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JEANS-LIKE INSTABILITIES FOR STRINGS IN COSMOLOGICAL BACKGROUNDS

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ABSTRACT

We study quantum-string propagation in cosmological backgrounds pointing out the possible emergence of Jeans-like instabilities. We also determine under which conditions the universe expands, when distances are measured by stringy rods. Examples include Friedman-Robertson-Walker (FRW) and de Sitter metrics, as well as a linear-in-time dilaton (constant background charge). A time-dependent dilaton does not give an expanding universe in the above sense, while FRW or de Sitter metrics usually do. For the latter case (inflation), instabilities occur for large enough Hubble constant.

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1. INTRODUCTION AND PHYSICAL ARGUMENTS

When, in General Relativity (GR), we say that Friedman-Robertson-Walker (FRW) or inflationary (de Sitter) metrics correspond to an expanding universe, the implicit assumption is made that our rods (or clocks) do not expand (slow down), at least not at a comparable rate. In other words, the distance between two stars grows relative to their own sizes or to a standard atomic meter. But is this indeed always the case?

In string theory it is possible to study this question from a new perspective. By solving (at least in some limit) for the quantum behaviour of strings in a given external metric we can compute both the way the distance between two strings varies with time and the way each one's size changes. We are thus able, in principle, to determine under which conditions a physical expansion occurs in the sense that strings become more and more diluted with time.

It is well known that, at large distances, string theory reduces to an effective Brans-Dicke theory [1,2] containing, besides the graviton, at least one extra spin-0 field, the dilaton ϕ (further scalars may come from compactification). This follows either from the low energy limit of string scattering amplitudes [3] or from the requirement of conformal invariance [4] of the non-linear σ -model. The latter is defined by the action:

$$S = -1/2 \int d\sigma d\tau \gamma^{1/2} \{ \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X) + {}^{(2)}R \phi(X) \} \quad (1)$$

where $\gamma^{\alpha\beta}$ and ${}^{(2)}R$ are the two-dimensional metric and scalar curvature, respectively.

A standard argument for determining which combination of the two background fields corresponds to the GR metric $g_{\mu\nu}$ rescales $G_{\mu\nu}$ by:

$$G_{\mu\nu} \longrightarrow G_{\mu\nu} \exp(4\phi/(D-2)) = G'_{\mu\nu} \quad (2)$$

so that the curvature term in the effective action takes the standard Einstein form. One then naturally identifies the rescaled metric $G'_{\mu\nu}$ with $g_{\mu\nu}$. It was recently proposed [5], for instance, that a dilaton with a linear time dependence (so-called constant background charge) corresponds to a linearly expanding universe. This procedure would be correct if the rescaled metric $G'_{\mu\nu}$ were to appear everywhere in the action in the way $g_{\mu\nu}$ does in GR. However this is not the case: for instance, for a massive scalar field A , the free term in the effective action has the form:

$$\exp(2\phi) G^{1/2} \{ G^{\mu\nu} \partial_\mu A \partial_\nu A + A^2 \} \quad (3)$$

and cannot be recast into an expression which contains just $G'_{\mu\nu}$.

A similar difficulty is known to exist [6] for the usual BD theory: if one tries to reabsorb the BD scalar field into the metric, particle masses as well as the periods of atomic clocks become time dependent (if the BD scalar is) and problems of interpretation arise.

In order to answer this problem in string theory we shall compute certain physical quantities as a function of some (generally variable) $G_{\mu\nu}$ and/or ϕ and determine under which conditions the universe, as measured in stringy rods, actually expands.

Our results are based on the study of quantum strings in FRW and inflationary metrics, as well as on the study of the dilaton-based model of ref.[5]. We reach the following conclusions:

- i) In FRW metrics distances among strings grow, as a function of time, relative to their characteristic size, precisely as it is implicitly assumed in GR. Thus distances grow in atomic units.
- ii) In an inflationary metric of Hubble constant H two possibilities occur according to whether H is smaller or larger than

the frequencies of the string's harmonic oscillators (to be defined more precisely below). In the first case, strings evolve as in FRW, while, in the second, some of the string modes inflate with the universe, get excited and, presumably, decay. Such a scenario could be very interesting in the context of inflation in the early universe, and could help to clarify previous ideas on the subject[7,8].

iii) Finally, if only the dilaton field is time dependent, distances among strings and string sizes expand exactly at the same rate i.e. the universe does not expand in string-size units. We shall comment more on this case later on.

Before presenting the actual calculations we would like to make some physical remarks which should make the results more intuitive.

Let us recall the roles played by $G_{\mu\nu}$ and ϕ in string theory. As repeatedly emphasized by one of us [9], $G_{\mu\nu}$, if constant, represents the Planck constant of string theory provided geometric string units for energy are used ($E \rightarrow \alpha' E = \text{length}$):

$$\pi G_{\mu\nu} = \lambda_s^{-2} \eta_{\mu\nu}, \quad \lambda_s^2 \equiv 2\alpha', \quad c=1 \quad (4)$$

On the other hand, as Witten has pointed out [10], a constant ϕ or better, a suitable exponential of it:

$$\alpha = \exp(-2\phi) \quad (5)$$

represents the string loop expansion parameter. By the closed-string unifying relation [11], α also represents, at the same time, the fine structure constant and Newton's constant G via:

$$\lambda_p / \lambda_s = \alpha^{1/2}, \quad \lambda_p^2 \equiv G\alpha' \quad (6)$$

Suppose now that $G_{\mu\nu}$ and/or ϕ vary with time: we can think of the resulting theory as one in which two of the fundamental constants of Nature, G and α , change with time.

Theories with a time-dependent G have been proposed long ago by Dirac [12] in order to explain the coincidence of some very large numbers in physics and have been subsequently investigated by many authors. There have been proposals that two distinct metrics are relevant, one for macroscopic and astronomical considerations and one for describing atoms, rods, and atomic clocks. Only a very slow variation of G with time, $|G/G| < 4 \cdot 10^{-11}/\text{year}$, looks acceptable [13] with stronger bounds arising if α is related to G as in eq. [14].

As for theories with varying α , they have not been considered explicitly, as far as we know. The point is that one can always keep α (or λ_s) fixed and put all the time dependence in the action. However, some of our results can simply be understood as if α had become time-dependent.

For instance, this point of view immediately suggests associating a time-dependent λ_s with a real expansion of the universe. Indeed, λ_s controls the typical size of a string and, as a result of (4), this size behaves as the inverse of the scale factor of the FRW metric. Thus strings are becoming smaller as time goes on and all that remains to be checked is that the distance between two strings does not follow the same trend. From this point of view the universe expands simply because the Planck constant controlling the size of physical systems shrinks! The universe which, long ago, was perhaps as large as a small multiple of λ_s has simply kept its size while everything else (including our meter) has shrunk^{*)}. This is how the universe becomes more and more classical with time.

The explicit calculations presented in this paper basically support the above arguments. As we shall see, they are valid in the

^{*)} It is amusing that a similar description of an expanding universe can also be found in Dirac's theory [15]: the context is however very different, since here we are discussing the case of a constant G and we have a single metric for microscopic and macroscopic physics.

"adiabatic limit" in which the rate of change in $G_{\mu\nu}$ is small compared to the frequencies of the oscillators in the string (or, for a "composite" object like a nucleus or an atom, of its own frequencies). In the opposite, "impulsive" limit the lowest string modes do not react to the fast change of the metric (of λ_S), remain frozen and their proper size expands with the scale factor.

The above analogy "explains" why the distance of two non-interacting strings does not react to the change in λ_S : that distance is a zero frequency mode and therefore, for this degree of freedom, the expansion is always impulsive, not adiabatic. Thus the distance of two strings does not react to the change of λ_S and grows in terms of their sizes. This is what one means by a physical expansion.

Finally, a word of warning on the question of conformal invariance in the metrics discussed in this paper. It is well known [4] that, for a single string moving in a background, the conditions of conformal invariance (vanishing β -functions) coincide with the vacuum Einstein equations (modulo string corrections). As such they require, at tree level, $R_{\mu\nu}=0$. Certainly, the backgrounds we shall consider do not satisfy such a condition.

Our interpretation of this difficulty is that these backgrounds we consider are simply not candidate string ground states (vacua). Yet these metrics can play the role of effective backgrounds felt by a single string as it moves in the presence of many others. An example of a situation of this kind is the effective Aichelbourg-Sexl (AS) metric felt by one string as it collides with another at very high energies [16]. Here too one would not identify the AS metric as a possible string vacuum: nonetheless, this metric is extremely valuable for providing an accurate, if not exact, description of the high energy process.

The price to pay for simplifying the true many-body problem into that of a single string moving in an effective, external metric will be indeed some (hopefully small) violation of unitarity. The main physics conclusions should, however, retain their validity at some

semi-quantitative level, especially in the region of validity of a semiclassical approximation (since the conformal anomaly is an $O(\hbar^2)$ effect).

In this paper we shall present our method and results in the order outlined above, leaving to a future publication further details of the calculations and their possible applications.

2. GENERAL CONSIDERATIONS FOR CONFORMALLY FLAT METRICS

In this section we shall consider in detail the case of conformally flat σ -model metrics. This includes the two physically interesting examples of FRW and inflationary universes.

The background metric is written, in conformal coordinates and after appropriate rescaling as:

$$g_{\mu\nu} = \pi \lambda_S^2 G_{\mu\nu} = C(T)^2 \eta_{\mu\nu} \quad (7)$$

where T is the conformal time. The problem of quantizing a string in this kind of metrics has been studied in ref.[17]. We shall now recall some of the results obtained there and reinterpret them. The classical equations of motion and constraints read:

$$\begin{aligned} \partial^2 X^0 + 1/C \partial C / \partial X^0 [- (\partial_\alpha X^0)^2 + (\partial_\alpha X^i)^2] &= 0 \\ \partial^2 X^i + 2/C \partial C / \partial X^0 \partial_\alpha X^0 \partial_\alpha X^i &= 0 ; \partial^2 = \partial_\sigma^2 - \partial_\tau^2 \end{aligned} \quad (8)$$

$$\begin{aligned} T_{\alpha\beta} &= C (X^0)^2 [- \partial_\alpha X^0 \partial_\beta X^0 + \partial_\alpha X^i \partial_\beta X^i - \\ &- 1/2 \eta_{\alpha\beta} (\partial_\gamma X^0 \partial_\gamma X^0 - \partial_\gamma X^i \partial_\gamma X^i)] = 0 \end{aligned} \quad (9)$$

where $X^0 = T$ and we have chosen to work in the conformal gauge:

$$\gamma^{\alpha\beta} = \rho(\sigma, \tau) \eta^{\alpha\beta} \quad (10)$$

At the classical level all gauge choices give the same physics, while, quantum mechanically, we can expect in general some gauge dependence due to the conformal anomaly. This point was already discussed in sect.1.

The exact solution of (8,9) is, of course, unknown. In ref.[17] the proposal was made to look for solutions of the type:

$$X^\mu = q^\mu + \eta^\mu \quad (11)$$

where q^μ defines the motion of a point particle in the above metric and η^μ describes finite-string-size effects. One can solve eqs.(8,9) up to second order in η , which turns out to be necessary and sufficient in order to get the physical input of both the equations of motion and of the constraints.

Under which conditions does one expect to get in this way a reasonable description of the actual string propagation? Obviously, one will have a good approximation if the metric is not changing too much (modulo general coordinate transformations) over distances of order of the string extent.

Classically, there is no characteristic string size: one can start with a small enough string and trust the approximation as long as the string remains small relative to the curvature radius R . At the quantum level strings acquire a minimal size, which is controlled by the string length parameter λ_s of eq.(4). We shall have to start therefore with metrics which are smooth over distances $O(\lambda_s)$.

The solution for the point-particle motion in the metric (7) is exactly known [17]. It is given implicitly by:

$$\alpha' \tau = \int_{q^0(0)}^{q^0(\tau)} dq^0 C(q^0) (m^2 + p^2 / C^2(q^0))^{-1/2}$$

$$q^i(\tau) = q^i(0) + \alpha' p^i \int_0^\tau d\tau' / C^2(q^0(\tau')) , \quad p^2 = \sum p_i^2 \quad (12)$$

and depends, of course, on the particle's mass m , initial position $q(0)$ and initial momentum p . Note for future reference that, in all cases, the proper string time τ and the cosmic time t defined by:

$$dt = C(\tau) d\tau \quad (13)$$

become proportional in the limit $C \rightarrow \infty$:

$$dt \rightarrow \alpha' m d\tau \quad (14)$$

To first order, the string fluctuations satisfy the linear equations:

$$\partial_0^2 (\eta^0 \pm \eta^1) - 2h(q^0) (\dot{q}^0 \pm \dot{q}^1) (\dot{\eta}^0 \pm \dot{\eta}^1) - \partial h / \partial q^0 [(\dot{q}^0)^2 \pm (\dot{q}^1)^2] \eta^{0=0} \quad (15a)$$

$$\partial^2 \eta^i - 2h(q^0) \dot{q}^0 \dot{\eta}^i = 0, \quad h \equiv C^{-1} \partial C / \partial q^0, \quad i=2, \dots, D-1 \quad (15b)$$

where we have separated the oscillations into those parallel to the point-limit motion (defined to be along the 1-axis) and those perpendicular to it.

For the transverse motion the solution can be written as:

$$\eta^i = 1/C \chi^i \quad (16)$$

where χ^i satisfies the simpler wave equation:

$$\partial^2 \chi^i = - (\ddot{C}/C) \chi^i \quad (17)$$

We can study the solution to (17) by Fourier analysis obtaining, for each mode:

$$\ddot{\chi}^i_n = [-n^2 + (\ddot{C}/C)] \chi^i_n \quad (18)$$

Obviously any particular (non-zero) mode oscillates in time as

long as $\dot{C}/C$ remains <1 and, in particular, if $\dot{C}/C < 0$. We shall see in a moment under which conditions this is the case for the various cosmological models.

A time-independent amplitude for χ is obviously equivalent to a fixed proper (invariant) size of the string perturbation η (the factor C in the definition (16) compensates the factor C in the metric).

As soon as $\dot{C}/C$ is positive and larger than 1 (N^2), the mode $n=1$ ($n=N$) becomes unstable and the corresponding proper amplitude grows with time. In the limit in which one can neglect the proper frequency (and, of course, always for the zero modes) one finds:

$$\chi(\tau) = \chi(0) C(\tau) / C(0) \quad (19)$$

i.e. in this limit the mode grows just like the scale factor C .

Before turning to the various cases we have to discuss the longitudinal motion and the evolution of η^0 . By a straightforward calculation one can recast eqs.(15a) for the proper amplitudes $\chi = C\eta$ as

$$\begin{aligned} \partial^2 \chi^1 &= -(\ddot{C}/C) \chi^1 + 2p(\dot{C}/C) (m^2 + p^2/C^2)^{-1/2} \dot{\eta}^0 + \\ &2p/C^2 (m^2 + p^2/C^2)^{-1/2} \{ (\ddot{C}/C) + (\dot{C}/C)^2 p^2/C^2 (m^2 + p^2/C^2)^{-1} \} \chi^0 \\ \partial^2 \chi^0 &= 2p(\dot{C}/C) (m^2 + p^2/C^2)^{-1/2} \dot{\eta}^1 + p^2/C^2 (m^2 + p^2/C^2)^{-1} \\ &\{ (\ddot{C}/C) + (\dot{C}/C)^2 (m^2 + 2p^2/C^2) (m^2 + p^2/C^2)^{-1} \} \chi^0 \end{aligned} \quad (20)$$

A careful look at these equations shows that the longitudinal mode behaves similarly to the transverse ones while the time coordinate never exhibits instabilities. Indeed, for very large C , the above equations simplify to give:

$$\begin{aligned} \partial^2 \chi^1 &= -(\ddot{C}/C) \chi^1 \\ \partial^2 \chi^0 &= 2p/m (\dot{C}/C) \eta^1 \end{aligned} \quad (21)$$

This justifies, a posteriori, our identification, at large C and up to a proportionality constant, of the cosmic time t with the world sheet time τ . It will be also sufficient, in the subsequent discussion of some particular cosmological models, to restrict our attention to the transverse coordinates.

3. EXAMPLES

a) Radiation dominated FRW universe.

In this cosmological model one has:

$$C(T) = a_0 T, \quad h(T) = 1/T, \quad t = 1/2 a_0 T^2 \rightarrow \alpha' m \tau \quad (22)$$

In this case $\dot{C}/C$ is always negative so that none of the modes are unstable. In particular, for large t (or τ), eq. (18) becomes:

$$\ddot{\chi}_n + [n^2 + 1/4 \tau^{-2}] \chi_n = 0 \quad (23)$$

whose solution:

$$\chi_n = (\tau)^{1/2} J_0(\pm n\tau) \quad (24)$$

gives, asymptotically:

$$\eta = (\tau)^{-1/2} \sum (2/n\pi)^{1/2} [\gamma_n e^{in(\sigma-\tau)} + \tilde{\gamma}_n e^{-in(\sigma+\tau)}] \quad (25)$$

where, upon quantization, the operators $\gamma_n, \tilde{\gamma}_n$ are normalized as:

$$[\gamma_n, \gamma_m] = \delta_{n+m} \quad (26)$$

Obviously, the proper size of the string amplitude remains constant in time.

b) Matter dominated FRW universe

This case is very similar to the previous one. Instead of (22) one has:

$$C(T) = b_0 T^2, h(T) = 2/T, \quad t = 1/3 b_0 T^3 \longrightarrow \alpha' m \tau \quad (27)$$

and:

$$\ddot{\chi}_n + [n^2 + 2/9 \tau^{-2}] \chi_n = 0 \quad (28)$$

This gives:

$$\eta = (\tau)^{-2/3} \sum_n (2/n\pi)^{1/2} [\gamma_n e^{in(\sigma-\tau)} + \tilde{\gamma}_n e^{-in(\sigma+\tau)}] \quad (29)$$

which means, again, constant proper amplitudes, i.e. no instabilities.

c. de Sitter (inflation)

In the case of de Sitter, things change in a qualitative way. Since:

$$C(T) = -R_0 / T, h(T) = -1/T, \quad t = -R_0 \ln (-T/R_0) \longrightarrow \alpha' m \tau \quad (30)$$

$\ddot{C}/C$ turns out to be positive and time-independent:

$$\ddot{C}/C = (\alpha' m)^2 R_0^{-2} = (\alpha' m)^2 H^2 \quad (31)$$

As a result, we now have, instead of (23):

$$\ddot{\chi}_n + [n^2 - (\alpha' m)^2 R_0^{-2}] \chi_n = 0 \quad (32)$$

whose solution gives:

$$\eta = \exp(-\tau/R_0) \sum_n [\gamma_n e^{in(\sigma-\omega_n \tau)} + \tilde{\gamma}_n e^{in(\sigma+\omega_n \tau)}] \quad (33)$$

where

$$\omega_n = [n^2 - (\alpha' m)^2 R_0^{-2}]^{1/2} \quad (34)$$

and the operators $\gamma_n, \tilde{\gamma}_n$ are now normalized to obey:

$$[\gamma_n, \gamma_m] = \omega_n^{-1} \delta_{n+m} \quad (35)$$

Eqs. (33,34), which had already been obtained in ref.(17), show that, for a large enough Hubble constant $H = R_0^{-1}$, the low frequency modes become unstable in the sense that the frequencies ω_n become imaginary. The interpretation is now clear: an imaginary ω_n means a growing proper amplitude χ . For $n \ll \alpha' m R_0^{-1}$ the proper amplitude grows like the scale factor and therefore η stays constant.

d) Linearly expanding universe.

We deal with this case for sake of comparison with the background charge model of ref. (5). Take:

$$C = \exp(QT), \quad t = Q^{-1} \exp(QT), \quad C(t) = Qt, \quad \ddot{C}/C = 0 \quad (36)$$

The equation for χ_n becomes asymptotically trivial leading to:

$$\eta = (\tau)^{-1} \sum_n (2/n\pi)^{1/2} [\gamma_n e^{in(\sigma-\tau)} + \tilde{\gamma}_n e^{-in(\sigma+\tau)}] \quad (37)$$

showing that, even in this case, there is no instability.

e) Constant background charge.

Let us compare our result (37) with that of ref.[5] for the constant background charge (linear-in-time dilaton). In the latter case one has an exact solution:

$$X = \sum_n (2/n\pi)^{1/2} [\gamma_n e^{in(\sigma-\tau)} + \tilde{\gamma}_n e^{-in(\sigma+\tau)}] + \text{zero modes} \quad (38)$$

The crucial difference between (37) and (38) resides in the factor $(\tau)^{-1}$ sitting in front of the sum in (37). It is this factor that guarantees that, upon quantization, the size of our string does not grow while the universe expands. By contrast, in the case of eq. (38), string sizes grow precisely as the scale factor leaving apparently no physical expansion. However, since physical rods and clocks are made out of atoms i.e. of bound states of strings and their properties depend on the (dilaton controlled) coupling constant, it is very difficult to decide if and in which sense there is a physical expansion (or a red shift) in this case.

4. SECOND-ORDER PERTURBATIONS

As pointed out in [17], in order to discuss the constraints one has to go to second order in the string fluctuations η . As expected, nothing surprising happens at second order whenever the first-order solution shows no instability. However, in the case of inflation (de Sitter) where instabilities can occur, something amusing happens, solving an apparent paradox.

The paradox is that, for large enough Hubble constant, we have seen that some of the modes grow exponentially. We would then expect that the invariant mass of the string would itself grow with the same law. On the other hand, the centre of mass of the string appears to follow the point particle motion typical of a fixed mass m . This can be checked to be the case up to first-order corrections.

Fortunately things change at second order. What happens is that the centre of mass motion gets a large second-order correction each time some of the (non-zero) modes become unstable at first order. This can be seen explicitly in the second-order solution given in [17], as we shall now discuss.

Define the second-order perturbation of the centre-of-mass motion by:

$$Q^\mu(\tau) = \int d\sigma X^\mu(\sigma, \tau) = q^\mu(\tau) + \eta^\mu(\tau) + \xi^\mu(\tau) \quad (39)$$

The solution given in ref. [17] for a de Sitter universe shows that ξ^μ can develop an instability. Indeed, in the notations of [17]

$$\xi^0 = R \exp(-\alpha' m H \tau) \quad \xi^1 = R \exp(-2\alpha' m H \tau) \quad (40)$$

where R satisfies an equation of the type:

$$\partial^2 R \simeq \int d\sigma (\chi'^2 + \chi^2) \quad (41)$$

The r.h.s. of (41) grows indefinitely for $\alpha' m H > 1$. For the sake of simplicity let us consider for the moment the case $\alpha' m H \gg 1$. In this case one finds:

$$R \simeq \exp(2\alpha' m H \tau) \quad \text{i.e.} \quad \xi^0 \simeq \exp(\alpha' m H \tau) \quad \xi^1 \simeq \text{const.} \quad (42)$$

In order to extract physical implications from (42) one would need a good definition of the mass of an extended object in GR which is just based on its motion. This is, to say the least, not easy. An approximate, coordinate-dependent definition could be:

$$M^2 = g_{\mu\nu}(Q(\tau)) \dot{Q}^\mu(\tau) \dot{Q}^\nu(\tau) \equiv m^2 + \delta m^2 \quad (43)$$

With this definition (42) would imply:

$$\delta m^2(\tau) \simeq \exp(2H\tau) \quad (44)$$

i.e. both the mass and the size of the string would grow precisely like the scale factor, in accordance with the intuitive arguments made in sect. 1. Thus, in the case of instability, the parameter m appearing in the solution loses its meaning of mass of the string.

Of course it becomes impossible to study the solution by linearized methods in this case, but it is quite likely that, once a mode becomes unstable and forces the mass to grow, the criterium

for subsequent instabilities will become $\alpha' M(\tau)H > 1$. This makes it easier for other modes to get destabilized as well so that, eventually, all modes become unstable and a new, interesting regime sets in.

We think that much could be learned about this regime from the many numerical and analytical studies made in the equivalent case of Jeans instabilities.

5. DISCUSSION

1. Let us now interpret the physical meaning of our results from the point of view of determining under which conditions a physical expansion takes place.

Let us consider not one but two strings moving independently in one of our cosmological metrics. Neglecting their mutual interactions, we can determine their relative motion as well as their own internal evolution.

In those cases in which we have found no instability, the centre of mass of each string follows the point-like motion while the intrinsic, proper size of each string stays constant. The centres of mass of the two strings move apart with their distance, measured in string-size units, growing like C itself. The string density thus decreases, for increasing C , like C^{-3} (in $D=4$).

This is not the case, however, for the background charge model of ref.[5]. Here the distance of the centers of mass of two strings stays fixed if distances are measured with stringy rods, while the way it actually changes in atomic units depends on how one relates fundamental strings to complicated, composite objects such as atomic meters or clocks.

In the case of de Sitter with a Hubble constant H , the centre of mass motion depends, as we have explained, on whether or not one has an instability. If $\alpha' mH < 1$, there is no instability and the motion of each string's centre of mass is that of a point particle. A horizon

appears at a proper distance $R_0 = H^{-1}$ in the sense that two strings that were separated by more than R_0 initially can never meet later on. Strings remain of a given size as the universe expands exponentially (inflation).

If, however, the condition of instability $\alpha' mH > 1$ is met, the strings stretch while the universe expands, starting with the lowest frequencies. This produces an increase of the effective mass, which, in turn, induces instabilities in the higher frequencies as well. Strings will eventually become larger than the horizon and then, presumably, break up and decay into shorter, causally disconnected strings which, again, start growing and so on.

The possibility that a process of this kind can take place and continue for a long time has been suggested by some authors [7,8] and might lead to a sort of self-sustained inflation. We hope that our results will help to find out if this possibility is indeed viable.

It is interesting to look at this situation from the thermodynamical point of view. Let us recall that de Sitter space has an associated Hawking temperature given by $T_{Haw} = \hbar H/2\pi$, while the Hagedorn temperature T_{Hag} is a multiple of $(\hbar/\alpha')^{1/2}$. Our instability criterium thus also reads, up to numerical constants:

$$T_{Haw} / T_{Hag} > T_{Hag} / m \quad (45)$$

For small T_{Hag}/m (which can be easily achieved for highly excited strings) instabilities can occur even at $T_{Haw} \ll T_{Hag}$. This is important since, at $T_{Haw} = T_{Hag}$, a more complete thermodynamical treatment is certainly needed.

It has been argued [18] that the string tension may have, near the Hagedorn temperature a dependence typical of a phase transition:

$$\lambda_s^2(T) = \lambda_s^2(0) (1 - (T/T_{Hag})^2)^{-1/2} \quad (46)$$

corresponding to a "melting" of the metric ($G_{\mu\nu} \rightarrow 0$) and to

restoration of general covariance in a purely topological phase [19].

2. Our results bear some resemblance with the instabilities occurring for massless scalar (or gravitational) fields in de Sitter space [20]. There are differences, though, which we shall now illustrate.

The minimally coupled scalar field in an expanding universe satisfies the equation:

$$(\partial_x^2 - \partial_{x^0}^2) \Phi - 2 C^{-1} \partial C \partial x^0 \partial_0 \Phi = 0 \quad (47)$$

This looks very similar to our eq. (15b) after converting from conformal time to cosmic time (or to τ).

In de Sitter space, after a rescaling analogous to our eq. (16):

$$\Phi = H X_0 \Xi \quad (48)$$

eq.(47) becomes:

$$\ddot{\Xi}_k + [k^2 - 2/X_0^2] \Xi_k = 0 \quad (49)$$

where we have gone to momentum space. Eq. (49) shows that, for sufficiently late times ($X_0 \rightarrow 0$ is $\tau \rightarrow \infty$), Ξ becomes unstable and starts growing like the scale factor C , precisely as we have found to be the case in sect. 3c.

There is an important difference, however, between the string and the scalar field. For the latter the momentum is continuous, while for the string it is quantized. As a result, the scalar field always exhibits an instability at sufficiently late times, while the string is perfectly stable if the minimal, non-zero frequency is larger than the critical one, $\alpha' m H$.

3. We want to go back, finally, to the analogy with Jeans instability to which we were referring in the introduction. In the comoving frame (where all wavelengths expand with the universe),

the fundamental equation for the density fluctuation $\delta(k)$ is [21] (for compressional modes):

$$\ddot{\delta} + 2 \dot{C}/C \dot{\delta} + [v_s^2 |k|^2 C^{-2} - 4 \pi G \rho_0] \delta = 0 \quad (50)$$

where v_s is the speed of sound and ρ_0 is the initial, unperturbed density. The Jeans frequency k_J is usually defined [21] as the frequency at which the bracket in (50) is zero i.e.

$$k_J = C [4 \pi G \rho_0 v_s^{-2}]^{1/2} \quad (51)$$

The instability is controlled by the original energy density ρ_0 and is little affected by the expansion of the universe. In our case the energy density term is absent, since we consider a single string and neglect the gravitational effects of the string on itself. However, unlike in the usual treatment, we do not neglect the $\dot{C}/C$ term and therefore, when we go over to the proper fluctuation χ , we get a term similar to that in the bracket of eq. (50). It is thus H which plays the role of $G \rho_0$, which is not so surprising after all, given the relation between H and the vacuum energy.

The other difference is that in Jeans' instability the critical original frequency is increased by the expansion, as indicated by the factor C appearing in eq. (51). This does not seem to be the case, at first sight, for our instability criterion. However, if one takes into account the fact that the condition of instability most likely involves M rather than m and that M changes with time like the scale factor itself (see eq. (44)), the usual red-shift effect on the Jeans frequency is nicely recovered.

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