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A Light Zeldovich-Konopinski-Mahmoud Neutrino with a Large Magnetic Moment

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Abstract

We propose a non-Abelian extension of a Zeldovich-Konopinski-Mahmoud lepton number symmetry which gives rise to a naturally light Dirac neutrino with a magnetic moment of $O(10^{-11}\mu_B)$. The neutrino mass appears first at the two-loop level and is well below the experimental upper bound.

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two contributions to m_ν by introducing a symmetry between the two generations. To this end, we will extend the ZKM lepton number symmetry to a non-Abelian group yielding the desired result.

The simplest extension of $U(1)_{ZKM}$ to a non-Abelian group involves a single additional group element S of order 2. The resulting group EL (extended lepton number symmetry) is defined by the following composition rules:

$$\begin{aligned} g(\alpha) \circ g(\beta) &= g(\alpha + \beta), & g(\alpha + 2\pi) &= g(\alpha) \\ S^2 &= g(0) = e, & S \circ g(\alpha) \circ S &= g(\alpha)^{-1} = g(-\alpha) \end{aligned} \quad (3)$$

$$g(\alpha) \in U(1)_{ZKM}, \quad n \in \mathbb{Z}.$$

The irreducible representations of this group are either one- or two-dimensional:

$$\begin{aligned} \mathbb{1}^+ : g(\alpha) &\rightarrow 1, & S &\rightarrow 1 \\ \mathbb{1}^- : g(\alpha) &\rightarrow 1, & S &\rightarrow -1 \\ \underline{2}^{(k)} : g(\alpha) &\rightarrow \begin{pmatrix} e^{ik\alpha} & 0 \\ 0 & e^{-ik\alpha} \end{pmatrix}, & S &\rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & k \in \mathbb{N}. \end{aligned} \quad (4)$$

The Clebsch-Gordan decomposition of tensor products of irreducible representations is easily worked out.

The concrete model we are going to investigate in detail is a two-generation model with the usual gauge group $SU(2) \times U(1)_Y$. For the moment, the second generation can be either μ or τ . In addition to the left-handed doublets L_{iL} there are three (instead of the usual one) $SU(2)$ singlets $\ell_{iR}, f_{iL}, f_{iR}$ with $q = -1$ ($Y = -2$) for each generation $i = 1, 2$. The Higgs sector consists only of doublets: three doublets $\Phi_I, \Phi_{II}, \Phi_R$ with the usual hypercharge $Y = 1$ and an unconventional doublet Φ_L with $Y = -3$ (containing Φ_L^-, Φ_L^+ as component fields). With respect to EL , we postulate the following representation content:

$$\begin{aligned} \begin{pmatrix} L_{iL} \\ L_{iR} \end{pmatrix}, \begin{pmatrix} \ell_{iR} \\ f_{iR} \end{pmatrix}, \begin{pmatrix} f_{iL} \\ f_{iR} \end{pmatrix} &\sim \underline{2}^{(1)} \\ \Phi_I, \Phi_L &\sim \mathbb{1}^+, \quad \Phi_{II}, \Phi_R &\sim \mathbb{1}^-. \end{aligned} \quad (5)$$

To suppress some unwanted couplings, we introduce an additional Z_2 symmetry. Under the action of the generating element of this Z_2 , Φ_I, Φ_{II} and ℓ_{iR} are multiplied by -1 , while all other fields remain unchanged.

The most general fermion mass and Yukawa Lagrangian with gauge symmetry $SU(2) \times U(1)_Y$ and global symmetry $EL \times Z_2$ has the form

$$\begin{aligned} \mathcal{L}_m + \mathcal{L}_Y &= -m_f \bar{f}_{iR} f_{iR} - g_I \bar{L}_{iL} \Phi_I - g_{II} \bar{L}_{iL} \Phi_{II} - g_R \bar{L}_{iL} \Phi_R - \\ &\quad - g_L \bar{L}_{iL} B_{ij}(f_{jL})^c \Phi_L + h.c. \\ A &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \end{aligned} \quad (6)$$

A large magnetic moment $\mu_\nu \simeq (1-10) \cdot 10^{-11} \mu_B$ ($\mu_B = e/2m_e$ is the Bohr magneton) of a Dirac electron neutrino provides an elegant solution [1] of the solar neutrino problem, accounting especially for the apparent anticorrelation between the solar neutrino flux and the number of sun spots [2]. The standard procedure to generate Dirac neutrinos introduces right-handed neutral gauge singlets¹. These right-handed sterile neutrinos have a tendency to disturb the conventional neutrino astrophysics and cosmology. In particular, from the observed neutrino burst of the supernova SN1987A [3] an upper bound $\mu_\nu \lesssim 10^{-12} \mu_B$ was derived by several authors [4]. This bound would seem to exclude the suggested explanation of the solar neutrino puzzle unless the right-handed neutrinos have sufficiently strong non-gauge interactions [5] to get trapped in the supernova core.

Another scenario without right-handed neutral singlets avoiding the stringent supernova bound on μ_ν invokes a large transition magnetic moment between nearly degenerate Majorana neutrinos [6]. Here we propose to investigate the synthesis of the two approaches where two degenerate Majorana neutrinos coalesce into a Dirac neutrino due to a Zeldovich-Konopinski-Mahmoud (ZKM) lepton number [7]. All known astrophysical and cosmological constraints are consistent with $\mu_\nu \simeq 10^{-11} \mu_B$ for a ZKM neutrino [8].

The economy in the number of neutral fermionic degrees of freedom and the absence of sterile neutrinos are very strong arguments in favour of the ZKM scenario. Another motivation is the search for a natural mechanism to combine a large magnetic moment with a small neutrino mass. This requirement has turned out to be very difficult to satisfy for the simple reason that any helicity-changing mechanism giving rise to a large μ_ν will in general also produce a big m_ν . A natural coexistence between big μ_ν and small m_ν seems to call for an underlying symmetry. Of several proposed models for a large magnetic moment, only a few have provided a natural explanation for the small neutrino mass, both for Dirac [9] and for Majorana [10] neutrinos.

To emphasize the basic mechanism for a naturally light ZKM neutrino with a large magnetic moment, we forget about the gauge structure for the moment. The main ingredients are a charged scalar field Φ^+ and two lepton generations, each consisting of a neutral Weyl field ν_{iL} and a massive Dirac fermion f_i with charge $q_f = -1$ ($i = 1, 2$). The theory is assumed to possess a conserved ZKM lepton number $L_{ZKM} = L_1 - L_2$ (i.e., $L_{ZKM}(\nu_{1L}, f_1) = -L_{ZKM}(\nu_{2L}, f_2) = 1$) yielding a Dirac neutrino

$$\nu = \nu_{1L} + (\nu_{2L})^c. \quad (1)$$

Therefore, the fermions have the following Yukawa couplings with Φ^+ :

$$\mathcal{L}_Y = \bar{\nu}(h_{1L} f_{1L} + h_{1R} f_{1R}) \Phi^+ + \bar{\nu}^c(h_{2L} f_{2L} + h_{2R} f_{2R}) \Phi^+ + h.c. \quad (2)$$

Consider now the one-loop contribution with Φ^+ exchange to both μ_ν (shown in Fig. 1 are the actual diagrams for the model of Eq. (6)) and to m_ν (same diagrams without the photon). Depending on the relative signs of the Yukawa couplings h_{iL} and h_{iR} , the amplitudes for f_1 and f_2 exchange interfere constructively for μ_ν and destructively for m_ν or vice versa. Our strategy will therefore be to achieve a cancellation between the

¹We only consider the low-energy gauge group $SU(2) \times U(1)_Y$.

Neutrino masses are of course forbidden by the gauge symmetry.

Although (6) is the most general Lagrangian with global group $EL \times Z_2$ it actually exhibits a larger global symmetry $EL \times U(1) \times U(1)'$ with

$$\begin{aligned} U(1) &: l_R \rightarrow e^{i\gamma} l_R, \quad \Phi_{I,II} \rightarrow e^{-i\gamma} \Phi_{I,II} \\ U(1)' &: f_i \rightarrow e^{i\delta} f_i, \quad \Phi_L \rightarrow e^{i\delta} \Phi_L, \quad \Phi_R \rightarrow e^{-i\delta} \Phi_R \end{aligned} \quad (7)$$

$$\gamma, \delta \in \mathbf{R}.$$

Continuous global symmetries are either broken explicitly by the remaining Lagrangian or they must remain unbroken to avoid Goldstone bosons. Since the singly charged fields in Φ_R and Φ_L will have to mix to produce $\mu_\nu \neq 0$, the Higgs potential necessarily contains terms of the form (and similar ones)

$$\tilde{\Phi}_I^\dagger \Phi_L \tilde{\Phi}_I^\dagger \Phi_R \quad (8)$$

which break $U(1)$ in (7) to the originally assumed Z_2 . On the other hand, the Higgs potential can either be $U(1)'$ invariant or it can break $U(1)'$ to Z_2' . The global symmetry of our model is therefore $EL \times Z_2 \times G'$ with $G' = U(1)'$ or Z_2' . The Higgs potential can always be chosen to give $\langle \Phi_R^0 \rangle_{\text{vac}} = 0$ so that G' is an exact symmetry. The corresponding conserved quantum number forbids mixing between f_i and l_i on the one hand and between Φ_R , Φ_L and Φ_I , Φ_{II} on the other hand.

The non-Abelian group EL must be spontaneously broken. Consider in particular the element $S \in EL$ which is represented on the neutral fermions as

$$\nu_L = \nu_{1L} \xrightarrow{S} \nu_{2L} = \nu^c_L \quad (9)$$

and which therefore forbids both a magnetic moment and a charge form factor:

$$\begin{aligned} \bar{\nu} \sigma_{\mu\nu} \nu &= \bar{\nu}_{1L} \sigma_{\mu\nu} (\nu_{2L})^c + \overline{(\nu_{2L})^c} \sigma_{\mu\nu} \nu_{1L} \xrightarrow{S} \bar{\nu}_{2L} \sigma_{\mu\nu} (\nu_{1L})^c + \overline{(\nu_{1L})^c} \sigma_{\mu\nu} \nu_{2L} = \\ &= -\bar{\nu}_{1L} \sigma_{\mu\nu} (\nu_{2L})^c - \overline{(\nu_{2L})^c} \sigma_{\mu\nu} \nu_{1L} = -\bar{\nu} \sigma_{\mu\nu} \nu \end{aligned} \quad (10)$$

and, similarly,

$$\bar{\nu} \gamma_\mu \nu \xrightarrow{S} -\bar{\nu} \gamma_\mu \nu. \quad (11)$$

The symmetry S must be distinguished from Voloshin's symmetry [9]

$$\nu_L \leftrightarrow (\nu_R)^c \quad (12)$$

which forbids both a neutrino mass term and a charge form factor [11], but allows for a non-vanishing magnetic moment. However, in contrast to our case the Voloshin symmetry does not commute with $SU(2) \times U(1)_Y$ because ν_R is a gauge singlet. The standard gauge group must therefore be extended to a larger gauge group [9], whereas our model is strictly based on $SU(2) \times U(1)_Y$.

The global symmetry $EL \times Z_2 \times G'$ is broken down to $U(1)_{ZKM} \times G'$ by

$$\langle \Phi_I^0 \rangle_{\text{vac}} \cdot \langle \Phi_{II}^0 \rangle_{\text{vac}} \neq 0 \quad (13)$$

giving masses to the charged leptons l_i and inducing mixing between $\Phi_I^\dagger$ and $\Phi_R^\dagger$ via (8). The mass eigenstates $\Phi_j^\dagger$ with masses M_j ($j = 1, 2$) are related to the weak eigenstates by a unitary two-dimensional mixing matrix V :

$$\Phi_R^\dagger = V_{Rj} \Phi_j^\dagger, \quad \Phi_L^\dagger = V_{Lj} \Phi_j^\dagger. \quad (14)$$

For convenience, we write down the neutrino Yukawa couplings from Eq. (6):

$$\begin{aligned} -\mathcal{L}_Y(\nu) &= \bar{\nu} [g_I l_{IR} \Phi_I^\dagger + g_{II} l_{IR} \Phi_{II}^\dagger + g_I^*(l_{2R})^c \Phi_I^- - g_{II}^*(l_{2R})^c \Phi_{II}^-] + \\ &\quad + g_R f_{IR} \Phi_R^\dagger + g_L^* f_{1L} \Phi_L^\dagger - g_R^*(f_{2R})^c \Phi_R^- + g_L^*(f_{2L})^c \Phi_L^-] + h.c., \end{aligned} \quad (15)$$

$$\nu = \nu_{1L} + (\nu_{2L})^c.$$

Due to the exact global symmetry G' , only the one-loop diagrams shown in Fig. 1 with internal fermions f_i and scalars $\Phi_j^\pm$ give rise to a non-zero μ_ν . Neither the usual charged leptons l_i nor the scalar fields Φ_I , Φ_{II} contribute because they cannot induce the necessary transition $\nu_{1L} \rightarrow (\nu_{2L})^c$.

The magnetic moment μ_ν of the ZKM neutrino ν due to the diagrams of Fig. 1 is calculated as

$$\mu_\nu = 1.6 \cdot 10^{-7} \mu_B \text{Re}(g_I g_R V_{L1}^* V_{R1}) \frac{M_W}{m_f} \left[f\left(\frac{m_f^2}{M_2^2}\right) - f\left(\frac{m_f^2}{M_1^2}\right) \right] \quad (16)$$

$$f(x) = \frac{x(x-1-\ln x)}{(1-x)^2}.$$

The monotonically increasing function $f(x)$ with $f(0) = 0$, $f(1) = 0.5$ approaches 1 for $x \rightarrow \infty$. Although $\mu_\nu \neq 0$ requires non-degenerate scalar fields $\Phi_I^\dagger$, $\Phi_{II}^\dagger$, Eq. (16) can naturally yield $\mu_\nu \simeq 10^{-11} \mu_B$ if m_f , M_1 and M_2 are all $O(M_W)$. Eq. (16) also exhibits the interesting feature that $\mu_\nu \rightarrow 0$ both for $m_f \rightarrow 0$ and for $m_f \rightarrow \infty$, keeping the M_i fixed. The Fermi scale (or M_W) is the natural scale for a big μ_ν in this model.

However, the real problem is not so much getting a sufficiently big μ_ν , but keeping m_ν small at the same time. Since m_ν also requires a transition $\nu_{1L} \rightarrow (\nu_{2L})^c$, the only relevant one-loop diagrams are the ones of Fig. 1 without the photon, because all other couplings are flavour diagonal. Although $EL \times Z_2$ is spontaneously broken, this breaking is not effective for m_ν at the one-loop level. Independently of the mixing between $\Phi_R^\dagger$ and $\Phi_L^\dagger$, the two amplitudes with f_1 and f_2 exchange cancel exactly. As a side remark, we note that if the global symmetry $G' = Z_2'$, a possible spontaneous breaking via $\langle \Phi_R^0 \rangle_{\text{vac}} \neq 0$ would induce m_ν at one loop, the magnitude depending, of course, on $\langle \Phi_R^0 \rangle_{\text{vac}} > \mu_\nu$.

The present model is the first example of a gauge model with the standard gauge group $SU(2) \times U(1)_Y$ where at the one-loop level a massless Dirac neutrino has a magnetic moment of the order $10^{-11} \mu_B$. In order to verify the stability of our model, we must estimate the natural order of magnitude of m_ν which arises at the two-loop level. Once again, the only flavour changing Yukawa coupling g_L must be involved. Moreover, the symmetry between the two generations must be broken in the second loop, which requires exchange of the charged leptons l_i . Altogether four diagrams contribute, two of which are

shown in Fig. 2 and there are two similar ones with Φ_L^{--} exchange. As expected, the sum of the two diagrams in Fig. 2 (and similarly for Φ_L^{--} exchange) is finite and proportional to $m_2^2 - m_1^2$. A standard estimate yields

$$m_\nu \sim \frac{\text{Re}(g_L g_R V_{L1}^* V_{R1})}{(4\pi)^4} (m_2^2 - m_1^2) m_f \frac{(M_1^2 - M_2^2)}{M^2} \left(\frac{|g_R|^2}{M_0^2} + \frac{|g_L|^2}{M_{--}^2} \right), \quad (17)$$

where $\bar{M}$ is some average of M_1 , M_2 and M_0 , M_{--} denote the masses of Φ_R^0 and Φ_L^{--} , respectively. Note that Φ_R^0 is a mass eigenstate (trivially true for Φ_L^{--}) because Φ_R does not mix with Φ_I , Φ_{II} due to the global G' . Comparing Eqs. (16) and (17), we find

$$m_\nu \sim 0.1 \text{ eV} \cdot \frac{m_2^2}{m_f^2} \cdot \frac{10^{11} \mu_\nu}{\mu_B} \cdot \frac{(M_1^2 - M_2^2) m_f^2}{M^2 [f(x_2) - f(x_1)]} \left(\frac{|g_R|^2}{M_0^2} + \frac{|g_L|^2}{M_{--}^2} \right) \quad (18)$$

with $x_i = m_i^2/M_f^2$. As a consequence, even for $m_2 = m_\tau$, i.e., for $L_{ZKM} = L_e - L_\tau$, the neutrino mass is naturally below the experimental upper bound of 18 eV [12]. For $L_{ZKM} = L_e - L_\mu$, m_ν is even expected to be several orders of magnitude lighter than the present bound.

In a recent paper [11], Liu has considered a special three-loop contribution to m_ν shown in Fig. 3. The two vertices stand for the magnetic moment form factor $\mu_\nu(q^2)$ and the charge form factor $R_\nu(q^2)$. Taking $\mu_\nu(q^2) = \mu_\nu(0) = \mu_\nu$, employing the standard model result [13] for $R_\nu(q^2)$ and cutting the integral off at M_W^2 , Liu obtains a natural bound

$$\mu_\nu \lesssim 10^{-11} \mu_B (m_\nu/\text{eV}) \quad (19)$$

by demanding that m_ν is not smaller than the contribution from the diagram in Fig. 3. This bound is surprisingly strict and should be confronted with our two-loop estimate (18) which would lead to a less stringent bound.

The solution of this seeming puzzle is related to the global symmetry EL which forbids a charge form factor as indicated by Eq. (11). More explicitly, for a ZKM neutrino the charge form factor $R_\nu(q^2)$ is given by

$$R_\nu(q^2) = R_\nu(q^2) - R_\nu(q^2). \quad (20)$$

The fermions f_i do not contribute to R_ν at one loop for the same reason as for m_ν . $R_\nu(q^2)$ is therefore given by the usual standard model result [13]. Compared to it, Eq. (20) furnishes a suppression factor $(m_2^2 - m_1^2)/M_W^2$. Consequently, the right-hand side of Liu's bound (19) has to be multiplied by $M_W^2/(m_2^2 - m_1^2)$ in our case. Thus, the bound becomes trivial irrespective of whether m_2 is m_μ or m_τ . In the present model, the neutrino mass is indeed naturally small.

Concerning the magnetic moments of charged leptons, the situation is essentially the same as for a two-Higgs-doublet version of the standard model. Due once again to the global symmetry G' , of the four scalar doublets in our model only Φ_I , Φ_{II} are relevant for the magnetic moments of charged leptons while Φ_R , Φ_L induce the neutrino magnetic moment. Therefore, the magnetic moments of charged and neutral leptons are completely independent and there are no reciprocal constraints.

We have presented the first example of an $SU(2) \times U(1)_Y$ gauge model where a naturally light Dirac neutrino has a large magnetic moment of $O(10^{-11} \mu_B)$. The crucial ingredient is a non-Abelian extension of a ZKM lepton number symmetry which guarantees a vanishing neutrino mass at the one-loop level. The neutrino mass arises at the two-loop level and is well below the present experimental bound irrespective of whether the conserved ZKM lepton number is $L_e - L_\mu$ or $L_e - L_\tau$.

Figure Captions

Fig. 1: 1-loop diagrams for μ_ν for the model of Eq. (6). There are two additional diagrams where the photon is emitted from the scalar lines. Note that $\nu_L \equiv \nu_{1L}$ and $\nu_R \equiv (\nu_{1L})^c$.

Fig. 2: 2-loop diagrams for m_ν . There are two similar diagrams with Φ_L^- (instead of Φ_R^0) exchange.

Fig. 3: Liu's diagram [11] for m_ν with magnetic moment form factor $\mu_\nu(q^2)$ and charge form factor $R_\nu(q^2)$.

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Fig.1

Fig.2

Fig. 3