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MULTIPLICITY DISTRIBUTIONS IN HADRON INTERACTIONS DERIVED FROM
THE STATISTICAL BOOTSTRAP MODEL

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A B S T R A C T

We give a self-contained (though sketchy) derivation of the charged multiplicity distribution of final hadrons in a simplified (only pions) version of the Statistical Bootstrap Model (SBM).

The analytical form of this distribution is determined by two prominent features of SBM: a singularity at some temperature T_0 (related to the phase transition hadrons $\rightarrow$ quark-gluon plasma) and the production of Poisson-distributed hadron clusters with well-defined decay structure. The multiplicity distribution of final hadrons has two free parameters: an average temperature T and the mean number $\bar{N}$ of clusters, both depending on the collision energy $\sqrt{s}$ such that for $\sqrt{s} \rightarrow \infty$, the temperature tends to the limit T_0 while $\bar{N}$ may grow (very slowly) without bound. With these two free parameters we obtain reasonable to good fits to experimental data at energies ranging from $\sqrt{s} = 7.87$ to 900 GeV (full and in some cases cut pseudorapidity). The average cluster sizes found in our fits are compatible with those inferred from (pseudo) rapidity correlations in the data.

We discuss the relation of this model to the Negative Binomial Distribution (NBD).

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1. - INTRODUCTION

It was shown in 1987 [1] that the charged multiplicity distribution in high energy $p\bar{p}$ collisions, as derived from the statistical bootstrap model (SBM), agrees reasonably well with the experimental data at $\sqrt{s} = 546$ GeV.

We call this distribution the statistical bootstrap distribution (SBD). It is of a similar shape as the negative binomial distribution (NBD), which is known to give reasonable to good fits to data at various energies, in several different reactions and also for various cuts in (pseudo) rapidity [2].

Our work has been motivated by that of A. Giovannini and L. Van Hove [3], who presented an analysis of NBD in terms of a convolution of the logarithmic multiplicity distributions of partons belonging to "clans" (a clan comprises all partons having a common "ancestor") weighted with the Poisson distribution for the number of clans produced in a collision. A similar idea was independently proposed by EKSpong [3e].

This picture is conceptually similar to that of SBM, in which after a first stage of parton interaction (on which SBM makes no statements) hadronization leads to clusters, which move along the collision axis, become heavier and more numerous with increasing $\sqrt{s}$ and which decay in a well-defined way.

SBM predicts that the number N of clusters is Poisson distributed and that its mean value $\bar{N}$ should be slowly varying with $\sqrt{s}$. It further predicts how a single cluster of given mass decays and how an average cluster, taken from a cluster gas in thermal and hadro-chemical equilibrium, decays. The final multiplicity distribution is then also here a convolution of the cluster decay multiplicity distributions weighted with the Poisson distribution for the number of clusters produced in a collision. Our final distribution has two free parameters: the average number $\bar{N}$ of clusters and the average temperature T ; the latter should, with increasing $\sqrt{s}$, monotonously tend to some upper limit T_0 of order of 200 MeV. This temperature T_0 is a singular point in SBM, where the model breaks down (because the partition function becomes complex) and which is interpreted as the transition temperature where hadron matter undergoes a phase transition into a quark-gluon plasma. The existence of such a T_0 governs all predictions of the model, among them the multiplicity distributions.

In spite of the kinematical resemblance there is a fundamental difference between SBM and the above-mentioned clan model: the latter assumes that formation, structure and decay of clans are determined by QCD at the quark-gluon level, so that the final multiplicity distribution is that of quarks and gluons just before hadronization (and not significantly changed by the latter), while in SBM everything happens on the hadron side: clustering of hadrons in dense hadronic matter

and subsequent re-decay of such clusters into hadrons being the essential dynamical picture of SBM. At the very end of this paper we shall speculate how both pictures might be reconciled.

In this paper we derive the results of [1] in a self-contained way from SBM and confront SBD with further data. Section 2 contains a sketchy introduction to SBM and the derivation of all formulae necessary here; Section 3 derives the SBM multiplicity distribution SBD; Section 4 compares SBD to experimental data; Section 5 compares SBD to NBD and presents some speculative interpretation.

Our notation is similar to that in [1]. We use units where $\hbar = c = k(\text{Boltzmann}) = 1$. In our Lorentz-invariant formulation: v^μ is the four-velocity of the observed (sub-) system, $\beta^\mu = 1/T v^\mu$ is the "inverse temperature four vector"; the temperature $T = 1/\beta = 1/\beta_\mu \beta^\mu$ is a Lorentz scalar; m denotes the pion mass throughout the paper. Average values are written in either way: $\bar{n}$ or $\langle n \rangle$ as convenient.

2. - THE ESSENTIALS OF SBM

2.1. Qualitative summary

SBM [4] is a statistical model of strong interactions based on the observation that hadrons form bound and resonance states. This has led to the concept of an unlimited sequence of heavier and heavier bound and resonance states, each self-consistently composed of lighter ones [4a]. We call all of them "clusters" (the pion being the lightest "one-particle cluster") and label them by their masses. The number of different cluster states in the mass interval $[M^2, dM^2]$ is called the "SBM mass spectrum" (level density) $\tau(M^2)dM^2$. If introduced as new, independent particles, these bound and resonance states simulate the interactions due to which they exist (hence: bootstrap) [4b]; to simulate all (attractive) interactions, we need all of them, also the yet undiscovered ones - that is: the complete mass spectrum; repulsive forces, on the other hand, are most conveniently represented by proper cluster volumes, leading to a Van der Waals type volume correction [4j,k,l,m].

In order to obtain the complete mass spectrum one imposes a self-consistency condition: the "SBM bootstrap equation" (BE), which, expressed in plain words, simply says that "clusters consist of clusters". This BE is an integral equation for the SBM mass spectrum [4c,e]; its solution implies that:

- the proper volumes of clusters are proportional to their masses [4j];
- the mass spectrum grows exponentially like $\tau(M^2) \sim M^{-a} \exp(M/T_0)$.

The values of a and T_0 depend on details; the simplest BE yields $a = 3$ and,

depending on the chosen set of "input particles" ($\pi, K, N, \dots$) $150 \lesssim T_0 \lesssim 200$ MeV [4a,b,c,d,h]. The mass-volume proportionality implies that all clusters have the same energy density which is then identified with the MIT-bag energy density [4k,l,5b].

T_0 is a singular temperature (branch point in the complex T -plane) beyond which SBM thermodynamics breaks down, the partition function becoming complex. Existence and value of T_0 are corroborated by the slope of the experimental hadron mass spectrum (below ~ 1.5 GeV), by the low (thermal) part of p_T -distributions, by the findings of other models (string, MIT-bag, QCD lattice calculations) and by the present multiplicity distribution.

The main power of SBM derives from the postulate that strong interaction - as far as needed in a statistical model - is completely simulated by the presence of clusters with an exponential mass spectrum and with mass-proportional proper volumes; this postulate implies that:

in SBM the strongly interacting hadron gas is formally replaced by a non-interacting (i.e., ideal) infinite-component cluster gas with Van der Waals volume corrections and exponential mass spectrum

which can be handled analytically without recourse to perturbative methods [4m]. The above postulate is justified by the self-consistent construction (via the BE) of the SBM mass spectrum, which makes the latter essentially complete: it grows exponentially with about the steepest possible slope (T_0 is of the order of the lowest hadron mass, m_π); on the other hand a spectrum richer than exponential destroys thermodynamics since the partition function diverges at any temperature. Any residual interactions, forgotten or deliberately neglected (e.g., Bose-Einstein/Fermi-Dirac statistics, internal symmetries, angular momentum conservation), have so far proved to leave the essentials of SBM intact, while in general slightly diminishing the mass spectrum by pushing up the values of a and of T_0 [4a,c,d,f,h,5a,d,e,f,6b].

SBM has been constructed on the basis of hadron phenomenology. Hadron phenomenology and with it SBM should in principle be derivable from QCD, similarly as chemistry is in principle derivable from QED.

Keeping this in mind, it is reasonable to interpret the singularity at T_0 as indicating a phase transition from the hadron gas (with net baryon number zero) to a quark-gluon plasma [4l]. This is also suggested by the actual behaviour of the SBM cluster gas when T_0 is approached; namely, clusters grow bigger and bigger (with constant energy density) and eventually all coalesce into a single, infinite supercluster when $T \rightarrow T_0$ just as droplets do near condensation [4,k,i,m], [Fig. 1.6a of Ref. 4n]. Although quarks and gluons do not explicitly appear in SBM, it is

natural to interpret this infinite supercluster as a deconfined quark-gluon phase, because SBM clusters share all thermodynamically relevant properties (mass spectrum, mass-volume relation) with MIT-bags [5b,f], so that the SBM cluster gas has the same thermodynamics as a MIT-bag gas [5c]. Furthermore QCD lattice calculations yield a phase transition from hadrons to a deconfined quark-gluon plasma at a temperature of the order of our T_0 [5g,h,i,j,k].

We cannot go here beyond these sketchy remarks. An easily readable introduction to SBM, not supposing any prior knowledge is found in [6a]; as it was written in 1971 some of its details and conclusions are now obsolete (e.g. T_0 was then claimed to be an ultimate temperature, not a transition temperature) but it may still serve as a detailed exposition of the main ideas and their motivations. A review of SBM up to 1984 containing a fairly complete list of references is given in [6b] while [6c] gives a general outline of the relation of SBM to the S-matrix approach to strong interactions.

2.2. SBM and high energy collisions

SBM as sketched above has been mainly conceived and developed in the framework of the grand canonical equilibrium situation; however, even the phase space formulation tacitly supposes equilibrium. How then can SBM be applied to collisions in which we have a microcanonical non-equilibrium situation (four-momentum conservation; violent collective motions along the collision axis), limited to space-time regions of order 10^{-23} sec. and 10^{-13} cm, where a thermodynamic limit is difficult to imagine?

Assuming that a thermal description is possible at all, we first observe that all we can measure are momentum distributions and multiplicities of secondary particles produced in these collisions. Particle production would be governed by Boltzmann factors $\sim \exp(-M/T)$, where T is limited from above by T_0 , and M from below by m_π , so that appreciable particle production requires $T/T_0 \gtrsim 0.5$ (remember T_0 and m_π are of the same order). Indeed the SBM equation of state predicts that the energy density of a hadron gas with net baryon number zero is still very small at $T/T_0 \lesssim 0.5$, but, above $T/T_0 \approx 0.8$, rises steeply to its (finite) maximum value at T_0 [Fig. 1.8 of 4n]. It follows that significant thermal particle production comes necessarily from regions with $T/T_0 \gtrsim 0.5$ and most of it from those regions with the highest temperatures attained in a given collision. Finally, strong interactions have a time scale of the same order as the collision time.

This leads one to expect that while an overall thermal equilibrium is excluded, local equilibrium might be approached in co-moving Lorentz frames;

namely, in small neighbourhoods Δy of given rapidities y along the collision axis. There SBM might be locally valid.

From the mentioned steep rise of the energy density above $T/T_0 \approx 0.8$ we conclude that even if only an (unknown) small fraction of the total $/s$ is locally thermalized, the temperature would be of order $0.8 \lesssim T/T_0 < 1$ over most parts of the rapidity range, except at the ends. The local hot SBM gas would produce clusters with temperature dependent masses, moving along the collision axis with their corresponding local rapidities and cascade-decaying into the final particles.

Much effort has gone into the formalization of this picture in the so-called thermodynamical models (TM) [7], which were useful in predicting momentum spectra and production rates of secondaries, mainly in pp collisions. The above considerations are thus justified a posteriori by the successes of TM's based on SBM.

We therefore expect that one may apply SBM to the problem of multiplicity distributions; the more so, as multiplicities are Lorentz invariant and therefore insensitive to longitudinal collective motions. As the temperature is (at sufficiently large $/s$) always of order T_0 , we do not consider variations of T along the rapidity axis, but rather use an effective (average) temperature which only slightly changes with $/s$.

It should be clear that we cannot claim to give more than some rough, overall picture; all finer details, in particular those linked to hard QCD processes, will escape this description. While we believe that SBM is a good model for equilibrium hadron thermodynamics, the conditions for its applicability might less and less be fulfilled when collisions at higher and higher energy are considered; but so far (up to $/s = 900$ GeV) it seems to work.

2.3. Basic equations of SBM

We adopt the latest version of the model $(4k, l, m, n)$ with the simplification that we here neglect baryons and strange particles, so that the pion is the only input particle - and the only one which can be produced. We use Boltzmann statistics, which can be justified [4a, j, k, l, 6a].

The BE $[4c, e, l]$ for the mass-spectrum $\tau(m^2)$ reads in this simplest possible case

$$H\tau(p^2) = H\delta(p^2 - m^2) + \sum_{k=2}^{\infty} \frac{1}{k!} \int \delta^4(p - \sum_{i=1}^k p_i) \prod_{i=1}^k H\tau(p_i^2) d^4p_i \quad (2.1)$$

where m is the pion mass, δ_0 is the positive energy part of the δ -function and H is a constant to be fixed later. This equation states that the level density (mass spectrum) $\tau(p^2)$ of a cluster with mass $M = \sqrt{p^2}$ (unless the cluster is itself a pion) equals a convolution of the level densities of its constituents, integrated and summed over all possible ways of composing it from subclusters, which themselves have the same internal structures characterized by $\tau(p_i^2)$.

The BE can be solved for the unknown mass spectrum $\tau(p^2)$ by Laplace transforming it ("putting it in a heat bath at temperature T ") [4e, f, g, h]. Define (K_1 is the first modified Hankel function):

$$z(\beta) := \int e^{-\beta p^2} H\delta_0(p^2 - m^2) d^4p = 2\pi H/m T K_1(m/T) \quad (2.2)$$

$$\Phi(\beta) := \int e^{-\beta p^2} H\tau(p^2) d^4p \quad (2.3)$$

The physical interpretation of $z(\beta)$ and $\Phi(\beta)$ is that they are essentially equal to the one-pion and one-cluster partition functions respectively [4j]. Both z and Φ depend only on β ; therefore Φ is a function of z and vice versa; thus we introduce the "bootstrap function"

$$G(z) := \Phi(\beta) \quad (2.4)$$

Applying this Laplace transform to the BE and using (2.4) we obtain an implicit equation for $G(z)$, which is called the Laplace transformed BE:

$$G(z) = z + \exp[G(z)] - G(z) - 1 \quad (2.5)$$

Forgetting about the thermodynamic significance (2.2) of $z(\beta)$ and considering z simply as a complex variable, we have the mathematical problem of solving the equation

$$z = 2G(z) - \exp[G(z)] + 1 \quad (2.6)$$

for $G(z)$. Knowing $G(z)$ leads, via inverse Laplace transform, to the knowledge of $\tau(p^2)$. The most complete solution of (2.6) is known as an integral representation [4, o] of all the (infinitely many) branches of the complex function $G(z)$; it does not interest us here.

Most instructive is the graphical solution, Fig. (2.1). The maximum values for which a real, physical solution exists [4e], are

$$z_0 = \ln 4 \sim 1$$

$$G_0 \equiv G(z_0) = \ln 2 \quad (2.7)$$

Given the constant H in Eq. (2.2), the maximum temperature T_0 follows from $z(\beta_0) = z_0$. If several input masses (pion, K, nucleon, hyperons) are considered, $z(\beta)$ becomes a sum of similar terms, each representing one input mass and T_0 will, for fixed H , depend on which input masses are taken into consideration. Neglecting here everything except pions, we keep the T_0 value (190 MeV) corresponding to a more complete model [4k,1] and assume H to be adjusted correspondingly to fulfil Eq. (2.2) at T_0 . Since $G(z)$ has at z_0 a square root singularity [4d] (Fig. 2.1b) inverse Laplace transforming gives for the asymptotic mass spectrum [4d,h]

$$\tau(M^2) \sim M^{-3} \exp(M/T_0) \quad (2.8)$$

For us, the most useful solution of Eqs. (2.5) and (2.6) is by a power expansion proposed by Yellin [4e, see also 4h]:

$$G(z) = \sum_{l=1}^{\infty} g_l z^l$$

$$g_0 = 0 ; \quad g_1 = 1$$

$$l g_l = -(l-1) g_{l-1} + l \sum_{k=1}^{l-1} g_k g_{l-k}$$

$$g_l \sim z_0^{-l} e^{-3/2} \sqrt{\frac{z_0}{4\pi}} \quad (2.9)$$

The recursion relation follows from differentiating the BE and eliminating $G(z)$, then inserting the power series [5a]. The convergence radius of this series is $z_0 = \ln 4 \sim 1$; it still converges at z_0 , as seen from the asymptotic behaviour of the coefficients g_l .

The coefficients g_l are of physical significance for multiplicity distributions, since, as we shall see now, they are directly related to the number of possible decay trees of a given cluster leading to n final pions.

Namely, Yellin [4e] only independently rediscovered Eqs. (2.6) and (2.9), which were already known in 1870 when they were first discovered by E. Schröder [8] in solving a purely combinatorial problem, which he described in the following terms (free translation from the original German):

"...Given n elements, e.g., material points freely movable in space, one divides the set of n elements arbitrarily into subsets; some possibly containing only one element, some with two, three... elements; each set of

more than one element is to be surrounded by a closed surface, which, together with its content, shall be considered as a new element on an equal footing with all the others. This new set of (obviously less than n) elements is submitted to the same procedure as long as one likes or until it ends because the next step would result in one single element. We wish to find the number of different compositions which can be generated by this procedure..."

This, indeed, is the abstract, purely combinatorial, content of SM; where a cluster is composed in all possible ways of subclusters, which are themselves composed in the same way. The number of different possible ways to compose a cluster out of n pions is, obviously, equal to the number of its different possible decay trees leading back to n pions, and this number was found by Schröder to be

$$S_n = n! g_n \quad (2.10)$$

He, however, did not attach any physical significance to the g_n or to the argument of the generating function $G(z)$, for which he wrote down Eq. (2.6) and from which he calculated the g_n more than hundred years ago.

The power series solution of Eqs. (2.5) and (2.6) allows us to write the mass spectrum $\tau(p^2)$ as a series in l -pion invariant momentum space integrals $\Omega_l(p^2)$:

$$\Omega_l(p^2) = \int \delta^4(p - \sum_{i=1}^l p_i) \prod_{i=1}^l H \delta_0(p_i^2 - m^2) d^4 p_i \quad (2.11)$$

Its Laplace transform is (see (2.2))

$$\int e^{-\beta p^2} \Omega_l(p^2) d^4 p = [z(\beta)]^l \quad (2.12)$$

Thus

$$G(z) = \sum_l g_l z^l = \int e^{-\beta p^2} \left[\sum_l g_l \Omega_l(p^2) \right] d^4 p \quad (2.13)$$

Comparing with the definition (2.3) and (2.4) of $G(z)$ gives [4e]

$$H \tau(p^2) = \sum_{l=1}^{\infty} g_l \Omega_l(p^2) \quad (2.14)$$

This explicit solution of the BE in terms of l -pion momentum space integrals will be the key to our problem of calculating multiplicity distributions.

3. - MULTIPLICITY DISTRIBUTIONS IN SBM

Intuitively the decay of a SBM cluster will go as depicted in Fig. 3.1; there are very many different possible decay trees, one of them is that of Fig. 3.1a, another one that of Fig. 3.1b, where - as it happens in reality [4c,h] - two- and three-body decays dominate.

Schröder [8] told us in 1970 how many different decay trees exist, leading to exactly n pions: $S_n = n!g_n$ (if $M > nm_\pi$). Indeed his description, given above, corresponds to reading the decay tree in the reverse sense leading from the final n pions back to the original cluster. What is still missing is the probability of the decay into just n pions.

Since SBM considers decay and composition as inverses of each other, both are fully determined by the BE and by other expressions derived from it. To calculate probabilities of particular realizations (trees), multiplicities, etc., it suffices to suppress sums and/or integrations in a way corresponding to the question posed, to properly normalize and to interpret the resulting expressions as decay probabilities instead of "composition probabilities". The following applications of this rule will make clear what is meant.

3.1. Basic ingredients of the multiplicity distribution (SRD)

In SBM the total multiplicity distribution results from a superposition of the multiplicity distributions arising from individual clusters. We therefore need three basic ingredients (elementary probability distributions):

- the probability distribution $R(\lambda; M^2)$ of the numbers λ of pions coming from the decay of a single cluster of mass M ,
- the probability $W(M^2; T) dM^2$ that a cluster, randomly chosen from our SBM cluster gas at temperature T , has a mass in the interval $[M^2, M^2 + dM^2]$;
- the probability distribution $Q(N; \bar{N})$ of the number N of clusters at given $\bar{N}$'s.

The probability distribution $P(n; \bar{N}, T)$ to find n final pions is then found by first combining a) and b) to calculate the average multiplicity distribution $R(\lambda; T)$ of pions coming from clusters at temperature T and then convoluting N such distributions with weight $Q(N; \bar{N})$ under the condition $\sum_{i=1}^N \lambda_i = n$, finally summing over N (δ_K is the Kronecker δ):

$$P(n; \bar{N}, T) = \sum_N Q(N; \bar{N}) \sum_{\{\lambda_i\}} \delta_K(n - \sum_{i=1}^N \lambda_i) \prod_{i=1}^N R(\lambda_i; T) \quad (3.1)$$

3.2. Generating functions

Weighted convolutions of probability distributions are most easily dealt with by using generating functions. Forget for the moment the special meaning of the above distributions P , Q and R and consider them as arbitrary, normalized probability distributions. Write

$$\begin{aligned} P_m &:= P(m; \dots) \\ Q_N &:= Q(N; \dots) \\ R_\ell &:= R(\ell; \dots) \end{aligned} \quad (3.2)$$

and define the generating functions (g.f.)

$$\begin{aligned} P(\lambda) &:= \sum_m P_m \lambda^m \\ Q(\lambda) &:= \sum_N Q_N \lambda^N \\ R(\lambda) &:= \sum_\ell R_\ell \lambda^\ell \end{aligned} \quad (3.3)$$

Normalization implies $P(1) = Q(1) = R(1) = 1$. What is $P(\lambda)$ when $Q(\lambda)$ and $R(\lambda)$ are given and the convolution (3.1) holds? Multiply (3.1) by λ^n and sum over n to obtain

$$P(\lambda) = \sum_N Q_N \prod_{i=1}^N R_{\lambda_i} \lambda^{\ell_i} = \sum_N Q_N R(\lambda)^N = Q[R(\lambda)] \quad (3.4)$$

Thus P_n is the coefficient of λ^n in the expansion of $Q[R(\lambda)]$ in powers of λ .

3.3 The elementary probability distributions of SBM

a) Fixed cluster mass M

We rewrite Eq. (2.14) as

$$1 = \frac{\sum_\ell g_\ell \Omega_\ell(M^2)}{4\pi T(M^2)} \quad (3.5)$$

and interpret it, according to the general rule given at the beginning of this section, as the sum of probabilities for the decay of $M + \lambda$ pions. Hence

$$R(\ell; M^2) = \frac{g_\ell \Omega_\ell(M^2)}{4\pi T(M^2)} \quad (3.6)$$

This is a very transparent result: according to Schröder [8] the total number of possible decay trees leading to λ pions is $S_\lambda = \lambda! g_\lambda$; on the other hand, $\Omega_\lambda(M^2)/\lambda!$

is the statistical weight imposed, by four-momentum conservation, on the final λ pion state. Thus the probability to create just λ pions is proportional to $S_{\lambda} Q_{\lambda} / \lambda! = g_{\lambda}^2 Q_{\lambda}$.

b) Probability distribution of cluster masses M at fixed T

The probability that a cluster, chosen randomly from the SBM cluster gas at temperature T has a mass in the interval $[M^2, dM^2]$ follows from Eqs. (2.3) and (2.4) which we rewrite as

$$1 = \frac{\int dM^2 \delta_0(p^2 - M^2) H_T(M^2) e^{-\beta_0 p^2} d^4p}{G(z)} \quad (3.7)$$

Interpreting this as the sum of probabilities for $M^2 \in [M^2, dM^2]$ we find

$$W(M^2, T) dM^2 = \frac{dM^2 \tau(M^2) \int d_0(p^2 - M^2) e^{-\beta_0 p^2} d^4p}{G(z)} \quad (3.8)$$

This, again, is a very transparent result: if one evaluates the d^4p -integral [see Eq. (2.2)] and uses the asymptotic formula $K_1(x) \sim \sqrt{\pi/2x} \exp(-x)$, one finds for $M \gg T$

$$W(M^2, T) dM^2 \approx \text{const} \cdot [dM^2 \tau(M^2)] \cdot [\sqrt{MT^3} e^{-M/T}] \quad (3.8')$$

where the first factor is the number of states available in $[M^2, dM^2]$ and the second is proportional to the Boltzmann probability for creating a fixed mass M at temperature T.

Before doing the d^4p -integration, however, we combine (3.6) and (3.8) to find the probability that a cluster, chosen randomly from the SBM cluster gas at temperature T, decays into exactly λ pions:

$$R(\lambda, T) = \int dM^2 W(M^2, T) R(\lambda, M^2) = \frac{g_c \int e^{-\beta_0 p^2} \Omega_c(p^2) d^4p}{G(z)} = \frac{g_c z^c}{G(z)} \quad (3.9)$$

where Eq. (2.12) has been used.

While the explicit mass spectrum $\tau(M^2)$ has cancelled in this integral, it is still implicitly present in the g_c and the normalizing $G(z)$.

The generating function for $R(\lambda, T)$ is obviously

$$R(\lambda, T) = G(\lambda z) / G(z) \quad (3.10)$$

c) The probability distribution for the number N of clusters

In thermo-chemical equilibrium the strongly interacting SBM hadron gas is formally an infinite-component ideal cluster gas, with the number N of clusters being Poisson distributed; its mean value $\bar{N}$ equals the "one-cluster partition function" (see, e.g., [6a], sect. 6.3). After applying the Van der Waals volume correction the mean cluster number density $\bar{N}/V$ first grows, as a function of T, up to some maximum and then, when $T \rightarrow T_0$, decreases to zero, because all clusters coalesce into one infinite supercluster [4.1,m].

In a collision no overall equilibrium is possible; following the philosophy of the well-tested old thermodynamical models [7], we assume that the interacting lump of matter splits into a number of pieces moving with different rapidities along the collision axis, separating from each other with relativistic velocities and soon becoming independent, while at the same time approaching internally hadron-chemical and thermal equilibrium according to SBM. The cluster number distribution in each of these subsystems is Poisson; the total distribution is then a simple convolution of the individual ones, hence again a Poisson distribution with mean value $\bar{N}$ equal to the sum of the individual $\bar{N}_i$, which tend to zero when $T \rightarrow T_0$.

On the other hand, the number of subsystems will grow roughly proportional to the total rapidity width, because the width Δy of a subregion in which equilibrium could be approached should be less than one unit: $\Delta y \lesssim 1$. Therefore there will be about $\text{const} \times \ln s$ independent regions, each contributing one $\bar{N}_i$, giving a total $\bar{N}$ of order

$$\bar{N} \sim \text{const} \cdot \langle \bar{N}_i \rangle \sim \ln \sqrt{s} \quad (3.11)$$

Here $\langle \bar{N}_i \rangle$ is the average of the individual mean values; the latter should decrease when $T \rightarrow T_0$ ($s \rightarrow \infty$) according to SBM. Therefore, finally, $\bar{N}$ in (3.11) should grow less than $\ln s$, which indeed we find (Section 4.4).

Our guess that the total cluster number is Poisson distributed with very slowly growing $\bar{N}$ is also found in other models (e.g., multiperipheral cluster [9] and bremsstrahlung [10] models).

In view of the complicated interplay of two mechanisms:

- splitting into local subsystems (with various energy contents),
 - approach to SBM equilibrium in each subsystem,
- we cannot calculate $\bar{N}$; the guess (3.11) is the best we can do; we therefore must treat $\bar{N}$ as a free parameter.

The probability distribution of the number N of clusters is then

$$Q(n; \bar{n}) = \frac{\bar{n}^n}{n!} e^{-\bar{n}} ;$$

$$Q(\lambda; \bar{n}) = \exp[\bar{n}(\lambda - 1)] .$$

(3.12)

3.4. The final total multiplicity distribution (SBD)

According to Eq. (3.4) the resulting total multiplicity distribution is, with (3.10) and (3.12), given by the generating function (belonging to the class called in the statistical literature "infinitely divisible" or "compound Poisson" distributions)

$$P^{SBD}(\lambda) = Q[R(\lambda; T)] = e^{-\bar{N}} \exp[\bar{N} G(\lambda z)/G(z)] . \quad (3.13)$$

Obviously, on account of Eq. (2.9), we have for $z \rightarrow 0$ $G(\lambda z)/G(z) = \lambda$, and (3.13) reduces to the generating function of a Poisson distribution; this is because for $z \rightarrow 0$ no cluster can decay - they are all just pions (remember the physical significance of z and $G \equiv \Phi$ given below Eq. (2.3)).

Our wanted distribution $P^{SBD}(n; \bar{n}, T)$ is now the coefficient of λ^n in the power expansion of $P^{SBD}(\lambda)$. The usual methods - Cauchy integral or differentiation - are too complicated to determine the coefficient of λ^n . We find it easily, however, from a recursion relation which is derived as follows: we differentiate $P(\lambda)$ and find

$$P'(\lambda) = P(\lambda) \cdot \frac{\bar{n}}{G(z)} \cdot z \frac{dG(z')}{dz'} \Big|_{z'=\lambda z} = \lambda z \quad (3.14)$$

insert $P(\lambda) = \Sigma P_n \lambda^n$ and $G(z) = \Sigma g_k z^k$ and compare coefficients. The result is

$$P^{SBD}(n; \bar{n}, T) = e^{-\bar{N}} C_n(\bar{n}, T) \cdot z(\tau)^n$$

$$C_0 = 1$$

$$C_n(\bar{n}, T) = \frac{\bar{n}}{G(z)} \cdot \frac{1}{n} \sum_{k=1}^n k g_k C_{n-k} \quad (3.15)$$

The g_k are known from (2.9).

Mean value $\bar{n}$ and dispersion $D^2 = \langle n^2 \rangle - \langle n \rangle^2$ are easily found to be

$$\bar{n} = \bar{N} \bar{\ell}$$

$$D^2 = \langle n^2 \rangle - \langle n \rangle^2 = \bar{N} \bar{\ell}^2 \quad (3.16)$$

with $\bar{\ell}$ = mean multiplicity per cluster, etc:

$$\bar{\ell} = z G'/G$$

$$\bar{\ell}^2 = (z G' + z^2 G'')/G \quad (3.17)$$

$$G' \equiv dG/dz \text{ etc.}$$

3.5. Limited rapidity intervals

Equation (3.15) would be the complete solution of our problem if one could measure total multiplicities (including neutrals) and if one would consider only multiplicities in full rapidity. Neither is the case.

We shall first consider limited rapidity^{*} intervals

$$\Delta = \{y_1, y_2\} \quad (3.18)$$

and adopt the notation

$$n = \text{multiplicities in full rapidity}$$

$$\lambda = \text{multiplicities in limited rapidity} \quad (3.19)$$

We define for a large set of events with fixed total multiplicity n

$$p(u) = \frac{\int_{\Delta} \frac{dn}{dy} dy}{\int_{-\infty}^{\infty} \frac{dn}{dy} dy} ; \quad q(u) = 1 - p(u) \quad (3.20)$$

Suppose we knew

$$w(\lambda, [n]) = \text{probability to have exactly } \lambda \text{ particles in } \Delta \text{ when there are } n \text{ in full rapidity;}$$

$$(3.21)$$

^{*}) Data often refer to pseudorapidity η , sometimes to rapidity y . For simplicity we speak here always of rapidity.

then for a given (correct) multiplicity distribution $P(n)$, the distribution in Δ would be calculable as

$$P_{\Delta}(l) = \sum_{n=l}^{\infty} w(l, [n]) P(n) \quad (3.22)$$

and no fits in Δ would be needed. In reality the $w(l, [n])$ are unknown and so is $P_{\Delta}(l)$.

An often used procedure is then: one takes a distribution function $P(n, \alpha)$ which has successfully been fitted to multiplicities in full rapidity (by adjusting its parameters α) and tries the same function to fit multiplicities in Δ ; if one obtains reasonable fits with parameters α_{Δ} , one compares them to the parameters α and tries to interpret physically their behaviour for various energies and rapidity windows Δ . It is obvious, however, that in general the distribution $P_{\Delta}(l)$ given by (3.22) will not have the same mathematical form as $P(n)$ and that therefore the mentioned procedure need neither yield good fits nor be physically meaningful. Indeed it was shown in a simple model with identical decaying clusters distributed over full rapidity that the distribution in Δ is non-trivially different from that in full rapidity [11].

We therefore try here an approach different from the usual one: instead of fitting in Δ with the same function P as in full rapidity, we fit with a formula of the type (3.22) after having replaced the unknown $w(l, [n])$ by its simplest approximation

$$w(l, [n]) \rightarrow \binom{n}{l} p^l q^{n-l} \quad (3.23)$$

which would hold if, for each multiplicity n , the particles were statistically equidistributed over the whole rapidity interval (one-dimensional box with a gas). Using (3.22) with the approximation (3.23) amounts to "separate off phase space effects".

As the above $p(n)$ are in general not known we simplify once more by replacing $p(n)$ by some constant p of the order of $p(\langle n \rangle)$ as would follow from the definition (3.20) when n is replaced by $\langle n \rangle$.

We therefore propose to fit the expression

$$P_{\Delta}(l) = \sum_{n=l}^{\infty} \binom{n}{l} p^l q^{n-l} P(n; \alpha_{\Delta}) \quad (3.24)$$

to the data in Δ when $P(n; \alpha)$ was a reasonable fit function in full rapidity.

This will, firstly, improve the chances to obtain good fits in Δ , because we have removed the trivial deformation of P due to phase space reduction and it will therefore, secondly, make the resulting parameters α_{Δ} more likely to reflect the true (average and approximate) physical conditions prevailing to Δ , e.g., in our model a higher temperature.

There is another reason to use (3.24) - namely, we shall see that the same formula can be used to project from total to charged multiplicities (with charge conservation on the average) - and this time with good accuracy. Both projections can then be combined into one single formula.

Before doing that we illustrate the meaning of Eq. (3.24) by looking at its effect on the generating function:

$$P_{\Delta}(\lambda) = \sum_{l=0}^{\infty} \lambda^l P_{\Delta}(l) = P(p\lambda + q; \alpha_{\Delta}) \quad (3.25)$$

This is interesting because there are a number of distributions which are form-invariant under the substitution

$$\lambda \rightarrow p\lambda + q = 1 + p(\lambda - 1) \quad (3.26)$$

i.e., they change only their parameters. Examples are the Poisson distribution and NBD; in the latter case,

$$P^{NBD}(\lambda; \bar{n}, k) = \left[1 + (1-\lambda) \frac{\bar{n}}{k} \right]^{-k} \rightarrow \left[1 + (1-\lambda) \frac{p\bar{n}}{k} \right]^{-k} \quad (3.27)$$

implying that under the projection (3.24)

$$k \rightarrow k; \quad \bar{n} \rightarrow p\bar{n} \quad (3.28)$$

It is known [2], that for limited rapidity intervals $\bar{n}_{\Delta}$ indeed grows roughly proportional to the width Δ , but k is not constant as expected from (3.28), showing that the projection (3.24) cannot be more than a crude approximation. As, on the other hand, we show in the next Section that the same projection formula (3.24) holds for transforming total into charged multiplicities - and in that case with good accuracy - Eq. (3.28) can serve to relate NBD parameters found in fits to charged multiplicities to those one would find for total multiplicities (in that case $p = 2/3$).

The g.f. of the SBM single-cluster decay distribution (3.10)

$$R(\lambda; T) = G(\lambda z)/G(z) \quad (3.29)$$

is not form-invariant under (3.25)^{*} - one more reason to avoid fitting SBD directly in Δ ; using (3.24) one at least gets rid of the trivial phase space deformation which in this case suffices already to make $P_\Delta(i)$ different from SBD.

The physical interpretation of using the projection (3.24) in fits to data referring to Δ is the following: if the projection (3.24) were correct, the parameters α_Δ resulting from fits to Δ would be those pertaining to full rapidity if the local average conditions prevailing in Δ were to apply to full rapidity; as (3.24) is a crude approximation, this interpretation is only qualitative.

3.6. Charge projection

SBD as given by Eq. (3.15) refers to total multiplicities including neutrals. We have to project out of (3.15) the charged multiplicities which are measured. Isospin projection is not practicable - not only because it is complicated [12,13,14, see also 5a], but also as often the initial state is not an isospin eigenstate and as, moreover, we consider only pions while in reality particles of other isospins are present and many will escape unobserved.

In the absence of any detailed knowledge the simplest ansatz is a statistical weight [15]. Let

$$\{k_+, k_0, k_-\} = \text{number of particles with charge } \{+1, 0, -1\} \quad (3.30)$$

(and neglect others).

If there are n particles altogether, n_c of them charged and if the total net charge (initial state) is c , then

$$\begin{aligned} n &= k_+ + k_0 + k_- \\ n_c &= k_+ + k_- \\ c &= k_+ - k_- \end{aligned} \quad (3.31)$$

the (purely combinatorial) probability for having $k_+ k_0 k_-$ is then (δ_k is the Kronecker delta)

^{*}) Neither is the single-clan logarithmic decay distribution, but its non-invariance can be absorbed into the Poisson parameter N upon convolution, yielding the invariant NBD. Nothing like that happens in SBD.

$$\begin{aligned} W(k_+, k_0, k_-; n, c) &= \left(\frac{1}{3}\right)^{k_+ + k_0 + k_-} \frac{N!}{k_+! k_0! k_-!} \times \\ &\times \int_K (n - k_+ - k_0 - k_-) \delta_K(c - k_+ + k_-). \end{aligned} \quad (3.32)$$

where N is a normalization factor. If we are only interested in the total number of charged particles, n_c , then we add another $\delta_K(n_c - k_+ - k_-)$ and obtain

$$W(n_c; n, c_i) = \frac{\left[\frac{(n_c + c_i)}{2}\right]! \left[\frac{(n - c_i)}{2}\right]! (n - n_c)!}{\sum_{m_c} [numerator]} \quad (3.33)$$

for the normalized probability to find n_c if there are n particles and if c_i is the initial net total charge (in general 0, 1, 2).

The charged probability distribution is then

$$P_\Delta(n_c; c_i) = \sum_{n=n_c}^{\infty} W(n_c; n, c_i) P(n) \quad (3.34)$$

This formula with $P = \text{pSBD}$ taken from (3.15) was used for our fits in full rapidity.

In limited rapidity intervals Δ the charge is not fixed. If we would try to use a projection like (3.33), we would need to know a probability distribution for the charge c_Δ in Δ and average (3.33) over it. In order to calculate such a probability distribution, we would need the probabilities (3.21) separately for all charges; if we knew them, however, the problem would not arise, because then the charged multiplicity distribution would already be fixed.

The simplest way out is to relax on exact charge conservation and require charge conservation on the average only. Then it is also a reasonable assumption that the average net charge in Δ is zero. If Δ is symmetric to the centre ($y = 0$) and not too large, this is a good approximation even when $c_i = 1$ or 2. We restrict our formulae to this case; generalizations are straightforward.

The probability to have n_c charged particles when n particles are present is, under the present assumptions,

$$W(n_c; n) = \binom{n}{n_c} \left(\frac{2}{3}\right)^{n_c} \left(\frac{1}{3}\right)^{n - n_c} \quad (3.35)$$

and then

$$P_{ch}(n_c) = \sum_{n_c=n_c}^{\infty} \binom{M}{n_c} \left(\frac{2}{3}\right)^{n_c} \left(\frac{1}{3}\right)^{M-n_c} P(n) \quad (3.36)$$

This is the same as Eq. (3.24) with $p = 2/3$, $q = 1-p = 1/3$. It follows at once [15]

$$P_{ch}(\lambda) = P\left(\frac{2}{3}\lambda + \frac{1}{3}\right) \quad (3.37)$$

Writing the substitution $\lambda \rightarrow p\lambda + q = 1+p(\lambda-1)$ as $S_{p,\lambda} = 1+p(\lambda-1)$, one sees that

$$S_{p_1} \cdot S_{p_2} = S_{p_2} \cdot S_{p_1} = S_{p_1 p_2} \quad (3.38)$$

We therefore can combine projection onto Δ and onto charged multiplicities in one single formula (3.24) with

$$p \rightarrow p_c = \frac{2}{3}p \quad ; \quad q_c = 1-p_c \quad (3.39)$$

Finally then,

$$P_{\Delta, ch}(\ell_c) = \sum_{n_c=\ell_c}^{\infty} \binom{M}{\ell_c} \left(\frac{2}{3}p\right)^{\ell_c} \left(1-\frac{2}{3}p\right)^{M-\ell_c} P(n) \quad (3.40)$$

This has been used to fit charged multiplicities in limited pseudorapidity intervals Δ with $P = P^{SBD}$ from (3.15).

Applying (3.40) with $p = 1$ to full rapidity fits we found that it gives parameters and χ^2 values practically equal to those obtained with exact charge conservation (3.33).

3.7. The parameters used in fits

In all our preceding discussions we have used the parameters $\bar{N}$ and T because of their obvious physical interpretation. In calculations it is much more convenient to use $z(T)$ as a parameter instead of T , since z appears explicitly [Eq. (3.15)], while T appears only as an argument of z [Eq. (2.2)]. Another advantage of using z is that in this variable the singularity is always at $z_0 = \ln 4 - 1$ [Eq. (2.7)] independently of how the constant H [Eq. (2.2)] and/or the set of input masses is chosen, while the value of T_0 depends on this choice [see the discussion between Eqs. (2.7) and (2.8)].

We thus fitted P^{SBD} to data using $\bar{N}$ and z as free parameters and we show in Section 4.4 the behaviour of these parameters as functions of the collision energy and of the rapidity window; this representation is free from the uncertainty of the

value of T_0 , which is not too well known (somewhere between 150-200 MeV; probably ≈ 190 MeV).

Given a (fitted) value z and chosen a value of T_0 , one can easily calculate the T corresponding to $z(T)$ via the exact relation following from Eq. (2.2):

$$\frac{z}{z_0} = \frac{T}{T_0} \cdot \frac{K_1\left(\frac{M}{T}\right)}{K_1\left(\frac{M}{T_0}\right)} \quad (3.41)$$

If z is very near to z_0 , one may expand $K_1(M/T)$ at T_0 and keep the first term only; one obtains

$$\frac{T_0 - T}{T_0} \approx \frac{z_0 - z}{z_0} \cdot \frac{T_0}{m} \cdot \frac{K_1\left(\frac{M}{T_0}\right)}{K_2\left(\frac{M}{T_0}\right)} \quad (\text{valid for } z \rightarrow z_0) \quad (3.42)$$

4. - COMPARISON WITH EXPERIMENTAL DATA

We have fitted P^{SBD} to eleven sets of pp and $\bar{p}p$ collision data at energies ranging from $\sqrt{s} = 7.87$ GeV to $\sqrt{s} = 900$ GeV.

4.1. Fits to raw data or to corrected data?

Our fits were done to raw data where available (UA5 only), otherwise to corrected data. This needs explanation. Let

$T(n)$ = "true" multiplicity distribution (unknown),

$O(n)$ = "observed" multiplicity distribution $\equiv$ raw data.

These two are related by

$$O(n) = \sum_m P_{mn} T(m) \quad (4.1)$$

where the matrix P_{mn} depends on experimental conditions: acceptance of detectors, trigger efficiency, energy, ...; it is in general found from Monte Carlo simulations of the histories of all particles from their generation in the collision ("event") through the whole set-up down to their detection and recording. This simulation uses not only known properties of the apparatus but also a mixture of experimental knowledge and theoretical model assumptions (at the level of the "event generator"), the whole being iteratively improved as overall experimental information accumulates [16].

As $O(m)$ is measured and $T(n)$ sought for, one would like to invert P_{mn} to find the true distribution. However, even if P_{mn} has a mathematical inverse, this does not serve to find $T(n)$, because the statistical nature of all members of Eq. (4.1) would often lead to senseless results; e.g., some negative $T(n)$. One has therefore invented various methods to find the "most likely true" data obeying some reasonable conditions as smoothness, positivity and others; the result of such a "smooth inversion" are the "corrected data $C(n)$ ".

Let us call the procedure of smoothly transforming $O(n) \rightarrow C(n)$ "correction" (in quotation marks); it may be interpreted as an operator C mapping the set $O(n)$ onto $C(n)$:

$$C[O(n)] = C(n) \neq T(n) \quad (4.2)$$

In general C is not a point transformation: the mapping of each point depends on the whole set, in particular on its fluctuations. "Correction" is a compromise and by its very nature imperfect; different methods have different advantages and disadvantages. In a recent UAS collaboration publication [17] the so-called "Maximum Entropy Method" [18] was employed.

The errors of $C(n)$ - apart from the trivial statistical ones - originate from two independent sources: the inaccuracy with which P_{mn} is known (4.1) and the inherent imperfection of the "correction" C .

Thus we have, in principle, the choice between two methods of fitting SBD to experimental data. Define "observed SBD":

$$O^{SBD}(m) = \sum_n P_{mn} P^{SBD}(n) \quad (4.3)$$

O^{SBD} would be the observed data if P^{SBD} were the true ones. We can then either fit

$$O^{SBD}(m) \text{ to } O(n) = \text{"fit to raw data"} \quad (4.4a)$$

or else

$$P^{SBD}(n) \text{ to } C(n) = \text{"fit to corrected data"} \quad (4.4b)$$

The first is to be preferred, since there only the uncertainties of P_{mn} enter, while in the second method we have in addition the ambiguities of the "correction". Thus whenever P_{mn} and raw data were available to us, we used the first method; in general, however, only corrected data are published, forcing us to use the second procedure.

We illustrate the difference between the procedures (4.4a) and (4.4b) in Figs. 4.1a and b where the first shows O^{SBD} with raw data and the second P^{SBD} with corrected data (900 GeV pp).

After having fitted $O^{SBD}(n)$ to raw data, one can produce nice-looking smooth figures by plotting the "corrected" $C(n)$ together with $P^{SBD}(n)$ (which is, of course, the exact true SBD corresponding to O^{SBD}). As the mappings $O(n) \rightarrow C(n)$ and $O^{SBD} \rightarrow P^{SBD}$ are different, it may (and does) happen that points $O(n)$ lie on the curve $O^{SBD}(n)$ while their images $C(n)$ do not lie on $P^{SBD}(n)$. Compare Figs. 4.21 with 4.3 and 4.2j with 4.4. In such cases the original fit $O^{SBD}(n)$ to $O(n)$ is the trustworthy one.

4.2. Results of fits in full rapidity

We present the results of our fits in Figs. 4.2a-k and in Table 4.1. Data were taken from Refs. [17,19-25].

Comments to the Figures and to Table 4.1

a) For most of the figures we use scaled (KNO) variables, i.e., we plot $\langle n \rangle P(n) / \langle n \rangle^2$ vs $n_c / \langle n \rangle$. The different shapes of the curves reflect KNO scaling violation;

b) In two cases the data refer to full inelastic processes including the diffraction component (Figs. 4.2b,d) while all others concern "non-single diffraction". As SBM has nothing to say about diffraction, we excluded in the two cases mentioned the low multiplicities (dominated by diffraction dissociation) from the fits. As a consequence the corresponding points clearly stick out. Our curves show what the data probably would be after subtracting the diffraction contribution.

c) The model has a tendency to overestimate the high multiplicity tail above $n_c / \langle n \rangle \approx 3$. This failure is inherent in the version used here of the model in which only pions can be produced and where the mass spectrum grows according to Eq. (2.8). Among other possible variants of the model, the one in which the mass spectrum grows like

$$T_{5/2}(M^2) \sim M^{-5/2} e^{M/T_0} \quad (4.5)$$

would have a different asymptotic behaviour (similar to that of NBD) and would not show this deficiency [26]. Although we knew this [1] before starting the present work, we have decided to keep to the standard SBM version because of its transparency and because there are other reasons which explain the discrepancy; in particular allowing for the production of heavier masses (K,N,hyperons...)

would have the effect of diminishing large multiplicities, where such particles are most frequently produced (at the expense of pions: a few pions less for each heavy particle). Having kept the model as simple as possible by considering only pion production, it would have been inconsistent to pay the prize for this simplification at the wrong place - namely by ad hoc changing the mass spectrum.

d) All fits to UA5 data were done with ϕ_{SBD} on raw data; all others with p_{SBD} on corrected data.

e) A real disagreement which we cannot discuss away is gradually developing from 200 to 900 GeV: a shoulder in the region $n_c / \langle n_c \rangle \sim 1.5$ to 2.5. Neither SBD nor NBD can reproduce such a shoulder. However, several models can obtain oscillating $P(n)$ [27] and might adjust to this new phenomenon. Another possibility is discussed in [28]: if each given event arises from either the one or the other of two classes of production mechanisms, then the resulting total distribution is the weighted sum (instead of a convolution) of the distributions proper to either class. Surprisingly good fits including the shoulder at 900 GeV are then obtained with two NBD's having physically meaningful parameters.

f) As done by the UA5 collaboration [17], we excluded data points with only one or two observed particles from the fits at 200, 546 and 900 GeV, because they have large systematic errors estimated to be greater than the statistical ones.

g) The errors on the fit parameters $\{z, \bar{n}\}$ have been defined as those changing the value of χ^2 by 1.

4.3. Results of fits to limited pseudorapidity intervals

We have fitted SBD with the projection (3.40) to limited pseudorapidity intervals Δ (symmetric about $\eta = 0$) at 200 and 900 GeV [17,24].

Our results are summarized in Table 4.2 and one case is shown in Fig. 4.5.

Comments to Table 4.2

a) At small rapidities more kinetic energy is thermalized than at large ones. Therefore the average temperature in Δ should increase when Δ becomes narrower; simultaneously the number of clusters per unit rapidity is expected to decrease

at small rapidities since there the temperature is higher and relative collective motions are less violent (see Section 3.3c). Although the projection formula (3.40) is a very crude approximation, it is sufficient to allow these features to appear in the fit parameters (except when Δ becomes smaller than a cluster decay width). It is noteworthy that when SBD is directly [i.e., without the projection (3.40)] fitted in Δ , the temperature does the opposite of what it should: it decreases when Δ shrinks. This can be understood as a purely kinematic effect, namely, if one uses the relations (5.6) between SBD and NBD parameters to calculate the variation of T resulting from the variation of k and $\bar{n}$ in the model [11] of moving clusters, one finds just this unphysical behaviour which does therefore not reflect the different intrinsic situation at small rapidities but rather the kinematic effect of imposing phase space constraints. This can be shown even more simply in a numerical gedanken experiment, using the relations derived in Section 5 between SBD and NBD parameters and the simple behaviour of NBD parameters under the projection (3.40): suppose we fit a given full rapidity distribution $\{SBD: z, \bar{n}; NBD: \bar{n}, k \text{ with Eq. (5.6) approximately fulfilled}\}$; then we apply the projection (3.40) to it and fit the result directly with SBD and NBD thus obtaining parameters $z_\Delta, \bar{n}_\Delta$ and $\bar{n}_\Delta, k_\Delta$ respectively. Since, according to (3.28) $\bar{n}_\Delta = p\bar{n}$, $k_\Delta = k$ and since the relations (5.6) would again be fulfilled, it follows that $z_\Delta < z$ (i.e., the temperature $T_\Delta < T$), contrarily to what it should be.

We have been so explicit on this point because it shows that a direct fit to data in limited rapidity intervals obscures the physical interpretation of the resulting parameters: it cannot be the same as for full rapidity.

b) As suggested by the above considerations, the cluster sizes (n_c /cluster) do indeed grow when Δ decreases; this, of course, comes to an end when the size of Δ becomes smaller than the decay width of SBN clusters, which is of the order of 1 in rapidity.

c) Remember the interpretation [after Eq. (3.29)] of the parameters obtained in fits to cut (pseudo) rapidities when using the projection (3.40): $p\bar{n}$ and not $\bar{n}$ is the number of clusters in Δ .

d) The fits reported in Table 4.2 were made using in the projection formula the value $p(\langle n \rangle)$ as defined in Eqs. (3.20) with $n \rightarrow \langle n \rangle$. As $p = p(\langle n \rangle)$ is not necessarily the best average p value to be used, we made a few fits with leaving p as a free parameter. Starting from $p = p(\langle n \rangle)$ the p -value then consistently settled near to its starting value. Starting, however, from $p = 0$, it converged to a much lower p -value with greatly improved χ^2 . For instance, at 200 GeV, $\ln l < 5$ we obtain $\chi^2/NDF = 56/42$ with $p = 0.37$ while with $p = p(\langle n \rangle) = 0.96$ we

get $\chi^2/\text{NDF} = 98/43$. Obviously there are several χ^2 minima in the parameter space $\{z, \bar{N}, p\}$. We discarded, however, the good fits because they were obtained with p-values inconsistent with those following from their physical definition.

e) When going from full rapidity to $|\eta| < 5$ and further to $|\eta| < 3$ the fits become less good than at full rapidity but then improve again upon further reduction of Δ . We have no explanation for this phenomenon; it could have to do with the decrease of the rapidity distribution towards its ends. It should also be remembered that the acceptance of the UA5 detector is limited to $|\eta| < 5.0$ and that data pertaining to "full rapidity" contains some extrapolation based on the UA5 pp simulation program [16].

4.4. Trends of parameters; extrapolation to higher energies

In Fig. 4.6 we show the variation of $z(T)$ as a function of $\sqrt{s}$ and in Fig. 4.7 the corresponding variation of $\bar{N}$. While z cannot, in this model, grow beyond $z_0 = \ln 4 - 1$, there is no limitation on $\bar{N}$, though it seems to grow very slowly.

We have tried to fit the trends of both parameters with various simple functional forms, having no theoretical justification (except that $z \rightarrow z_0$) and chosen just for convenience. We find as one possibility

$$z = z_0 - A / [(\ln \sqrt{s})^B + C] \quad (4.6)$$

$$A = 6.6; B = 3.2; C = 14.4;$$

$$\bar{N} = a (\ln \sqrt{s})^b.$$

$$a = 4.5; b = 0.71. \quad (4.7)$$

There might be better choices, but these allow at least some moderate extrapolation to higher energies. For instance, at $\sqrt{s} = 1.8$ TeV we thus get

$$z = 0.3761$$

$$\bar{N} = 18.8 \quad (4.8)$$

Figure 4.8 shows the expected multiplicity distribution (charged) at this energy. We predict $\langle n_{ch} \rangle = 41$, $\langle n_{ch} \rangle / \text{cluster} = 2.2$ and $\sqrt{\langle n^2 \rangle - \langle n \rangle^2} = 23$.

5. - DISCUSSION

5.1. Comparison with NBD

NBD and SBD are both two-parameter distributions fitting the data approximately (with varying success). In addition they are similar in conception (if NBD is associated with the parton clan picture [3]). Therefore it might be useful to relate them to each other as follows: when both are fitted to the same data, the fit-parameters are mainly determined by the regions where most events are found; such fits will therefore lead to roughly the same mean values and dispersions for both distributions. We then expect that (see Ref. [3a] for D_{NBD}^2)

$$\bar{m}_{\text{SBD}} \approx \bar{m}_{\text{NBD}} (\text{total}) = \frac{3}{2} \bar{m}_{c, \text{NBD}} \quad (5.1)$$

$$D_{\text{SBD}}^2 \approx D_{\text{NBD}}^2 (\text{total}) = \left(\frac{3}{2} \bar{m}_c + \frac{9}{4} \frac{\bar{m}_c^2}{k} \right)_{\text{NBD}} \quad (5.2)$$

SBD parameters refer to total, NBD parameters to charged multiplicities. As shown in Eq. (3.27), NBD is form-invariant under charge projection [(3.36), (3.37)], but changes parameters according to (3.28) with $p = 2/3$:

$$k_c = k$$

$$\bar{m}_c = \frac{2}{3} \bar{m} (\text{total}) \quad (5.3)$$

hence the factor $3/2$ in Eq. (5.1). In fits of NBD normally $\bar{m}_c$ and k are determined. Combining Eqs. (5.1) with (3.16) and (3.17) we have

$$\bar{m}_{\text{SBD}} = \bar{N} z G' / G \approx \frac{3}{2} \bar{m}_{c, \text{NBD}} \quad (5.4)$$

$$D_{\text{SBD}}^2 = \bar{N} (2G' + z^2 G'^2) / G \approx \left(\frac{3}{2} \bar{m}_c + \frac{9}{4} \frac{\bar{m}_c^2}{k} \right)_{\text{NBD}}$$

which gives after simple manipulations

$$\bar{m}_{c, \text{NBD}} \approx \frac{2}{3} \bar{N} z G' / G$$

$$k_{\text{NBD}} \approx \bar{N} G'^2 / G G'' \quad (5.5)$$

From the BE (2.6) it follows that near the singularity z_0

$$G(z) = G_0 - \sqrt{z_0 - z} - \dots$$

$$G_0 = \ln 2$$

$$z_0 = \ln 4 - 1 \quad (5.6)$$

which can be used to derive an approximate inverse of (5.4):

$$\frac{z}{z_0} \approx \left(\frac{3\bar{n}_c}{k+3\bar{n}_c} \right) NBD \quad (5.6)$$

$$\bar{n}_c / G_0 \approx \sqrt{k(k+3\bar{n}_c)} / z_0 - k$$

At 200, 546 and 900 GeV, these relations are fulfilled within 0.1% for z/z_0 and 6% for $\bar{n}$. Asymptotically (5.4) gives for $\sqrt{s} \rightarrow \infty$

$$\bar{n}_c / G_0 \approx \frac{2}{3} \bar{n} \frac{z_0}{2G} \sqrt{z_0 - z} \rightarrow \infty \left(a_0 \frac{\ln \sqrt{z_0 - z}}{\sqrt{z_0 - z}} \right) \quad (5.7)$$

$$k \approx \frac{\bar{n}}{G} \sqrt{z_0 - z} \rightarrow 0 \left(a_0 \ln \sqrt{z_0 - z} \right)$$

hence

$$k \bar{n}_c / G_0 \approx \bar{n} \frac{z_0}{3} / G^2 \approx \frac{\bar{n}^2 z_0}{3 G_0^2} (1 + 2 \sqrt{z_0 - z} / G_0 + \dots) \quad (5.8)$$

which is fulfilled within 15% between 200 and 900 GeV. If this trend remains true for fits at still higher energies, it follows that for $\sqrt{s} \rightarrow \infty$

$$k \bar{n}_c / G_0 \rightarrow \ln^2 \cdot \bar{n} \quad (5.9)$$

Finally we mention without proof (which employs the asymptotics of SBD and NBD) that the tail of SBD lies always above that of NBD when both are fitted to the same data.

5.2. Physical interpretation

Leaving aside the many other models which give acceptable fits to the data, we have to discuss the problem that the rather similar fits obtained with NBD and SBD respectively could mean that the parton clan picture and the SBD cluster picture might both be true or at least overlap to some extent. Multiplicity distributions are by-products of both models, which indeed explain many other features of particle production.

The QCD-based parton picture is the more fundamental one, but SBD has the advantage to be a consistent summary of non-perturbative phenomenological properties of strongly interacting hadron matter in thermal equilibrium up to the phase transition. The two descriptions cover very different regions of basically the same physics, which meet in the neighbourhood of the phase transition, just where the final stages of the processes take place which end with the observable particle distributions.

In the multiplicity distributions derived here, some of the more prominent properties of SBD are instrumental:

- the existence of a singularity at some temperature T_0 ;
- the formation of Poisson-distributed massive clusters with a well-defined temperature dependent mass distribution (3.8);
- a completely fixed decay law (3.9) for these clusters;
- the obvious need to convolute the decay multiplicity distributions of these clusters with Poisson weight.

If any of these would be changed, a rather different multiplicity distribution would result. It is therefore satisfying that our results are consistent with these general properties:

- the average temperature increases when $\sqrt{s}$ grows, but seems to tend to a limit ($z \rightarrow z_0$);
- the mean charged multiplicity per cluster as follows from the SBD fits to the data (listed in Table 4.1) agrees with what is inferred (not unambiguously) from UA5 pseudorapidity correlations in these data [29], thus giving a rather explicit support to our cluster picture;
- while we have assumed for simplicity that all clusters come from the same average temperature (at fixed $\sqrt{s}$), in reality there would be a temperature distribution with higher temperatures being associated with the more central regions of rapidity and with more central collisions, leading in both cases to fewer and heavier clusters. This is consistent with our findings in fits to limited (symmetrical) rapidity windows, where T increases and $\bar{n}$ decreases when Δ becomes narrower.

As apparently both pictures - parton clans and SBD clusters - contain some truth, there seems to be some overlap region where partons and clans coexist with preformed hadrons and clusters. Preconfinement [30] as well as the similarity of MIT bags (which are QCD objects) with SBD clusters (see Section 2.1 and Refs. [3b,f]) suggest that in this region around the phase transition the two pictures are complementary. Just when passing this region, distributions are frozen to what is actually observed. It seems then natural that both pictures give meaningful interpretations of the data.

Assuming this to be true, it is tempting to speculate that upon hadronization clans become clusters. Local parton-hadron duality [31] was invoked by Giovannini and Van Hove to explain why NBD fits parton distributions as well as hadron distributions [3d] if one puts

$$\bar{n}_{hadron} \approx 2 \bar{n}_{parton} \quad (5.10)$$

$$k_{hadron} \approx k_{parton}$$

On the other hand, NBD fits result in an approximately energy independent average

number $\bar{N}_{clans}^{SBD}$ of parton clans

$$\bar{N}_{clans}^{SBD} \approx 8 \quad (5.11)$$

while we find that the number $\bar{N}_{cluster}^{SBD}$ grows very slowly such that

$$\bar{N}_{clusters}^{SBD} \approx 15 \text{ to } 18 \text{ between } 200 \text{ and } 900 \text{ GeV} \quad (5.12)$$

so that

$$\bar{N}_{clusters}^{SBD} \approx f(\sqrt{s}) \cdot \bar{N}_{clans}^{SBD} \quad (5.13)$$

with a slowly growing factor $f(\sqrt{s}) \approx 2$ in the region 200 to 900 GeV, which, if really clans become clusters, implies that on the average a clan gives about two clusters, similar to (5.10).

Finally we remark that - independent of these speculations - the ability of SBM to fit multiplicity distributions gives further (indirect) evidence in favour of the existence of a phase transition; all the above-mentioned prominent properties of SBM are linked to this phase transition and they are all necessary to give SBM its actual shape and variation of parameters.

Table 4.1 - Fits in full rapidity

$\sqrt{s}$	Type	z	N	χ^2/NDF	$n_c/cluster$	Fig.	Ref.	χ^2/NDF	n_c	Remarks
7.87	pp	0.124	7.56	5.29	0.70	4.2a	[19]	8.6/6	2-18	Includes diffraction first few excluded
13.8	pp	0.235	8.6	6.97	0.81	4.2b	[25]	4.4/5	6-20	Significant difference between pp and pp
13.8	pp	0.199	9.3	7.09	0.76	4.2c	[20]	16.9/7	2-20	Includes diffraction first few excluded
22	pp	0.262	10.5	9.02	0.86	4.2d	[21]	9.2/8	8-28	Includes diffraction first few excluded
30.4	pp	0.290	11.4	10.60	0.93	4.2e	[22]	24.5/14	2-34	
44.5	pp	0.316	11.8	12.16	1.03	4.2f	[22]	7.5/16	2-38	
52.6	pp	0.323	11.81	12.59	1.07	4.2g	[22]	23.4/18	2-42	
62.2	pp	0.320	12.9	13.61	1.06	4.2h	[22]	59.8/17	2-40	Very bad fit; irregular in several respects
200	pp	0.3591	14.7	21.38	1.45	4.2i	[17]	76.9/43	3-64	Shoulder develops; just tolerable fit
546	pp	0.3697	16.81	29.72	1.77	4.2j	[23]	106/62	3-99	Shoulder stronger; bad fit
900	pp	0.3739	17.6	35.12	2.0	4.2k	[24]	154/73	3-114	Shoulder very strong; very bad fit

χ^2/NDF	fit	Remarks	ρ	$n_c/\text{cluster}$	ρ	N	z	$ n <$	χ^2/NDF	fit	Remarks
76.9/43	3-max	Same as in Table 4.1	1	1.46	1.56	13.7	0.3627	5	13.7	2-max	98.6/43
98.5/40	2-max		0.724	1.75	1.75	12.2	0.3692	3	12.2	2-max	46.3/24
17.6/9	1-max	Fig. 4.5	0.117	1.66	1.75	12.2	0.3712	1.5	12.2	2-max	17.6/9
154.7/3	3-max	same as in Table 4.1	1	2.02	2.24	15.9	0.3739	8	17.6	3-max	194.7/5
148.6/4	2-max		0.629	2.56	2.56	13.9	0.3795	3	13.9	2-max	72.1/38
23.2/15	2-max		0.101	2.47	2.70	13.2	0.3801	1.5	13.2	2-max	23.2/15
0.3806	2-max	Nearest to	0.101	2.47	2.70	13.2	0.3801	0.5	13.2	2-max	0.3806
14.4	2-max		0.101	2.47	2.70	13.2	0.3801	0.5	13.2	2-max	14.4

Table 4.2 - Fits in limited pseudorapidity ($|\eta| < 2.4$)

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FIGURE CAPTIONS

Fig. 2.1 a) The curve $z(G)$ as given by Eq. (2.5);
b) Its graphical inverse by exchanging z and G .

Fig. 3.1 Decay trees from initial cluster of mass M to final particles:
a) general,
b) if two- and three-body decays dominate.
SBM considers the totality of all possible decay trees.

Fig. 4.1 a) Fit of "observed" SBD to raw UA5 data (900 GeV);
b) Original SBD vs. corrected UA5 data (900 GeV).

Fig. 4.2 a)-k)
Fits to eleven sets of data plotted vs. corrected data. Figures 4.2b,d contain single diffraction contribution (first few points excluded from fit). The wiggles in experimental data of Fig. 4.2j are due to an old (now abandoned) correction method [23].

Fig. 4.3 Fit of "observed" SBD vs. raw UA5 data (200 GeV).
Fig. 4.4 Fit of "observed" SBD vs. raw UA5 data (546 GeV).
Fig. 4.5 Example (neither the best nor the worst) of fit in limited pseudorapidity $|\eta| < 1.5$ (UA5, 200 GeV).
Fig. 4.6 Variation of z as function of $\sqrt{s}$. For $\sqrt{s} \rightarrow \infty$, we expect $z \rightarrow z_0 = \ln 4 - 1$.
Fig. 4.7 The average number $\bar{N}$ of clusters as functions of $\sqrt{s}$.
Fig. 4.8 "Predicted" multiplicity distribution at $\sqrt{s} = 1.8$ TeV calculated with parameters found by extrapolation [Eqs. (4.6) to (4.8)].

Fig. 2.1

Fig. 3.1

Fig. 4.1a,b

Fig. 4.2a-k

Fig. 4.3

Fig. 4.4

Fig. 4.6

Fig. 4.5

Fig. 4.7

Fig. 4.8