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Thinning out of Relevant Degrees of Freedom in Scattering of Strings

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Abstract

By comparing the scattering of point-like and string-like particles in a massless shock wave, we show that only the former process is sensitive to ripples in the metric living on scales shorter than $\lambda_s \equiv \sqrt{2\alpha'\hbar}$. We take this result as evidence for the irrelevance of very-short-distance degrees of freedom in string theory.

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1. The conjecture [1] that in string theory distances less than the fundamental length $\lambda_s = \sqrt{2\alpha'\hbar}$ are not relevant has been suggested by arguments based, independently, on high-energy scattering at fixed angle [2,3], on high-temperature behaviour of the free energy [4] and on duality of closed strings [5]. The existence of such a minimal explorale length, if proved true, would imply a thinning out of the number of dynamical degrees of freedom of the theory because short-distance details would not contribute. In other words, strings would neither couple nor be at all sensitive to all those features of an external field the characteristic scale of which lies below λ_s .

While a formal proof of such a conjecture seems rather premature, we found it possible to provide an example that is both non-trivial and completely soluble, in which the above decoupling of short-distance degrees of freedom is explicitly shown to occur.

2. We take as our starting point the elastic scattering of a massless state of a (closed) string in a shock-wave background, the metric of which is

$$ds^2 = -2du dv + f(b) \delta(u) du^2 + db^2. \quad (1)$$

In eq. (1), the impact parameter b represents the $D - 2$ transverse coordinates, $u = (t - z)/\sqrt{2}$ and $v = (t + z)/\sqrt{2}$. The splitting of space-time in longitudinal and impact parameter parts makes sense because we work in the high energy small-angle approximation in which the transverse momentum $|\mathbf{q}|$ is taken to be much smaller than the longitudinal momentum p^u . This scattering problem has the advantage of being soluble [6], with a scattering matrix given by:

$$S(\mathbf{q}) = \int d^{D-2}b \langle \exp i \left[\mathbf{q} \cdot \mathbf{b} + \frac{p^u}{2\pi} \int_0^\pi d\sigma f(\tilde{\mathbf{X}}(\sigma, 0) + \mathbf{b}) \right] \rangle. \quad (2)$$

In eq. (2), as well as in the rest of the paper, $\hbar = 1$, the brackets stand for a functional average over the transverse string coordinates

$$\tilde{\mathbf{X}}(\sigma, \tau) = \alpha' \tau \mathbf{q} + \frac{i}{2} \lambda_s \sum_{n \neq 0} \frac{1}{n} (\tilde{\alpha}_n \exp(-2in\tau) - \alpha_{-n} \exp(2in\tau)) \exp(-2in\sigma) \quad (3)$$

weighted by the free action, and normal ordering is implied. Fermionic degrees of freedom are not affected by the shock-wave background and therefore all our results can be readily extended to the scattering of superstrings as well.

Among the possible shock-wave backgrounds, we choose the simplest, a planar (axi-symmetric) shell of finite extension $R \gg \lambda_s$. This can be thought of as a homogeneous beam of gravitons and it generalizes the Aichelburg-Sexl metric [7]. In such a case,

$$f(\mathbf{x}) = f_0(\mathbf{x}) \equiv -\beta \mathbf{x}^2 \quad |\mathbf{x}| \leq R \quad (4)$$

where, for a beam of energy E_b ,

$$\beta \equiv \frac{16\pi G_N E_b}{(D-2)\Omega_{D-2} R^{D-2}} \quad (5)$$

with $\Omega_D \equiv 2\pi^{D/2}/\Gamma(D/2)$. For instance, in $D = 4$, $\beta \equiv 4G_N E_b/R^2$.

For such a background eq. (2) becomes

$$S(\mathbf{q}) = S_0 \int d^{D-2}b \exp i \left[\mathbf{q} \cdot \mathbf{b} - \frac{p^u \beta}{2} \mathbf{b}^2 \right], \quad (6)$$

where

$$S_0 \equiv \left\langle \prod_{n=1}^{\infty} \exp \left[-\frac{ip^u \beta \lambda_s^2}{4n^2} (\tilde{\alpha}_n - \alpha_{-n}) \cdot (\tilde{\alpha}_n - \alpha_{-n}) \right] \right\rangle \quad (7)$$

comes from the part quadratic in the string coordinates and represents the absorption due to the diffractive excitations of massive string states [2].

The integral over b is dominated by a stationary phase at

$$\mathbf{q} = p^u \beta \mathbf{b} \quad (8)$$

with the condition $|\mathbf{b}| < R$. Eq. (8) thus gives a maximum scattering angle

$$\theta \simeq \frac{|\mathbf{q}|}{p^u} = \beta R \quad (9)$$

and, in order to have $|\mathbf{q}| \ll p^u$, we must impose $\beta R \ll 1$. Moreover, in order to be able to compare the scattering of the string with the one for the particle, we must restrict ourselves to

$$|\mathbf{q}| < \frac{1}{\lambda_s}. \quad (10)$$

In fact, if instead

$$|\mathbf{q}| \lambda_s = p^u \beta R \lambda_s > p^u \beta \lambda_s^2 > 1 \quad (11)$$

we will have (in the elastic channel of the massless state of the string) [2]:

$$|S_0| \simeq (D-3) \left(\frac{\pi p^u \beta \lambda_s^2}{4} \right)^{D-2} \exp \left[-\frac{\pi(D-3)}{2} p^u \beta \lambda_s^2 \right] \ll 1 \quad (12)$$

and the string amplitude will be negligible, making the comparison with the particle case meaningless.

3. We add now a gravitation ripple to the planar shell, that is an arbitrary (small) perturbation in the homogeneity of the beam. In order to control the scale λ of such a perturbation, we decompose it into Fourier components as

$$f(\mathbf{x}) = \int d^{D-2}\mathbf{k} \, \epsilon(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{x}), \quad (13)$$

where the infinitesimal amplitude $\epsilon(\mathbf{k})$, besides satisfying the Fourier integrability condition, is exponentially damped for $|\mathbf{k}| < 1/\lambda$. If it is true that distances below λ_s are irrelevant to string theory, we should find that for $\lambda \ll \lambda_s$ this perturbation in the planar shell metric, while modifying the particle scattering, does not affect that of the string.

The scattering matrix can be evaluated by making a Taylor expansion of the functional in powers of $\epsilon(\mathbf{k})$. Accordingly, we have

$$S(\mathbf{q}) = \sum_{\mathbf{n}} \frac{1}{\mathbf{n}!} S_{\mathbf{n}}(\mathbf{q}), \quad (14)$$

where

$$S_{\mathbf{n}}(\mathbf{q}) = \int d^{D-2}\mathbf{b} \langle \exp i \left[\mathbf{q} \cdot \mathbf{b} + \frac{p^u}{2\pi} \int_0^\pi d\sigma f_0(\tilde{\mathbf{X}}(\sigma) + \mathbf{b}) \right] \times \left(\frac{ip^u}{2\pi} \right)^n \prod_{i=1}^n \int d^{D-2}\mathbf{k}_i \epsilon(\mathbf{k}_i) \exp i \left\{ \mathbf{k}_i \cdot [\tilde{\mathbf{X}}(\sigma_i) + \mathbf{b}] \right\} \rangle. \quad (15)$$

The particle scattering matrices can be obtained from eq. (15) by putting $\tilde{\mathbf{X}}(\sigma_i) = 0$. In this case, all terms in (14) can be estimated by means of the stationary phase approximation. This gives, for the first term,

$$S_1(\mathbf{q}) \simeq \frac{ip^u}{2} \epsilon(-\mathbf{q}) \quad (16)$$

that is vanishingly small for $\lambda \ll \lambda_s$ because of the exponential cut-off in $\epsilon(\mathbf{k})$. The next term, however, is sizeable and equal to

$$S_2(\mathbf{q}) \simeq \left(\frac{ip^u}{2} \right)^2 \int d^{D-2}\mathbf{k}_1 \epsilon(\mathbf{k}_1) \epsilon(-\mathbf{k}_1 - \mathbf{q}). \quad (17)$$

The result we find for $S_2(\mathbf{q})$ is readily generalized to the remaining $S_{\mathbf{n}}(\mathbf{q})$'s to show that they do not vanish. Therefore, for the point, they can be resummed to an equation like (6) with $\beta\mathbf{b}^2 \rightarrow \beta\mathbf{b}^2 + f(\mathbf{b})$ and the overall deflection can be estimated:

$$\mathbf{q} = p^u \beta \mathbf{b} - \frac{ip^u}{2} \int d^{D-2}\mathbf{k} \, \epsilon(\mathbf{k}) \mathbf{k} \exp(i\mathbf{k} \cdot \mathbf{b}). \quad (18)$$

We conclude that a ripple affects the particle trajectory; this conclusion is based on semiclassical (saddle point) approximation and is expected to be valid for large enough p^u (small Compton length limit). Notice that the $\mathbf{k}$'s must have a configuration such that $|p^u \beta \mathbf{b} - (ip^u/2) \int d^{D-2}\mathbf{k} \, \epsilon(\mathbf{k}) \mathbf{k} \exp(i\mathbf{k} \cdot \mathbf{b})| < \lambda_s^{-1}$ in order for condition (10) to hold.

We consider now the string. $S_1(\mathbf{q})$ is, apart from an inessential constant factor due to the Gaussian integration, the same as for the particle and it vanishes in the limit we are

interested in. The next term,

$$S_2(\mathbf{q}) = \int d^{D-2}\mathbf{k}_1 \int d^{D-2}\mathbf{k}_2 \int d^{D-2}\mathbf{b} \exp i \left[\mathbf{q} \cdot \mathbf{b} - \frac{p^u \beta}{2} \mathbf{b}^2 + (\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{b} \right] \times \left(\frac{p^u}{2\pi} \right)^2 \int d\sigma_1 \int d\sigma_2 \epsilon(\mathbf{k}_1) \epsilon(\mathbf{k}_2) \times \left(\prod_{n=1}^{\infty} \exp \left[-\frac{ip^u \beta \lambda_s^2}{4n^2} (\tilde{\alpha}_n - \alpha_{-n}) \cdot (\tilde{\alpha}_{-n} - \alpha_n) \right] \right) \times \exp i \left[\mathbf{k}_1 \cdot \tilde{\mathbf{X}}(\sigma_1) + \mathbf{k}_2 \cdot \tilde{\mathbf{X}}(\sigma_2) \right] \quad (19)$$

is evaluated to give

$$S_2(\mathbf{q}) = \int d^{D-2}\mathbf{k}_1 \int d^{D-2}\mathbf{k}_2 \int d^{D-2}\mathbf{b} \exp i \left[\mathbf{q} \cdot \mathbf{b} - \frac{p^u \beta}{2} \mathbf{b}^2 + (\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{b} \right] \times S_0 \left(\frac{p^u}{2} \right)^2 \epsilon(\mathbf{k}_1) \epsilon(\mathbf{k}_2) \exp \left[-\frac{\lambda_s^2}{2} \mathbf{k}_1 \cdot \mathbf{k}_2 \log 2 \right] \times B \left(\frac{\lambda_s^2}{4} \mathbf{k}_1 \cdot \mathbf{k}_2 + \frac{1}{2}, \frac{1}{2} \right). \quad (20)$$

where $B(x, y)$ is Euler's beta function. This function has poles on the negative axis of its first argument; to avoid them we assume that the momentum transferred $\sqrt{-t} \equiv |\mathbf{q}|$ lies slightly above the real axis. Under this assumption, as long as λ , the characteristic scale of the ripple, is smaller than λ_s and in the regime for which $|\mathbf{q}| = |p^u \beta \mathbf{b} - \mathbf{k}_1 - \mathbf{k}_2| < 1/\lambda_s$,

$$S_2(\mathbf{q}) \simeq S_0 \left(\frac{p^u}{2} \right)^2 \int d^{D-2}\mathbf{k}_1 \epsilon(\mathbf{k}_1) \epsilon(-\mathbf{k}_1 - \mathbf{q}) \times \exp \left[-\frac{\lambda_s^2}{2} (\mathbf{k}_1^2 + \mathbf{k}_1 \cdot \mathbf{q}) \log 2 \right] B \left(-\frac{\lambda_s^2}{4} (\mathbf{k}_1^2 + \mathbf{k}_1 \cdot \mathbf{q}) + \frac{1}{2}, \frac{1}{2} \right). \quad (21)$$

is exponentially depressed compared to the particle one in eq. (17). A similar behaviour governs all the other $S_{\mathbf{n}}(\mathbf{q})$'s and therefore details of the perturbation that are finer than λ_s are not felt by the string; that is the result we wanted to prove.

4. Any lingering doubt concerning the influence of the assumption $|\arg t| < \pi$ on the final result can be dispelled by showing that the poles we found in (20) are an artifact of the Fourier decomposition (13) of the perturbation. This is done by decomposing the ripple into a more suitable set of elementary waves. For instance, we can use Gaussian wavelets [8], that is

$$f(\mathbf{x}) = \int d^{D-2}\mathbf{w} \int d^{D-2}\mathbf{c} \, \epsilon(\mathbf{w}, \mathbf{c}) \exp \left(i \frac{\mathbf{a}}{\mathbf{w}} \cdot \mathbf{x} \right) \exp \left[-\frac{(\mathbf{x} - \mathbf{c})^2}{2\mathbf{w}^2} \right]. \quad (22)$$

Such wavelets are an orthonormal system of waves of frequency $\mathbf{a}/|\mathbf{w}|$, with $\mathbf{a}$ large; their amplitudes are centred around $\mathbf{c}$ with a Gaussian spread given by $|\mathbf{w}|$. They are nicely apt

to describe a perturbation which is localized both in position and in momentum space. As in the case of the Fourier decomposition, the $\epsilon(\mathbf{w}, \mathbf{c})$ is chosen vanishingly small for all $|\mathbf{w}| > \lambda$ with λ setting the scale of the perturbation.

By using (22) instead of (13) and retracing our steps from (2) to (14), we write

$$S_n(\mathbf{q}) = \left(\frac{ip^\mu}{2\pi} \right)^n \int d^{D-2}\mathbf{b} \exp i \left[\mathbf{q} \cdot \mathbf{b} + \frac{p^\mu}{2\pi} \int_0^\pi d\sigma f_0(\dot{\mathbf{X}}(\sigma) + \mathbf{b}) \right] \times \prod_{i=1}^n \int d^{D-2}\mathbf{w}_i \int d^{D-2}\mathbf{c}_i \int_0^\pi d\sigma_i \epsilon(\mathbf{w}_i, \mathbf{c}_i) \times \exp \left\{ i \frac{a}{\mathbf{w}_i} \cdot [\mathbf{b} + \dot{\mathbf{X}}(\sigma_i)] - \frac{1}{2\mathbf{w}_i^2} [\mathbf{b} + \dot{\mathbf{X}}(\sigma_i) - \mathbf{c}_i]^2 \right\} \quad (23)$$

which is Gaussian in the string coordinates and can be evaluated. In order for the wavelets to interact with the string at all, their centres must be around the trajectory of the string itself. Therefore we make the approximation $\epsilon(\mathbf{w}_i, \mathbf{c}_i) \approx \epsilon(\mathbf{w}_i) \delta(\mathbf{c}_i - \mathbf{b})$ and then integrate over the string coordinates to obtain

$$S_n(\mathbf{q}) = \int d^{D-2}\mathbf{b} \exp i \left[\mathbf{q} \cdot \mathbf{b} + \frac{p^\mu \beta}{2} \mathbf{b}^2 \right] \times \prod_{i=1}^n \int d^{D-2}\mathbf{w}_i \int d^{D-2}\mathbf{c}_i \epsilon(\mathbf{w}_i) \delta(\mathbf{b} - \mathbf{c}_i) \exp \left[i \sum_i \frac{a}{\mathbf{w}_i} \cdot \mathbf{b} \right] F(\mathbf{w}_i), \quad (24)$$

where

$$F(\mathbf{w}_i) \equiv S_0 \left(\frac{1}{2\pi\mathbf{w}_i^2} \right)^{D/2-1} \int_0^\pi d\sigma_i \prod_{j < i} \exp \left[\frac{1}{2} \frac{a^2}{\mathbf{w}_i \mathbf{w}_j} \lambda_s^2 G(\sigma_i, \sigma_j) \right] \times \int d^{D-2}\eta_i \exp \left[-\frac{\mathbf{w}_i^2}{2} \eta_i^2 \right] \times \exp \left[\frac{1}{2} \lambda_s^2 [\eta_i \cdot \eta_j + a \frac{\mathbf{w}_i \cdot \eta_i + \mathbf{w}_j \cdot \eta_j}{\mathbf{w}_i \cdot \mathbf{w}_j}] G(\sigma_i, \sigma_j) \right], \quad (25)$$

and $G(\sigma_i, \sigma_j) = \log \mu |e^{i\sigma_i} - e^{i\sigma_j}|$ is the Green function for the 0-genus world-sheet. The additional integrations over the η_i come about because we used the trick of introducing Gaussian integrals in order to linearize the part quadratic in $\dot{\mathbf{X}}(\sigma_i)$ in the exponent of the last line of eq. (23).

We consider the various terms in turn:

$$S_1(\mathbf{q}) = S_0 \frac{a}{q^2} \epsilon \left(\frac{a}{q} \right) \quad (26)$$

gives no contribution as long as $\lambda \ll \lambda_s$ and $|\mathbf{q}| < 1/\lambda_s$;

$$S_2(\mathbf{q}) = \int d^{D-2}\mathbf{w}_1 \int d^{D-2}\mathbf{w}_2 \int d^{D-2}\mathbf{b} \exp i \left[\mathbf{q} \cdot \mathbf{b} - \frac{p^\mu \beta}{2} \mathbf{b}^2 + (\mathbf{w}_1 + \mathbf{w}_2) \cdot \mathbf{b} \right] \times \epsilon(\mathbf{w}_1) \epsilon(\mathbf{w}_2) \times F(\mathbf{w}_1, \mathbf{w}_2); \quad (27)$$

and similarly for the other $S_n(\mathbf{q})$'s. For the particle, $F(\mathbf{w}_1, \mathbf{w}_2)$ in eq. (27) is equal to one and the perturbation modifies the trajectory in the same manner as we found in the Fourier case. Instead, for the string,

$$F(\mathbf{w}_1, \mathbf{w}_2) = S_0 \left(\frac{1}{2\pi\mathbf{w}_1^2} \right)^{D/2-1} \left(\frac{1}{2\pi\mathbf{w}_2^2} \right)^{D/2-1} \int d\sigma_1 \int d\sigma_2 \times \exp \left\{ -\frac{a^2 \lambda_s^2}{2\mathbf{w}_1 \cdot \mathbf{w}_2} \left[\frac{1}{1 - \frac{\mathbf{w}_1 \cdot \mathbf{w}_2}{\lambda_s^2 G(\sigma_1, \sigma_2)}} - 1 \right] G(\sigma_1, \sigma_2) \right\}. \quad (28)$$

In the limit $\lambda_s^2 \gg |\mathbf{w}_1 \cdot \mathbf{w}_2|$ eq. (28) becomes

$$F(\mathbf{w}_1, \mathbf{w}_2) \simeq \pi^2 \left(\frac{1}{2\pi\mathbf{w}_1^2} \right)^{D/2-1} \left(\frac{1}{2\pi\mathbf{w}_2^2} \right)^{D/2-1} \exp(-a^2), \quad (29)$$

so that $S_2(\mathbf{q})$ turns out to be exponentially small for a large. This is also true for all the other $S_n(\mathbf{q})$'s. This time there are no poles to prevent us from taking the limit $\lambda \ll \lambda_s$. Even though wavelets may be less familiar than the usual Fourier components, they seem to be better when used for a perturbation, like ours, that is sharply confined below a given scale.

For p^μ large, fixed-angle scattering of strings is always exponentially small due to the diffractive excitation of higher string states; eq. (12) is an example of this. Only at small $|\mathbf{q}|$ might we expect the amplitude for the string to be comparable to the one for the particle. Nevertheless, we find that, even in the forward direction, a perturbation, the scale of which lies below λ_s , is not detected by strings.

5. This conclusion may be expressed in a slightly more general manner. The coupling between strings and the metric of the space-time in which they move is given by the partition function

$$Z(g) = \int d[X^\mu(\sigma, \tau)] \exp \left[\frac{i}{\lambda_s^2} \int d\sigma d\tau \partial X^\mu(\sigma, \tau) \partial X^\nu(\sigma, \tau) g_{\mu\nu}(X) \right]. \quad (30)$$

The example we have considered teaches us that a small perturbation $\delta g_{\mu\nu}(X)$ of the metric tensor $g_{\mu\nu}(X)$ has no effect whatsoever as long as

$$\int_V \sqrt{g} \delta g(x) d^D x \approx 0 \quad \text{for} \quad \int_V \sqrt{g} d^D x = \lambda_s^D. \quad (31)$$

It is as if in string theory some sort of enlarged equivalence principle is operating to give us a generalized version of Einstein's general relativity, in which dynamics is not only independent of coordinate transformations but also of structures occurring at distances shorter than λ_s . Such a principle would seem to be behind the miracles by which string theory provides a finite quantum theory of gravity.

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