CERN-TH.5505/89

THE DECAY $Z \rightarrow \pi^0 \gamma$

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ABSTRACT

The rate for Z decay into $\pi^0 \gamma$ is calculated using the pion dominance of the divergence of the axial vector current, which is intimately related to the chiral anomaly. The branching ratio can be sizeable. The relatively frequent observation of such spectacular events would give a test of PCAC in a new regime. This $Z \rightarrow \pi^0 \gamma$ is the first example of a new class of Z and W decays.

*) Work supported in part by the United States Department of Energy under grant No. DE-FG02-84ER40158.

With LEP beginning to operate, the properties of the Z boson will be intensely investigated in the near future. The known decay modes of Z are lepton pairs and quark pairs, with possibly additional gluons and photons. In the case of quark pairs, hadronization usually leads to many hadrons in the final state. For example, in the first Z event seen in the ALEPH detector, there are over twenty hadrons. For hadronic events, isolated hadrons are believed to be very rare, because a jet is unlikely to leave practically all its momentum to a single hadron, the suppression factor being about 10^{-3} [1]. Isolated π^+ are expected to come almost exclusively from τ^+ decays.

It is the purpose of this paper to initiate the study of possible decays of Z into very few hadrons and photons. Such events, if present, are easy to observe, easy to analyze, and spectacular.

The Z boson is a spin-1 particle with both vector and axial vector couplings. Its decays into $\gamma\gamma$ and into $\pi^0\pi^0$ are both forbidden because of the identity of the final-state particles and the conservation of angular momentum [2]. Therefore, an extreme example of Z decay into very few hadrons and photons is $Z \rightarrow \pi^0\gamma$. This is the subject of study here. Note that, unlike π^+ , there is no known source of isolated π^0 . Experimentally, it is not necessary to distinguish π^0 and γ , i.e., it is sufficient to observe two back-to-back clusters of electromagnetic energy.

The analysis of $Z \rightarrow \pi^0\gamma$ decay is similar to the well-known case of $\pi^0 \rightarrow \gamma\gamma$ decay. Even though anomalies cancel each other in the standard model, the AVV (axial-vector-vector, or Adler-Bell-Jackiw) anomaly [3] nevertheless contributes to these two processes because the presence of π^0 allows only the u and d quark loops, not the electron loop. We argue that the Z γ coupling is determined by the anomaly contribution, and therefore calculable.

We consider the γZ transition in an external axial field, retaining for our purpose the neutral isovector part only. The transition is taken at a low momentum transfer $q^2 = (k-k')^2$, where k and k' are the photon ($k^2 = 0$) and Z ($k'^2 = M^2$) momenta respectively.

While the Z has both vector and axial couplings, only the vector part of the Z coupling contributes. In perturbation theory the loop with axial coupling cancels when summing over the two different orderings, or going clockwise and anticlockwise around the loop. The two terms are equal in the case of vector coupling, and the amplitude is then symmetric under the exchange of the momenta and polarization vectors of the Z and of the photon. The form of the amplitude is fixed by Lorentz invariance and parity. As is well known, there are six independent tensors. Vector current conservation imposes two relations among them.

Since the pion is the only low mass isovector particle, one should distinguish at low momentum transfer the one-pion contribution from a remainder corresponding to a direct coupling. We therefore write the transition amplitude as

$$R_{\mu\nu\tau}(k, k') = \tilde{R}_{\mu\nu\tau}(k, k') - f_\pi \frac{q_\tau}{q^2 - m_\pi^2} F(k, k') \Sigma_{\mu\nu\alpha\beta} k^\alpha k'^\beta \quad (1)$$

Here μ, ν and τ are the polarization indices of the Z, the photon and the axial field respectively. The pion decay constant f_π is defined in the usual way, with $f_\pi \approx 92$ MeV. The $\pi Z\gamma$ coupling is the value of F when mass-shell conditions are imposed.

According to PCAC, one imposes that the axial current should be conserved in the zero pion mass limit. The divergence of the axial amplitude (1) should therefore be zero in that limit [4]. This reads:

$$q^\tau R_{\mu\nu\tau} = 0 \quad (2)$$

and, using (1),

$$q^\tau \tilde{R}_{\mu\nu\tau} - f_\pi F \Sigma_{\mu\nu\alpha\beta} k^\alpha k'^\beta = 0 \quad (3)$$

While, as previously said, the most general term for $\tilde{R}$ involves six independent tensors, one finds only one term taking the divergence, namely

$$q^\tau \tilde{R}_{\mu\nu\tau} = \tilde{\gamma} \Sigma_{\mu\nu\alpha\beta} k^\alpha k'^\beta \quad (4)$$

This gives

$$F = \frac{1}{f_\pi} \tilde{\gamma} \quad (5)$$

The $\pi Z\gamma$ coupling is therefore simply related to the value of the direct axial coupling, as is the case in the standard derivation of the Goldberger-Treiman relation. While this holds in the zero pion mass limit, it is usually assumed to remain a good approximation when the pion is on its mass shell. We shall first assume this to hold. However, in view of the very large Z mass, we shall return to this point later. The mass-shell conditions fix the values of all the invariants on which F and $\tilde{\gamma}$ depend.

Our next step is to assume that the large value of the Z mass justifies a calculation of $\tilde{r}$ to lowest order in perturbation theory, considering only the couplings of the Z , photon and axial field to quarks. There are only two diagrams, given by the Figure. The W loop does not contribute in this case. For the reasons already mentioned, only the vector part of the Z contributes.

The one-loop Feynman graph calculation of such an amplitude (see Figure) raises important but well-known difficulties. The linear divergence of each term taken separately is such that the circulating momentum cannot be translated. As a result, the chiral symmetry of the Lagrangian is lost when including such a radiative term. The calculation procedure is to obtain those pieces of the amplitude which are well behaved and generate the remaining ones through the current conservation condition [3]. This is how the $\pi\pi\gamma$ decay rate was calculated. Comparison with experimental data gave one of the first instances of evidence for colour [5].

In the π^0 decay case, all unknown terms which are not determined by either the pion contribution or the anomaly contribution can be eliminated by going to the $k = k' = 0$ limit. The decay rate is thus obtained as a "low-energy" theorem. In our present case we rely on perturbation theory to calculate the amplitude $\tilde{r}$, and therefore do not need to go to the $k = k' = 0$ limit.

The explicit one-loop calculation of $\tilde{r}$ gives two terms. Factorizing out the product of the relevant coupling written as $e^2 S$ times a factor $(2\pi)^{-4}$ associated with the loop amplitude, we get

$$\tilde{r} = 2 m \nu_0 + 8 \pi^2 \frac{e^2 S}{(2\pi)^4} \quad (6)$$

The first term is the standard one, in which the quark mass has been factored out. The second one is the anomaly contribution. With a massive Z , the loop integral appearing in ν_0 is only logarithmically divergent when the quark mass goes to zero, whereas it includes an m^{-1} factor in the low-energy ($k = k' = 0$) limit. In our case we can take the zero quark mass limit in (6), since the isovector current couples to u and d quarks only. This gives

$$\tilde{r} = \frac{2}{f_\pi} \frac{\alpha}{\pi} S \quad (7)$$

Here S corresponds to the sum of a product of couplings over the u and d quark loops relevant for the pion. Including colour, one finds

$$S = \frac{1}{g} \frac{1 - 4 \sin^2 \theta_W}{\sin \theta_W \cos \theta_W} \quad (8)$$

As is easily seen from (8), such an anomalous contribution to the axial coupling is, as already mentioned, cancelled by the electron loop. However, when extracting the pion coupling we should keep only the u and d quark loops and retain the anomaly.

One can calculate the $Z\pi\gamma$ rate:

$$\Gamma = \frac{1}{12\pi} \left(\frac{M}{Z} \right)^2 F^2 \quad (9)$$

and, taking the numerical value of F [(7) and (8)], one obtains $\Gamma = 4.3$ MeV. This corresponds to a branching ratio of $(1.7 \pm 0.7) \times 10^{-3}$. The value of $\sin^2 \theta_W$ used [6] is $\sin^2 \theta_W = 0.2283 \pm 0.0045$. The uncertainty in this value is responsible for the bulk of the error quoted in the branching ratio. With such a branching ratio, this decay mode would be well within reach of the first LEP experimental runs. The signature for this decay $Z \rightarrow \pi^0 \gamma$ is spectacular, and the background due to the process $e^+e^- \rightarrow \gamma\gamma$ can be separated out because of its forward peaking.

Exciting as it seems, this branching ratio raises three serious problems. It should therefore not be taken at face value, but rather as an indication that the $\pi^0 \gamma$ mode may have a sizeable branching ratio which makes it worth looking for in the first LEP runs.

The first question stems from the fact that the branching ratio of 0.2%, which looks surprisingly large, is actually already suppressed by the value of $\sin^2 \theta_W$, which is close to 0.25. The branching ratio includes a factor $(1 - 4 \sin^2 \theta_W)^2$. If $\sin^2 \theta_W$ were 0.5, which is not excluded by any purely theoretical consideration, this radiative branching ratio would be about 13%, which is embarrassingly large.

This brings up the second question, which comes from the extension of our result to the decay $W^\pm \rightarrow \pi^\pm \gamma$. This decay is especially interesting, because its observation would give an excellent way of determining the mass of the W . In this case, the vector coupling is not reduced and the expected branching ratio would be of the order of 10%. This is again surprisingly and embarrassingly large, although it does not seem to be experimentally excluded yet [7].

The third problem is even more serious. Since it is the vector coupling of Z and $W^\pm$ that is responsible for the decays $Z \rightarrow \pi^0 \gamma$ and $W^\pm \rightarrow \pi^\pm \gamma$, they may be

replaced by the photon itself. In other words, the present consideration may be extended to the scattering process $e^+e^- \rightarrow \gamma^* \rightarrow \pi^0\gamma$, which is actually even more closely related to π^0 decay. Similar to (9), the cross-section for this process at high energies is found to be

$$\sigma = \frac{\alpha^3}{24 \pi^2 f_\pi^2} \quad (10)$$

This cross-section, unlike those for any other one-photon annihilation processes, does not decrease as S^{-1} . It corresponds to one unit of R at 34 GeV already, and cannot have been missed as an extra contribution to the $\gamma\gamma$ channel. The $\pi^0\gamma$ angular distribution is $1+\cos^2\theta$, whereas the two-photon annihilation process is strongly peaked forward. However, if it were reduced by a factor of about 5, then it may not have been seen at PETRA and TRISTAN [8]. At least as large a suppression factor should apply also to $Z \rightarrow \pi\gamma$ and $W \rightarrow \pi\gamma$.

In view of these three problems, let us review the assumptions used in arriving at the decay rate for $Z \rightarrow \pi^0\gamma$. The first assumption is PCAC in the form of Eq. (2), which expresses the conservation of the axial current in the limit where the pion mass is zero. The second assumption is that this PCAC can be extrapolated to the actual pion mass. The third assumption is that the Feynman diagrams of the Figure lead to the main contributions to the axial coupling. In our view, the third assumption is almost certainly correct; it is the second assumption that is suspicious at multi-GeV energies. The needed extrapolation may involve a sizeable factor.

The branching ratio of 0.17%, even if suppressed by one or two orders of magnitude, is still very much larger than the previous estimate based on the presence of a form factor of the order of $(m_\pi/M_Z)^2$ and a resulting branching ratio at the level of 10^{-11} [9]. The reason is that anomalies come from integration over extremely large loop momenta and are therefore effectively point interactions. As such, considerations on the pion size and on possible form factors are not relevant. Cancellation of anomalies does not mean absence of the effects of anomalies.

The observation of the $Z \rightarrow \pi^0\gamma$ mode would be the first example of a large class of interesting events, two further examples being $Z \rightarrow \pi^+W^-$ and $Z \rightarrow \eta\gamma$.

ACKNOWLEDGEMENTS

We are grateful to Sau Lan Wu for enquiring about the special property of such radiative modes. We would like to thank John Bell and John Ellis for helpful discussions. T.T.W. thanks the CERN Theory Division for its hospitality.

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FIGURE CAPTION

Quark-loop contributions to the Z axial photon coupling.