases towards the anode source. The radial electric field directed outwards from the anode plasma is stronger than the corresponding field on the cathode side which is inwardly directed. The equipotential surfaces mainly wrap around the high potential anode plasma. There is a consequent radial loss of ions so that the axial ion current, j_i , should decrease away from the anode source. For a given total current, which for strong DL's is close to the electron current, j_e , the Langmuir condition, $j_i/j_e = \sqrt{m_e/m_i}$, may therefore be satisfied by a change of the DL position. This explains the response of the DL to a change in the anode source density: It moves towards the source if the density is reduced in order to maintain the value of j_i required by the Langmuir condition. Similarly for increasing applied potential the

radial ion losses should be increased accounting for the DL shift towards the anode source. The Langmuir condition is also tested by varying the source densities in such a way that the DL remains at the same position. The estimated ion to electron current ratio is found to be approximately constant as required. Using a specially designed screened probe the presence of an accelerated electron beam is verified, as well as an ion flow towards the DL. Foot point oscillations are found on the cathode side, which may be suppressed by increasing the anode source density, whereas a decrease leads to large scale relaxation oscillations. Finally if the neutral gas density is allowed to increase the DL potentials become limited to 20 – 30 V and become independent of the anode source density, implying that the anode plasma is produced by impact ionisation as in previous double plasma machine experiments (Torvén and Lindberg 1980 [126], Andersson 1981 [128]).

Locally induced current disruptions have been studied in laboratory experiments on current sheet formation and dynamics (Stenzel *et al* 1983 [144]). These are produced by raising the potential of a central section of the anode. Spontaneous disruptions occur when a critical current density is exceeded, displacing the current to the sides of the sheet. An inductive voltage is then generated through the inductance of the external circuit and the electric field is concentrated into a DL. Particles are accelerated and some background plasma heating is found. A physical model is proposed in which the electron drift velocity is limited when it approaches the thermal velocity, so that the interrupted current cannot be restored directly. Once the DL forms it reduces the ion density on the anode side, hence reducing the electron density and further reducing the current. There is an effective negative resistance since the increasing electric field leads to a current reduction. An unstable feedback is set up leading to an impulsive loss of current.

Investigations of DL formation in a magnetised plasma column when the external electric circuit is dominated by an inductance, L , have been reported (Torvén *et al* 1985 [145]). The experiments are performed in the previously described triple plasma machine (Torvén 1982 [143]) now including an inductance, $L = 100mH$, in the external circuit. Periodic current disruptions are found when the applied voltage, U_0 , exceeds a critical value. The sources are operated asymmetrically so

that in the main chamber $T_e = 10\text{eV}$ and $n_e = 1.5 \cdot 10^{16}\text{m}^{-3}$ near the cathode source, and $T_e = 8\text{eV}$ and $n_e = 0.5 \cdot 10^{16}\text{m}^{-3}$ near the anode source. For small applied voltages there is a flat minimum in the potential profile, which develops as the voltage increases, current disruptions and inductive voltage pulses appearing for $U_0 \geq 30\text{V}$. Typically, for $U_0 = 100\text{V}$ and current $I = 105\text{mA}$, 400V overvoltages appear. The pulse durations are constant but their frequency of occurrence increases, becoming nearly periodic for $U_0 \geq 100\text{V}$. Probe measurements indicate that the whole inductive voltage is taken up by a DL. The current drops below 20% during a disruption indicating that 95% of the inductively stored energy is released as particle energy by the DL. The potential minimum deepens during a disruption and can reduce the current by reflection of electrons. At maximum overvoltage there are strong radial electric fields and the potential surfaces are of the same general form as observed previously (Torvén 1982 [143]) taking a rounded V-shaped form on the anode side.

Further studies of DL formation in the same triple plasma machine as used for studies of periodic disruptions (Torvén *et al* 1985 [145]) have been presented (Carpenter and Torvén 1987[a] [18], Carpenter and Torvén 1987[b] [92]). The overvoltage seen at the same time as the current disruptions is given exactly by LdI/dt and is much greater than the applied voltage. The plasma current, I , in turn is controlled by the potential minimum and there is a close qualitative agreement. To test the relationship quantitatively the inductance is removed and the current-voltage ($I - V$) characteristics of the device found by slowly varying the applied voltage U_0 . The magnitude of the potential minimum V_m is also measured. This leads to truncated velocity distributions for both electron populations originating at the sources. The cut off velocities $v_{a,c}$ are determined by the condition that an electron from a particular source is just reflected at the potential minimum. Thus if a distribution is given at a potential $V_{a,c}$, the cut off velocity there is,

$$v_{a,c} = \pm \sqrt{2e(V_{a,c} - V_m)/m_e} \quad (7.1)$$

The sign being negative for a source on the left. The reflected electrons form symmetric distributions and the electron current I_e is carried by the tails. The net current density, j_e , is given by,

$$j_e = -e \int_{|v_c|}^{\infty} v f_{e,c} dv - e \int_{-\infty}^{-|v_a|} v f_{e,a} dv \quad (7.2)$$

where by convention the “cathode” source (subscript “c”) is here to the left of the “anode” (“a”). Since the minimum is on the low potential side of the DL, the anode cut off velocity becomes large as the DL strength increases so that the electron current from the cathode source “c” dominates. Assuming Maxwellian distributions and including an ion current determined by the potential difference $V_a - V_c$ alone the total current may be evaluated (Carpenter and Torvén 1987[a] [18]) and is found to give good agreement with the measurements. To fit the measurements higher source temperatures than those measured must however be assumed. This suggests that the tails of the distributions, which carry the electron current, are overpopulated probably consisting of primary ionising electrons in the sources.

The dependence of the potential minimum on $U_0 = V_a - V_c$ may be understood by introducing the requirement of charge neutrality at the extended potential minimum, which should be quasi-neutral. Calculations based on assumed distributions from the sources as for the evaluation of the current give a good qualitative agreement with observations. (Radial ion losses which depend on U_0 are not included.) As U_0 increases the minimum weakens i.e. V_m increases, due to the reflection of anode electrons requiring a compensating reduced reflection of cathode electrons. Once the anode electrons are virtually all reflected, the accelerated ion beam density dominates and V_m increases to maintain neutrality as the ion beam density decreases. This results in a decrease of the electron current as U_0 increases, and explains the negative resistance regime of the I-V characteristic.

Oscillations appear in the experiment, for the negative resistance regime, if the resonance frequency of the external circuit is reduced by a parallel LC -circuit. Introducing a normalised time $\tau = t/\sqrt{L/C}$ the voltage drop between the sources V satisfies,

$$\frac{d^2 V}{d\tau^2} + g(V) \frac{dV}{d\tau} + V = U_0 \quad (7.3)$$

where the experimentally determined function $g(V) = \sqrt{LC} dI/dV$ (Carpenter and Torvén 1987[b] [92]). This implies exponential growth for small perturbations if $g(V) < 0$ corresponding to the observed oscillations. These appear beyond the maximum of the I-V characteristic and disappear when the negative resistance dI/dV becomes so small that the neglected resistances in the circuit become important. In general the equation is non-linear through the factor $g(V)$, but is amenable to analytical studies.

Studies of the dynamics (Carpenter and Torvén 1987[a] [18]) with carefully designed probes for quick response reveal a backward and forward jitter motion of the DL at around the ion-acoustic speed. The formation process is studied by abruptly applying a constant potential with a $1\mu s$ rise time. Initially a nearly constant field is set up along the plasma column. The potential profile then steepens locally to form the DL within somewhat less than the ion acoustic time scale. The time resolution has been improved in more recent experiments and the three dimensional potential structure found (Bohm and Torvén 1987 [93])

The formation of ion-acoustic DL's starting from the growth of ion-acoustic instability has been demonstrated in two separate experiments (Chan *et al* 1984 [146], Sekar and Saxena 1985 [147]). In the second case a modified double plasma machine was used, in which the ionisation in the source chamber was provided by electron impact ionisation using heated tungsten filaments and the plasma in the main chamber was produced by energetic electrons from the source chamber (Sekar and Saxena 1985 [147]). A steady beam of electrons is injected into the main chamber by an applied voltage difference. Above a critical voltage strong monotonic DL's are formed. The experiments are performed below this voltage, and the development is triggered by adding a negative step to the applied voltage. Ion-acoustic fluctuations spatially growing away from the separating grids reach an amplitude which is sufficient to locally trap ions. A persistent negative potential dip is formed which partially reflects the streaming electrons. The dip becomes asymmetric, with a potential jump on the high potential side as for ion-acoustic DL's, and with a net potential difference $e\phi_{DL}/kT_e \leq 0.4$. There is also

an associated density depletion $\delta n/n \leq 0.8$ with a density spike on the low potential side. If a positive voltage step is instead applied a local density enhancement is produced, but this decays at the same time as a small density depletion grows and a DL develops. The DL development closely follows the scenario of numerical simulations (Sato and Okuda 1980 [33]) and the structures resemble asymmetric ion holes (Schamel 1982 [31] and 1986 [32]).

Experiments in a double plasma machine have demonstrated the formation of a DL by injection of an ion beam into a magnetic mirror (Stenzel *et al* 1980 [148] and 1981 [149]). The ion beam is created by raising the potential of the source chamber with respect to the target chamber. There is an externally applied uniform magnetic field, on which is superimposed the field of an oppositely aligned permanent magnet in the target chamber. There are electrodes at the faces of the magnet, which are positively biased so as to draw an electron saturation current, producing electrode sheathes. Both hydrogen and argon plasmas are used to compare the effects of having magnetised and non-magnetised ions. A DL is formed if the electrode bias at the magnet V_m is sufficient to reflect the ion beam with energy eV_b i.e. if $V_m > V_b$. The DL is limited to the central current carrying channel, and its position is very stable. Using an energy analyser, the electron distribution is found to consist of accelerated electrons and trapped low energy electrons, which must be supplied by wave-particle scattering. Ionisation effects are shown to be negligible. The ion population on the anode side consists of decelerated and reflected beam ions. The evolution is studied by a pulsed injection of the ion beam. A weak DL emerges from the initial electrode sheath, propagates (at less than the ion-acoustic velocity) and grows in amplitude at the same time as the potential profile steepens. The final position and strength of the DL are experimentally found to be determined by the need to establish charge neutrality on the high potential side. The equipotential contours wrap around the low potential region to give a V shaped DL (for argon the DL is wider and more planar). If the external magnetic field is varied, the position of the separatrix between it and the field of the magnet changes. The electron supply to the DL

depends sensitively on the field topology, and if the bias field is reduced the enhanced electron supply pushes the DL back into the sheath. The unimportance of ionisation effects is demonstrated by forming weak DL's in an afterglow plasma. It is suggested that magnetospheric DL's may be formed, in a way analogous to that in the experiment, as a result of the precipitation of energetic ions from the magnetotail.

The effects of a non-uniform magnetic field on DL formation and related phenomena have been studied in two double plasma machines of discharge type (Alport *et al* 1986 [150], Plamondon *et al* 1988 [151]). The basic configurations are similar in both cases. An argon plasma is produced in a source chamber by a DC discharge, enters a relatively long main chamber ($\approx 1\text{ m}$) with a constant uniform magnetic field, followed by an end chamber with diverging magnetic field and an anode plate. In both cases the experiments are performed so that the electron current drift is away from the magnetic mirror formed by the diverging magnetic field.

In the first case, a directly heated conical helical cathode is used in the source chamber and the plasma enters the main chamber through an anode mesh. The plasma densities in the middle of the end chamber are in the range $10^{14} - 10^{16}\text{ m}^{-3}$ and increase with the background neutral gas density. The neutral gas pressure is 13 – 130 mPa and hence the degree of ionisation is typically small $n_e/n_n < 10^{-3}$. The magnetic field of several kilogauss is sufficient to strongly magnetise electrons but for the ions the collision and gyrofrequencies are comparable. Electron temperatures are 1 – 3 eV i.e. an order of magnitude greater than the expected ion temperatures. Stable DL's are produced by increasing the voltage applied to the anode plate (Alport *et al* 1986 [150], Cartier and Merlino 1987 [152], Merlino and Cartier 1987 [153]). After an initial rise the current-voltage dependence shows a current limitation, which is broken at $\approx 40\text{ V}$ when electrons accelerated through the anode sheath become energetic enough to ionise the background gas (ionisation energy $\approx 15.8\text{ eV}$) producing a new plasma which can carry a higher current. This plasma expands detaching the sheath from the anode to form a three dimensional DL. For higher neutral densities the current-voltage characteristic exhibits

a negative resistance at the appearance of the DL. The DL region is visible as a conical region of enhanced light-emission which matches the three-dimensional potential structure. Strong DL's may be produced ($\phi_{DL}/kT_e \sim 5 - 15$). The DL moves further away from the anode as the voltage is increased until the region of uniform field is reached. The DL then tends to be unstable. The DL also moves in the same direction if the neutral gas density is increased. The thickness ΔL decreases corresponding to the increased electron density so that $\Delta L \sim 10 - 100\lambda_{De}$.

Waves at the ion-cyclotron frequency and its harmonics are found to be associated with the DL (Alport *et al* 1986 [150]). Phase relations indicate radially inward propagation on the high potential (anode) side. On the low potential side there is a central standing wave component, and outwardly directed propagation in the outer region. The electron density has a minimum value on the axis and on the anode side of the DL there is a large region of density depletion. The DL produces an electron beam, detected via the characteristics of a Langmuir probe, which decays within the anode plasma over a few *cm* possibly due to electron-neutral elastic collisions. A possible mechanism for wave excitation is the current-driven ion cyclotron instability (Drummond and Rosenbluth 1962 [154]). This is in line with the maximum drift velocity, which is located in the maximum density at the outer edge of the plasma, and the observed inward wave propagation. However the frequency appears to be locked to the ion-gyrofrequency at the DL. In this way there is a resemblance to experiments on ion cyclotron instabilities where the frequency is determined by the magnetic field near the exciting electrode (Hatakeyama *et al* 1980 [155]). In this respect it may be remarked here that a relaxation instability could also be involved as has been suggested for ion cyclotron instability experiments (Popa *et al* 1985 [156]).

In the experiments carried out in the second plasma machine (Plamondon *et al* 1988 [151]), an indirectly heated oxide cathode is used in the source chamber and the plasma enters the main chamber through an 'iris diaphragm' with adjustable diameter. The electron density in the main chamber without current is measured in several ways and is $n_e \approx 5 \cdot 10^{15} \text{m}^{-3}$. The low neutral gas pressure 40 – 70 mPa is maintained by two diffusion pumps and is in the same range as in the other device.

The electron temperatures are also comparable 4 – 4.6 eV. The magnetic field is weaker, 350 G, decreasing to 100 G at the anode surface. Stable DL's are again produced by increasing the anode voltage. The current–voltage characteristic exhibits a very sharp peak at an anode voltage ≈ 4 V (floating potential, i.e. for no plasma current, -15 V) after which the current drops abruptly and remains constant until the voltage reaches ≈ 33 V and a DL with an approximately equal potential forms. The DL moves towards the uniform field region as the voltage is increased, finding no equilibrium there for fields ≥ 200 G.

There is a potential minimum on the low potential side of the DL which controls the current by reflecting electrons (Plamondon *et al* 1988 [151]). If the magnetic moment of an electron is $\mu = mv_{\perp}^2/2B$ the resulting mirror force leads in adiabatic theory to an 'effective potential',

$$\psi(x) = \phi(x) - (\mu/e)B(x) \quad (7.4)$$

with a minimum depending on μ so that the transmitted current depends on the spatial distribution of the axial field $B(x)$ strength and potential $\phi(x)$. If however the position of the minimum is approximately assumed to have the same position, the current is found to be,

$$j = j_0 \exp(-\bar{\phi}_m) \left(1 + \frac{1 - \exp(-\alpha \bar{\phi}_m)}{\alpha} \right) \quad (7.5)$$

where $\bar{\phi}_m = e(\phi_s - \phi_m)/kT_e$, ϕ_s is the potential at the source where a half Maxwellian distribution of electrons is assumed, $\alpha = B_m/(B_0 - B_m)$ and the subscript m refers to values at the minimum point. Using experimental values for the minimum values the theoretical value of the current j broadly matches the measured current–voltage characteristic. This helps to confirm the significance of the potential minimum in controlling the current.

Recently, evidence has been found for the existence of very strong (3 – 5 kV) propagating DL's in high density fully ionised hydrogen and argon plasmas (Lindberg 1988 [157]). The initial plasma is created in a conical theta–pinch plasma gun (Lindberg 1978 [158]), expands along an extended length of tubing, enters

a wider main chamber and finally reaches a grounded end plate. The plasma production is pulsed and the velocity in the main chamber is given by the time of flight. There is a uniform axial magnetic field of 1000 G. The electron density (from microwave transmission) approaches $n_e = 2 \cdot 10^{19} \text{m}^{-3}$, and for measured temperatures $T_e = 5 - 10 \text{eV}$ the Debye length is $\lambda_{De} \approx 0.01 \text{mm}$. DL's and current disruptions are produced when a critical plasma current is reached. This current is produced by applying large positive or negative voltages to a grid at the entrance to the main chamber. The voltage source is designed so that if it was short circuited the current would correspond to a plasma current with drift velocity exceeding the electron thermal velocity. For a current antiparallel to the flow a current disruption of 80 – 90% is found. Probes show the passage of several DL's propagating at around or slightly below the plasma expansion velocity. For parallel current the DL's first move upstream before reversing direction and moving with the plasma. A very high positive pulse close to the DL is argued to be due to the loss of electrons energised by interaction with the electron beam produced by the DL. Microwave emission is often seen at the first (strongest) disruption.

7.2 Q Machine Experiments

Stationary DL's were produced in a strongly magnetised ($B = 2 - 4 \text{kG}$) fully ionised Q machine plasma (Sato *et al* 1981 [159], Sato *et al* 1983 [160]). The position of the DL produced by externally applying a potential difference is controlled by adjusting the densities of the plasmas produced by contact ionisation of potassium vapour at the two hot tantalum plates located at the ends of the plasma column. The DL moves away from a given source if the density is increased and is located near the centre of a broad density minimum. The strengths, determined by the applied potential, may be varied from weak, $e\phi_{DL}/kT_e \approx 1$ to very strong, $e\phi_{DL}/kT_e \approx 2 \cdot 10^3$, where the electron temperature $T_e \approx 0.2 \text{eV}$ ($\approx T_i$). As the applied voltage is increased the DL moves towards the high potential source. The plasma current is limited by the DL and develops low frequency fluctuations. Fluctuations also appear locally at the low potential edge of the DL, where an ion beam is detected. The dissipation of the electron beam seen on the high potential side is ascribed to the development of finite plasma Langmuir wave modes.

The formation of the DL was studied by applying the potential abruptly as a step function of time and using sampling techniques to build up a picture of the space-time evolution of the plasma parameters over many repetitions. Initially the whole plasma was raised to the positive potential applied to one source except for a narrow sheath at the other grounded source. The localised potential drop then moves as a DL towards the source on the high potential side followed by a plasma flow from the grounded source, to establish an equilibrium configuration.

DL layer dynamics were investigated in a Q machine with a caesium plasma (Iizuka *et al* 1982 [161], Iizuka *et al* 1983 [162]). For single ended operation, the plasma is supplied by a grounded source and the DL is produced by applying a positive potential pulse at a cold collector terminating the plasma column at the opposite end. The DL forms at the grounded source and propagates towards the collector at a speed of $\approx 4\sqrt{kT_e/m_i}$. The electric field and the potential drop decrease as the DL approaches the collector where it becomes an ordinary electrode sheath. The DL lies at the moving front of plasma expanding from the source, and a negative potential dip on the low potential side, moving with the plasma, inhibits the electron current by reflection. When the DL reaches the collector the dip vanishes and the current increases to a maximum. For a constant applied potential plasma current oscillations are produced related to the repeated formation and propagation of the DL. For double ended operation with two sources, the initial response is similar but the DL comes to rest at an intermediate position depending on the density ratio between the two sources. The low potential tail then shows back and forth motions correlated to oscillations in the current. The current is a minimum when the low potential edge of the DL is steepest.

8. Global Effects

The local plasma properties strongly influence the structure and dynamics of a DL. However the current carried by the DL links it to an extended region, and the potential difference ϕ_{DL} must be externally maintained. These “global” conditions are clear in laboratory experiments, where an external circuit provides the current and applied voltage. Naturally occurring DL’s are also part of a global plasma

with an extended current system. The energy released by the DL in the form of accelerated particles must be taken from the external region, and may be in the form of kinetic, thermal or magnetic energy. In the latter case the energy is stored inductively in the large scale current system, and the release of this energy results in an inductive voltage maintaining the DL potential, and supplies the dissipated energy $I\phi_{DL}$. This process has been studied in the laboratory by including a large inductance in the external circuit (Torvén *et al* 1985 [145]).

The release of magnetic energy in natural plasmas is complicated by the fact that there is no simple external fixed circuit, although circuit models for natural plasma systems may be devised to contain some of the essential physical properties. On a large scale the magnetised plasma may be described in terms of magnetohydrodynamics. The appearance of a DL results in localised breakdown of a simple Ohm's law relation between the current density and the electric field. As a result a large scale MHD relaxation occurs, and the magnetic energy release in part goes to the DL.

As a simple illustration a cylindrically symmetric situation may be considered (Carlqvist 1979 [163], Raadu 1984 [164]) as shown in Figure 8. Within a flux tube of radius r_0 there is a total distributed current, I , which together with an axial magnetic field leads to a twisted field line configuration. Externally, $r > r_0$, there is only a constant axial field, so that there must be a return current, $-I$, on the surface of the twisted flux tube. If the current density is constant the available stored magnetic energy per unit length (which may be released by a complete untwisting of the field) is then $\mu_0/16\pi I^2$ i.e. the inductance per unit length is $\mu_0/8\pi$. Everywhere except at the DL it is assumed that the length and time scales are such that the resistive effects are negligible, so that,

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \quad (8.1)$$

Cylindrical coordinates, r, θ, z , are taken with the DL at $z = 0$, and the angular velocity $\Omega(r, z)$ is antisymmetric about the plane of the DL so that there is no net angular momentum.

A steady state is possible (Figure 8(a)) where all the plasma to the left of the DL within the flux tube ($r < r_0$) rotates as a solid body with an angular velocity Ω_0 . It is assumed that the surface $r = r_0$ always has zero potential, so that the surface return current, $-I$, does not release energy. Then from Equation (8.1) there is a potential $\phi_1(r)$ given by,

$$\phi_1(r) = \int_r^{r_0} r \Omega_0 B_z dr \quad (8.2)$$

within the rotating cylinder. To the right where the angular velocity is $-\Omega_0$ there is an equal and opposite potential distribution. The potential across the DL is therefore $\phi_{DL}(r) = 2\phi_1(r)$ with a maximum at the axis. The DL then releases energy at a rate,

$$\dot{W}_{DL} = \int_0^{r_0} j_z \phi_{DL}(r) 2\pi r dr = 2 \int_0^{r_0} r \Omega_0 \frac{B_\theta B_z}{\mu_0} 2\pi r dr \quad (8.3)$$

using Equation (8.2) and the relation between the axial current density j_z and B_θ in the MHD approximation. If free boundaries were to be introduced at say $z = \pm L$, this means that the energy dissipated by the DL is exactly equal to the total mechanical work which must be done against the surface magnetic stresses on the boundaries. For the sun this may correspond to a twisted magnetic loop in the corona containing a DL, where the dissipated energy is supplied by photospheric mass motions maintaining the twist.

If field line tying is now assumed on fixed boundaries at $z = \pm L$, the angular velocity must be zero there $\Omega(\pm L) = 0$ (Figure 8(b)). The field line twisting is no longer maintained. For a slow untwisting (i.e. for a time scale $\tau \gg L/v_A$ where $v_A = B_z/\sqrt{\mu_0 \rho}$ is the velocity of Alfvén modes along the flux tube) a constant gradient of the angular velocity along the axis may be assumed with amplitude $\pm\Omega_0$ on either side of the DL. The DL potential $\phi_{DL}(r)$ and energy release $\dot{W}_{DL}$ (Equation (8.3)) are as in the previous case. The magnetic field is unwinding uniformly so that,

$$\frac{\partial B_\theta}{\partial t} = -\frac{\Omega_0}{L} |B_z| \quad (8.4)$$

leading to a rate of change of the stored magnetic energy,

$$\dot{W}_B = 2L \frac{\partial}{\partial t} \int_0^{r_0} \frac{B_\theta^2}{2\mu_0} 2\pi r dr \quad (8.5)$$

which using Equation (8.4) is equal to $-\dot{W}_{DL}$ i.e. the energy dissipated by the DL is supplied from that stored in the field line twist.

The dynamics of the plasma within the twisted flux tube sets an upper limit to the DL potential (Carlqvist 1979 [163]) and to the rate of energy release at the DL. This may be seen by considering the response of the twisted flux tube to the sudden introduction of a DL (Figure 8(c)). The energy release is supplied by the untwisting of the magnetic flux tube on either side of the DL. The length which can be untwisted at any time is limited by the finite propagation velocity, the Alfvén velocity v_A , of the wave fronts starting at $z = 0$ when the DL is formed. For a uniform axial current j_z , i.e. for an inductance $\mu_0/8\pi$, the maximum energy release rate, given by complete untwisting, is therefore,

$$\dot{W}_{max} = \frac{\mu_0}{8\pi} I^2 v_A \quad (8.6)$$

The released energy goes to the DL and to the kinetic energy of the continuously increasing length of is reduced accordingly. The axial field is now assumed to be constant

and the dynamics can be shown to lead to a propagation velocity of the wave fronts equal to the Alfvén velocity $B_z/\sqrt{\mu_0\rho}$. Only a fraction κI of the initial current crosses the DL and the energy release rate there is,

$$\dot{W}_{DL} = \frac{1}{2} \kappa I \phi_{DL}(0) = \frac{1}{2} \kappa I \Omega_0 B_z r_0^2 \quad (8.7)$$

where B_z is a constant. The energy input to the rotational motions is at a rate,

$$\dot{W}_\Omega = \frac{\pi}{2} \rho r_0^4 \Omega^2 v_A \quad (8.8)$$

The condition that this must be less than the maximum total energy release rate, i.e. $\dot{W}_\Omega < \dot{W}_{max}$ gives an upper limit to the DL potential (Carlqvist 1979 [163]),

$$\phi_{DL}(0) = \Omega_0 B_z r_0^2 < \frac{\mu_0 I}{2\pi} \frac{B_z}{\sqrt{\mu_0 \rho}} \quad (8.9)$$

In the case of a maximum potential the current through the DL κI is zero. To find the maximum energy release rate at the DL it is necessary to consider a partial untwisting, $\kappa > 0$. The energy release rate is then,

$$-\dot{W}_B = \frac{\mu_0}{8\pi} (1 - \kappa^2) I^2 v_A \quad (8.10)$$

This must equal the sum of the dissipation at the DL and the input to the rotational kinetic energy (Equations (8.7) and (8.8)) from which it follows that the DL potential is,

$$\phi_{DL}(0) = \frac{\mu_0}{2\pi} (1 - \kappa) I \frac{B_z}{\sqrt{\mu_0 \rho}} \quad (8.11)$$

and the energy release rate at the DL is,

$$\dot{W}_{DL} = \frac{1}{2} \kappa I \phi_{DL}(0) = 2\kappa(1 - \kappa) \dot{W}_{max} \quad (8.12)$$

and for the rotational motions,

$$\dot{W}_\Omega = (1 - \kappa)^2 \dot{W}_{max} \quad (8.13)$$

The dissipation is maximum at the DL for $\kappa = 0.5$ and an equal energy input is then delivered to the mass motions. For a flux tube with boundaries at $z = \pm L$ this description is valid for times less than L/v_A after which the boundary conditions and the reflection properties of the wave fronts must be taken into account. The three examples discussed here clearly show the interdependence between the DL and the resultant MHD relaxation that it produces on a global scale. Such considerations are of great importance for astrophysical applications where the microphysics of the DL is hardly accessible to direct observation, whereas the large scale properties of the ambient plasma are often quite well known. Limits may thus be set on the possible DL parameters.

The particular example of the response of a twisted magnetic flux tube to the introduction of a DL illustrates the close coupling that is to be expected between the internal dynamics of a DL and the response of the external system. The complexity of this response may be simplified by introducing equivalent circuits, modeling the volume distribution of electrical currents by a limited number of currents passing through circuit elements corresponding to components of the physical system.

This has for example been done in the case of geomagnetic substorms (Boström 1974 [165]). An equivalent circuit is found for the coupling between the solar wind, magneto-tail and ionosphere. The substorm is associated with a diversion of a part of the neutral sheet current down into the ionosphere by Birkeland currents. The triggering mechanism is an anomalous increase of the neutral sheet resistivity. The appearance of a DL would have an equivalent effect (Alfvén 1977 [166]). Circuits have also been proposed for magnetic energy storage in the solar atmosphere and for the release by solar flares (Spicer 1982 [167]). Such energy storage may be accomplished through an expansion of the current system leading to an increasing inductance with little change in the total current (Zuccarello *et al* 1987 [168]).

Global effects, represented by equivalent circuits have been included in some simulation studies of DL formation (Sato and Okuda 1981 [96], Smith 1982 [85]). In one case (Sato and Okuda 1981 [96]) the circuit was chosen to model the coupling between the magnetosphere and the ionosphere, which acts as a resistive load. In the second case (Smith 1982 [85]) the external circuit included inductance, resistance and an applied voltage, so that the effects of magnetic energy storage and release are included.

A further simplification may be introduced to study the mutual interaction between the DL and the ambient plasma by representing the DL itself by an equivalent circuit element. Such a representation is only possible in a restricted physical range e.g. for a particular frequency range. On a time scale intermediate between that of electron and ion dynamics, an analysis is possible where the ions are assumed to be fixed and the electrons continuously respond to external changes through a series of steady states (Silevitch 1981 [169]). In some cases the DL can exhibit a negative resistance. The method is refined in a later paper (Raadu and Silevitch 1983 [170]), where it is shown that a particular magnetospheric DL model (Kan and Lee 1980 [24]) can show a negative resistance. If the external resistance is less than the magnitude of this negative resistance an instability can develop, eventually modified by the ion response. Relaxation oscillations (Burger 1965 [171]) may then be set up.

9. Magnetospheric Double Layers

The observation of accelerated electrons in a rocket experiment was one of the first indications that electrostatic acceleration could occur in the magnetosphere (McIlwain 1960 [172]). Barium cloud experiments have given direct evidence of electric fields parallel to the magnetic field at altitudes ≥ 5000 km (Haerendel *et al* 1976 [173], Wescott *et al* 1976 [174]). Barium ions were abruptly accelerated upwards to energies of several KeV, indicating the existence of an efficient mechanism for the energisation of auroral electrons. A similar interpretation may be placed on the observation of O^+ ions, presumably originating from the ionosphere, moving up along auroral magnetic field lines (Shelley *et al* 1976 [175]). Parallel electric fields may also be argued to be responsible for the “inverted V” events observed in the spatial structure of electron energy spectra (Ackerson and Frank 1972 [176]). Parallel electric fields are clearly an important aspect of the auroral phenomenon (Fälthammar 1983 [177]). One possible means of sustaining the parallel field is through anomalous resistivity (Papadopoulos and Coffey 1974 [178]) produced by current driven instabilities. However DL’s are an equally viable mechanism (Block 1972[a] [20], 1972[b] [27], 1978 [179]).

An early indication of magnetospheric DL’s with potentials of several kilovolts was provided by an interpretation of the visible motion of auroral forms (Carlqvist and Boström 1970 [180]). If there is a DL of finite width on a set of auroral field lines, and if the large scale field is electrostatic, there must also be electric fields perpendicular to the magnetic field in addition to the parallel fields within the DL. These lead to an $\mathbf{E} \times \mathbf{B}/B^2$ drift motion of the magnetospheric plasma, mapping down along the field lines to give a shearing motion within the aurora. The observed shears then lead to the estimate of kilovolt potentials over a possible DL. This is an example of the way in which a DL can reveal its presence through the global response it produces in the surrounding plasma (see Section 8)

Analytical models of oblique plane parallel electrostatic DL’s (“shocks”) have been constructed (Swift 1975 [11]) and presented as possible acceleration mechanisms for the energetic electrons producing the discrete structure of auroral arcs. Highly oblique DL’s, where the angle between the magnetic field and the DL electric field $\theta \approx \pi/2$, could give a thickness across an auroral field line comparable to

a gyroradius for an extension parallel to the magnetic field on a magnetospheric scale.

The models are constructed by solving the time independent Vlasov–Poisson equations. Solutions of the Vlasov equation are given by arbitrary functions of the constants of motion. Particle densities are then found by integration over velocities. Cartesian coordinates are introduced with the y -axis in the DL plane and the z -axis parallel to the magnetic field. All quantities are assumed to only be functions of $\eta = z - \alpha x$ where $\alpha = \tan \theta$, and hence depend only on the distance, $\eta \cos \theta$, to the DL reference plane.

The particles are assumed to move adiabatically in a constant magnetic field and a varying electric field perpendicular to the DL. The electrons are assumed to be so strongly magnetised that they respond to the electric field by an acceleration $\dot{v}_{\parallel}$ along the magnetic field and an $\mathbf{E} \times \mathbf{B}$ drift in the y direction. The distribution function over the parallel velocity is then a function of the constant of motion $K_e = 1/2m_e v_{\parallel}^2 - e\phi$ and defined piecewise for positive and negative velocities as for simple one-dimensional DL's. An integration over velocity then gives the electron density. The ion-motion is influenced by finite gyroradius effects. To find constants of motion a guiding centre description is used, treating the gyroradius as a small compared to spatial scales. Analogously to the electron case one such constant is found to be $K_i = 1/2m_i v_{\parallel}^2 + e\phi + 1/2m_i E_x^2/B^2$ were the kinetic energy of an $\mathbf{E} \times \mathbf{B}$ drift motion has now to be included. For the electrons the adiabatic invariant, $\mu_e = v_{\perp}^2/B$, was not needed since the magnetic field B is constant. However for the ions there is a first order correction to this invariant, which in the present case with constant B takes the form that the gyroradius ρ_i is an adiabatic invariant. The gyrofrequency is modified by finite gyroradius effects so that the phase of the ion gyromotion ψ is given by

$$\dot{\psi} = \omega_{ci} \left(1 - \frac{e}{2m_i \omega_{ci}^2} \frac{\partial E_x}{\partial x} \right) \quad (9.1)$$

and the perpendicular velocity is $v_{\perp} = \rho_i \dot{\psi}$. The theory is based on the requirement that the correction term, which is due to the variation of the electric field over one ion gyration, is very small i.e. $(e/2m_i \omega_{ci}^2) |(\partial/\partial x) E_x| \equiv \delta^2 \ll 1$. A solution of the

ion Vlasov equation is now given by any function of the two adiabatic invariants, K_i and ρ_i . Integration over velocities gives the ion density. An expansion is made to first order in δ^2 and $m_i E_x^2/B^2$. Substituting the ion and electron densities in Poisson's equation finally gives,

$$(1 + \alpha^2 \kappa) \frac{d^2 \phi}{d\eta^2} + \frac{\alpha^2}{2} \frac{d\kappa}{d\eta} \frac{d\phi}{d\eta} = -\epsilon_0^{-1} e [n_i(\phi) - n_e(\phi)] \quad (9.2)$$

where the first order terms due to the finite ion gyroradius are all contained in $\kappa = 1 + \omega_{pi}^2/\omega_{ci}^2$ and $n_i(\phi)$ is the zero order ion density corresponding to strong magnetisation. A standard linear analysis with the same approximations gives an identical plasma dielectric coefficient, κ , and corresponds exactly with the linearised form of the equation.

Here instead of the original analysis (Swift 1975 [11]) a more general treatment is indicated. The equation may be rewritten in a different form,

$$\frac{d}{d\phi} \left[\left(\frac{d\phi}{ds} \right)^2 \left(1 + \frac{\omega_{pi}^2 \sin^2 \theta}{\omega_{ci}^2} \right) \right] = -2\epsilon_0^{-1} e [n_i(\phi) - n_e(\phi)] \quad (9.3)$$

where here $s = \eta \cos \theta$ the distance to the DL has been used. In this form it becomes clear that in the limit of strong magnetisation, $\omega_{pi}/\omega_{ci} \ll 1$, the DL structure becomes independent of the direction of the magnetic field. Also since the space derivative only occurs once it may be seen that the finite magnetisation effects give a local stretching of the structure compared with the strong case. Thus if there is a solution $\phi(s) = \Phi_\infty(s)$ for infinite magnetisation, solutions for finite magnetisation are given by $\phi(s) = \Phi_\infty(s'(s))$ where the coordinate stretching is given by

$$\frac{ds}{ds'} = \sqrt{1 + \frac{\omega_{pi}^2 \sin^2 \theta}{\omega_{ci}^2}} \quad (9.4)$$

where ω_{pi}^2 is proportional to the local ion density, $n_i = n_i(\Phi_\infty(s'))$. Here it should be noted that the solution $\Phi_\infty(s)$ is also the solution for $\theta = 0$, the non-oblique case. Typically the thickness of such DL's scale as the Debye length λ_D . In the oblique case with weakly magnetised ions ($\omega_{ci} \ll \omega_{pi}$) the coordinate stretching transformation shows that the DL thickness ΔL scales as $\bar{\rho}_i \sin \theta$ where $\bar{\rho}_i$ is a mean value of the ion gyroradius over the DL. However the analysis requires

that $\bar{\rho}_i \ll \Delta L$ corresponding to a thick, $\Delta L \gg \lambda_D$, non-oblique DL. Also the requirement that $\delta^2 \ll 1$ may be written as

$$\left| \frac{\text{div}(\epsilon_0 \mathbf{E})}{n_i e} \right| < \frac{1}{2} \frac{c^2 \sin^2 \theta}{v_A^2} \quad (9.5)$$

where the ratio between the velocity of light c and the Alfvén velocity v_A is normally very large. This means that the departure from charge neutrality within the DL must be small. In particular strong oblique DL's lie outside the range of the theory as pointed out in the original analysis (Swift 1975 [11]). A DL with beam like transmitted particles, i.e a strong DL, was considered for the non-oblique case.

A case with both a hot and a cold ion component together with an electron beam was found to fall within the range of applicability of the theory (Swift 1975 [11]) for magnetospheric parameters. The condition that the net charges at the edges of the DL have the appropriate sign, the Bohm conditions, gives lower limits to the energy of the injected electron beam W_{e0} . A necessary condition for charge neutrality on the low potential side gives a lower limit to the hot ion temperature, $T_{i,h}$, which may be written

$$kT_{i,h} > \frac{e\phi_{DL}}{\log(1 + e\phi_{DL}/W_{e0})^{1/2}} \quad (9.6)$$

and depending on the choice of parameters requires that the hot ion temperature is comparable to the DL potential, ϕ_{DL} .

Provided that adiabatic theory still holds, in particular if $\rho_i \ll \Delta L$, oblique DL solutions can be found even for cases where $\delta^2 \simeq 1$ and $U_E^2 \simeq v_{\parallel}^2$. The method is to calculate the ion density from the continuity equation, using the polarisation drift velocity,

$$v_z = U_E = \frac{e}{m_i \omega_{ci}^2} v_{\parallel} \frac{\partial^2 \phi}{\partial z \partial x} \quad (9.7)$$

where $v_{\parallel}$ is found by assuming K_i invariant (Swift 1975 [11]). The analysis may be simplified by assuming quasi-neutrality, so that Poisson's equation is not used. A second order differential equation is found (the derivatives come from the expression for U_E) and integrated to find DL potential profiles.

The energy of precipitating electrons in a discrete aurora varies in a characteristic way over a narrow region, typically 50 km in longitudinal extent, with a

sharp central peak. From the appearance of a plot of energy against position, this precipitation pattern is known as an “inverted V” structure. Optical observations reveal very thin substructures, “auroral arcs”, a few 100 m wide, which may be associated with fine substructures within an inverted V. At distances of an earth radius oppositely directed electric field structures have been detected on auroral field lines (Mozer *et al* 1977 [181]). The paired electric fields are inwardly directed (Temerin *et al* 1981 [139]), within a thin layer a few tens of kilometers thick. All three features appear to be associated (Torbert and Mozer 1978 [182]). Since the ≈ 1 V/m paired electric fields do not map down the field lines to the ionosphere, if they are electrostatic there must be an intervening region with electric field parallel to the magnetic field. The net potential difference could account for the ≈ 5 keV energies of inverted V electrons. There is now an accumulation of evidence for the association of parallel electric fields, particle acceleration and auroral structures (cf Fälthammar 1977 [177]).

According to one theory for auroral arcs and inverted V structures the perpendicular field is primary, being caused by a field aligned electron current sheet (Kan and Akasofu 1979 [183]). The electric field region is limited in width by the shielding effect of plasma sheet ions, which have a temperature of typically 5 keV, around 10 times that of the electrons. Pedersen currents close the current in the ionosphere, requiring only weak electric fields due to the high conductivity. The potential difference of the paired field region, given by the plasma sheet ion temperature is then taken up by parallel electric fields.

The theory has been developed to give two-dimensional numerical models (Kan *et al* 1979 [184]). These are found by finding solutions of the Vlasov–Poisson equations for given distributions of plasma sheet, ionospheric and trapped particles. The trapped electrons below the acceleration region originate from electron scattering by neutrals and from beam plasma instabilities. Auroral arcs are regarded as thin structures comparable to an ion Debye length, $\lambda_{D,i}$, wide and much thinner than an ion gyro-radius, ρ_i . One consequence is that ions are non-magnetised. Also since the structure is so thin it is assumed that in the Poisson equation the Laplace operator need only contain the perpendicular spatial variation $\nabla^2 \approx \nabla_{\perp}^2$.

The Poisson equation is then integrated perpendicular to the magnetic field, and a linear dependence on height is assumed for the minimum potential at the centre. In this way the two-dimensional potential pattern is found. The minimum potential at the upper boundary is determined via the perpendicular boundary conditions. For a particular example with 5 keV ions this minimum gives an accelerating potential of 6.35 keV. A model of an inverted V structure is found assuming a thickness much greater than an ion gyro-radius. Solutions are sought for an acceleration region with a vertical extent comparable to a Debye length, $\lambda_{D,i}$. The magnetic field can then be assumed to be constant, and the Laplacian is assumed to operate along the magnetic field only, $\nabla^2 \approx \nabla_{\parallel}^2$. At the upper boundary charge neutrality is assumed, so that the potential profile is given as a function of the spatial distribution of the particle densities. Integration on each individual field line gives a two-dimensional DL structure with total potential difference again close to that given by the temperature of plasma sheet ions.

A general result of these studies of auroral arc and inverted V models was that the maximum potential difference is set by the ion thermal energy, with a weak dependence on the density of the electron current sheet. The energy for the region as a whole is supplied by the plasma sheet ions, whereas for the current sheet there is an assumed constant current driving source maintaining the injected electron velocity distribution. The thermal energy of the ions may be maintained by magnetospheric convection at typical velocities, 15 km s^{-1} , across the current sheet above the acceleration region over a length of $\approx 10 R_E$. The structures, i.e. several thin auroral arcs with distributed parallel fields and an inverted V in the form of a DL, are thought of as superimposed. Electrons passing only through the inverted V produce a faint broad aurora, and those accelerated by both structures produce the embedded thin bright auroral arcs.

It has been proposed that oblique DL's are responsible for the origin of ion conics in auroral regions (Borovsky 1984 [185]). These consist of upwardly directed beams of ions having a pitch angle distribution with a minimum in the direction of the magnetic field. They were first seen in 1 keV O^+ ion beams at $\approx 7600 \text{ km}$ in the northern dayside polar cusp (Sharp et al 1977 [186]). Subsequently they have

been observed in high energy (≥ 500 eV) ion beams (mainly for O^+ ions) at great altitudes on auroral field lines (> 5000 km) (Ghielmetti *et al* 1978 [187]). Low energy (≤ 500 eV) conics have also been observed from a high altitude sounding rocket (at 400 – 500 km) (Whalen *et al* 1978 [188]) in regions of auroral electron precipitation. Correlations have been found between field-aligned ion beams, ion conics and electrostatic hydrogen cyclotron waves at great altitudes (> 5000 km) (Kintner *et al* 1979 [189]) suggestive of a single source for both types of ion distributions.

It has been proposed that ion cyclotron waves can heat ions transverse to the magnetic field and that they are subsequently accelerated by the earth's diverging magnetic field to produce the observed ion conics (Ungstrup *et al* 1979 [190]). However it is equally possible that they are a consequence of the ion-conics rather than the cause. Thus in two-dimensional simulations of oblique auroral DL's (Borovsky and Joyce 1983 [191]) ion cyclotron waves were found in association with phase bunched ion conics. It was found that the ions in fact became more field aligned as the waves grew, suggesting that the waves were a product of the conics. The conics were produced by the ion acceleration in the DL, so that the mechanism can only apply on the low potential side above the auroral DL. Perpendicular heating by resonant interaction with lower-hybrid modes is a possible mechanism for low altitude conics (Chang and Coppi 1981 [192]).

The simulation studies of oblique DL's (Borovsky and Joyce 1983 [94]) indicated that the conics are formed due to the tendency of weakly magnetised particles to be accelerated in the direction of the DL electric field, strongly magnetised particles being instead accelerated to form magnetic field aligned beams. A possible criterion for this was sought in a continuation of these studies (Borovsky 1984 [185]). The trajectories of variously magnetised test particles (ions) were followed using a steady state DL potential profile found in the simulation studies (Borovsky and Joyce 1983 [94]). Since these studies and analytical results (Borovsky 1983 [193]) have shown that the DL structure is practically invariant to the magnetic field strength and obliquity θ ($\leq 86^\circ$) the same potential profile, with a total difference $\Delta\phi = 50 kT_i/e$, was used for the various imposed magnetic

fields. The ions were assumed to be emitted as Maxwellians (with temperature T_i) from a wall, which absorbed any gyrating back to it in the magnetic field. Those reaching the DL thus had a non-isotropic distribution. The test ions with gyrofrequency ω_{ci} were characterised by a magnetisation parameter $\omega_{ci}\tau_{CR}$, where the crossing time τ_{CR} is that for an ion entering the DL with the mean velocity of a half Maxwellian distribution and moving parallel to the magnetic field. For an oblique DL at $\theta = 60^\circ$ a transition between electric field alignment and magnetic field alignment was found in the range $2.18 \leq \omega_{ci}\tau_{CR} \leq 8.72$. Also for three magnetic field inclinations, $\theta = 30^\circ, 60^\circ, 80^\circ$, the variation of the pitch angles averaged over 2000 test particles was found for varying magnetisation $\omega_{ci}\tau_{CR}$ by changing the magnetic field strength. In all cases alignment transition occurred in the range $1 \leq \omega_{ci}\tau_{CR} \leq 10$. These results were compared with a previous analysis of particle motions through an oblique DL (Borovsky and Joyce 1983 [191]). For a magnetic field aligned injection velocity the analytical results give an approximate upper limit to the perpendicular velocity $v_\perp$ of the accelerated ion, $v_\perp \leq \sqrt{2} E \sin \theta / B$, where E is the average DL electric field. This limit due to the gyromotion becomes effective when $\omega_{ci}\tau_{CR} > \pi$ in agreement with the test particle results. Also an analytical estimate of the pitch angle was found using a guiding centre approximation giving good agreement with the numerical results. The magnetisation parameter $\omega_{ci}\tau_{CR}$ was applied to the weak DL's observed in the auroral zone (Mozer *et al* 1977 [181]). For a range of injection energies it was found that accelerated H^+ ions should be magnetic field aligned, but O^+ ions should form conics. In regions where both magnetic field aligned beams and ion conics are observed for different species of ions, the parameter $\omega_{ci}\tau_{CR}$ can be used to give an estimate of the crossing time τ_{CR} and hence the thickness of the acceleration region. This may be used as a remote indication of the presence of DL's, which can accelerate particles with intense electric fields over short distances.

A two-dimensional particle simulation has been used to simulate the many magnetospheric plasma processes observed in auroral regions (Singh *et al* 1987 [194]). A current sheet acts as a driving mechanism. Electric fields perpendicular to the magnetic field develop at the edges, and a DL forms inside, accelerating electrons

downwards. Outside the current sheet an oppositely directed DL is produced. The properties of the accelerated ions inside the sheet depend on the thickness, beams being formed for thin sheets, whereas for sheets much thicker than an ion gyroradius heating predominates. The ion pitch angle distribution indicates acceleration perpendicular to the magnetic field as observed in ion conics. The accelerated electrons show a spread in both energy and pitch angle. A low frequency relaxation oscillation is found to be associated with electron plasma waves. Waves are also seen at low frequencies, in the range of the ion-cyclotron and lower hybrid frequencies.

The first observations of weak DL's in the magnetosphere were made with a three component electric field probe on the S3-3 polar orbiting satellite (Temerin *et al* 1982 [195]). The DL's were detected in the electric field component parallel to the magnetic field and appeared as localised regions of oppositely directed field. For a presumed upward motion these corresponded to a potential minimum with a net potential difference such as to accelerate electrons downward. Such a potential profile is observed for ion acoustic DL's in plasma simulations. Solitary waves with no net potential were also seen. The DL's and solitary waves occurred in an altitude range 6 000 – 8 000 km in regions with upgoing ion beams and enhanced downward electron flux. The forms of the particle distributions indicated a potential drop ≈ 0.5 keV below and a parallel electric field region above the satellite. Estimates of the DL velocities, ≥ 50 km/s , together with the typical duration of an event give a length of ≥ 200 m. The net potentials over a DL were then a few volts. From typical plasma parameters for such regions, a DL thickness was typically $40 \lambda_D$ and the average separation was $800 \lambda_D$. It was argued that if such DL's with a similar separation were distributed over an altitude of 1 000 km they could account for the full ≈ 0.5 kV potential needed to produce the observed particle distributions. There was a close association between the DL's and oscillations in the perpendicular field components (within a few seconds) interpreted as electrostatic ion cyclotron waves, which are commonly observed in acceleration regions. This low frequency turbulence may be important for the formation of the DL's.

A series of papers have considered the effects of low frequency turbulence, and in particular localised ion density depressions, on the response of electrons to the electric field component parallel to a magnetic field (Rowland *et al* 1981[a] [196], Rowland *et al* 1981[b] [197], Rowland and Palmadesso 1983 [198], Rowland and Palmadesso 1987 [199]). Quasi-neutrality for the reduced density, $n_0 - \delta n$, at an ion cavity requires the formation of a potential minimum, ϕ_{min} , to reduce the electron density, and for a Maxwellian distribution, $\phi_{min} = (kT_e/e)\log(1 - \delta n/n_0) < 0$. Such a minimum is also expected for situations where the relative drift velocity is small. (If the velocity exceeded the electron thermal velocity, the electrons would have to be accelerated to reduce their steady state density requiring a potential maximum (Alfvén and Carlqvist 1967 [7])). Electrons are then trapped between adjacent ion cavities, and for example cavities with a 20% density depression would trap around half the electrons for a Maxwellian distribution (Rowland *et al* 1981[a] [196]). When a uniform external electric field is applied and the bulk of the electrons are trapped, these electrons act to shield out the applied field, so that the applied potential drop is distributed over the cavities (Rowland and Palmadesso 1983 [198], Rowland and Palmadesso 1987 [199]). The resulting potential profile then resembles that for ion-acoustic DL's and the weak DL's observed in the magnetosphere (Temerin *et al* 1982 [195], Boström *et al* 1988 [200]). This mechanism may also operate on the electron dynamical time scale during the formation of ion-acoustic DL's, since these are also associated with local density depletions.

The formation of ion cavities as a direct result of current driven instabilities has been discussed in the case of high relative drift velocity in the linear (Carlqvist 1973 [57]) and non-linear regimes (Raadu and Carlqvist 1981 [59]) assuming quasi-neutrality. It is also a general consequence of current instabilities in the long wavelength, non-dispersive regime (Raadu and Carlqvist 1981 [59]). Another possibility is the production of "cavitons" in the presence of a high level of Langmuir turbulence, produced for example by electron beams (for a brief outline see reference, Sagdeev 1979 [201]). The electron plasma waves tend to concentrate in areas of low density and their radiation pressure, the ponderomotive force, pushes the

plasma away reducing the density further (Vedenov and Rudakov 1965 [202]). The non-linear development (Zakharov 1972 [203]) depends on the dimensionality of the system; stabilisation is possible in one-dimension, but not in three-dimensions, the two-dimensional case being marginal. Computer simulations have also shown cavitation as a result of electron beam produced Langmuir waves and by externally imposing a Langmuir wave with a given wavelength (Rowland 1980 [204]). In the case of the magnetosphere, cavities may be produced by the observed strong ion-cyclotron waves. The current driven waves grow at large angles to the magnetic field and give a separation of low density regions comparable to that between the observed weak DL's along field lines (Rowland and Palmadesso 1987 [199]). The threshold for the electrostatic ion-cyclotron instability is below that for the ion-acoustic instability.

When an electric field is applied, some of the non-trapped electrons moving against the field are directly accelerated, others initially travelling in the direction of the field are decelerated, and, if this deceleration is sufficient, reflected by the potential minima and then accelerated in the reverse direction. In this way a beam is formed with the peak of the distribution corresponding to those electrons originally just above the trapping velocity which have been freely accelerated (Rowland *et al* 1981[a] [196]). These runaway electrons can drive an instability through the anomalous Doppler resonance (Kadomsev and Pogutse 1968 [205]). They see a perpendicular field in their rest frame varying at the gyrofrequency ω_{ce} leading to resonant heating and pitch angle scattering. This only occurs for velocities $v_{\parallel} > v_c \approx 3(\omega_{ce}/\omega_{pe})\sqrt{kT_e/m_e}$ and may be seen in the $2\frac{1}{2}$ -dimensional simulations where a peak is visible in the parallel velocity distribution at $\approx v_c$ (Rowland *et al* 1981[b] [197]). The effect is lost in the one-dimensional Vlasov simulations (Rowland and Palmadesso 1987 [199]) which were used to study the electric field distribution for a relatively long system, $L = 512\lambda_D$, with several localised fixed ion density wave packets. These were chosen to represent the effect of ion-cyclotron waves, and the resulting structures showed features in common with the observed magnetospheric DL's (Temerin *et al* 1982 [195]).

Recent observations from the Viking satellite at altitudes of around 10 000 km (Boström *et al* 1988 [200]) have confirmed the earlier report (Temerin *et al* 1982 [195]) of the existence of solitary waves and weak DL's in the magnetosphere. The structures, which it is argued probably represent different evolutionary phases of a single phenomena, are detected simultaneously by density and electric field measurements. Measurements with a pair of probes biased to measure plasma density show localised rarefactions of up to 50%. An analysis of the delay between the signals at the probes shows that the phase velocities are upwards along the magnetic field and in the range 5–50 km/s, comparable to the ion acoustic velocity for the measured electron temperatures ($T_e = 1 - 10$ eV). The lengths of around 100 m then correspond to $5-50 \lambda_D$ for densities $10^6 - 10^7 \text{ m}^{-3}$. It is also concluded that the transverse size is larger than the probe separation (80 m), and that the evolution time is greater than the typical delay between probe signals ($10^{-2} \text{ s} > 10 \omega_{pi}^{-1}$). For the upward phase velocities, the electric field measurements imply a potential minimum ($|\phi| \leq 2 \text{ V}$) sometimes with a net upward potential drop ($\leq 1 \text{ V}$). Thus the measurements indicate the presence of ion holes or, when there is a net potential difference, ion acoustic DL's as seen in simulation studies. The correlation between maximum potential and density variations is consistent with a Boltzmann relation for 2 – 20 eV electrons. The measurements were made in an acceleration region with ≈ 1 KeV upwardly directed ion beams and downward electron beams. The motion of the thermal ions was uncertain but could be upward so that the direction of the motion of the holes and DL's with respect to these ions could be downward, corresponding to the initial motion seen in simulations.

DL's may also be present in the magnetosphere of Jupiter (Hubbard *et al* 1974 [206], Shawhan 1976 [207], Smith and Goertz 1978 [208]). The Jovian decametric radiation is modulated by one of its satellites, Io. This is argued to be a consequence of particle acceleration to several 100 keV by a DL. The driving potential of ≈ 580 kV is set up across Io by the relative motion with respect to the Jovian magnetic field. A closed current system is set up linking the ionospheres of Io and Jupiter along connecting magnetic field lines. The driving potential is

assumed to fall over a DL at the top of Io's ionosphere (Shawhan 1976 [207]). In this context it is argued that the DL formation may result from the action of the ponderomotive force of unstable waves produced by current driven instabilities (Smith and Goertz 1978 [208]).

10. Solar Flares

Solar flares are the result of a sudden irreversible release of energy in the corona and chromosphere of the sun (Švestka 1976 [209]). They lead to particle acceleration observed mainly in a range $10^4 - 10^9$ eV, and to a transient heating of the chromosphere (to $\approx 10^4$ K) and corona (to $\approx 10^7$ K). Electromagnetic radiation is produced over an extremely broad range from radio up to X- and sometimes even γ -ray frequencies. A typical flare passes through a sequence of phases. There is an initial preflare or preheating phase lasting around 10 min characterised by a slow optical brightening with an accompanying rise in the extreme ultraviolet (e.u.v.) and soft X-ray emission. This is followed by a short explosive or impulsive phase (≈ 1 min) during which there are impulsive emissions of hard X-rays, e.u.v. and microwaves i.e. the most energetic non-thermal events. There is a rise to maximum brightness and emission in soft X-rays and e.u.v. during the 5 – 15 min flash phase, followed by a slow decline in the main phase (≈ 1 hour). Flares occur mainly in active regions, containing sunspots and a generally enhanced magnetic field, and are believed to derive their energy through this field. There are strong arguments for the storage of energy in the magnetic field i.e. as inductive energy in an extended current system (Giovanelli 1947 [210] and 1948 [211], Syrovatsky 1972 [212]). Estimates of the energy release due to changes in the magnetic field topology are of the right order of magnitude (Tanaka and Nakagawa 1973 [213]). The total energy release in a flare is in a range extending over more than three orders of magnitude from $\approx 10^{22}$ J for a typical sub-flare to $\approx 3 \times 10^{25}$ J in the most energetic cases. For large flares this energy is roughly equally divided between electromagnetic radiation, energetic particles and shock waves propagating outwards into the solar wind region (Priest 1982 [214]). Flare “areas” are reported for the extent of the $H\alpha$ emission at maximum intensity and range from less than

$3 \cdot 10^8 \text{ km}^2$ for subflares, and from $7.5 \cdot 10^8 \text{ km}^2$ to over $3.6 \cdot 10^9 \text{ km}^2$ for major flares. However high resolution observations reveal small (diameter $\approx 3000 - 6000 \text{ km}$) impulsive flare kernels with enhanced brightness associated with the X-ray bursts (Vorpahl 1972 [215]). Small scale “compact” flares may take the form of simple loops accompanied in some cases by surges. The much larger “two-ribbon” flares, so called from the pair of $\text{H}\alpha$ emission regions in the form of long ribbons observed during the flash phase, are closely associated with a prior filament destabilisation and eruption.

A model for a flare has been proposed, where the energy is stored in a current loop in the solar corona and released by the creation of a DL (Jacobsen and Carlqvist 1964 [216], Alfvén and Carlqvist 1967 [7]) (Figure 9(a)). The model has been analysed in some detail in order to make comparisons with flare related parameters (Carlqvist 1969 [8]). It is argued that due to the high conductivity of the ambient coronal plasma only the internally stored energy $L_i I^2/2$ is available for release as flare energy W_f . For a loop of radius R and a parabolic distribution of current density within the current channel, the internal inductance is,

$$L_i = \frac{11\mu_0 R}{24} \quad (10.1)$$

from an inductance $11/6 (\mu_0/8\pi)$ per unit length (the factor $11/6$, as compared to 1 in Section 8, is due to the non-uniform current distribution assumed here). The inductance is independent of the width of the current channel. For a radius $5 \cdot 10^6 - 5 \cdot 10^7 \text{ m}$, $L_i \approx 3 - 30 \text{ H}$ and typical flare energies $W_f \approx 10^{21} - 10^{25} \text{ J}$ may be stored for currents $I \approx 10^{10} - 10^{12} \text{ Amp}$. In order that the time scale for energy release at the DL,

$$\tau_0 = W_f / I \phi_{DL} \quad (10.2)$$

should be comparable to the impulsive or flash phase, the DL has to be highly relativistic so that,

$$id^2 \approx \frac{4\epsilon_0 c}{m_i c^2 / e} \phi_{DL}^2 \quad (10.3)$$

(Equation (2.9)) and according to the relativistic Langmuir condition $i_i \approx i_e$ so that equal numbers of ions and electrons are accelerated. The parameters are such that the dissipation rate $I \phi_{DL}$ can be maintained by the rate of field line

untwisting. It is argued that the one-dimensional description of the DL breaks down when the thickness d exceeds $\approx r_0/2$ where r_0 is the radius of the current channel. The potential should saturate at this limit, and a maximum voltage for a given current $I = i\pi r_0^2$ may be estimated from Equation (10.3) by substituting $d = r_0/2$. Thus for $I = 3 \cdot 10^{11}$ Amp the maximum value of ϕ_{DL} is $\approx 5 \cdot 10^{10}$ V. Now for a given loop internal inductance L_i and stored flare energy W_f , the current is given by $I = \sqrt{2W_f/L_i}$. Hence the maximum possible DL potential and the minimum of τ_0 the DL energy release time (Equation (10.2)) are also determined. Thus for $W_f = 10^{23}$ J and for an inductance $L_i = 20$ H it is found that $I = 10^{11}$ Amp, $\phi_{DL,max} = 3 \cdot 10^{10}$ V, $\tau_{0,min} = 30$ s, and the total number of primary accelerated ions and electrons are both 10^{31} . It is also concluded that, for flares with energies $10^{21} - 10^{25}$ J to release energy from a circuit carrying $10^{10} - 10^{12}$ Amp in ≈ 100 s, potentials $\phi_{DL} = 10^9 - 10^{11}$ V are necessary.

The primary electrons at coronal heights initially lose their energy through synchrotron emission (within < 0.1 s for a 10 GeV electron in 1000 Gauss field). The primary ion energies are comparable to those of solar cosmic rays observed at the Earth. The energy spectra are difficult to estimate since the DL is expected to be time dependent. The magnetically trapped ions will lose energy by causing the configuration to expand, leading to regions of compressional heating. It is argued that such regions should give rise to optical radiation.

In a further development of the model the velocities associated with the magnetic energy release were estimated (Carlqvist 1979 [163]). According to the arguments, some of which have already been presented in Section 8, the magnetic energy, which is released by the introduction of a DL to a twisted magnetic flux rope, goes both to the particles accelerated by the DL and to plasma mass motions. Since the plasma density is low, particles will initially begin to move with the $\mathbf{E} \times \mathbf{B}/B^2$ drift velocity. This velocity is parallel to the Poynting vector $\mathbf{E} \times \mathbf{H}$, and since the DL is a region of electromagnetic energy dissipation there will be a mass flow directed towards the DL. This motion may explain the ejection of material seen in solar surges. It is found that for large filament currents ($I \approx 10^{12}$ Amp) the main energy release is through the DL acceleration of particles, and this can

represent an important flare. For weaker currents ($I \approx 10^{10}$ Amp), corresponding in energy to minor flares, it is possible that most of the energy goes to mass motions resulting in a surge. This surge model also predicts a significant rotational motion accompanying the ejection of matter.

Models of DL's, found by solving the steady state Vlasov-Poisson equations, with power law distributions of free particles have been applied to the DL theory of flares (Hasan and ter Haar 1978 [217]). The hardness of the particle spectrum for strong flares is argued to be less easy to reconcile with observations. However there may in fact be several weaker DL's within the current circuit (Hénoux 1986 [218]). This would also provide an alternative way of producing a spectrum of particle energies

There is a possible relation of the DL flare model to periodic pulsations observed both in microwave and X-ray emission, with periods in the range 0.5 – 50s. Several explanations have been proposed, including modulations of the emission from energetic particles by whistler (Chiu 1970 [219]) or standing Alfvén (Rosenberg 1972 [220]) waves. Alternatively the pulsations may be a result of a particle stream instability in a pulsating regime (Zaitsev 1971 [221]). However such pulsations have even been observed in the absence of flares, and a model has been suggested where the acceleration mechanism itself is modulated (Tapping 1986 [222]). Such a model naturally accounts for the wide frequency range of the emissions for which pulsations are observed , and a DL is proposed since its small spatial size favours rapid time variations. The configuration is as for the flare model (Figure 9(a)), assuming that the longitudinal field and the flux tube radius are kept constant at the ends. As the current is increased a local force-free equilibrium is assumed so that the tube contracts in the central section enhancing the required current density and leading to a magnetic mirror near the top of the loop due to the compression of the longitudinal component. At the mirror only the electrons in the loss cone can contribute to the current. The combination of these effects leads to a situation where there are insufficient current carriers at the top of the loop. The inductance of the system acts to maintain the current and the inductive voltage leads to the formation of a DL near the magnetic mirror. The DL accelerates

electrons into the loss cone, enhancing the current. If the energy supply to the DL, which is limited by the finite Alfvén velocity (see Section 8), is insufficient to balance the dissipation the DL suffers a periodical collapse so that there is a mean energy balance. The time scale for the resulting pulsations is given by the transit time of Alfvén waves along the flux tube.

The topology of two-ribbon flares and their close association with an erupting filament, has lead to the development of a circuit model illustrated in Figure 9(b). An approximately two-dimensional structure is assumed with the structure lying along the observed line of zero vertical magnetic field in the photosphere. Due to the high conductivity and inertia of the photosphere the magnetic field lines are held fixed there, and the introduction of the filament current I_f leads to surface currents which are equivalent to an equal and opposite mirror current below the photosphere. This provides an upwardly directed force on the filament (Kuperus and Raadu 1974 [223]). The ambient field produces a force on the filament which may be in either direction, and if it is downward can result in a force-free equilibrium (Kuperus and van Tend 1981 [224]). Changes in the photospheric field due to motions converging below the filament or equivalently an increase of the filament current can lead to a loss of equilibrium. The filament rises and induced electric fields produce a current sheet at the neutral line separating the filament field from the underlying arches. The sheet current I_s direction is such as to try to preserve the flux linkages (Kaastra 1985 [225]). A fully dynamical version of this model has been recently presented, which reproduces the different phases in the flare development in considerable detail (Martens and Kuin 1989 [226]). The impulsive phase is identified with the rapid formation of the current sheet, through mutual induction with the filament current. Particle acceleration occurs within the sheet, and loss of particles onto the underlying magnetic arches produces the characteristic $H\alpha$ ribbons at the foot points. The models predict a maximum electric field in the current sheet of typically 450V/m, greatly exceeding (by more than five orders of magnitude) the Dreicer field for thermal runaway. For a total length parallel to the axis of symmetry of $1.2 \cdot 10^8$ m this leads to a total voltage, $5.4 \cdot 10^{10}$ V. Typically the sheet current is $I_s \approx 5 \cdot 10^{11}$ Amp. These parametrers

are close to those for the DL model of a flare, and in view of the high average electric field it may be proposed that the voltage drop may be concentrated over one or several DL's. A DL has been produced experimentally in a current sheet configuration (Stenzel *et al* 1983 [144]). There is little theoretical work for such a DL, although some calculations of the structure of collisionless current sheets lead to a significant local concentration of electric field along the current direction and potential contours resembling those for two-dimensional DL's (Cowley 1971 [227] and 1973 [228]).

The analysis of observations from the Solar Maximum Year (1979–1981) have lead to an improved understanding of the build up phase of solar flares (Gaizauskas and Švestka 1987 [229]) and of the energy release (Rust and Batchelor 1987 [230]). Hard X-ray and microwave bursts have been found to have fine structure on time scales of a few 10ms, possibly related to discrete emission events from flare kernels (Kaufmann *et al* 1984 [231]). During the impulsive phase hard X-ray emission is observed at the foot of loops with microwave emission from the tops (Duijveman *et al* 1982 [232], Hoyng *et al* 1981 [233]). Of particular interest for models with acceleration by strongly relativistic DL's is the observation of energetic neutrons (50 – 500MeV, where the upper limit is uncertain and may be much greater) produced at the impulsive phase (Chupp *et al* 1982 [234], Chupp 1983 [235]). The DL model is able to meet the rather stringent requirements for the acceleration mechanism set by the observations, both as regards the time scale and the number of high energy primary protons which produce the neutrons (Carlqvist 1986 [236]). Further evidence of acceleration to very high energies comes from observations of γ -ray emission due to protons (Ryan *et al* 1983 [237]). In the context of these most recent results the DL has the advantage of accounting for the acceleration to very high energies and allowing extremely rapid time variations, since its structure can vary on the local ion transit time scale .

11. X-Ray Pulsars

Binary X-ray pulsars, first discovered in 1971 (Giacconi *et al* 1971 [238], Lewin *et al* 1971 [239]), are largely accepted as being powered by the kinetic energy

of free falling matter accreting onto the magnetic poles of rotating neutron stars, this matter being supplied by a normal binary companion (Joss and Rappaport 1984 [240]). The magnetic field, estimated to be $\geq 10^{12}G$, funnels the flow to a small area and the radiative processes result in a directed emission of X-rays. The spectra are typically broadband over 2 – 20 keV with a rapid falloff above $\approx 20 \text{ keV}$ (White *et al* 1983 [241]). The observed periodicities, in the range 0.1 – 1000 s, are a consequence of the non-alignment of the rotational and magnetic axes. The deceleration of the accreting matter and the thermalisation of its kinetic energy ($\approx 100 \text{ MeV/proton}$) may be achieved in several ways (Joss and Rappaport 1984 [240]). For high luminosities a “photon cushion” due to radiation pressure can decelerate the electrons and hence via electrostatic coupling the ions. Otherwise particles may reach the surface at their free-fall velocities and be stopped over a short distance by binary collisions. Another possibility is that a strong shock may form largely thermalising the incoming matter before it settles onto the surface.

Recently it has been suggested that the formation of a DL may be responsible for the deceleration of the accreting matter (Williams *et al* 1986 [242]). It is argued that the electrons will be stopped more rapidly than the ions in the neutron star atmosphere tending to set up a proton current and leading to a charge separation. The charge separation electric field decelerates the ions and in a steady state the resultant structure is that of a DL. The DL has no net current, and might be regarded as a laminar shock; it decelerates an incoming high velocity ion beam. However the DL accelerates electrons producing a downwardly directed highly relativistic beam, which it is argued ultimately produces the observed X-rays. The essential rôle of the DL is to provide a highly effective means of transferring ion beam energy to electrons. A model is presented with six particle populations comprising ions and electrons from the infalling matter and the ambient plasmas entering from above and from below. The relativistic form of the steady state Vlasov equation is used for the distribution function, $f_e(x, p_x)$, of the accelerated

atmospheric electrons. Solutions are found by choosing the electron distribution function to be a suitable function of the relativistic energy, w_e , given by,

$$w_e = \sqrt{p_x^2 c^2 + m_e^2 c^4} - m_e c^2 - e\phi \quad (11.1)$$

Integration of $f_e(x, p_x)$ over the relativistic momentum p_x then gives the electron density as a function of the potential ϕ . The other particle densities are also found as functions of ϕ . In the case of the atmospheric electrons trapped below the DL, it is assumed that the distribution is a superposition of a waterbag and a Maxwellian component. The parameters of these components are adjusted to give the best approximation to a solution of Poisson's equation, where the potential profile is assumed to have a predetermined form with hyperbolic tangent profile. This approximate procedure is a simplified form of the BGK method (Knorr and Goertz 1974 [16]) avoiding the inversion of an integral equation. The trapped electron distribution determined in this way for a particular example of a DL potential profile ($\phi_{DL} = 1.3 \cdot 10^8$ V, width $d = 1$ cm) is physically reasonable, supporting the hypothesis that the DL may act as the deceleration mechanism for the accreting matter at the surface of a neutron star. Here it should be noted that the typical net charge density inside the DL expressed as $e\delta n$ where $\delta n = \epsilon_0 \phi_{DL} / ed^2$ is small. For the particular example reported $\delta n \approx 7.2 \cdot 10^{13} \text{ cm}^{-3}$, which, for a neutron star radius $R_* = 10^6 \text{ cm}$ and assuming that the accretion areas occupy a fraction $s \ll 1$ of the surface area, gives a surface integrated line density of $\delta N = s 4\pi R_*^2 \delta n \approx s 9.0 \cdot 10^{26} \text{ cm}^{-1}$. This δN is much smaller than the line densities of the accreting ions at the upper edge of the DL, $4.05 \cdot 10^{28}$. Since the density of these ions increases as they are decelerated by the DL, δN is also relatively small inside. The DL is therefore quasi-neutral.

12. Extragalactic Jets

Extragalactic jets are observed by radio techniques connecting the active nuclei of radio galaxies to pairs of extended lobes of emission, and it is believed that they feed the energy required by the lobes from the nuclei. In several cases the narrow radio jets are also visible optically (van Breugel and Miley 1980 [243]).

As well as transporting kinetic energy, jets may also transmit energy by electrical currents. Current pinching can account for their excess internal pressure (Benford 1978 [244]). The possibility of DL formation within extragalactic jets has been analysed in some detail (Borovsky 1986 [245]) following earlier suggestions (Alfvén 1977 [166], Alfvén 1978 [246]). For typical jet parameters the relative drift velocity between ions and electrons, $v_0 \approx 10^{-4} \text{ m s}^{-1}$, is extremely small compared to the electron thermal velocity, $v_{th,e} \approx 4 \times 10^5 \text{ m s}^{-1}$ for $T_e = 10^4 \text{ K}$. Buneman, ion-acoustic and ion-cyclotron instabilities are not therefore possible, and the wavelengths of possible electromagnetic kinklike instabilities are too large compared to a jet radius, $R_J = 10^2 - 10^3 \text{ pc}$. Thus anomalous resistivity effects directly produced by current driven instabilities are unlikely. The drift velocities are also too small to satisfy the Bohm criteria for strong DL's. However it is possible for DL's and anomalous resistivity effects to exist simultaneously in a symbiotic relationship. A strong DL emits high energy ion and electron beams on the low and high potential sides respectively. These can drive two-stream Buneman instabilities producing growing plasma waves propagating away from the DL on both sides, with a quiet region over a growth length close to the DL. The waves can reach potential amplitudes $\approx kT/e$ capable of reflecting all except the high energy tails of the distributions and the beams. Effective collision frequencies then approach the plasma frequency, ω_{pe} . The region of wave turbulence strongly inhibits the particle transport and can maintain a high density plasma. Particles leak out of the turbulent regions at very low density and with a drift velocity comparable to their thermal velocities. They can then satisfy the Bohm criteria when they enter the DL. In this way the DL maintains a density cavity by the pressure of the accelerated beams allowing the Bohm criteria to be met. Within the turbulent regions quasi-static fields are needed to drive the current, and density gradients are associated with the diffusive flows. There are experiments providing evidence of DL-driven cavities (Guyot and Hollenstein 1983 [136]). The concept resembles Langmuir's (Langmuir 1929 [4]) model for collisional plasmas where a thin sheath with net charges is matched to a pre-sheath with ambipolar diffusion driven by density gradients. This senario also allows multiple strong DL's. The

separation must be sufficient to thermalise the particles which then form sources for adjacent DL's. High energy particles may form a runaway component which can be accelerated by a series of DL's, and some may acquire energies comparable to the total potential along the jet. The radio emission from jets has been assumed to be produced by cold electron beams (Benford 1978 [244]), and such beams are produced on the high potential sides of DL's. The observed polarised radiation may be produced by synchrotron emission from plasma wave scattering or, more efficiently, by collective emission from charge clumps produced by beam-plasma instabilities. An unresolved filamentary structure would have important consequences for the modeling of extragalactic jets. If the volume filling factor was extremely small ($\leq 10^{-9}$) then for the same total current the relative drift velocities would be sufficient to directly drive instabilities and to satisfy the Bohm criteria. However here one should recall the Bennet pinch relation used to deduce the total current, I ,

$$\frac{\mu_0}{8\pi} I^2 = P A \quad (12.1)$$

where $A = \pi R_j^2$ is the cross sectional area of the jet and P the particle pressure. We see that if several threads are present with the same total area, then $\sum I^2$ would be the same but the total current $\sum I$ would be smaller. A filling factor would imply a further reduction of the total current. The simple Bennet pinch relation is probably an over simplification in the present case and the effects of gravity and a longitudinal magnetic field should be considered (Carlqvist 1988 [247]). Also it was pointed out (Borovsky 1986 [245]) that the estimates of the internal pressure of jets assume that the radio emission is produced by synchrotron emission in a uniform magnetic field. Compressional MHD waves or turbulent MHD motions can however produce inhomogeneities in the density or magnetic field, modifying both the intensity and spectrum of the radio emission. Consequently the pressure estimates may be systematically biased and not be so reliable. The model of multiple DL's mutually sustaining regions of plasma turbulence for plasmas with very low current drift velocities is very significant for other astrophysical applications, since the low drift velocities deduced observationally are often used as a counter argument to the existence of DL's.

13. Cosmic Rays

Large scale current circuits, in which DL's may exist, have been proposed both for the heliosphere and for the galaxy (Alfvén 1978 [246]). In the galactic case currents of $10^{17} - 10^{19}$ Amp have been estimated (Alfvén and Carlqvist 1978 [248]). The driving mechanism is the dynamo action of the galactic rotation, setting up an axisymmetric current system. Currents flow in the plane of the galaxy, entering or leaving it along the axis of rotation and closing through a distributed system to the galactic plane, about which the currents are symmetric. The axial currents are compressed by pinching, and the current density further enhanced by filamentation. These regions are the most favourable for DL formation.

A detailed analysis has been made of the possibility of accelerating high energy cosmic ray particles using the energy stored in the galactic circuit (Carlqvist 1986 [236]). For circuit dimensions $\approx 3 \cdot 10^4 \text{ pc} \approx 10^{21} \text{ m}$ it is argued that the inductance is $\approx 10^{15} \text{ H}$, so that the stored magnetic energy $W_B \approx 5 \cdot 10^{49} \text{ J}$ for a current $I_0 = 3 \cdot 10^{17} \text{ Amp}$. Strongly relativistic DL's are considered for which,

$$\phi_{DL} = \sqrt{\frac{m_i c^2 / e}{4 \epsilon_0 c}} i d^2 \quad (13.1)$$

(from Equation (2.8)), and it is noted that the potential, ϕ_{DL} , increases with the square root of the mass to charge ratio, $\sqrt{m_i / e}$, and hence depends on the dominating ion species. If the ions are replaced by positively charged interstellar dust grains with a typical value of $m_g / Ze \approx 6 \cdot 10^4 m_p / e$ (for grains of mass 10^{-20} kg charged up to a potential of 10V) the DL voltage is increased by a factor ≈ 250 compared to that in a hydrogen plasma. The charges within the relativistic DL are strongly separated. There is a concentration of positive charge on the low potential boundary, but the net negative charge is uniform over the rest of the DL. Consequently it is argued that the DL loses its one-dimensional character if the thickness d becomes comparable to the radius of the current channel r_0 , and by analogy with a capacitor the maximum potential for given charges is given by the one-dimensional approximation with $d \approx r_0$. The maximum potential is then, using Equation (13.1),

$$\phi_{DL,m} \approx \sqrt{\frac{m_i c^2 / e}{4 \pi \epsilon_0 c}} I_0 \quad (13.2)$$

and depends on the total current I_0 . The corresponding maximum DL dissipation, $I_0\phi_{DL}$, is well below the maximum rate at which the circuit can deliver energy (see Section 8) and therefore Equation (13.2) sets the upper limit on the DL potential. For $I_0 = 3 \cdot 10^{17}$ Amp the galactic DL can therefore accelerate particles up to $\approx 10^{14}$ eV for a hydrogen plasma. If there are a few particles with higher charge Ze they will reach energies a factor Z higher. In cases where there are large amounts of heavy ions or charged dust grains the energies will be higher still.

The DL naturally produces a spectrum of energies, since the potential varies across the current channel (see Section 8), and moreover may be expected to be time dependent. The mechanism can produce particles with energies up to $10^{14} - 10^{16}$ eV, and it is argued that it also may provide the injection mechanism for acceleration to still higher energies by other processes. There is a knee in the cosmic ray energy spectrum at 10^{15} eV that may well be an indication of a transition between different acceleration processes e.g. from the DL mechanism to some other.

14. Conclusions

This review has indicated the many facets to DL research. Since the basic phenomena are complex, involving strongly non-linear dynamics and large scale coupling through the plasma current system, interrelated studies are of great importance. Models of steady state DL's enable a first understanding of laboratory results, the acceleration of particles and the resulting high frequency fluctuations in the ambient plasmas. Computer simulation studies have reproduced much of the dynamics of pulsed experiments, and have been of great importance in revealing the observable consequences of three-dimensional DL's in the magnetosphere. Since weak DL's are more amenable to analytical studies, their dynamics is rather well understood and this even provides a basis for a greater understanding of strong DL's, although much remains to be done.

As opposed to the magnetosphere, where there is much detailed evidence, particularly for weak DL's, more distant areas of astrophysical application must rely heavily on an analysis of the global aspects of DL's. The storage and maximum

release rate of the energy dissipated by DL's is determined by such aspects and this provides one of the crucial tests for the applicability of DL's to a particular phenomenon. Although the internal structure of an astrophysical DL is not directly observable, there are only a limited number of free parameters, particularly in the case of strong DL's, and the relation between these parameters places further constraints on the application. A particular example is the upper limit on the DL potential set via the requirement that the thickness should not exceed the diameter for one-dimensional analysis.

Finally it should again be stressed that DL's are essentially dynamical in nature, so that deductions drawn from simple time independent models need to be made against the background of laboratory and simulation studies of dynamical DL's. In particular the local ambient plasma conditions at the edges of a DL may be strongly perturbed, so that the matching conditions are more complex than normally assumed for analytical models. Even if the plasma is not turbulent, the local conditions may be very different from those in the distant plasma due to intervening pre-sheaths.

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Figure Captions

Figure 1. The general structure of a monotonic double layer (DL), with potential difference ϕ_{DL} and thickness d . The potential profile $\phi(x)$ is shown on the right. The areas of phase space occupied by different populations of ions and electrons are shown to the left. The hatched areas indicate accelerated populations. The reflection of ions and electrons lying within the separatrices (broken and symmetrically placed curves) is indicated by the arrows showing velocity reversal. The upper and lower boundaries should not be regarded as sharp, and only indicate the range of the bulk of the distributions, particles in the high velocity tails lying beyond them.

Figure 2. The properties of relativistic strong double layers (DL's). The solid curve gives the relation between id^2 (i = total current; d = thickness), shown on the right vertical boundary, as a function of the DL potential ϕ_{DL} . Approximate forms of this relation for non-relativistic, semi-relativistic and highly relativistic DL's are shown as light broken curves ($u_i = m_i c^2 / e \approx 9.38 \cdot 10^8 \text{ V}$; c_j are constants). The heavy broken curve gives the electron current i_e as a percentage of the total current i (for a hydrogen plasma). In the non-relativistic limit $i_e = i / (1 + \sqrt{m_e/m_i})$ so that the electron current dominates, but in the highly relativistic case $i_e \approx i_i$ (i_i = ion current), so that equal numbers of ions and electrons are accelerated.

Figure 3. Ion and electron phase space diagrams for various types of double layers (DL's). Potential profiles $\phi(x)$ are shown to the right. The hatched areas indicate the areas of significant phase space density. The separatrices for the particle motions are shown as dashed lines. *First row:* Strong DL with distinct populations of reflected and transmitted particle populations. The DL potential is so high that the reflected populations only penetrate the outer edges. *Second row:* Slow ion acoustic DL (SIADL). The “tuning fork” distribution of the ions resembles one half of an ion-hole. For the example shown the current is carried by the electrons. *Third row:* Ion acoustic DL, with a distinctive relatively large potential dip on the low potential edge. The ion distribution is that of an

asymmetric ion hole, here identified with an ion acoustic DL. Simulations (cf Figure 6) often show only the beginning of ion trapping, so that the DL is in a dynamical state. *Fourth row:* Cold-ion DL characterised by a single cold ion population forming a beam in the reference frame of the DL. The electrons have a non-Maxwellian distribution. For example, if hot and cold Maxwellian distributions are superimposed, the phase space density on the high potential is enhanced (indicated by double hatching).

Figure 4. The Sagdeev (classical) potential $V(\phi)$ and to the left the corresponding potential profile $\phi(x)$. For the case illustrated the weak DL depends on a discontinuity in the electron distribution function. The Sagdeev potential takes the normalised form, $\bar{V}(\phi) = \frac{2}{5} \phi^{3/2} \left(\phi_{DL}^{1/2} - \phi^{1/2} \right)^2 / \phi_{DL}^{5/2}$. The solution exhibited is: $\phi(x)/\phi_{DL} = \tanh^4(x/d)$ for $x > 0$, $= 0$ for $x \leq 0$.

Figure 5. A quasi-steady strong DL, from a particle simulation (Yamamoto 1985 [91]) with an imposed potential difference, $50 kT_{1e}/e$, formed after a time, $t = 600/\omega_{pe}$. The sources are isothermal. The cathode source electrons have a temperature T_{1e} , 10 times that for the anode source. The potential profile, to the right of the phase space diagrams, shows a small ambipolar potential dip on the right hand edge, which follows the DL as it propagates to the left.

Figure 6. A DL of ion-acoustic type, from a periodic particle simulation with an imposed constant current condition. The initial temperature ratio is $T_e/T_i = 20$, and the drift velocity $v_d = 2 c_e$ ($c_e = \sqrt{kT_e/m_e}$). The phase space diagrams and normalised potential distribution ($\bar{\phi} = e\phi/kT_e$) are shown for two times, $t = 80/\omega_{pe}$ ((a) to the left) and $t = 120/\omega_{pe}$ ((b) to the right). DL formation is triggered by an initial density depression at the centre ($x/\lambda_{De} = 64$) and the DL is slowly propagating to the right, with a potential dip on the low potential edge. At the later time (b) the accelerated beam and low energy reflected electrons are clearly visible to the right, and an ion hole structure is just beginning to form at the potential dip. A large positive potential pulse, with an associated vortex structure, which is close to the high potential edge of the DL at the earlier time, moves ahead of it and becomes well separated.

Figure 7. Wave phenomena associated with a strong DL (Torvén and Lindberg (1980) [126], Lindberg (1982) [127]). The distributions of the electrons over energy, $\mathbf{W}$, (curves (a),(b) and (c)) are estimated from the second derivatives of a probe current. The spacial distribution of the amplitudes, $\mathbf{G}$ ($\mathbf{G}_{max}$ = maximum level), of the high frequency ($500\text{MHz} \approx f_{pe}$) and low frequency (50kHz) fluctuations are shown at the bottom left together with the approximate locations of the energy distribution measurements. These locations are also indicated for the potential profile, $\phi(x)$, on the right. The potential dip near the anode source helps to confine a trapped distribution of electrons on the high potential side of the DL. The electron energy measurements show a thermal distribution on the cathode side (a), and a high energy beam population on the anode side close to the DL (b), which becomes diffuse further away (c). A background low energy component is also visible on the anode side. The high frequency fluctuations are associated with the beam diffusion. The low frequencies are mainly a result of the DL motion and changes in amplitude.

Figure 8. The magnetohydrodynamic relaxation of a twisted magnetic flux tube (I = axial current) as the result of the formation of a DL. The field configuration and DL location are shown to the left and the distribution of angular velocity $\Omega(z)$ (z = axial coordinate) to the right. *Case (a):* A steady state where the plasmas within the flux tube (extending to $z = \pm\infty$) rotate as solid cylinders with equal and opposite angular velocities, $\pm\Omega_0$, on either side of the DL. *Case (b):* Slow relaxation with uniform unwinding (see $\Omega(z)$) of the field. Line tying is assumed at boundaries, $z = \pm L$. *Case (c):* Partial unwinding triggered by the abrupt formation of a DL. Two fronts propagate away from the DL at the Alfvén velocity v_A setting the initially stationary plasma ($\Omega = 0$) into rotation. Behind the fronts the plasmas rotate as solid cylinders with $\Omega = \pm\Omega_0$.

Figure 9. Models of solar flares. The lower boundary in both cases is the high density photosphere. *To the left:* Energy stored in a twisted magnetic flux loop in the corona is released by a DL (Alfvén and Carlqvist (1967) [7]). The power dissipated at the DL (potential, ϕ_{DL}) is $I\phi_{DL}$, where I is the current along the flux rope, and goes directly to equal numbers of high energy ions and

electrons. *To the right:* Model of a two ribbon flare (Kaastra (1985) [225]). The configuration is plane symmetric. The flare results from the eruption of a filament (with current I_f) into the corona forming a current sheet (with current I_s). High voltages develop along the current sheet, accelerating particles perpendicular to the illustrated cross-section. Particles scattered out of the acceleration region follow field lines down to the photosphere, on both sides, producing the typical two ribbon $H\alpha$ emission regions.

Figure 1.

Figure 2.

Figure 3.

Figure 4.

Figure 5.

Figure 6 (a).

Figure 6 (b).

Figure 7.

Figure 8.

Figure 9.

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THE PHYSICS OF DOUBLE LAYERS AND THEIR ROLE IN ASTROPHYSICS

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Research on electrostatic double layers (DL's) is surveyed and astrophysical applications are reviewed. Strong highly relativistic DL's directly accelerate equal numbers of ions and electrons, whereas in the non-relativistic limit electron acceleration predominates. The existence criteria for DL's are discussed. There is a close relationship between the Bohm criteria, the plasma dispersion relation and the Penrose stability condition. Current driven instabilities can lead to DL formation, and account for the fluctuations associated with strong DL's. A class of weak DL's may be described using the modified Korteweg-de Vries equation, and the conditions for the emergence of a DL from arbitrary initial conditions can be specified. Simulation studies of strong DL formation reveal important dynamical features. Weak DL's are found in simulations of ion-acoustic instabilities. Laboratory DL's have been studied in great detail. There is a strong coupling between a DL and the global current system. The results of theoretical, simulation and observational investigations of magnetospheric DL's are presented. High energy observations of solar flares agree well with a DL model. DL's may be a feature of many high energy astrophysical phenomena.

Key words: Double layers, Existence criteria, Electrostatic instabilities, Dynamics, Computer simulations, Experiments, Global effects, Particle acceleration, Magnetosphere, Solar flares, Astrophysics