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**The Proton Mass, the $\pi - N$ Sigma term and
the Scale Anomaly**

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Abstract

It is argued that the scale anomaly of QCD makes the nucleon mass more strongly dependent on the sea quark masses than the naive inference.

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Our recent lattice Monte Carlo simulations [1] have shown that the proton mass is more sensitive to variation of the sea quark mass than to variation of the valence quark mass, in the region roughly corresponding to the strange quark mass, i.e., $m_{sea} = m_{val} \approx m_s$. The lattice result is only at a qualitative stage. It is beset by unknown systematic errors, and is nowhere close to the scaling region from where it would be possible to make a prediction for the continuum theory. Yet it is significant enough to make one wonder about what it is that is actually going on.

A closer inspection reveals two features embedded in the results. The first is a global effect. All hadron masses are significantly affected by variation of m_{sea} , while the variation of dimensionless mass ratios with m_{sea} is smaller. This suggests that changing m_{sea} while keeping the bare gauge coupling fixed is actually changing the scale of the theory. The second is a differential effect. The sea quarks do not affect all hadrons in an identical fashion; in particular, the difference between the rho and the nucleon stands out. This is an important clue in trying to understand the dynamics of QCD.

The global effect is after all an expected result; the scale of the theory depends on the β -function, which in turn depends on the value of the sea quark mass. In the “mass-independent” dimensional regularisation scheme, one often writes down at an energy scale μ

$$\beta(g(\mu)) \equiv \frac{\partial g}{\partial(\ln\mu)} = -b_0 g^3 + \mathcal{O}(g^5), \quad b_0 = \frac{33 - 2N_f}{3(4\pi)^2}. \quad (1)$$

However, this result is meaningful only when used (a) in conjunction with matching conditions following from the decoupling theorem [2], and (b) when μ is not close to the scale where heavy particles are integrated out. The matching conditions are imposed by replacing N_f in eq.(1) by $\sum_{i=1}^{N_f} \theta(\mu - \mu_i)$. The step function θ is only a crude approximation of what might be happening when the scale μ is in the vicinity of the boundary μ_i characterising the effective mass of the i^{th} quark. (In momentum space regularisation, which is much closer to lattice regularisation than dimensional regularisation is, the step function gets replaced by a smoother function.) In any case, changing m_{sea} from 0 to ∞ amounts to decreasing the number of quark flavours, N_f , by one:

$$\int_0^\infty dm_i \frac{\partial \beta(g, m_i)}{\partial m_i} = -\frac{1}{24\pi^2} g^3 + \mathcal{O}(g^5). \quad (2)$$

It is common practice to trade off the scale-dependent gauge coupling $g(\mu)$ for a dimensionful parameter Λ_{QCD} characterising the scale of the theory. All dimensionful quantities in QCD can only be determined in units of this scale parameter. QCD by itself can only predict what the dimensionless ratios would be, and not what the scale of the theory is. (For example, the proton does not weigh 938 MeV; rather whatever the proton weighs, we call it 938 MeV.) Therefore in phenomenological discussions, when one talks

about changing the mass of a quark, one considers only its differential effects, i.e., m_{sea} is changed while holding Λ_{QCD} fixed. Any global effect of changing m_{sea} , which can be absorbed in the renormalisation of the bare coupling constant g , is not directly accessible to measurements. The experimental data can provide information only on the differential effects; the global scale Λ_{QCD} essentially remains a free parameter.

In the lattice formulation, the quark mass appearing in the fermion determinant is designated m_{sea} , while the mass of the quarks coupled to external sources is labelled m_{val} . In the physical world, on the other hand, there is no such distinction between the sea and the valence quarks. What is labelled as sea or valence contribution is actually scale-dependent, and we have to define a convention for separating the two.

A convenient way to parametrise our ignorance is to use the language of chiral Lagrangians. For simplicity I restrict myself to the case of 3 quark flavours, exact isospin symmetry ($m_u = m_d = m$) and first-order symmetry breaking formalism. The pseudo-scalar Goldstone boson fields are represented by the special unitary matrix $\Sigma = \exp(2i\pi^a T^a/f)$, which transforms as the $(3, 3)$ representation of the chiral $SU(3)_L \otimes SU(3)_R$ symmetry. The mass term for these fields defines the quark mass matrix M

$$M_{PS} = v \, Tr(M\Sigma + M\Sigma^\dagger). \quad (3)$$

In the quark language, the chiral limit is taken by letting the valence quark masses go to zero. In this effective theory, the chiral limit corresponds to letting M go to zero. So it is natural to associate the matrix M with the valence quarks. Parameters such as v and f are determined in terms of the QCD scale, and depend on the gluonic interactions as well as on m_{sea} . The sea quarks change M too, but only through a multiplicative renormalisation so as to leave the chiral limit unaffected. In phenomenological discussions, one typically varies M_{PS} by varying M while holding v and f fixed.

The baryon octet fields can be represented by a 3×3 traceless matrix B , but due to spontaneous breakdown of chiral symmetry, B (or any other field which remains massive in the chiral limit) is not restricted to transform as a particular representation of the chiral symmetry. The baryon mass term in the leading order can be written in the form:

$$M_B = (M_0 + \frac{S}{3} Tr(M)) Tr(\bar{B}B) + F Tr(\bar{B}[M_{NS}, B]) + D Tr(\bar{B}\{M_{NS}, B\}), \quad (4)$$

$$M_{NS} = M - \frac{1}{3} Tr(M). \quad (5)$$

The first term on the r.h.s. of eq.(4) stands for the singlet contribution common to all the members of the multiplet, M_0 and $Tr(M)$ denoting the gluonic and quark parts respectively. The non-singlet part of the mass matrix, M_{NS} , together with the reduced matrix elements F and D , governs the mass splittings within the baryon octet. The non-singlet

contribution is associated with the valence quarks, while the singlet matrix element S depends on both the valence and the sea quarks.

One can hide the sea quark contribution by separating S into its valence and sea components, and then absorbing the sea quark contribution into a redefinition of the quark mass matrix:

$$S = S_{val} + S_{sea}, \quad M' = M + \frac{S - S_{val}}{3S_{val}} Tr(M). \quad (6)$$

Such a rearrangement results in a new quark mass matrix for the baryon octet, M' , which is shifted with respect to the mass matrix M appearing in the pseudo-scalar mass term by a singlet piece. Though a non-zero value for $M' - M$ violates the OZI rule, it is not forbidden by any symmetry. Letting $M' \neq M$ alters the derivatives of the baryon mass with respect to the various quark masses [3]. But this is perfectly all right once one realises that the singlet and the non-singlet terms are not renormalised the same way, i.e., the quark mass ratios are not independent of the scale. (It is worthwhile to note that the only practical determination of quark mass ratios comes from the pseudo-scalar meson sector; all the other mass relations in meson and baryon multiplets refer to quark mass differences which remain unaffected by a shift proportional to $Tr(M)$ [4].) The question is only about understanding the magnitude of this effect.

The sigma term appearing in $\pi - N$ scattering amplitude is defined as [5]

$$\sigma = \langle N | \frac{1}{3} \sum_{a=1}^3 [X^a, [X^a, H_{SB}]] | N \rangle, \quad (7)$$

where H_{SB} is the symmetry breaking term in the baryon Hamiltonian and X^a are the axial vector charge operators. Formal evaluation of the commutators in QCD yields

$$\sigma = \langle N | m(\bar{u}u + \bar{d}d) | N \rangle = m \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right) M_N, \quad (8)$$

where the derivatives with respect to quark masses are to be evaluated at a fixed value of the gauge coupling. The valence quark component of σ can be estimated by a fit to the baryon octet masses:

$$\sigma = \sigma_{val} + \sigma_{sea}, \quad \sigma_{val} = m(3F - D) \approx 26 \text{ MeV}. \quad (9)$$

On the other hand, the experimental data on $\pi - N$ scattering is consistent with $\sigma \approx 50$ MeV [6]. This leaves ample room for contribution from the sea quarks: $\sigma_{sea} \approx \sigma_{val}$. A sea quark matrix element of this size produces a large value for $\partial M_N / \partial m_s$ (the strange quark is present in the proton only in the sea). This apparently hard to swallow feature

can be reconciled with one's quark model intuition by making the constituent quark mass include the effect of the sea quarks [7].

I stress that the validity of this standard first-order flavour $SU(3)$ breaking analysis is by no means compelling. One can keep $M' = M$, and still explain both the baryon octet mass splittings and the experimental σ -term, by complicated higher order chiral symmetry breaking effects [8]. The first-order flavour $SU(3)$ breaking assumption is necessary only if one wants to estimate the influence of the strange quark by converting the σ -term value to $\partial M_N / \partial m_s$. The situation in the real world is likely to be somewhere in between, i.e., $\partial M_N / \partial m_s > 0$, but the non-linear effects are important as well.

Now let us turn to arguing how a sizeable sea quark contribution may arise in QCD. The leading perturbative diagram where a sea quark mass contributes to the renormalisation of a valence quark mass occurs at 2 loops. As shown in fig. 1a, it has a closed quark loop connected to a valence quark line by gluons. Evaluation of this diagram produces chiral logarithms and infra-red divergences. This problem can be bypassed by considering 1-loop operator mixing between the sea quarks and the gluons. The infra-red catastrophe is then dumped into the vacuum expectation values of the gluon condensate $\langle [FF] \rangle$ and the quark condensates $\langle [m\bar{q}q]_i \rangle$.

The triangle quark loop of fig. 1b is finite and is easily evaluated to be

$$\Delta_{\mu\nu} = -\frac{ig^2}{12\pi^2} (p^2 g_{\mu\nu} - p_\mu p_\nu) A\left(\frac{m^2}{-p^2}\right), \quad (10)$$

$$A\left(\frac{m^2}{-p^2}\right) = \int_0^1 dx \frac{6x(1-x)m^2}{m^2 - x(1-x)p^2}. \quad (11)$$

For a heavy quark, $A(\infty) = 1$, and the sea quark contribution can be replaced by an insertion of the operator $[FF]$ [9]. As necessitated by the matching condition, this result is equivalent to changing the number of quark flavours in the β -function by one. The next term in the heavy quark mass expansion is a 4-Fermi interaction $(\bar{q}\gamma_\mu T^a q)^2$. Instead of using such an expansion, let us crudely estimate the effect of a sea quark of finite mass by calculating $A(m^2 / -p^2)$. As shown in fig. 2, this function is monotonic and crosses over from 0 to 1 in the region $-p^2 \approx 4m^2$. Taking the relevant scale for the gluons to be $-p^2 = m_0^2 \approx \Lambda_{QCD}^2$, the contribution of a sea quark can be estimated as a shift in the number of quark flavours by $A(m^2 / m_0^2)$. It is not unreasonable to take $A(m = m_s) = 1/2$. The strange quark then falls into a region where A is significantly different from zero and varying with the quark mass.

How the QCD scale changes with the number of quark flavours is accomplished by considering the renormalisation group scaling, $\Lambda_{QCD} \sim \exp(-1/(2b_0 g^2))$, implies that

$$\frac{\Delta \Lambda_{QCD}}{\Lambda_{QCD}} = \left(\frac{\Delta b_0}{b_0} \right) \left(\frac{1}{2b_0 g^2} \right). \quad (12)$$

It is not at all obvious what effective value of g^2 should be used to characterise the scale where the σ -term is measured. However, for $g^2 = \mathcal{O}(1)$, the second factor on the r.h.s. is of $\mathcal{O}(10)$. This is indeed the size of the effect seen in lattice calculations — changing the number of sea quark flavours from 0 to 2 while holding $g^2 = 1$ fixed, can alter the scale of the theory by a factor of 2. From this point of view, a sizeable shift (of the order of 20%) in the scale, and consequently in the hadron masses, is not unexpected when the strange quark mass is varied from 0 to m_s . In other words, the $S \text{Tr}(M)$ term in eq.(4) is a mere parametrisation. It turns out to be large because it carries the burden of the scale change while the other parameters are held fixed.

The above discussion only concerns the overall effect of the sea quarks on the QCD scale. One also needs to understand how the influence of sea quarks can vary from hadron to hadron. This is a strictly non-perturbative phenomenon, requiring one to understand how the quark operators are embedded in hadrons. Fortunately, the scale anomaly provides the basic ingredients for understanding both the global and differential effects. The trace of the stress-energy tensor consists of two separately renormalisation group invariant (at least up to two loops) pieces [10] :

$$\theta_{\mu\mu} = O_1 + O_2 , \quad (13)$$

$$O_1 = \frac{\beta(g)}{g} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \sum_i \gamma_m [m\bar{q}q]_i , \quad O_2 = \sum_i [m\bar{q}q]_i . \quad (14)$$

The lower dimension operator $[m\bar{q}q]_i$ mixes into the higher dimension operator $[FF]$. The combination O_1 is responsible for giving hadrons (other than the Goldstone bosons) non-zero masses in the chiral limit. The chiral symmetries prevent the pseudo-scalar masses from getting any contribution from this operator. By definition:

$$\langle \pi | \theta_{\mu\mu} | \pi \rangle = m_\pi^2 , \quad \langle N | \theta_{\mu\mu} | N \rangle = m_N . \quad (15)$$

On the other hand current algebra dictates that (at least in the leading order)

$$\langle \pi | \sum_i [m\bar{q}q]_i | \pi \rangle = m_\pi^2 . \quad (16)$$

It immediately follows that

$$\langle \pi | O_1 | \pi \rangle = 0 . \quad (17)$$

This result is peculiar to the case of pions; the operator O_1 has non-zero expectation values in all other hadronic states. The result is related to the fact that the pion, being a pseudo-Goldstone boson, has a vanishing scaling dimension [11]. This discussion brings out the crucial feature that the operator contributing to $\partial M_N / \partial m$ differs from the operator

contributing to M_π^2/m , through the dependence of O_1 on the quark mass. In general, for any hadron state having a non-zero expectation value for operator O_1 , its mass has a distinct quark mass contribution which is different from that in the case of the pion, i.e., $M' \neq M$.

The quark masses affect the operator O_1 in two ways. The sea quark influence enters through the β -function. If $M_N^{(0)}$ is the mass of the proton in the limit where the up and down quark masses vanish, then the fractional contribution of the strange quark to the proton mass can be estimated as

$$\Delta M_N / M_N^{(0)} = \Delta b_0 / b_0 . \quad (18)$$

This is a much smaller fraction than the result of eq.(12).

The additional contribution provided by the anomalous dimension γ_m further enhances the quark mass dependence of the hadron masses, since (at least for sufficiently large scale μ)

$$\gamma_m \equiv \frac{\partial(\ln m)}{\partial(\ln \mu)} = -\frac{1}{2\pi^2} g^2 + \mathcal{O}(g^4) < 0 , \quad (19)$$

The expectation values of both O_1 and $[m\bar{q}q]$ in a hadron are independent of the energy scale, but $(-\gamma_m)$ increases as the scale decreases. Therefore, upon decreasing the scale, the fractional contribution of the $[m\bar{q}q]_i$ part of O_1 goes up, while the contribution of the $[FF]$ part goes down.

The arguments in this paper have been by and large at a qualitative level, because it is unclear what the appropriate values of β and γ_m are for use in these low energy on-shell matrix elements. Nonetheless, to reiterate the distinction I have tried to make, the variation of a hadron mass when a sea quark mass is changed is large, but the fractional contribution of a sea quark to the hadron mass is small. The former corresponds to taking a quark mass derivative at fixed g , while the latter corresponds to taking a quark mass derivative at fixed Λ_{QCD} . The large difference between the two is caused by the scale breaking inherent in QCD. Eqs. (11,12,17,18) help us understand the features seen in the lattice calculations and in the analysis of the σ -term. In the standard model, the quark mass term arises from the couplings of the Higgs boson to the quarks. The above considerations can be readily extended to the case of Higgs-hadron couplings along the lines of refs. [9][11].

Finally, it is hard to miss the parallel between the analysis here and the other instance where contribution of sea quarks has turned out to be important, namely the coupling of the singlet axial current to the nucleon [12] [13] [14]. In both cases, it is the mixing of quark operators into the gluonic sector, caused by the well-known anomalies, that is responsible for giving rise to large sea quark effects.

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Figure Captions

- [1] (a) The 2-loop diagram for sea quark contribution to the valence quark mass renormalisation, and (b) the 1-loop diagram mixing the sea quarks with the gluons.
- [2] The function A of eq.(11) illustrating the variation of the effective number of quark flavours with the sea quark mass.

Fig. 1

Fig. 2