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STRESS-TENSOR COMMUTATORS AND SCHWINGER TERMS
IN SINGLETON THEORIES

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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS**STRESS-TENSOR COMMUTATORS AND SCHWINGER TERMS
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ABSTRACT

We compute the commutators of the regularized quantum stress-tensor of singleton theories formulated on the boundary of a $(p+2)$ -dimensional anti de Sitter space (AdS_{p+2}). (These are superconformal field theories on $S^p \times S^1$.) We find that the algebra is not closed except in the case of AdS_3 . It does contain, however, the finite dimensional AdS_{p+2} algebra $SO(p+1,2)$. We also find divergent, field dependent as well as field independent Schwinger terms (i.e. the central extensions), which, however, do not lead to anomalies in the algebra of the AdS charges. We also give a simple derivation of the two-point functions for bosonic and fermionic singletons.

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1. INTRODUCTION

The proposition that the cosmological constant is not identically zero but very small (and negative) has led to several studies on the physics in anti de Sitter (AdS) space [1]. One then is inevitably led to consider the most fundamental representations of the AdS group. These are known as singletons, and were first discovered by Dirac [2] for the case of the AdS group $SO(3,2)$. Since then several remarkable properties of the singletons have been discovered. For example, the singleton field theory can be described on the $S^p \times S^1$ boundary of AdS_{p+2} alone, as opposed to the whole of AdS_{p+2} [3][4][5]. Moreover, it is known that one can construct, on purely kinematical grounds, infinitely many massless states of all spins out of just two singletons [6][4][5]. Flato and Fronsdal [7] consider these massless states to be bound states, and argue that in order to obtain meaningful interactions of these states in the whole of AdS space, one should quantize the singletons with an unusual spin-statistics. An alternative interpretation of singletons has been suggested, according to which the states obtained by repeated action of an arbitrary number of singleton creation operators on the vacuum are considered to be excitations of a supermembrane [8][9]. This is based on the conjecture that the supersingleton action may arise from the supermembrane action in a physical gauge upon expansion about a suitable supermembrane vacuum [10][8][9].

The purpose of this paper is to explore further the (super)singleton field theories [3][11][12]. In particular, we shall study (at the quantum level) the algebra of the stress tensor, which in the case of two dimensions leads to the Virasoro algebra with a central extension. Of course, many of the features arising in two dimensions are rather special, and are not expected to carry over to higher dimensions, but we would like to determine to what extent they do. We shall find that the quantum commutators of the components of the stress tensor lead to divergent operator valued Schwinger terms, as well as field independent ones (i.e. the central extensions). The latter are expected to play a role in the anomaly structure of the theory, which, if the conjectured supersingleton-supermembrane connection actually exists, would shed light on the issue of the critical dimension for the supermembrane. We shall not pursue this problem here, but provide some of the tools which might facilitate its study. See also [13] for a study of several aspects of the stress-tensors of conformal field theories in higher dimensions.

The calculations are rather lengthy, though straightforward. The two point functions play an essential role. We shall provide a simple derivation of the bosonic, as well as fermionic two-point functions in this paper.

The plan of this paper is as follows. In Sec. 2, the improved stress-tensor for supersingletons is discussed. In Sec. 3, the quantum stress-tensor and a regularization scheme is defined and the bosonic as well as fermionic two-point functions are derived. In Sec. 4, the quantum stress-tensor commutators are given. In Sec. 5 we comment further on our results, and state some of the open problems. Some useful formulae are collected in the Appendix.

2. THE SUPERSINGLETON ACTION AND THE IMPROVED CURRENTS

Our starting point is the following Lagrangian for N_B scalar fields φ^I and N_F fermi fields λ^A which are defined on $S^p \times S^1$ [11][12]:

$$L = -\frac{1}{2}\sqrt{-h}(h^{ij}\partial_i\varphi^I\partial_j\varphi_I - i\bar{\lambda}^A\gamma^i\nabla_i\lambda_A + \frac{1}{4}(p-1)^2\varphi^I\varphi_I), \quad i = 0, 1, \dots, p \quad (2.1)$$

$$I = 1, \dots, N_B \quad A = 1, \dots, N_F.$$

Here λ_A^A is a spinor of $SO(p, 1)$ where α is the spinor index which will often be suppressed, h_{ij} is the metric of $S^p \times S^1$ with signature $(-, +, \dots, +)$ and h is the determinant of h_{ij} . The symmetries of the Lagrangian (2.1) and the possible values of p and N_B are given in Table 1, while the properties of the spinors λ_a^A are given in Table 2. Note that in all cases, the full bosonic symmetry is $SO(p+1, 2) \times SO(N_B)$.

TABLE 1

p	N_B	Bosonic Symmetry	Supersymmetry
1	1,2,4,8	$SO(2, 2) \times SO(N_B)$	$OSp(N_B 2)$
2	1,2,4,8	$SO(3, 2) \times SO(N_B)$	$OSp(N_B 4)$
3	2	$SO(4, 2) \times SO(2)$	$SU(2, 2 1)$
4	4	$SO(4, 2) \times SU(2) \times SU(2)$	$SU(2, 2 2) \times \widetilde{SU}(2)$
4	4	$SO(5, 2) \times SU(2) \times SU(2)$	$F(4) \times SU(2)$
5	4	$SO(5, 2) \times SU(2) \times SU(2)$	$OSp(6, 2 2) \times \widetilde{SU}(2)$

The bosonic transformations of the fields are given by [11]

$$\delta\varphi^I = \xi_{PQ}^I\partial^P\varphi^Q + (p-1)\Omega_{PQ}\varphi^I + \Lambda^I{}_J\varphi^J \quad P, Q = 0, 1, \dots, p+1, p+3 \quad (2.2)$$

$$\delta\lambda^A = \xi_{PQ}^A\nabla_i\lambda^A + \frac{1}{4}(\nabla_i\xi_j^A)\varepsilon^j{}_P\gamma^i\lambda^A + p\Omega_{PQ}\lambda^A + \frac{1}{4}\Lambda^{IJ}(\sigma_{IJ})^A{}_B\lambda^B, \quad (2.3)$$

where $\sigma_{IJ} \equiv \sigma_{IJ}^{\dagger}$ are the Clebsch-Gordan coefficients which satisfy (see Table 2)

$$\sigma^I{}_J\sigma^{IJ} + I \leftrightarrow J = 2\delta^{IJ}, \quad (2.4)$$

$$\sigma^{IJ}\sigma^J + I \leftrightarrow J = 2\delta^{IJ}.$$

The $\Lambda^I{}_J$ are the parameters of the $SO(N_B)$ transformations and the ξ_{PQ}^I are the $\frac{1}{2}(p+2)(p+3)$ conformal Killing vectors of $S^p \times S^1$ satisfying

$$\nabla_i\xi_j + \nabla_j\xi_i = 4\Omega_{ij}, \quad (2.5)$$

where $\nabla_i\xi_j = \partial_i\xi_j - \Gamma_{ij}^k\xi_k$ is the standard covariant derivative on $S^p \times S^1$.

The supersymmetry transformation rules are given by [11]

$$\delta\varphi^I = \sigma_{AB}^{IJ}\bar{\varepsilon}^A\lambda^B, \quad (2.6)$$

$$\delta\lambda_A = -i\sigma_{AB}^I(\gamma^I\partial_i\varphi^I\varepsilon^B + \frac{1-p}{2}\varphi^I\varepsilon^B), \quad (2.7)$$

where A, B label (possibly inequivalent) representations of $SO(N_B)$.

TABLE 2

N_B	p	$\delta\varphi^I$ $I = 1, \dots, N_B$	Spinor Properties
8	p=1,2	$\sigma_{AB}^I\bar{\varepsilon}^A\lambda^B$	Majorana $\alpha = 1, 2, \quad A = 1, \dots, 8$
4	p=5	$\sigma_{AB}^I\bar{\varepsilon}^A\lambda^B$	Symplectic Majorana-Weyl $\alpha = 1, \dots, 8, \quad A, \dot{A} = 1, 2$
	p=3,4	$\sigma_{AB}^I\bar{\varepsilon}^A\lambda^B$	Symplectic Majorana-Weyl $\alpha = 1, \dots, 4, \quad A, \dot{A} = 1, 2$
	p=1,2	$\gamma_{ab}^I\bar{\varepsilon}^a\lambda^b$	Majorana $\alpha = 1, 2, \quad a, b = 1, \dots, 4$
2	p=3	$\bar{\varepsilon}\sigma^I\lambda$	Majorana $\alpha = 1, \dots, 4$
	p=1,2	$\sigma_{AB}^I\bar{\varepsilon}^A\lambda^B$	Majorana $\alpha = 1, 2, \quad A = 1, 2$
1	p=1,2	$\bar{\varepsilon}\lambda$ ($\sigma_{11}^1 = 1$)	Majorana $\alpha = 1, 2$

As shown in Table 2, the equivalent representations of $SO(N_B)$ occur for $N_B = 1, 2$ and the $p = 1, 2$ cases for $N_B = 4$. Note that for $(N_B = 4; p = 1, 2)$ the σ -matrices are the $SO(4)$ -matrices and for $(N_B = 2; p = 1, 2)$ they are the $SO(2)$ matrices. Note also that for $N_B = 2, p = 3$ the σ -matrix must be in $SO(N_B) \times SO(p, 1)$ space and it involves the γ_5 -matrix.

The symbols ε^A and η^A in (2.7) are defined as the product of the constant supersymmetry parameters $C_A^{(\tau)}$ and the Killing spinors $\varepsilon^{(\tau)}, \eta^{(\tau)}$ on $S^p \times S^1$:

$$\varepsilon^A \equiv C^{A(\tau)}\varepsilon_{(\tau)}, \quad (2.8)$$

$$\eta^A \equiv C^{A(\tau)}\eta_{(\tau)}, \quad \tau = 1, \dots, 2 \lfloor \frac{p+1}{2} \rfloor. \quad (2.9)$$

The index τ labels a spinor representation of $SO(p+1, 2)$. This is due to the fact that $G^{A(\tau)}$ must transform like the supercharge (see below) which must carry the representation of the bosonic symmetries (the last column of Table 2, excluding the $\tilde{SU}(2)$ factors). The Killing spinor equations for $\varepsilon^{(\tau)}$ and $\eta^{(\tau)}$ are given by:

$$\begin{aligned}\nabla_\mu \varepsilon &= \frac{1}{2} \gamma_\mu \eta^{(\tau)}, \\ \gamma^i \nabla_i \eta^{(\tau)} &= \frac{1}{2} (1-p) \varepsilon^{(\tau)}.\end{aligned}\quad (2.10)$$

The Noether currents corresponding to the transformations (2.2), (2.3), (2.6) and (2.7) are given by

$$\begin{aligned}J_{PQ}^i &= \sqrt{-h} h^{ij} (T_{ik} \xi_{PQ}^k + \frac{p-1}{2} \varphi^2 \partial_j \Omega_{PQ}) \\ &\quad + \partial_j (c_1 \sqrt{-h} \xi_{PQ}^{i1} (\nabla^j \varphi^2) + c_2 \sqrt{-h} (\nabla^i \xi_{PQ}^j) \varphi^2), \\ J_{IJ}^i &= \sqrt{-h} h^{ij} \varphi_{(i} \partial_j \varphi_{j)} + \frac{i}{4} \sqrt{-h} \bar{\lambda}^A \gamma^i (\sigma_{IJ})_A^B \lambda_B, \\ J_{A(\tau)}^i &= i \sqrt{-h} (\bar{\varepsilon}_{(\tau)} S_A^i + \frac{1-p}{2} \varphi^i \bar{\eta}_{(\tau)} \gamma^i \sigma_{BA}^i \lambda^B) \\ &\quad + i c_3 \partial_j (\sqrt{-h} \bar{\eta}_{(\tau)} \gamma^j \varphi_j \sigma_{BA}^i \lambda^B),\end{aligned}\quad \begin{aligned}(2.11) \\ (2.12) \\ (2.13)\end{aligned}$$

where the energy momentum tensor T_{ij} , and the supercurrent S_A^i are given by

$$\begin{aligned}T_{ij} &= -\partial_i \varphi^j \partial_j \varphi_i + \frac{1}{2} h_{ij} (\partial_k \varphi^k \partial^l \varphi_l + \frac{1}{4} (p-1)^2 \varphi^l \varphi_l) - \frac{i}{2} \bar{\lambda}^A \gamma_{(i} \nabla_{j)} \lambda_A \\ S_A^i &= \gamma^j \partial_j \varphi_i \gamma^i \sigma_{BA}^i \lambda^B\end{aligned}\quad \begin{aligned}(2.14) \\ (2.15)\end{aligned}$$

The terms in (2.11) and (2.13) proportional to the arbitrary constants c_1 , c_2 and c_3 are the so-called improvement terms.

The conserved charges corresponding to the currents (2.11)-(2.13) are given by

$$\begin{aligned}M_{PQ} &= \int d^p x J_{AB}^{i=0} \\ T_{IJ} &= \int d^p x J_{IJ}^{i=0} \\ Q_{A(\tau)} &= \int d^p x J_{A(\tau)}^{i=0}\end{aligned}\quad (2.16)$$

Note that the improvement terms drop out in the expression for M_{PQ} and $Q_{A(\tau)}$.

In this paper we prefer to work with currents which do not contain derivatives of ξ^i and $\varepsilon^{(\tau)}$ or, equivalently, in which there are no terms involving Ω and $\eta^{(\tau)}$. This can be achieved by taking the coefficient c_1 , c_2 and c_3 to be

$$c_1 = \frac{1}{2} c_2 = -\frac{1}{2} c_3 = -\frac{p-1}{2p}.\quad (2.17)$$

For this choice of coefficients the $SO(p+1, 2)$ current and the supercurrent are given by

$$J_{PQ}^i = \sqrt{-h} h^{ij} \bar{T}_{jk} \xi_{PQ}^k, \quad (2.18)$$

$$J_{A(\tau)}^i = i \sqrt{-h} \bar{\varepsilon}_{(\tau)} \tilde{S}_A^i, \quad (2.19)$$

where $\bar{T}_{ij}$ is an improved energy momentum tensor which is not only conserved but also traceless, i.e. $\nabla_i \bar{T}^{ij} = \bar{T}^i_i = 0$ (modulo field equations) and $\tilde{S}_A^i$ is an improved supercurrent which is not only conserved but also gamma-traceless, i.e. $\nabla_i \tilde{S}_A^i = \gamma_i \tilde{S}_A^i = 0$ (modulo field equations):

$$\bar{T}_{ij} \equiv T_{ij} - \frac{p-1}{4p} (h_{ij} \nabla^k \nabla_k - \nabla_i \nabla_j + R_{ij}) \varphi^2. \quad (2.20)$$

$$\tilde{S}_A^i \equiv S_A^i + \frac{p-1}{p} \gamma^{ij} \nabla_j (\varphi_i \sigma_{BA}^i \lambda^B). \quad (2.21)$$

The advantage of working with the improved energy momentum tensor and supercurrent is that, their algebra now manifestly contains the finite dimensional superconformal algebra via (2.18), (2.19) and (2.16).

3. REGULARIZATION AND THE TWO-POINT FUNCTIONS

A) BOSONS

The covariant quantization of the action (2.1) is in principle straightforward and has already been performed for the case ($p=2$; $N=8$) in [9]. In order to quantize the bosons one proceeds as follows. The most general solution of the field equations that follows from (2.1) is given by the following expansion of the N scalar fields φ^I on S^p :

$$\varphi^I = \sum_{l=0}^{\infty} \sum_m (a_{lm}^I \varphi_{lm} + a_{lm}^{I*} \varphi_{lm}^*), \quad (3.1)$$

where

$$\varphi_{lm} = \left[\frac{d_l}{V_p (2l+p-1)} \right]^{\frac{1}{2}} e^{-i(l+\frac{1}{2}p-\frac{1}{2})t} D_{lm}^l(L^{-1}) \quad p \geq 2 \quad (3.2)$$

$$\varphi^I = \frac{1}{\sqrt{\pi}} \left[q^I + \frac{p^I}{2} t + \sum_{l=1}^{\infty} \frac{1}{2\sqrt{l}} (a_l^I e^{-il(t+\theta)} + \bar{a}_l^I e^{-il(t-\theta)} + h.c.) \right] \quad p=1 \quad (3.3)$$

Note that for $p=1$ there are the zero modes q^I and p^I . In the above equations $V_p = \frac{2\pi}{\Gamma(\frac{p+1}{2})}$ is the volume of S^p and $D_{lm}^l(L^{-1})$ denotes the unitary representation matrix of $SO(p+1)$ for the representation labelled by l . Some basic relations of the D-functions are given in the Appendix. The range of the indices m is equal to the dimension d_l of the representation labelled by l . Since the scalar fields φ^I are defined on a coset space $S^p = \frac{SO(p+1)}{SO(p)}$ the expansion (3.2) is somewhat

restricted. To be precise, the expansion (3.2) only includes the representations of $SO(p+1)$ which on restriction to $SO(p)$ contain at least one singlet. These representations are exactly all symmetric traceless representations of $SO(p+1)$. The dimension d_l of these representations is given by the formula $d_l = \frac{(2l+p-1)(l+p-2)!}{l!(p-1)!}$.

One next imposes the following commutation relations:

$$\begin{aligned} [a_{lm}^I, a_{lm'}^{IJ}] &= \delta^{IJ} \delta_{lm} \delta_{mm'}, & p \geq 2, \\ [q^I, p^J] &= i\delta^{IJ}, [a_l^I, a_p^{IJ}] = [\bar{a}_l^I, \bar{a}_p^{IJ}] = \delta^{IJ} \delta_{lp}, & p = 1. \end{aligned} \quad (3.4)$$

The other commutators vanish. The a_{lm}^I , and $a_l^I, \bar{a}_l^I$ are now operators in a Fock space whose vacuum $|0\rangle$ is defined by:

$$\begin{aligned} a_{lm}^I |0\rangle &= 0, & p \geq 2, \\ a_l^I |0\rangle &= \bar{a}_l^I |0\rangle = 0, & p = 1. \end{aligned} \quad (3.5)$$

The quantum energy momentum tensor is given by the expression (2.20) where the φ^I are now assumed to be operators which act in a Fock space with vacuum defined by (3.5). This expression is supposed to be normal ordered with respect to this vacuum. We use a regularization in which the equal time commutator of two such normal ordered energy momentum tensors is given by:

$$[:T(t) :, :T'(t-i\epsilon) :] \equiv \lim_{\epsilon \rightarrow 0} :T(t-i\epsilon) : - :T'(t-i\epsilon) : :T(t+i\epsilon) : \quad (3.6)$$

for positive ϵ . Using this regularization it is clear that in order to calculate the commutator we first must calculate the following basic two point function:

$$\delta^{IJ} G(\gamma, \epsilon) \equiv \langle 0 | \varphi^I(t-i\epsilon, x) \varphi^J(t+i\epsilon, x') | 0 \rangle, \quad (3.7)$$

where it is understood that for $p = 1$ the zero modes are omitted. In eq. (3.7) the $x = (\theta_1, \dots, \theta_p)$ are p angular variables parametrizing S^p and γ is a function of x and x' which is given below.

We will first calculate the two point function G for $p \geq 2$. Substituting the expansion (3.2) into (3.7) and using the product formula for the D-functions given in the Appendix we arrive at the following formula:

$$G(\gamma, \epsilon) = \sum_l \frac{d_l}{V_p(2l+p-1)} e^{-2\epsilon(l+\frac{1}{2}p-\frac{1}{2})} D_{00}^l(L^{-1}L'). \quad (3.8)$$

The D-function on S^p satisfy the following differential equation

$$\nabla^2 D_{00}^l(L^{-1}L') = -l(l+p-1) D_{00}^l(L^{-1}L') \quad (3.9)$$

Since the Gegenbauer polynomials $C_l^{\frac{1}{2}(p-1)}$ also satisfy the same equation [14], it follows that (compare with [13]):

$$D_{00}^l(L^{-1}L') = \frac{l!(p-2)!}{(p+l-2)!} C_l^{\frac{1}{2}(p-1)}(\cos \gamma), \quad (3.10)$$

where $\cos \gamma$ is defined by

$$\begin{aligned} \cos \gamma &= \cos \theta_p \cos \theta'_p + \sin \theta_p \sin \theta'_p [\cos \theta_{p-1} \cos \theta'_{p-1} + \sin \theta_{p-1} \sin \theta'_{p-1} \{ \\ &\quad \dots + \sin \theta_2 \sin \theta'_2 \cos(\theta_1 - \theta'_1) \} \dots]. \end{aligned} \quad (3.11)$$

The coefficient in (3.10) is determined by comparing the known expressions for $D_{00}^l(1)$ and $C_l^{\frac{1}{2}(p-1)}(0)$. Some useful identities involving γ have been collected in the Appendix. Using the defining property of the Gegenbauer polynomials

$$\frac{1}{(1-2t\cos\gamma+t^2)^\lambda} = \sum_{l=0}^{\infty} t^l C_l^{\frac{1}{2}(p-1)}(\cos\gamma), \quad (3.12)$$

and setting $\lambda = \frac{1}{2}(p-1)$, $t = e^{-2\epsilon}$, from (3.8) we finally obtain the following expression for the two point function:

$$G(\gamma, \epsilon) = \frac{1}{(p-1)V_p} \times \frac{1}{[2(\cos 2\epsilon - \cos \gamma)]^{\frac{p-1}{2}}} \quad p \geq 2. \quad (3.13)$$

Note that

$$\lim_{\epsilon \rightarrow 0} \frac{\partial}{\partial \epsilon} G(\gamma, \epsilon) = -\delta^p(x, x'), \quad (3.14)$$

where $\delta^p(x, x')$ is the delta function on S^p .

For $p = 1$ a similar calculation yields the following result:

$$G(\gamma, \epsilon) = -\frac{1}{4\pi} \ln [2e^{-2\epsilon}(\cos 2\epsilon - \cos \gamma)], \quad p = 1. \quad (3.15)$$

$$\lim_{\epsilon \rightarrow 0} \frac{\partial}{\partial \epsilon} G(\gamma, \epsilon) = -\delta(\gamma) + \frac{1}{2\pi}, \quad (3.16)$$

where $\gamma = (\theta - \theta')$. Note that if we take the limit $p \rightarrow 1$ in (3.13) we obtain the result (3.14) plus an infinite constant.

For $p \geq 2$ the two point functions for $\partial_0 \varphi, \varphi \partial_0 \varphi$ and $\partial_0 \varphi \partial_0 \varphi$ are given in terms of G by:

$$\begin{aligned} \langle 0 | \partial_0 \varphi | 0 \rangle &= +\frac{i}{2} \frac{\partial}{\partial \epsilon} G, \\ \langle 0 | \varphi \partial_0 \varphi | 0 \rangle &= -\frac{i}{2} \frac{\partial}{\partial \epsilon} G, \\ \langle 0 | \partial_0 \varphi \partial_0 \varphi | 0 \rangle &= +\frac{1}{4} \frac{\partial^2}{\partial \epsilon^2} G. \end{aligned} \quad (3.17)$$

For $p = 1$ these two point functions are given by the same expressions up to a constant.

B) FERMIONS

In order to covariantly quantize the fermions we first consider the most general solution to the fermionic field equations, which is given by the following expansion on S^p :

$$\chi_\alpha^p = \sum_{j=\frac{1}{2}}^{\infty} \sum_m (d_{jm}^A \lambda_{j\alpha\alpha} + d_{jm}^{A\dagger} \lambda_{j\alpha\alpha}^\dagger), \quad (3.18)$$

where

$$\lambda_{j\alpha\alpha} = \frac{1}{\sqrt{2}} \left(\frac{d_j}{d_{\frac{1}{2}}} \right)^{\frac{1}{2}} e^{-i(j+\frac{1}{2}-\frac{1}{2})t} \begin{pmatrix} D_{\alpha\alpha}^j(L^{-1}) \\ D_{\alpha\alpha}^j(L^{-1}) \end{pmatrix} \quad p \geq 2 \quad (3.19)$$

For $p = 1$, the expansion is

$$\chi^A = \frac{1}{\sqrt{2}\pi} \sum_{j=\frac{1}{2}}^{\infty} (d_j^A e^{-i(j+\frac{1}{2}-\frac{1}{2})t} + d_j^{A\dagger} e^{i(j+\frac{1}{2}-\frac{1}{2})t}) \quad \partial = 1 \quad (3.20)$$

Note that for $p = 1$ there are no fermionic zero modes due to the anti-periodic boundary conditions. In eq. (3.19) $V_p = 2(\pi)^{\frac{p-1}{2}} / \Gamma(\frac{p-1}{2})$ is the volume of S^p . The index $\alpha = (\alpha', \alpha)$ refers to the spinor representation of $SO(p, 1)$. The indices α' and α refer to two inequivalent spinor representations of $SO(p)$ for $p = 2$ and to two equivalent spinor representations of $SO(p)$ for $p \neq 2$. The $D_{\alpha\alpha}^j(L^{-1})$ denote unitary representation matrices of $SO(p+1)$ for the representation labelled by j . The expansion (3.19) is restricted to include only those representations of $SO(p+1)$ which on restriction to $SO(p)$ contain at least the two spinors of $SO(p)$ labelled by α' and α . These representations are exactly all gamma-traceless symmetric tensor-spinors of $SO(p+1)$. The dimension d_j of such representations is given by the formula $d_j = \frac{(p+j-\frac{1}{2})!}{(j-\frac{1}{2})!(p-1)!} \times d_{\frac{1}{2}}$, where $d_{\frac{1}{2}}$ is the real dimension of the fundamental representation of $SO(p, 1)$, i.e. 1 for $p = 1, 2, 2$ for $p = 2, 4$ for $p = 3$, and 8 for $p = 4, 5$.

One next imposes the following anti-commutation relations on the d -oscillators:

$$\{d_{jm}^p, d_{j'm'}^{p\dagger}\} = \delta^{pq} \delta_{jj'} \delta_{mm'} \quad p \geq 2$$

$$\{d_{j'}^p, d_{j'}^{p\dagger}\} = \delta^{pq} \delta_{jj'} \quad p = 1 \quad (3.21)$$

The other anti-commutators vanish. The d_{jm}^A and $d_{j'}^{A\dagger}$ are now operators in a Fock space whose vacuum is defined by

$$d_{jm}^A |0\rangle = 0 \quad p \geq 2$$

$$d_{j'}^A |0\rangle = 0 \quad p = 1. \quad (3.22)$$

Like in the bosonic case we use the regularization defined in (3.6). In order to calculate the commutators of the currents we first must calculate the following basic fermionic two point function:

$$\delta^{AB} G_\alpha^F(\gamma, \varepsilon) \equiv \langle 0 | \lambda_\alpha^A(t - i\varepsilon, x) \lambda^{\dagger B}(\varepsilon, x') | 0 \rangle$$

$$= -(\gamma_0 C)_{\alpha\gamma} (C^{-1})_{\gamma_0} \delta^B < 0 | \lambda^{\dagger B}(\varepsilon, x) \lambda_\alpha^A(t + i\varepsilon, x') | 0 >$$

The second equation of (3.23) only holds if λ is a Majorana spinor. The $x = (\theta_1, \dots, \theta_p)$ in this equation are p angular variables parametrizing S^p and γ is a function of x and x' .

We will first calculate the two point function G^F for $p \geq 2$. Substituting the expansion (3.19) into (3.23) and using the product formula for the D-functions given in the Appendix we arrive at the following formula:

$$G_\alpha^F(\gamma, \varepsilon) = \frac{1}{2} \sum_j \frac{d_j}{V_p} e^{-2\varepsilon(j+\frac{1}{2}p-\frac{1}{2})} D_\alpha^j(L^{-1}L'). \quad (3.24)$$

By multiplying the r.h.s. of (3.24) with $D_\gamma^{\dagger B}(L'^{-1})$ and integrating over x' one can show that the following relation holds

$$\lim_{\varepsilon \rightarrow 0} G_\alpha^F(\gamma, \varepsilon) = \frac{1}{2} \delta_\alpha^\beta \delta^p(x, x'), \quad (3.25)$$

where $\delta^p(x, x')$ is the delta function on S^p .

We now calculate the two point function G^F for $p = 1$ and for λ a Majorana-Weyl spinor. Substituting the expression (3.120) into (3.23) we obtain

$$G(\gamma, \varepsilon) = \frac{1}{2\pi} \frac{e^{-\varepsilon(\frac{1}{2}+\frac{1}{2})}}{1 - e^{-2\varepsilon+\frac{1}{2}\gamma}}, \quad p = 1, \quad (3.26)$$

where $\gamma = (\theta - \theta')$. For λ a Majorana spinor one obtains the same expression plus its complex conjugate. Note that

$$\lim_{\varepsilon \rightarrow 0} G(\gamma, \varepsilon) = \frac{1}{2} \delta(\theta - \theta'), \quad (3.27)$$

where $\delta(\theta - \theta')$ is the delta function for anti-periodic functions.

4. QUANTUM STRESS-TENSOR COMMUTATORS AND SCHWINGER TERMS

A) BOSONS

Using the basic rules (3.17) and the expressions (3.13), (3.15) for the two point function G we are now in a position to calculate the quantum commutator algebra of the improved energy momentum tensor $\tilde{T}$. After a rather long but straightforward calculation we have obtained the following result which holds for both $p = 1$ and $p \geq 2$ (in the following equations (4.1-9), $\tilde{T}_{ij}$ refers to the *purely bosonic* part of the total improved energy momentum tensor $\tilde{T}_{ij}$):

$$[\tilde{T}_{00}(x), \tilde{T}_{00}(y)] = +i\nabla_\mu^2 \delta^p(x, y) [\tilde{T}_0^\mu(x) + \tilde{T}_0^\mu(y)], \quad (4.1)$$

$$[\tilde{T}_{0\alpha}(x), \tilde{T}_{0\alpha}(y)] = +i\nabla_\mu^2 \delta^p(x, y) \tilde{T}_{0\alpha}^\mu(x) + i\nabla_\mu^2 \delta^p(x, y) \tilde{T}_{0\alpha}^\mu(y), \quad (4.2)$$

$$[\tilde{T}_{00}(x), \tilde{T}_{0\alpha}(y)] = -i\nabla_\mu^2 \delta^p(x, y) \tilde{T}_{00}^\mu(y) - i\nabla_\mu^2 \delta^p(x, y) \tilde{T}_\alpha^\mu(y)$$

$$\begin{aligned}
& -i \frac{(p-1)^2}{4p} \nabla_x^2 [\delta^p(x, y) : \varphi^2(x) :] - i \frac{(p-1)^2}{4p^2} \Delta_x [\nabla_x^2 \delta^p(x, y) : \varphi^2(x) :] \\
& + i \frac{p-1}{4p} \nabla_x^2 [\nabla_x^2 \delta^p(x, y) \nabla_b : \varphi^2(x) : + \nabla_b^2 \delta^p(x, y) \nabla_a : \varphi^2(x) :] \\
& + C_{00,0a}(x, y), \tag{4.3}
\end{aligned}$$

$$\begin{aligned}
[\tilde{T}_{00}(x), \tilde{T}_{ab}(y)] &= +i\delta^p(x, y) \partial_0 \tilde{T}_{ab}(x) + 2i\nabla_a^T \delta^p(x, y) \tilde{T}_{0b}(y) + i \frac{p-1}{4p} \nabla_a^T \nabla_b^T [\delta^p(x, y) \partial_0 : \varphi^2(x) :] \\
& + i \frac{p-1}{4p^2} h_{ab} \Delta_x [\delta^p(x, y) \partial_0 : \varphi^2(x) :], \tag{4.4}
\end{aligned}$$

$$\begin{aligned}
[\tilde{T}_{ab}(x), \tilde{T}_{cd}(y)] &= -\frac{2i}{p} h_{ab} \nabla_c^T [\delta^p(x, y) \tilde{T}_{0d}(x)] + \frac{i}{p^2} h_{ab} h_{cd} \nabla_c^T [\delta^p(x, y) \tilde{T}_b^d(x)] \\
& + \frac{i}{p} h_{ab} \delta^p(x, y) \partial_0 \tilde{T}_{cd}(x) + i \frac{p-1}{4p^2} h_{ab} \nabla_c^T \nabla_d^T [\delta^p(x, y) \partial_0 : \varphi^2(x) :] \\
& - i \frac{p-1}{4p^3} h_{ab} h_{cd} \nabla_c^T [\delta^p(x, y) \nabla^e \partial_0 : \varphi^2(x) :] \\
& - (ab \leftrightarrow cd, x \leftrightarrow y), \tag{4.5}
\end{aligned}$$

$$\begin{aligned}
[\tilde{T}_{0c}(x), \tilde{T}_{ab}(y)] &= +i\delta^p(x, y) \nabla_c \tilde{T}_{ab}(x) - 2i\nabla_a^T [\delta^p(x, y) \tilde{T}_{b,c}(x)] \\
& - \frac{i}{p^2} \nabla_c N x [\delta^p(x, y) h_{ab} \tilde{T}_{00}(x)] + \frac{i}{p} \nabla_a^T [\delta^p(x, y) h_{ab} \tilde{T}_c^d(x)] \\
& - i \frac{p-1}{2p} \nabla_c^T [\delta^p(x, y) \tilde{T}_{ab}(x)] - \frac{i}{2p} \nabla_a^T [\delta^p(x, y) h_{b,c} \partial_0^2 : \varphi^2(x) :] \\
& + \frac{i}{2p^2} \nabla_c^T [\delta^p(x, y) h_{ab} \partial_0^2 : \varphi^2(x) :] + \frac{i}{2p} \nabla_a^T [\delta^p(x, y) h_{b,c} \Delta : \varphi^2(x) :] \\
& + i \frac{p-1}{4p^2} \nabla_c^T [\nabla_b^T \delta^p(x, y) \nabla_a^d : \varphi^2(x) : h_{ab}] - \frac{p+1}{4p^3} \nabla_c^T [\delta^p(x, y) h_{ab} \Delta : \varphi^2(x) :] \\
& - i \frac{p-1}{4p^2} \nabla_c^T [\delta^p(x, y) \nabla_a \nabla_b : \varphi^2(x) :] + i \frac{p-1}{4p^2} \nabla_c^T [\nabla_a^T \nabla_b^T \delta^p(x, y) : \varphi^2(x) :] \\
& - i \frac{p-1}{4p} \nabla_a^T \nabla_b^T [\nabla_c^T \delta^p(x, y) : \varphi^2(x) :] + i \frac{p-1}{4p} (h_{ab} \nabla_c^T - h_{ca} \nabla_b^T) \delta^p(x, y) : \varphi^2(x) : \\
& - i \frac{(p-1)^2}{2p} \nabla_c^T \delta^p(x, y) h_{b,c} : \varphi^2(x) : + \frac{(p-1)^2}{4p^2} h_{ab} \nabla_c^T \delta^p(x, y) : \varphi^2(x) : \\
& + i \frac{p-1}{2p} \delta^p(x, y) h_{ab} \nabla_c : \varphi^2(x) : - i \frac{p-1}{2} \delta^p(x, y) h_{ca} \nabla_b : \varphi^2(x) : \\
& + C_{0c,ab}(x, y), \tag{4.6}
\end{aligned}$$

$$[\tilde{T}_{00}(x), : \varphi^2(y) :] = +i\delta^p(x, y) \partial_0 : \varphi^2(x) :, \tag{4.7}$$

$$[\tilde{T}_{0a}(x), : \varphi^2(y) :] = +\frac{i}{p} \delta^p(x, y) \nabla_a^T : \varphi^2(x) : + i \frac{p-1}{p} \nabla_a^T \delta^p(x, y) : \varphi^2(x) :, \tag{4.8}$$

$$[\tilde{T}_{ab}(x), : \varphi^2(y) :] = +\frac{i}{p} h_{ab} \delta^p(x, y) \partial_0 : \varphi^2(x) :, \tag{4.9}$$

$$[: \varphi^2(x) :, : \varphi^2(y) :] = 0. \tag{4.10}$$

The central extensions $C_{00,0a}(x, y)$ and $C_{0c,ab}(x, y)$ are given by

$$C_{00,0a}(x, y) = \frac{2^{-(p+1)} i N}{V_p^2 (p-1) p^2 (p+2)} (\Delta_x + p) \nabla_a^T \frac{\partial}{\partial \epsilon} \frac{1}{(\cos 2i\epsilon - \cos \gamma)^{p-1}}, \tag{4.11}$$

$$\begin{aligned}
C_{0c,ab}(x, y) &= \frac{2^{-p} (p+1) N \sin 2i\epsilon}{p V_p^2 (\cos 2i\epsilon - \cos \gamma)^{p+3}} \nabla_a^T \cos \gamma \times \\
& (\nabla_b^T \cos \gamma \nabla_c^T \cos \gamma + \nabla_b^T \nabla_c^T \cos \gamma (\cos 2i\epsilon - \cos \gamma)) \tag{4.12}
\end{aligned}$$

It is instructive to consider the case $p = 1$ separately and to re-discover the known string results. First of all for $p = 1$ the $: \varphi^2(x) :$ terms in the commutator of two $\tilde{T}$'s drop out. So the algebra of $\tilde{T}$'s closes without the $: \varphi^2(x) :$ operators. For $p = 1$ the central extensions (4.11) and (4.12) are related by $C_{00,01} = C_{01,11}$ and they are given by

$$C_{00,01} = \frac{-i}{12\pi} (\nabla_1^3 + \nabla_1) \delta(\theta - \theta'), \tag{4.13}$$

where we have used the following representation of the delta function

$$\delta(\gamma) = \lim_{\epsilon \rightarrow 0} \frac{-i}{2\pi} \frac{\sin 2i\epsilon}{\cos 2i\epsilon - \cos \gamma} + \text{constant}. \tag{4.14}$$

Defining the Virasoro generators $L_n, \tilde{L}_n$ as follows

$$L_n = -\frac{1}{2} \int_0^{2\pi} d\sigma e^{in\sigma} \tilde{T}_{++}, \tag{4.15}$$

$$\tilde{L}_n = -\frac{1}{2} \int_0^{2\pi} d\sigma e^{in\sigma} \tilde{T}_{--} \tag{4.16}$$

it is straightforward to derive from the results given in eqs. (4.1)-(4.10) and (4.13) the standard string result:

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{N}{12} (m^3 - m) \delta_{m+n,0} \tag{4.17}$$

and the same for the $\tilde{L}_n$ generators.

B) FERMIONS

Eqs. (3.25) and (3.27) is all we need to calculate the operator algebra. The two point function $\langle \partial_0 \lambda \lambda \rangle$ is calculated by first converting the time derivative into a space derivative by

using the field equation of λ . After a long but straightforward calculation we have obtained the following result which holds, *up to central extensions*, for both $p = 1$ and $p \geq 2$ (In the following equations (4.18-22), $\tilde{T}_{ij}$ is the *total* improved energy-momentum tensor):

$$\{\tilde{S}_A^0(x), \tilde{S}_B^0(y)\} = \delta_{AB} \delta^p(x, y) (-\tilde{T}_{00} + \gamma^0 \gamma^0 \tilde{T}_{00}) (x) - \frac{1}{2} \sigma_{HA}^{IJ} \delta^p(x, y) \gamma^{ab} \partial_a J_b^{IJ} + (p-1) - \text{terms} + \text{bilinears in fermions} \quad (4.18)$$

$$\{\tilde{S}_A^0(x), \tilde{S}_B^0(y)\} = \delta_{AB} \delta^p(x, y) (\tilde{T}_0^0 - \gamma^c \gamma^0 \tilde{T}_c^0) (x) + \frac{1}{2} \sigma_{HA}^{IJ} \delta^p(x, y) (\gamma^{abc} \gamma^0 \partial_b J_c^{IJ} + \gamma^{ab} \partial_0 J_b^{IJ} - \gamma^{ab} \partial_b J_0^{IJ}) (x) + (p-1) - \text{terms} + \text{bilinears in fermions} \quad (4.19)$$

$$\{\tilde{S}_A^0(x), \tilde{S}_B^0(y)\} = \delta_{AB} \delta^p(x, y) ((\gamma^{abc} + h^{ab} \gamma^c) \gamma^0 \tilde{T}_{0c} - \gamma^a \gamma^b \tilde{T}_{00} + \gamma^b \gamma^c \tilde{T}_c^0 - \gamma^0 \gamma^c \tilde{T}_c^0 - \frac{1}{2} \gamma^{ab} \nabla^i \nabla_i \varphi^2 + \frac{1}{4} (p-1)^2 \gamma^{ab} \varphi^2) (x) - \frac{1}{2} \sigma_{BA}^{IJ} \delta^p(x, y) (\gamma^{1c} \gamma_a \gamma_b \gamma^d \partial_c J_d^{IJ} - \frac{1}{2} \gamma^{ab} \delta^p(x, y) (\partial_b J_c^{IJ} - \partial_c J_b^{IJ})) (x) + (p-1) - \text{terms} + \text{bilinears in fermions} \quad (4.20)$$

$$[\tilde{S}_A^0(x), \tilde{T}_{0a}(y)] = i \delta^p(x, y) \nabla_a \tilde{S}_A^0(x) - i \partial_a \delta^p(x, y) \tilde{S}_A^0(x) + \frac{i}{4} \partial_a \delta^p(x, y) \gamma^{ab} \tilde{S}_A^0(x) + \frac{i}{2} \partial_b \delta^p(x, y) \gamma^0 \gamma^a \tilde{S}_A^0(x) + (p-1) - \text{terms} + \text{bilinears in fermions} \quad (4.21)$$

$$[\tilde{S}_A^0(x), \tilde{T}_{0a}(y)] = i \delta^p(x, y) \nabla_a \tilde{S}_A^0(x) - \frac{i}{4} \partial_a \delta^p(x, y) \gamma^c \tilde{S}_A^0(x) - \frac{i}{2} \partial_a \delta^p(x, y) \tilde{S}_A^0(x) - \frac{i}{2} \partial^b \delta^p(x, y) \tilde{S}_{aA}(x) + \frac{i}{2} \partial_c \delta^p(x, y) \tilde{S}_A^c(x) \delta_a^b - \frac{i}{2} \partial^c \delta^p(x, y) (\gamma_c \gamma^b \tilde{S}_{0A}(x) + \gamma_c \gamma^b \gamma_a \gamma_d \tilde{S}_A^d(x)) + i \partial^c \delta^p(x, y) \gamma^0 (\delta_c^b \gamma_a + \delta_a^b \gamma_c - 2 h_{ca} \gamma^b) (\lambda^A \partial_0 \varphi^A) (x) \sigma_{AA}' + (p-1) - \text{terms} + \text{bilinears in fermions} \quad (4.22)$$

The $(p-1)$ -terms arise from the improvement terms in $\tilde{T}_{ij}$ and $\tilde{S}_A^0$, and they may in principal cancel on the RHS of the above (anti) commutators.

5. COMMENTS

From the results of the previous section we see that the stress-tensor commutators do not close unless $p = 1$. For $p > 1$, new operators such as $\partial_b \tilde{T}_{ab}$ occur on the RHS of the commutators. The commutators of these operators will presumably give rise to further new operators, thereby leading to an algebra of infinitely many local operators. This algebra contains the superconformal algebra, which in turn is known to admit an infinite dimensional extension [15]. It would be interesting, therefore, to find a relation between the two algebras.

In the stress-tensor commutators, we have found divergent, field dependent Schwinger terms. This does not necessarily imply inconsistency of the theory. In fact, we have found that they drop out in the computation of the superconformal algebra of the charges. One way to search for trace anomalies is to compute the variation of the stress-tensor under the (super)conformal transformations [13]. For example, $\delta_\xi \tilde{T}_{0a} = [\int \mathcal{P} \sigma \sqrt{-h} \tilde{T}_{0i} \xi^i, \tilde{T}_{0a}]$ will now involve the Schwinger terms occurring in $[\tilde{T}_{00}, \tilde{T}_{0a}]$. In this case, the trace anomaly, if present, would manifest itself as an anomalous term in $\delta_\xi \tilde{T}_{0a}$.

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APPENDIX

Some Useful Identities

$$\text{On } S^p: \quad R_{ab}{}^c{}_d = \partial_a \Gamma_{bd}^c + \dots = \delta_a^c h_{bd} - \delta_b^c h_{ad} \quad (a = 1, 2, \dots, p)$$

$$\{\nabla_a, \nabla_b\}V_c = h_{ac}V_b - h_{bc}V_a \quad \text{for any vector } V_c$$

Conformal Killing vectors (See (2.5)):

$$\nabla^i \nabla_i \Omega = (1-p)\Omega$$

$$\nabla^j \nabla_j \xi^i = (1-p)\xi^i + 2(1-p)\nabla^i \Omega$$

$$\nabla_i \xi^i = 2(p+1)\Omega$$

Some properties of $D_{mm'}^i$ and the angle γ (See (3.11)):

$$\Delta \cos \gamma = -p \cos \gamma$$

$$\nabla_a \cos \gamma \nabla^a \cos \gamma = \sin^2 \gamma$$

$$D_{mm'}^i(L^{-1}) = D_{m'm}^i(L)$$

$$D_{mm'}^i(1) = \delta_{mm'}$$

$$\int d^p x \sqrt{-h} D_{mm'}^i(L) D_{m'm}^j(L^{-1}) = \frac{2(\pi)^{\frac{p+1}{2}}}{d! \Gamma(\frac{p+1}{2})} \delta_{ij} \delta_{mm'} \quad \text{no sum over } m'$$

$$D_{0m}^i(L) = \alpha_{im} D_{m0}^i(L^{-1}) \quad \text{with } |\alpha_{im}|^2 = 1$$

$$\sum_m D_m^i(L^{-1}) D_{mm'}^i(L') = D_{m'm}^i(L^{-1}L')$$

$$\nabla_a \nabla^a D_{mm'}^i(L^{-1}) = -l(l+p-1) D_{mm'}^i(L^{-1})$$

$$\text{sign } h_{ij} = (-, \underbrace{+, \dots, +}_{p\text{-times}})$$

$$\{\gamma^i, \gamma^j\} = 2h^{ij}$$

$$\gamma_0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma_a = \begin{pmatrix} 0 & -i\sigma^a \\ i\sigma^a & 0 \end{pmatrix}$$

$$\text{with } \sigma^{ia}\sigma^b + \sigma^{ib}\sigma^a = 2\delta^{ab}, \sigma^a\sigma^{ib} + \sigma^b\sigma^{ia} = 2\delta^{ab}.$$

$$\bar{\psi} = \psi^\dagger \gamma_0 \quad \psi^c = C \bar{\psi}^T$$

$$(\sigma^a \nabla_a)_{\alpha'}^{\beta} D_{\beta m}^j(L^{-1}) = -(j + \frac{p}{2} - \frac{1}{2}) D_{\alpha' m}^j(L^{-1})$$

$$(\bar{\sigma}^a \nabla_a)_{\alpha'}^{\beta'} D_{\beta m}^j(L^{-1}) = +(j + \frac{p}{2} - \frac{1}{2}) D_{\alpha' m}^j(L^{-1})$$

$$\begin{aligned} (c_1 \Delta_z + c_2) \nabla_a^g \partial_{\epsilon} \frac{1}{(A-B)^{p-1}} &= \frac{\sin 2i\epsilon \nabla_a^g \cos \gamma}{(A-B)^{p+3}} 2ip(p-1) \times \\ &\times \left[(-pc_1 + c_2)(A^2 + B^2) - [2c_2 + (p^2 + p + 2)c_1]AB + (p+1)(p+2)c_1 \right] \end{aligned}$$

where $A \equiv \cos 2i\epsilon$ and $B \equiv \cos \gamma$, and c_1, c_2 are arbitrary constants.

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