BEAM-BEAM EFFECTS IN ELECTRON AND PROTON COLLIDERS

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Abstract This review of beam-beam effects in electron and proton colliders is divided into two main parts, the first devoted to electron positron colliders and the second to proton antiproton colliders. Both parts start with a discussion of recent observations in existing colliders. The part on electron positron colliders continues with a discussion of the frequencies of the coherent oscillations of the rigid bunches coupled by the beam-beam forces, and concludes with a discussion of several attempts to understand the nature of the beam-beam interaction theoretically. The part on proton antiproton colliders continues with an interpretation of the observations in terms of the tune spreads present in the colliding beams, and concludes with a discussion of the observed distribution functions.

INTRODUCTION

This review of beam-beam effects in electron and proton colliders consists of two main parts. Chapter contains the discussion of beam-beam effects in electron positron colliders, and Chapter the discussion of proton antiproton colliders. Chapter contains final remarks. This review includes material presented at the Advanced ICFA Beam Dynamics Workshop¹ on "Beam-Beam Effects in Circular Colliders", held from 29 May to 3 June 1989 at Novosibirsk, USSR.

BEAM-BEAM EFFECT IN ELECTRON-POSITRON COLLIDERS

For identical electron and positron bunch parameters, the luminosity L and the beam-beam strength parameters ξ_x for horizontal and ξ_y for vertical betatron oscillations are given by:

$$L = \frac{N^2 f k}{4\pi\sigma_x\sigma_y} \quad \xi_x = \frac{Nr_e\beta_x}{2\pi\gamma\sigma_x(\sigma_x + \sigma_y)} \quad \xi_y = \frac{Nr_e\beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)} \quad (1)$$

The symbols are: r_e classical electron radius, N number of particles in a bunch, γ is the relativistic factor, β_x and β_y horizontal and vertical amplitude functions, respectively, σ_x and σ_y horizontal and vertical rms beam radii, respectively, at the crossing point, f revolution frequency, and k number of bunches in one beam (fk is the bunch collision frequency). Both equations are derived under the assumption

that the variation of β_x , β_y , σ_x and σ_y along the beam direction can be neglected in the neighbourhood of the collision point. This assumption is well satisfied if the bunch length σ_z is small compared to β_x and β_y . Otherwise, the strength parameters ξ_x and ξ_y increase², and the luminosity L decreases³. For $\xi_x, \xi_y \rightarrow 0$, ξ_x and ξ_y describe the horizontal and vertical tune shift, respectively, for test particles with betatron oscillation amplitudes small compared to σ_x and σ_y , and are therefore often loosely called the horizontal and vertical beam-beam tune shift, respectively.

Recent Observations in Electron Positron Colliders

Two new electron positron colliders have recently come into operation, Tristan⁴ at the KEK Laboratory in Tsukuba, Japan, and BEPC⁵ at the Institute of High Energy Physics in Beijing, China. Their parameters are summarized in Table 1. It should be noted that the current quoted in⁴ is the total current in both beams.

Table 1: Parameters of electron colliders

	BEPC	Tristan
Energy E/GeV	1.6	26
Bunch current I/mA	16	2.2
Circumference C/m	240.4	3018
Vertical damping time τ_y/ms	46	3.2
Amplitude fct. β_y/m	0.085	0.1
Bunches/beam k	1	2
Luminosity $L/10^{30}\text{cm}^{-2}\text{s}^{-1}$	2	7.2
Beam-beam strength ξ_y	0.031	0.032

Comparing these data to a compilation⁶ of beam-beam strength limits ξ_y shows that the observations in the new machines fall into the range of the observations in earlier machines. This confirms the opinion that the beam-beam strength parameter is a good variable for describing the beam-beam effect because it shows relatively little variation over machines in a wide range of energy and size.

Coherent Beam-Beam Effect

The coherent beam-beam effect describes the changes to the motion of electron and positron bunches in a storage ring, caused by the fact that they are coupled by the beam-beam forces at the collision points. In particular, the coherent dipole effect deals with the motion of the centres of gravity of the bunches. Higher-order effects, dealing with changes to the bunch shape, have been studied in a water-bag model by Chao and Ruth⁷. Including effects of order n causes stopbands to grow out of all rational tunes $Q = p/q$ with $1 \leq q \leq n$, but the stability behaviour remains dominated by the first-order instability threshold. Since the dipole effect is used in the operation of electron-positron colliders to optimize the luminosity, the rest of this section will be devoted to it.

Linear Theory of the Dipole Mode In the usual approximation, it is assumed that the bunches perform linear oscillations between the collision points. Much confusion has arisen because different authors have made different assumptions about the ratio

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Figure 1: Frequency spectrum of coherent vertical beam-beam oscillations for $k = 1$, $n_x = 2$, $Q_x = 1.25$, $\xi_y = 0.0357$

Figure 2: Coherent horizontal bunch oscillation. Solid line: equilibrium Gaussian, dashed line: coherent oscillation with c.m. shift $0.1\sigma_x$

between the number k of bunches in one beam, assumed to be equal to the number of bunches in the other beam, and the number of collision points n_x , using either $n_x = k$ or $n_x = 2k$. My formulation in terms of n_x works in both cases.

If the beam-beam force is also linearized, the stability of the bunch motion can be investigated separately in the horizontal and vertical plane, and be reduced to a computation of the eigenvalues of matrices of order $4k$ which are products of $2k$ (2×2) block-diagonal matrices describing the bunch motion in the arcs, alternating with $2k$ kick matrices describing the bunch collisions. In the simplest case, with $k = 1$, one finds two modes. For equal bunch intensities, the σ -mode has the frequency Q of the betatron oscillations in the absence of the beam-beam collisions; the π -mode has a higher (lower) frequency for particle-antiparticle (particle-particle) collisions, approximately given by:

$$Q_\pi \simeq Q_\sigma \pm n_x \delta Q_y \quad (2)$$

Here δQ_y is the coherent beam-beam tune shift due to a single beam-beam collision. For $k > 1$ bunches there are $2k$ modes, but for equal populations, there remain a mode with the unperturbed frequency Q , and a mode with the largest frequency shift, given by Eq. 2.

The coherent beam-beam tune shift δQ_y is related to the kick which a test bunch receives when it travels through the opposite bunch, obtained by integration over the whole test bunch with a weight corresponding to the density distribution. Talman⁸, Hirata⁹ and Hofmann and Myers¹⁰ obtained a coherent beam-beam strength parameter δQ_y which is equal to the incoherent one, $\delta Q_y = \xi_y$, for $y_+ - y_- \ll \sigma_y$.

Nonlinear Theory of the Dipole Mode The forces due to the beam-beam collisions are used to construct the Vlasov equation of the particle distribution in action-angle variables. Averaging over the angles, an integral equation is obtained, in which the ratio of the coherent and incoherent beam-beam strength parameters appears as an eigenvalue $\lambda_y = \delta Q_y / \xi_y$. Expanding into orthogonal polynomials, the equation for λ_y becomes a matrix eigenvalue problem after truncation to finite order. Meller and Siemann¹¹ used Legendre polynomials and found $\lambda_y = 1.34$. Yokoya, Funakoshi, Kikutani, Koiso, and Urakawa¹² used Laguerre polynomials and obtained the values of $\lambda_x = \delta Q_x / \xi_x$ and $\lambda_y = \delta Q_y / \xi_y$ as functions of the axis ratio parameter $r = \sigma_y / (\sigma_x + \sigma_y)$. Their results can be fitted to better than a percent by the following expressions:

$$\lambda_x(r) = 1.330 - 0.370r + 0.279r^2 \quad \lambda_y(r) = 1.239 - 0.188r + 0.279r^2 \quad (3)$$

This leads to a result which is larger than that from the rigid bunch calculation^{8,9,10}:

$$Q_\pi - Q_\sigma \simeq \pm n_x \lambda_y(r) \xi_y \quad (4)$$

This can be understood by inspection of Fig. 2, which shows a coherent horizontal oscillation with an amplitude of $0.1\sigma_x$. The coherent dipole mode does not involve a shift of the Gaussian density distribution as a whole. Instead, the core of the beam oscillates and the tails remain practically stationary.

Experimental Results The frequency difference between the σ and π modes was obtained by Fourier analyzing the motion of the bunch centres in a multi-particle beam-beam simulation^{13,14}. An example of the results is shown in Fig. 1. The frequency difference $Q_\pi - Q_\sigma$ corresponds to $\delta Q_y / \xi_y \simeq 4/3$. The frequency difference between the σ and π modes was also observed in electron positron colliders⁸, and a careful comparison with Eq. 4 was done by Yokoya et al.¹², who report good agreement between their theory and experiment.

Beam-Beam Limit

In the early 1980's, multi-particle simulations of the beam-beam effect^{13,15,16,17,18} had obtained satisfactory agreement with experimental data. They all used the strong-strong model in which the beam-beam forces act on both beams and modify their motion. Recent work on beam-beam simulation was reviewed by Siemann¹⁹. In addition, there have been several attempts to obtain the beam-beam limit by analytical methods.

Maps of the Second Moments of Distribution Functions An ideal solution to the dynamics of two colliding beams must be self-consistent, i.e. the phase-space distribution functions of both beams must satisfy the Vlasov equation at all times, and the forces seen by any particle must be obtained from the distribution function by the Maxwell equation.

Hirata²⁰ and Furman, Ng and Chao²¹ have studied the one-turn maps for the second moments of the distribution functions in colliding beams. The ingredients of their calculations are:

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- Beam-beam kicks which are functions of the intensity size and shape – flat or round – of the opposite bunch. Hirata calculates the forces for a Gaussian density distribution. Furman, Ng and Chao assume that the force is linear. The intensity is expressed in terms of the unperturbed beam-beam strength parameter ξ_0 , calculated in the absence of changes to the rms beam radii.
- Radiation damping, described by the decrement δ between collision points, or the parameter $\lambda = \exp(-2\delta)$.
- Quantum excitation to compensate the radiation damping such that an equilibrium is reached – in the absence of beam-beam forces – at the unperturbed beam dimensions.
- Linear transformations with phase advance $2\pi\nu$ between the kicks. In both papers, one beam-beam collision is assumed in a revolution.

In either case, a map is obtained which describes the transformation of the canonical scaled variables $(q_{\pm}, p_{\pm})$ through one revolution. It is transformed into another map which describes the transformation of the lowest moments $\langle q_{\pm}^2 \rangle$, $\langle q_{\pm} p_{\pm} \rangle$ and $\langle p_{\pm}^2 \rangle$ through one revolution. This map is non-trivial because it not only operates on the moments, but also contains them in the coefficients.

Since Hirata applies a nonlinear force derived from a Gaussian distribution in phase space, he introduces an inconsistency with the Vlasov equation into his calculation because a Gaussian distribution in phase space cannot remain Gaussian under the influence of a nonlinear force. Furman, Ng and Chao avoid this difficulty by assuming a linear force, ignoring the Maxwell equations altogether. Their approximation only holds for particles with amplitudes small compared to the rms beam radii. It overestimates the force on all particles, and by a large factor that on particles in the tail of the distribution function at several rms beam radii.

Hirata finds the map for the moments in the case of round beams, Furman, Ng and Chao find it both for round and flat beams. They then proceed to obtain the fixed points with period one, i.e. stationary solutions for the moments which repeat on every revolution. Furman, Ng and Chao also linearize the map in the neighbourhood of the fixed point and analyze its stability. Both authors iterate the map for many revolutions and compare the results to their analytical results.

An example of results²¹ is shown in Fig. 3. It gives the rms beam-size for a flat beam as a function of ξ_0 . The abscissa is $\rho = 4\pi\xi_0/(1 + \lambda) = 6.722\xi_0$. For $0 < \rho < 0.4$, both beams have the same size which shrinks with increasing ρ , presumably due to the dynamic β effect which reduces β at the collision point while ξ_0 increases for the tune of the example. Between $0.4 < \rho < 1.4$, i.e. $0.06 < \xi_0 < 0.21$, the map is unstable. Iteration of the map just above $\xi_0 = 0.06$ shows chaotic behaviour, i.e. the beam radii vary from turn to turn with no apparent regularity. At $\rho = 1.4$, $\xi_0 = 0.21$, a bifurcation occurs, and the sizes of the two beams become vastly different, one is blown up, the other one shrinks. The beam size is stable between $1.5 < \rho < 2.0$. Hirata²⁰ observes a similar bifurcation, and relates it to the flip-flop effect, where one beam is indeed blown up and the other one shrinks, which has been observed in SPEAR²² and in computer simulations¹⁸. In SPEAR,

Figure 3: RMS beam sizes and stability for the period-1 fixed point solutions (solid = stable, dots = unstable). Flat-beam case, $\nu = 0.15$, $\lambda = 0.8694$

Figure 4: Particle distributions for $\nu = 0.08$ and $\xi = 0.16$

the effect was observed for $\xi_0 = 0.025$. In simulations, the flip-flop effect has been observed for $\xi_0 = 0.06$, i.e. a factor of 3.5 smaller than in Fig. 3.

By iterating the map, Furman, Ng and Chao also find fixed points with period two, three and four which Hirata does not find. This is believed to be due to the different assumptions about the beam-beam force.

Hirata^{23,24} has employed Stratonovich expansions²⁵ to include higher moments than the second. In²³ he used the proper gauge in a strong-strong calculation. He had to stop the expansion at the fourth moments because of numerical difficulties, and his results were not very different from earlier ones²⁰. In²⁴ he used the Gaussian gauge in a weak-strong calculation including moments up to 20-th order without approximation other than truncation. The agreement with multi-particle tracking was much improved.

Renormalized Theory A renormalized theory of the beam-beam interaction in electron positron colliders was developed and implemented by Chin^{26,27}, who calculated an approximate solution to the Fokker-Planck equation for the stationary particle distribution function. The formalism avoids the secular terms of conventional perturbation theory which cause it to diverge in the neighbourhood of resonances if the perturbation series is truncated after a few terms.

The formalism is derived in the approximation of one-dimensional motion, i.e. a round beam, and of collisions between a weak test beam with a strong beam with Gaussian density distribution. The result is the distribution function $P(I)$ in terms of the action I for the weak beam. In principle, the knowledge of the distribution function, including its tails, allows to calculate the beam lifetime in the presence of the beam-beam effect, and therefore to establish a relation between the observation in existing machines that the lifetime suddenly drops when the beam-beam strength parameter ξ exceeds a given value.

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The formalism is implemented in a computer program. An associated tracking program allows a comparison between theory and simulation. Fig. 4 shows an example of the results in a region of tune space with only high-order resonances and a rather high value of ξ . The density distribution resembles more an exponential than a Gaussian and the agreement between theory and simulation is very good.

Conclusions for Electron Positron Colliders

Two new machines, TRISTAN and BEPC, came into operation recently. From the point of view of beam-beam effects, they show no surprises. There now is a consensus how the coherent dipole beam-beam effect should be calculated, and agreement between theory and observation. Several interesting attempts to explain the beam-beam needed drastic simplifications, i.e. one rather than two dimensions, simple forces, lack of self-consistency, weak-strong model, to deal with the mathematics. This makes it difficult to compare theory with observations in existing colliders to check their validity, and to exploit their predictive power.

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The colliding beams in proton colliders are usually very nearly round. Assuming that this holds exactly, i.e. that the horizontal and vertical rms beam radii are equal, $\sigma_x = \sigma_y = \sigma$, and that the amplitude functions at the collision points are also equal, $\beta_x = \beta_y = \beta$, but allowing for different bunch populations $N_{\pm}$ and different rms beam radii $\sigma_{\pm}$, the equations for luminosity L and the beam-beam strength parameters $\xi_{\pm}$ may be written as follows, with the classical proton radius r_p :

$$L = \frac{N_+ N_- f k}{2\pi(\sigma_+^2 + \sigma_-^2)} \quad \xi_{\pm} = \frac{N_{\mp} r_p \beta}{4\pi\gamma\sigma_{\mp}^2} \quad (5)$$

Recent Observations

In this section I shall summarize recent observations in the SPS at CERN²⁸ and the Tevatron²⁹ at Fermilab. Table 2 shows the parameters of the two machines. It calls for a series of comments:

Table 2: Parameters of proton colliders

	SPS		Tevatron	
Energy E/GeV	315		900	
Amplitude fct. $\beta_x\beta_y/m$	1.0	0.5	0.5	0.5
Particle	p	$\bar{p}$	p	$\bar{p}$
Part./bunch $N/10^{10}$	12	8	7	2.5
Bunches/beam k	6	6	6	6
Norm. emittance $\epsilon/\mu\text{m}$	14	10	16	8
Bunch area A/eVs	0.7	0.5	3.0	3.0
BB strength $\xi_x/10^{-3}$	4.9	5.2	1.5	2.1
BB strength $\xi_y/10^{-3}$	3.5	3.7	1.5	2.1
Luminosity $L/\text{cm}^{-2}\text{s}^{-1}$	2.8×10^{30}		1.7×10^{30}	

There are 12 head-on beam-beam collisions around the Tevatron, but only 3 head-on collisions in the SPS because the beams are horizontally separated over most of its circumference.

The bunchlength is not small compared to β , hence the luminosity is 2/3 of the figure which can be calculated from Eq. (10) and the data in Table 2.

The proton bunches are more intense and have a larger emittance than the antiproton bunches, such that the beam-beam strength parameters ξ_+ and ξ_- differ by less than a factor 3/2 which brings both machines into the strong-strong configuration.

Interpretation

The observations in the SPS and the Tevatron agree that the tunes must be kept between two resonant values, and this must be done for all particles in the beam including any tune spread which they may have.

Table 3: SPS and Tevatron operating regions

Machine	Tune range	Width	Resonance
SPS	2/3 - 7/10	0.0333	11/16
Tevatron	2/5 - 3/7	0.0286	5/12

Tune Spreads Both machines operate in the neighbourhood of the coupling resonance $\{Q_x\} = \{Q_y\}$ where $\{..\}$ denotes the fractional part. The particular regions selected are shown in Table 3, together with their width and the lowest order resonance inside the region.

The beam-beam effect causes a tune spread of the order of ξ for each head-on collision, and a smaller value for each separated collision. In the Tevatron, the total tune spread ΔQ is approximately $\Delta Q \simeq 12\xi$. Accurate calculations, taking into account the amplitude functions and dispersions at the collision points yield slightly smaller values. The tune spread is independent of energy. Comparing the tune spread to the space between resonances leads to the beam-beam limit $\xi \leq 0.0024$, quite close to the observed values shown in Table 2. Indeed, it is found in the Tevatron that the beam emittance $\epsilon = 4\pi\gamma\sigma^2/\beta$ must be larger than a minimum value and/or the bunch intensity N must be smaller than a maximum value, such that their ratio N/ϵ which is proportional to ξ , stays below an upper limit. In the SPS, the total tune spread is $\Delta Q = 3\xi$. Comparing this to the space between resonances leads to the beam-beam limit $\xi \simeq 0.01$, while the actual observed value shown in Table 2 is about half as much.

A second source of tune spread is the direct space-charge effect (Laslett tune shift) which causes particles with large synchrotron oscillation amplitudes to experience the full tune shift when they are at the centre of the bunch and hardly any tune shift at all when they are at the head or tail of the bunch. In the SPS, this effect causes a tune spread $\Delta Q_{sc} \simeq 0.035$ at the injection energy, 26 GeV. It is comparable to the space between resonances and therefore believed to present an upper limit on the bunch population²⁸. A closer look reveals that the direct space-charge effect in the approximation of round beams, neglecting the contribution of

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the momentum spread to the beam size, is proportional to the ratio N/ϵ , called the beam brightness at Fermilab, as is the beam-beam strength parameter ξ . Since for given N/ϵ the direct space-charge effect scales like γ^{-2} it is unimportant in the Tevatron with injection at 150 GeV.

Density Distributions and Lifetime Observations at the Tevatron²⁹ show that the proton and antiproton density distributions are very well approximated by a Gaussian at injection. Once the beams are accelerated and the β -functions squeezed to a low value, the distribution of the protons remains a Gaussian, while the distribution of the antiprotons is higher in the core, narrower in the flanks, and higher in the tails than a Gaussian with the same area and standard deviation.

Observations at the SPS²⁸ show a clear dependence on the ratio of the proton and antiproton emittance. In 1987, the proton emittance was about four times the antiproton emittance. This not only reduced the proton lifetime at the beginning of the coast to less than 10 hours, but also created intolerably high background rates for the physics experiments. In 1988, the proton emittance was about a factor of two smaller than in 1987, and the emittance ratio less than two. Within minutes after the start of a coast, the proton lifetime increased to about 50 hours and the background became tolerable.

For the SPS, this behaviour is explained by the presence of high order non-linear resonances within the tune spread (5/12 in the Tevatron, 11/16 in the SPS). Their widths increase with the amplitude of the test particle, and the maximum width moves to higher amplitudes as the order increases³⁰. When the emittance of the test beam is larger than the emittance of the driving beam, more test particles reach dangerous amplitudes than in the case of similar emittances.

Conclusions for Proton Antiproton Colliders

Both proton colliders, SPS and Tevatron, operate with more intense and larger proton than antiproton beams, such that the beam-beam strength parameters acting on protons and antiprotons, ξ_+ and ξ_- , are almost balanced for the SPS, while for the Tevatron $\xi_- \simeq 3/2\xi_+$. Both machines exploit the tune space between resonances of moderate order: the SPS for accommodating the direct space-charge tune spread at injection, the Tevatron for accommodating the total tune spread due to twelve beam-beam collisions. Therefore, the Tevatron is limited by the beam-beam effect while the SPS is not. The SPS observes that the narrower antiproton beam causes losses from the tails of the wider proton beam and a lifetime reduction, while the opposite is observed in the Tevatron.

FINAL REMARKS

Beam-beam effects in electron and proton colliders are a field of active research. Much has been learned from observations. In electron colliders the beam-beam limit, $\xi_y \simeq 0.03$, is relatively independent of the number of collision points. In proton colliders the beam-beam limit ξ is reached when the total tune spread $n_x\xi$ fills

the tune space between resonances of moderate order. Novel theoretical techniques for studying the beam-beam effect in electron colliders have been tried on simplified models. They hold the promise of eventually explaining the beam-beam limit if the difficulties associated with more realistic models can be overcome.

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REFERENCES

1. G. Tumaikin, ed., Proc. Third Advanced ICFA Beam Dynamics Workshop, Novosibirsk 1989, to be published.
2. G.E. Fischer, SPEAR-154 (1972).
3. Spear Group, IEEE Trans Nucl.Sci. **NS-20** (1973) 838.
4. K. Satoh et al., Proc. 6th Symp. Accel. Sci. Technology (Tokyo, 1987) 18.
5. S.X. Fang, this conference.
6. E. Keil and R. Talman, Part. Acc. **14** (1983) 109.
7. A.W. Chao and R.D. Ruth, Part. Acc. **16** (1985) 201.
8. R. Talman, AIP Conf. Proc. **153** (1987) 789.
9. K. Hirata, Nucl. Instr. Meth. **A269** (1988) 7.
10. A. Hofmann and S. Myers, CERN LEP Note 604 (1988).
11. R.E. Meller and R.H. Siemann, IEEE Trans. Nucl. Sci. **NS-28** (1981) 2431.
12. K. Yokoya, Y. Funakoshi, E. Kikutani, H. Koiso and J. Urakawa, KEK Preprint 89-14 (1989).
13. E. Keil, Nucl. Instr. Meth. **188** (1981) 9.
14. E. Keil, CERN LEP Note 294 (1981).
15. A. Piwinski, Proc. 11th Int. Conf. High Energy Accel., Geneva (Birkhäuser, 1980) 751.
16. S. Peggs and R. Talman, *ibid.* 754.
17. S. Myers, IEEE Trans. Nucl. Sci. **NS-28** (1981) 2503.
18. S. Myers, Nucl. Instr. Meth. **211** (1983) 263.
19. R.H. Siemann, Cornell Report CBN 89-4 (1989).
20. K. Hirata, Phys. Rev. **D37** (1988) 1307.
21. M.A. Furman, K.Y. Ng and A.W. Chao, SSC-174 (1988).
22. M.H.R. Donald and J.M. Paterson, IEEE Trans. Nucl. Sci. **NS-26** (1979) 3580.
23. K. Hirata, CERN/LEP-TH/89-14 (1989).
24. K. Hirata, CERN/LEP-TH/89-35 (1989).
25. R.L. Stratonovich, *Topics in the Theory of Random Noise* (Gordon and Breach, New York-London 1963).
26. Y.H. Chin, LBL-25370 (1988).
27. Y.H. Chin, LBL-25983 (1989).
28. L. Evans, J. Gareyte, M. Meddahi and R. Schmidt, CERN SPS/89-12 (AMS) (1989).
29. G. Dugan, these proceedings.
30. L.R. Evans, CERN 84-15 (1984) 319.