On the Origin of Time-Dependent CP Asymmetries in Neutral B Decays

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ABSTRACT

CP asymmetries in neutral B decays may arise either as a result of CP violation in the decay of a pure B^0 ($\bar{B}^0$) state or as a consequence of $B - \bar{B}$ mixing. We show how these two mechanisms can be distinguished in a model-independent manner by their different characteristic time-dependence. We use this analysis to examine standard model predictions.

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1. INTRODUCTION

In recent years a large variety of ways have been proposed to observe CP violation in B decays¹. One of the methods is to look for CP asymmetries in the time-dependence of neutral B decays². The asymmetries require an interference between two weak amplitudes. There are two types of possible interference:

- Due to the large observed $B - \bar{B}$ mixing³ an initial B^0 may decay to a final state f by two chains $B^0 \rightarrow B^0 \rightarrow f$ and $B^0 \rightarrow \bar{B}^0 \rightarrow f$ which may interfere.
- The amplitudes of $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow f$ themselves may each obtain contributions from two separate terms of the low energy effective Hamiltonian which may interfere with each other.

The purpose of this note is to show how these two mechanisms of CP violation can be distinguished by their different time-dependent asymmetries. The first part of our study, in which this question will be answered, will be quite general and, besides some mild assumptions, will not depend on the KM mechanism of CP violation. In the second part we will examine weak decays in the standard model and identify the asymmetries expected in various cases with one (or two) of the above types. This kind of analysis may be used in the future to test the standard model or any other model of CP violation.

The emphasis of our work differs from previous studies². The latter were aimed mainly at estimates of the time-dependent asymmetries expected in the standard model, without trying to distinguish between their two possible origins⁴. Our purpose is, in the first place, to be able to identify an observed asymmetry as originating from either one of the above mechanisms, and then to compare with standard model predictions. We will study the general case of neutral B decays to hadronic states which are not CP eigenstates. A similar study of the specially

interesting case of decays to CP eigenstates¹ was recently carried out by us⁵.

2. TWO KINDS OF TIME-DEPENDENT CP ASYMMETRIES

To discuss the time-dependent asymmetries we use standard notations^{1,2}. The two physical neutral B mesons (either B_L^0 or B_H^0) are denoted by

$$\begin{aligned} |B_L\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle & (m_L = m - \frac{\Delta m}{2}, \Gamma) \\ |B_H\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle & (m_H = m + \frac{\Delta m}{2}, \Gamma) \end{aligned} \quad (1)$$

In the parentheses we give the corresponding masses and the very approximately equal widths which are taken to be equal to each other. We will assume $|q/p| = 1$ for the mixing parameter. This result, which holds in the standard model within a per cent¹, can be shown to follow from more general considerations⁶. The time-evolutions of initially pure B^0 and $\bar{B}^0$ states are, respectively,

$$\begin{aligned} |B_{p,q}^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle \\ |\bar{B}_{p,q}^0(t)\rangle &= \frac{p}{q}g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle \end{aligned} \quad (2)$$

where

$$\begin{aligned} g_+(t) &= \exp(-\frac{\Gamma}{2} - im)\cos(\frac{\Delta m}{2}t) \\ g_-(t) &= \exp(-\frac{\Gamma}{2} - im)i\sin(\frac{\Delta m}{2}t) \end{aligned} \quad (3)$$

The decay amplitudes of B^0 and $\bar{B}^0$ to a state f and its CP conjugate $\bar{f}$ are given by

$$\begin{aligned} A_f &\equiv A(B^0 \rightarrow f) = A_1 e^{i\phi_1} + A_2 e^{i\phi_2} \\ \bar{A}_f &\equiv A(\bar{B}^0 \rightarrow f) = \bar{A}_1 e^{i\phi_1} + \bar{A}_2 e^{i\phi_2} \\ A_{\bar{f}} &\equiv A(B^0 \rightarrow \bar{f}) = \bar{A}_1 e^{-i\phi_1} + \bar{A}_2 e^{-i\phi_2} \\ \bar{A}_{\bar{f}} &\equiv A(\bar{B}^0 \rightarrow \bar{f}) = A_1 e^{-i\phi_1} + A_2 e^{-i\phi_2} \end{aligned} \quad (4)$$

The two complex hadronic amplitudes appearing in each of these decays ($A_i, \bar{A}_i; i = 1, 2$) correspond to possible contributions of two separate terms of the low energy effective Hamiltonian. The phases in eqs.(4) are CP violating phases which appear in the Hamiltonian. In the standard model they are given by phases in the KM matrix. In the special case of CP eigenstates, $\bar{f} = \pm f$, one has $\bar{A}_i = \pm A_i$, $\bar{\phi}_i = -\phi_i$.

From eqs.(2)(3)(4) we find the time-dependent decay rates of initially pure B^0 and $\bar{B}^0$ decaying into f and $\bar{f}$ at time t ⁷

$$\begin{aligned}\Gamma_f(t) &\equiv \Gamma(B_{ph}^0(t) \rightarrow f) = |A_f|^2 e^{-\Gamma t} [\cos^2(\frac{\Delta m}{2}t) + |\xi|^2 \sin^2(\frac{\Delta m}{2}t) - Im\xi \sin \Delta m t] \\ \bar{\Gamma}_f(t) &\equiv \Gamma(\bar{B}_{ph}^0(t) \rightarrow f) = |A_f|^2 e^{-\Gamma t} [|\xi|^2 \cos^2(\frac{\Delta m}{2}t) + \sin^2(\frac{\Delta m}{2}t) + Im\xi \sin \Delta m t] \\ \Gamma_{\bar{f}}(t) &\equiv \Gamma(B_{ph}^0(t) \rightarrow \bar{f}) = |A_f|^2 e^{-\Gamma t} [\bar{\xi}^2 \cos^2(\frac{\Delta m}{2}t) + \sin^2(\frac{\Delta m}{2}t) + Im\bar{\xi} \sin \Delta m t] \\ \bar{\Gamma}_{\bar{f}}(t) &\equiv \Gamma(\bar{B}_{ph}^0(t) \rightarrow \bar{f}) = |\bar{A}_f|^2 e^{-\Gamma t} [\cos^2(\frac{\Delta m}{2}t) + |\bar{\xi}|^2 \sin^2(\frac{\Delta m}{2}t) - Im\bar{\xi} \sin \Delta m t]\end{aligned}\quad (5)$$

where

$$\begin{aligned}\xi &= \frac{q}{p} \frac{\bar{A}_f}{A_f} \\ \bar{\xi} &= \frac{p}{q} \frac{A_f}{\bar{A}_f}\end{aligned}\quad (6)$$

In deriving eqs.(5) we have used $|q/p| = 1$.

CP violation is manifested by $\Gamma_f(t) \neq \bar{\Gamma}_f(t)$ and by $\bar{\Gamma}_f(t) \neq \Gamma_{\bar{f}}(t)$. The difference between these time-dependent rates may follow from $|A_f| \neq |\bar{A}_f|$ and/or from $\xi \neq \bar{\xi}$ ⁸. The first condition corresponds to CP violation in the decay $B^0 \rightarrow f$ itself and holds only when two amplitudes with different CP phases contribute to this transition. The second inequality can hold also when CP is conserved in the direct decay and corresponds in this case to CP violation via mixing. It requires

that both B^0 and $\bar{B}^0$ decay to a common state f . This condition is fulfilled automatically when f is a CP eigenstate. These two distinct mechanisms of CP violation can be characterized as follows:

a. *CP violation via mixing*: $|A_f| = |\bar{A}_f|$, $|\xi| = |\bar{\xi}|$, $Im\xi \neq Im\bar{\xi}$. This holds when $A_1 \neq 0$, $\bar{A}_1 \neq 0$, $A_2 = \bar{A}_2 = 0$. In this case

$$\begin{aligned}\Gamma_f(t) &= |A_f|^2 e^{-\Gamma t} [\cos^2(\frac{\Delta m}{2}t) + |\xi|^2 \sin^2(\frac{\Delta m}{2}t) - Im\xi \sin \Delta m t] \\ \bar{\Gamma}_f(t) &= |A_f|^2 e^{-\Gamma t} [|\xi|^2 \cos^2(\frac{\Delta m}{2}t) + \sin^2(\frac{\Delta m}{2}t) + Im\xi \sin \Delta m t] \\ \Gamma_{\bar{f}}(t) &= |A_f|^2 e^{-\Gamma t} [|\xi|^2 \cos^2(\frac{\Delta m}{2}t) + \sin^2(\frac{\Delta m}{2}t) + Im\bar{\xi} \sin \Delta m t] \\ \bar{\Gamma}_{\bar{f}}(t) &= |A_f|^2 e^{-\Gamma t} [\cos^2(\frac{\Delta m}{2}t) + |\xi|^2 \sin^2(\frac{\Delta m}{2}t) - Im\bar{\xi} \sin \Delta m t]\end{aligned}\quad (7)$$

b. *CP violation in direct decay*: $|A_f| \neq |\bar{A}_f|$, $\xi = \bar{\xi} = 0$. This holds when $arg A_1 \neq arg A_2$, $\phi_1 \neq \phi_2$, $\bar{A}_1 = \bar{A}_2 = 0$. In this case

$$\begin{aligned}\Gamma_f(t) &= |A_f|^2 e^{-\Gamma t} \cos^2(\frac{\Delta m}{2}t) \\ \bar{\Gamma}_f(t) &= |A_f|^2 e^{-\Gamma t} \sin^2(\frac{\Delta m}{2}t) \\ \Gamma_{\bar{f}}(t) &= |\bar{A}_f|^2 e^{-\Gamma t} \sin^2(\frac{\Delta m}{2}t) \\ \bar{\Gamma}_{\bar{f}}(t) &= |\bar{A}_f|^2 e^{-\Gamma t} \cos^2(\frac{\Delta m}{2}t)\end{aligned}\quad (8)$$

The general situation, which combines these two phenomena, is described by eqs.(5).

In principle, to check the two conditions of CP conservation separately, $|A_f| = |\bar{A}_f|$ and $\xi = \bar{\xi}$, it seems sufficient to compare the time-dependence of $\Gamma_f(t)$ and $\bar{\Gamma}_f(t)$. One may expand the observed time-dependent rates in terms of the three functions appearing in eqs.(5) and look for differences between the corresponding

The CP asymmetry, which is induced by mixing, is an odd function of time whereas the direct decay asymmetry is even. This provides a way for projecting out either one of these asymmetries. To illustrate this possibility let us consider measuring CP asymmetries at the $\Upsilon(4S)$. Here observing one of the decaying neutral B mesons serves to project the other B instantaneously into an orthogonal flavour state. This follows from Bose statistics when applied to the p-wave state in which the two B mesons are produced. Therefore the rates of eq.(5) may describe events in which the flavour of a B^0 (or a $\bar{B}^0$) is identified by the decay of its partner at time $t = 0$ to an opposite-flavour tagging mode (say a leptonic mode), and in which it is observed to decay to f (or to $\bar{f}$) at a later time t . For instance, the rate of tagging a B^0 at time $t = 0$ by $B^0 \rightarrow l^+ + \dots$ and observing its partner to decay to f at time t is given by

$$\Gamma_{Bf} \equiv \Gamma(B^0(t=0), f(t)) = |A_f|^2 \bar{\Gamma}_f(t) \quad (13)$$

where A_f is the semileptonic decay amplitude.

The rates of the time-reversed processes, in which one of the neutral B 's decays to f (or to $\bar{f}$) at $t = 0$ and its partner decays to a flavour-specific mode at a later time t (e.g. Γ_{fB}), are obtained from the corresponding eqs.(5) simply by flipping the signs of the last terms. By adding and subtracting the rates of processes with reversed time-order, e.g. $\Gamma_{Bf} \pm \Gamma_{fB}$, one can then form t -even and t -odd CP asymmetries, respectively. Using definitions similar to eqs.(9) to describe asymmetries in the two neutral B decays at the $\Upsilon(4S)$, one may project out the constant term, the $\cos \Delta mt$ and the $\sin \Delta mt$ terms of the asymmetries.

constant coefficients. A further comparison of the rates of the two other processes, $\bar{\Gamma}_f(t)$ and $\Gamma_f(t)$, can be done to check consistency. In practice such a detailed study aimed at distinguishing between the two kinds of asymmetries is expected to be difficult mainly due to limited statistics. Note that to measure the above rates requires knowing whether the decaying B was born at $t = 0$ as a B^0 or as a $\bar{B}^0$.

To distinguish between the above two possible origins of CP violation let us define the following two time-dependent asymmetries:

$$a_{1,2}(t) = \frac{(\Gamma_f(t) - \bar{\Gamma}_f(t)) \pm (\bar{\Gamma}_f(t) - \Gamma_f(t))}{\Gamma_f(t) + \bar{\Gamma}_f(t) + \Gamma_f(t) + \bar{\Gamma}_f(t)} \quad (9)$$

where the subscripts 1,2 correspond to the $+$, $-$ signs, respectively. We find

$$\begin{aligned} a_1(t) &= \frac{|A_f|^2 + |\bar{A}_f|^2 - |A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2 + |A_f|^2 + |\bar{A}_f|^2} \\ a_2(t) &= \frac{(|A_f(t)|^2 - |\bar{A}_f(t)|^2 + |A_f|^2 - |\bar{A}_f|^2) \cos \Delta mt + 2(Im\bar{\xi} - Im\xi) \sin \Delta mt}{|A_f|^2 + |\bar{A}_f|^2 + |A_f|^2 + |\bar{A}_f|^2} \end{aligned} \quad (10)$$

The first asymmetry (a_1) is independent of time. The second one (a_2) contains a $\cos \Delta mt$ and a $\sin \Delta mt$ term. The constant asymmetry a_1 and the $\cos \Delta mt$ term in a_2 are the result of direct decay CP violation, whereas the $\sin \Delta mt$ term in a_2 follows from CP violation by mixing. This can be easily seen by substituting eqs.(7)(8) into eq.(9). One finds for the above two cases:

a. *CP violation via mixing:*

$$a_1(t) = 0 \quad a_2(t) = \frac{Im\bar{\xi} - Im\xi}{1 + |\xi|^2} \sin \Delta mt \quad (11)$$

b. *CP violation in direct decay:*

$$a_1(t) = \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2} \quad a_2(t) = a_1(t) \cos \Delta mt \quad (12)$$

Let us define

$$a_{1,2}^{\pm}(t) = \frac{[(\Gamma_{Bf} \pm \Gamma_{\bar{B}\bar{f}}) - (\Gamma_{Bf} \pm \Gamma_{\bar{B}\bar{f}})] \pm [(\Gamma_{Bf} \pm \Gamma_{\bar{B}\bar{f}}) - (\Gamma_{Bf} \pm \Gamma_{\bar{B}\bar{f}})]}{\Gamma_{Bf} + \Gamma_{\bar{B}\bar{f}} + \Gamma_{Bf} + \Gamma_{\bar{B}\bar{f}} + \Gamma_{Bf} + \Gamma_{\bar{B}\bar{f}} + \Gamma_{Bf} + \Gamma_{\bar{B}\bar{f}}} \quad (14)$$

where the subscripts 1,2 correspond to the $+$, $-$ signs between the square parentheses, respectively, and the superscripts $\pm$ correspond to the signs within the parentheses. One finds that in general a_1^+ vanishes, a_1^- is given by the constant asymmetry a_1 of eqs.(10), a_2^- is given by the $\sin \Delta mt$ term of a_2 and a_2^+ - by the $\cos \Delta mt$ term. These asymmetries distinguish between the two cases of a mixing-induced asymmetry and a direct decay asymmetry in a straightforward manner:

- a. *CP violation via mixing*: $a_1^+ = a_2^+ = 0$, $a_2^- \propto \sin \Delta mt$.
- b. *CP violation in direct decay*: $a_2^- = 0$, $a_1^+ = \text{const.}$, $a_2^+ = a_1^+ \cos \Delta mt$.

A t -odd asymmetry characterizes CP violation by mixing and a t -even asymmetry identifies a direct decay asymmetry. This is very much like in decays to CP eigenstates⁵. When the time-order between the two neutral B decays cannot be measured or when the two orders are summed over, one is sensitive only to direct decay CP violation. Any asymmetry due to mixing is washed out in these cases.

3. TIME-DEPENDENT ASYMMETRIES IN THE STANDARD MODEL

In the standard model weak hadronic decays of neutral B mesons may involve asymmetries of both kinds. In decays to CP eigenstates the mixing-induced asymmetry is expected¹ to be very small in some cases (e.g. KM-allowed B_d^0 decays) and to be potentially large in others (e.g. KM-allowed B_d^0 decays). A similar variety exists in direct-decay asymmetries⁹. The situation in decays to states which are not CP eigenstates is similar. Here it is possible either to have no asymmetry at

all, to have an asymmetry due to mixing alone, due merely to CP violation in the direct decay, or due to a combination of both origins. The expected asymmetries may vary in magnitude from case to case^{1,2}. This can be seen by considering the decays in terms of the relevant quark operators. For CP violation by mixing one requires that both B^0 and $\bar{B}^0$ can decay to f by two components of the low energy effective Hamiltonian. CP violation in the direct decay can occur only when the Hamiltonian has two terms, both of which lead to either $B^0 \rightarrow f$ or $\bar{B}^0 \rightarrow f$.

It is straightforward to consider any given decay in terms of the corresponding low energy effective quark Hamiltonian. The latter contains QCD-corrected left-handed four-fermion terms¹⁰ and W loop QCD-induced ("penguin") terms¹¹. For instance, consider $B_d^0 \rightarrow \bar{D}^- \pi^+$. In this case $\bar{B}_d^0 \rightarrow \bar{D}^- \pi^+$ occurs via $b \rightarrow u\bar{c}d$ with $|\bar{A}_1/A_1| \sim (\lambda^3 - \lambda^2)$, $\lambda = 0.22$, and one expects a rather small asymmetry due to mixing². Also $A_2 = \bar{A}_2 = 0$ and one expects no direct decay asymmetry at all. In another case $B_d^0 \rightarrow \pi^- K^+$, which has no CP violation due to mixing, one expects a sizeable direct decay asymmetry due to interference between a four-fermion amplitude and a penguin amplitude⁹. The various possible cases are summarized in the Table. The first column gives the fundamental quark processes, each characterizing a class of hadronic processes. The second column presents examples of corresponding decays of B_d^0 to states which are not CP eigenstates. These examples can be easily generalized to B_s^0 decays. The next two columns describe qualitatively the expected asymmetry due to direct decay (DD) CP violation and/or due to mixing (MI). The descriptions "large" (possibly as large as a few tens of per cents) and "small" (at most a few per cent) are based only on the known magnitudes of the KM matrix elements. It is rather difficult to make quantitative estimates due to the unknown magnitudes and phases of hadronic matrix elements and of phases in

the KM matrix. Nonetheless, the classification presented in the Table provides a systematic testing ground for the KM mechanism of CP violation.

4. CONCLUSION

We defined t-even and t-odd CP asymmetries, which distinguish in a model-independent way between two possible origins of CP violation in neutral B decays: asymmetries appearing in the decay of a pure state and asymmetries due to $B - \bar{B}$ mixing. Only the first kind, a constant- in-time asymmetry, occurs in charged B decays. We then specified which of the two asymmetries is expected to show up at various classes of hadronic decays in the standard model. Quantitative predictions of CP asymmetries in B decays are known generally to suffer from hadronic uncertainties, except in decays to certain CP eigenstates ^{1,9}. This may make the task of testing the KM origin of CP violation in B decays a hard one. Here we point out that, besides estimates of asymmetries, certain salient features of time-dependent asymmetries may be used for this purpose. Our study, which makes a clear distinction between two forms of time-dependent asymmetries, can be used to shed light on the mechanism of CP violation when such asymmetries are observed. This requires, of course, a substantial increase in the available sample of B decays and measurements of B decay-times ¹²

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Table

Direct-decay and mixing-induced CP asymmetries in B_d^0 decays in the standard model

Quark Process	Final State	DD Asymmetry	MI Asymmetry
$b \rightarrow c\bar{u}s, u\bar{c}s$	$\bar{D}^- K^+, D_s^+ \pi^-$	none	none
$b \rightarrow c\bar{u}d, u\bar{c}d$	$\bar{D}^- \pi^+$	none	small
$b \rightarrow c\bar{s}s$	$\bar{D}^- D_s^+$	very small	none
$b \rightarrow u\bar{u}s$	$\pi^- K^+$	large	none
$b \rightarrow c\bar{c}d$	$\bar{D}^- D^{*+}$	small	large
$b \rightarrow u\bar{u}d$	$\pi^- \rho^+$	small	large

REFERENCES

1. For two recent reviews, see I. I. Bigi, V. A. Khoze, N. G. Uraltsev and A. I. Sanda, in *CP Violation*, edited by C. Jarlskog (World Scientific, 1989); G. Balenger et al., in *Proceedings of the 1988 Summer Study on High Energy Physics in the 1990s*, Snowmass, 1988, to be published.
2. The importance of time-dependent CP asymmetries was particularly emphasized by I. Dunietz and J. L. Rosner, *Phys. Rev. D* **34** (1986) 1404. This paper contains references to previous work. A few updated examples of this analysis were recently given by F. Gilman, SLAC preprint SLAC-PUB-4955, April 1989.
3. H. Albrecht et al. (ARGUS collaboration), *Phys. Lett. B* **192** (1987) 245; M. Artuso et al. (CLEO collaboration), *Phys. Rev. Lett.* **62** (1989) 2233.
4. An attempt in this direction was made by L. L. Chau and H. Y. Cheng, *Phys. Lett. B* **165B** (1985) 429, however we do not agree with their conclusions.
5. M. Gronau, Max-Planck Institute preprint MPI-PAE/PTh 27/89, May 1989, to be published.
6. J.-M. Gérard, private communication. Similar estimates are obtained in: a four family model - I. I. Bigi and S. Wakaizumi, *Phys. Lett. B* **188** (1987) 501; a left-right symmetric model - G. Ecker and W. Grimus, *Z. Phys. C* **30** (1986) 293; a three-doublet-scalar model - J. F. Donoghue and E. Golowich, *Phys. Rev. D* **37** (1988) 2542.
7. The first and last eqs.(5) were obtained by Dunietz and Rosner, Ref.2.
8. Dunietz and Rosner, Ref.2.
9. Gronau, Ref. 5 and in *Proceedings of the XII International Workshop on Weak Interactions and Neutrinos*, Ginosar, April 1989 (to appear in *Suppl. Nucl. Phys. B*); D. London and R. D. Peccei, *Phys. Lett. B* **223** (1989) 257.
10. M. K. Gaillard and B. W. Lee, *Phys. Rev. Lett.* **33** (1974) 108; G. Altarelli and L. Maiani, *Phys. Lett. B* **52B** (1974) 351.
11. A. I. Vainshtein et al., *Sov. J. Nucl. Phys.* **22** (1975) 340.
12. For recent studies and proposals, see for instance D. G. Cassel, G. J. Feldman and R. Siemann, in *Proceedings of the 1988 Summer Study on High Energy Physics in the 1990s*, Snowmass, 1988, to be published.