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THE MUON $g - 2$ EXPERIMENTS

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1. INTRODUCTION

It is now thirty years since a group of experimental physicists at CERN, under Leon Lederman, started to study the problem of the muon g -factor. The g -factor relates the magnetic dipole moment to the intrinsic angular momentum of a charged system. Classically, the dipole moments can arise from either charges or currents. For example, the circulating current, which is due to an orbiting particle with an electric charge e and mass m , has associated with it a magnetic dipole moment $\vec{\mu}_L$ given by

$$\vec{\mu}_L = \frac{e}{2mc} \vec{L}, \quad (1.1)$$

where $\vec{L}$ is the orbital angular momentum. Alternatively, the electric dipole moment possessed by certain polar molecules is due to the relative displacement of the centres of the positive and negative charge distributions. Thus we have examples of a magnetic dipole moment and an electric dipole moment both having their origins in electric charge, and it is interesting to note that all electromagnetic phenomena are explained in terms of electric charges and their currents; there is no place, as yet, for magnetic charges. In particular, the intrinsic magnetic dipole moments of all particles can be considered, in the classical picture, to be made up of circulating electric currents and not of distributed magnetic charges [1]. This is just one aspect of the basic asymmetry between electricity and magnetism, which is apparent in Maxwell's equations. The argument, first proposed by Dirac [2], that the existence of magnetic charge would lead naturally to the quantization of both magnetic and electric charge, still stands as a challenge to physicists, both theoretical and experimental, to find a proper place for the magnetic monopole in the electromagnetic theory and to establish its physical reality.

The Dirac equation permits any value of g for the electron or muon, through the possible presence of the Pauli term, but the simplest version without Pauli term implies $g = 2$. To this must be added the corrections due to quantum electrodynamics which it is the purpose of the experiment to measure.

For a particle with both magnetic and electric dipole moments, the electromagnetic interaction Hamiltonian contains a part

$$H = -\vec{\mu}_m \cdot \vec{B} - \vec{\mu}_e \cdot \vec{E}, \quad (1.2)$$

where $\vec{B}$ and $\vec{E}$ are the magnetic and electric field strengths, and $\vec{\mu}_m$ and $\vec{\mu}_e$ are the magnetic and electric dipole moment operators. Following the general form of Eq. (1.1), we write

$$\vec{\mu}_m = g \frac{e}{2mc} \frac{1}{2} \left(\frac{h}{2\pi} \right) \vec{\sigma}, \quad (1.3)$$

$$\vec{\mu}_e = \eta \frac{e}{2mc} \frac{1}{2} \left(\frac{h}{2\pi} \right) \vec{\sigma}, \quad (1.4)$$

where the components of $\vec{\sigma}$ are the three Pauli spin matrices, and for the negative lepton we have to insert the charge $e = -|e|$. Making use of the Bohr magneton $\mu_0 = eh/4\pi mc$, these equations can be simplified to

$$\vec{\mu}_m = g\mu_0 \frac{\vec{\sigma}}{2} \quad (1.5)$$

and

$$\vec{d} \equiv \vec{\mu}_e = \eta\mu_0 \frac{\vec{\sigma}}{2}, \quad (1.6)$$

where we have taken the opportunity to introduce the conventional symbol $\vec{d}$ for the electric dipole moment (EDM).

The expectation value of the electric dipole moment $\vec{d}$ must be zero for a particle described by a state of well-defined parity. The polar molecules that we have referred to above are in a mixture of degenerate states with opposite parities, and so are not covered by this symmetry condition.

Arguments involving the time-reversal operation also require that the electric dipole moment should vanish as a consequence of the different symmetry properties of the magnetic and electric fields. Whilst $\vec{B}$ is an axial vector, $\vec{E}$ is a polar vector. Thus if the Hamiltonian equation (1.2) is to remain invariant with respect to parity inversion P and time reversal T , then $\vec{\mu}_m$ must transform like an axial vector and $\vec{\mu}_e$ must transform like a polar vector; an axial vector changes sign under T but not under P , whilst for a polar vector the opposite is true. From Eqs. (1.5) and (1.6) we see that the dipole moment operators should transform like the spin operator $\vec{\sigma}$. Since this latter behaves like an axial vector, all is consistent for $\vec{\mu}_m$; but in the case of $\vec{\mu}_e$, either of the operations P or T changes the relative sign of the two sides of the equation.

In order to have a satisfactory situation, η must be zero. These arguments can be generalized to show that for a system of definite parity, the odd electric (dipole, sextupole, etc.) and even magnetic (quadrupole, octupole, etc.) moments must be zero. Since the discovery that nature does not respect parity invariance, however, the invariance of interactions with respect to symmetry operations must always be underpinned by experiments.

2. SURVEY OF THE THEORY

If the muon obeys the simple Dirac equation for a particle of its mass (206 times heavier than an electron), then $g = 2$ exactly; but this is modified by the quantum fluctuations in the electromagnetic field around the muon, as specified by the rules of quantum electrodynamics (QED), making g larger by about 1 part in 800. The quantum fluctuations require further correction for the very rare fluctuations, which include virtual pion states and strongly interacting vector mesons.

At present, theory and experiment agree at the level of 1 part in 10^8 , and the muon g -factor, together with that of the electron and positron, R_∞ , c , and the frequency of the hydrogen maser, are the most accurately known constants in nature.

The muon ($g - 2$) value has played a central role in establishing that the muon obeys QED and behaves like a heavy electron. The experimental value of ($g - 2$) has been determined by three progressively more precise measurements at CERN, the latest one achieving a precision of 7.3 ppm (parts per million) in the anomaly [$a_\mu \equiv (g - 2)/2$]. The theoretical value for ($g - 2$) has improved steadily as higher-order QED radiative contributions have been evaluated, and as knowledge of the virtual hadronic contribution to ($g - 2$) has been improved both by further measurements of the experimental ratio $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ and by calculations.

The gyromagnetic ratio is increased from its primitive value of 2, arising from the Dirac equation, to $g = 2(1 + a_\mu)$, where a_μ is the anomalous magnetic moment or anomaly. In QED theory, the lepton g -factor may be expressed as a power series in α/π :

$$a_\mu^{\text{th}} = A(\alpha/\pi) + B(\alpha/\pi)^2 + C(\alpha/\pi)^3 + \dots \quad (2.1)$$

Typical Feynman diagrams, which contribute to the calculation of the theoretical value of a_μ for the electron and the muon, are shown in Fig. 1, whilst a complete set of diagrams up to order $(\alpha/\pi)^4$ is given by Kinoshita et al. [3].

Fig. 1 Feynman diagrams used in calculating a_μ . The solid line represents the muon, which interacts with the laboratory magnetic field at X. The zigzag line represents a virtual photon, which is emitted and later reabsorbed. In (d) and (f) an e^+e^- pair, created and then annihilated, gives rise to the closed loop (solid line).

The calculations [3] of the coefficients in Eq. (2.1) have given the following results

$$\begin{aligned}
A &= 0.5 \\
B &= 0.765\,858\,6 \\
C &= 24.067\,(3) \\
D &= 124.82\,(65) \\
E &= 570\,(140) .
\end{aligned} \tag{2.2}$$

Note that the value of B includes a small contribution from τ -loops. Using [4]

$$\alpha^{-1} = 137.035\,989\,5\,(61) \quad (0.045 \text{ ppm})$$

the result is

$$a_{\mu}^{\text{QED}} = 116\,584\,730\,(5)\,(5) \times 10^{-11} , \tag{2.3}$$

where the first error comes from the uncertainty in the coefficients C, D, E and the second reflects the error in α . Combining in quadrature the overall uncertainty in the QED value is 0.06 ppm.

So far, only the change in the gyromagnetic ratio due to the interaction of a particle with its own electromagnetic field has been mentioned. Any other field coupled to the particle should produce a similar effect, and calculations have been made for scalar, pseudoscalar, vector, and axial-vector fields, using a coupling constant κ (assumed to be small) to a boson of mass M [5]. For a vector field, for example,

$$\Delta a_{\mu}^{\text{V}} = \left(\frac{1}{3\pi} \right) \left(\frac{\kappa^2}{M^2} \right) m_{\mu}^2 . \tag{2.4}$$

A precise measurement of a_{μ} could therefore reveal the presence of a new field; but first, all known fields, including the weak and the strong interactions, must be taken into account. Strongly interacting particles do not couple directly to the muon, but if they are charged, they couple to the photon. Thus they can appear in the inner loops, as Fig. 1d, for example, with a pion pair replacing the e^+e^- pair. Because of the high mass of the pion, one would initially expect such amplitudes to be small, but there are strong resonances in the $\pi^+\pi^-$ system that enhance the effect. Only a vector resonance can contribute, because it alone can transform directly into the virtual photon which must have $J^{\text{PC}} = 1^{--}$ (one unit of angular momentum, negative parity, and negative charge conjugation).

To calculate this contribution it is necessary to specify the overall probability amplitude for a photon of a given q^2 to connect the two muon vertices shown in Fig. 1b, with the effect of virtual hadron loops fully included; that is, one requires the propagator function of Fig. 2a.

Fig. 2 The photon propagator is modified by the creation of virtual hadrons (a). This is related by the dispersion theory to real hadron production in e^+e^- collisions (b).

This cannot be calculated from theory, because not enough is known about hadrons. But fortunately the propagator in Fig. 2a can, in principle, be cut in half to obtain that of Fig. 2b, which shows an e^+e^- pair annihilating to give real hadronic states. By using dispersion theory, the cross-section for Fig. 2b as a function of $s = (\text{total energy})^2$ can be related to the propagator shown in Fig. 2a [6], and so to the anomalous moment arising from Fig. 1d with hadron loops:

$$\Delta a_\mu^{\text{had}} = \left(\frac{1}{4\pi^3} \right) \int_{4m_\pi^2}^{\infty} \sigma_H(s) K(s) ds, \quad (2.5)$$

where $\sigma_H(s)$ is the cross-section for hadron production via one-photon exchange in e^+e^- collisions, s is the square of the centre of mass energy, and

$$K(s) = \left\{ x^2 \left(1 - \frac{x^2}{2} \right) + (1+x)^2 \left(1 + \frac{1}{x^2} \right) \left[\ln(1+x) - x + \frac{x^2}{2} \right] + \frac{1+x}{1-x} x^2 \ln x \right\}, \quad (2.6)$$

where

$$x = \frac{1-\beta}{1+\beta}; \quad \beta = (1 - 4m_\mu^2/s)^{1/2};$$

$\sigma_H(s)$ is normally obtained by measuring the ratio

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)},$$

the denominator being known from QED.

The process $e^+e^- \rightarrow \text{hadrons}$ has been extensively studied in electron-positron colliding beams, and the cross-section is rather well defined from near threshold up to about 3 GeV centre-of-mass energy. Therefore the integration in Eq. (2.5) can be carried out with fair confidence. Three new evaluations were published in 1985 [7] with the following results:

	$\Delta a_\mu^{\text{had}}$
Kinoshita et al.	$703 (19) \times 10^{-10}$
Casas et al.	$705 (12) \times 10^{-10}$
Barkov et al.	$679 (12) \times 10^{-10}$

The first two of these papers are by theorists and use the same experimental data. They analyse separately the contributions of a wide variety of hadronic final states such as 2π , 3π , ..., 6π , K^+K^- , $K_S^0K_L^0$, $K^+K^-\pi^+\pi^-$, ρ , ω , etc., and include higher-order diagrams with two hadronic loops, and the scattering of light by light via a hadron loop. It is encouraging that these two independent evaluations, using different approaches, agree so well.

The main contribution of Barkov et al. is to present new very precise data on $\sigma_H(s)$ for the $\pi^+\pi^-$ final state, which were unfortunately not available to Kinoshita et al. nor to Casas et al. Barkov et al. use the new data to evaluate $\Delta a_\mu^{\text{had}}$, but they do not say whether all the multiparticle final states mentioned above are included. We cannot tell whether their result is lower because of the new experimental data, or because some states are omitted.

Further measurements are now planned by Barkov et al. and it is felt that a detailed comparison of the calculations should be deferred until their results are available.

Pending further clarification, it seems prudent to split the difference and increase the error to include the whole range of values above. In this case we can state with some confidence

$$\Delta a_\mu^{\text{had}} = 69 (2) \times 10^{-9}. \quad (2.7)$$

The contribution of the four-fermion weak interaction is illustrated in Fig. 3a. This is second order in the weak interaction and turns out to be negligibly small ($\sim 10^{-12}$). However, a larger weak contribution arises from single-loop Feynman diagrams involving W, Z, and ϕ , as shown in Figs. 3b-d. Although the W and Z particles have been discovered [8], the Higgs scalar particle ϕ predicted by the Standard Model has not been detected. Calculations based on the Glashow-Salam-Weinberg theory [9] predict this contribution to be

$$\Delta a_\mu^{\text{weak}} = 195 (1) \times 10^{-11}, \quad \text{or} \quad (1.7 \pm 0.01) \text{ ppm in } a_\mu^{\text{th}}. \quad (2.8)$$

Fig. 3 Contribution to a_μ by the weak interaction: (a) with four-fermion interaction; (b) by the virtual production of an intermediate boson $W^\pm$, (c), (d) single-loop diagrams contributing to the muon g-factor.

The two diagrams with a virtual W and a virtual Z particle contribute at about the same level, but the one involving the Higgs scalar ϕ is relatively negligible if $M_\phi > 7 \text{ GeV}/c^2$, as is now believed [10]. The uncertainty arises from the quoted error in $\sin^2 \theta_w$, but it does not include any estimate of higher-order weak interactions. The contribution of the weak interaction is a very small effect, at present masked by the uncertainty in the strong interaction contribution. So in order to utilize fully a higher-precision experimental value of a_μ to determine the electroweak contribution, it is necessary to improve our knowledge of a_μ^{had} , implying better measurements of $R(s)$, particularly in the low-energy region with invariant mass $< 1 \text{ GeV}/c^2$. Combining all the above values the overall theoretical prediction is

$$a_\mu^{\text{th}} = 116\,591\,8(2) \times 10^{-9}. \quad (2.9)$$

Finally, we must consider the effect on a_μ^{th} of a modification of QED. If the muon is not completely point-like in its behaviour, but has a form factor $F(q^2) = \Lambda_\mu^2/(q^2 + \Lambda_\mu^2)$, it can be shown that

$$\frac{\Delta a_\mu}{a_\mu} = \frac{-4m_\mu^2}{3\Lambda_\mu^2}, \quad (2.10)$$

implying, for example, a reduction in a_μ of 1 ppm if $\Lambda_\mu = 125 \text{ GeV}/c^2$. Similarly, a modification of the photon propagator by the factor $\Lambda_\gamma^2/(q^2 + \Lambda_\gamma^2)$ implies that

$$\frac{\Delta a_\mu}{a_\mu} = \frac{-2m_\mu^2}{3\Lambda_\gamma^2}. \quad (2.11)$$

This result was first obtained by Berestetskii et al. [5], who emphasized the value of experiments on the muon; the high mass m_μ implies a significant correction to a_μ even when Λ_γ is large. The violation of unitarity suggested by this modification of the photon propagator can be circumvented by introducing a negative metric [11].

It is worth while to emphasize in this brief introduction to the theory that over a period of many years the muon ($g-2$) has provided a very good constraint on theoretical constructs. Theories that attempt to generalize the Standard Model in one way or another all have their effect on the muon ($g-2$) value. Such theories include, for example, left-right symmetric theories, Kaluza-Klein theories. Supersymmetric theories introduce many new particles, with masses of the order of m_w , which would contribute to ($g-2$). Composite models treat the so-called elementary particles as composites of several fundamental entities [12]; the present precision of the muon ($g-2$) imposes a lower limit, of the order of 1 TeV, on the mass of such constituent particles.

3. MUON PRECESSION AT REST

If a positive muon is brought to rest in a metal, graphite, or other absorber it takes up an interstitial position and keeps its polarization. As the slowing down is accomplished by atomic electric fields which, on the average, are longitudinal, the Lorentz transformation to the rest

frame gives no average magnetic field in the rest frame, and so the spin direction does not change. The stationary μ^+ is then influenced by the local magnetic field; the spin rotates at frequency

$$f_s = (g/2)(eB/2\pi mc) \quad (3.1)$$

and the angular distribution of decay electrons must rotate at the same frequency. If decays are counted in a particular direction, the counting rate will be

$$N(t) \propto \exp(-t/\tau) [1 + A \cos(2\pi f_s t + \phi)] , \quad (3.2)$$

that is, an exponential decay, modulated by the spin frequency f_s . This may be recorded using a time analyser, which sorts the events according to the time delay t between the arrival of the μ^+ and the observation of the e^+ .

This precession frequency f_s has been measured many times in fields calibrated in terms of the proton spin precession frequency f_p . The ratio $\lambda = f_s/f_p = \mu_\mu/\mu_p$, where μ_μ and μ_p are the magnetic moments of the muon and proton, has been determined in this way, and also by measurements of the hyperfine splitting in muonium. The current best results are [13], [14]

$$\lambda = 3.183\,344\,(17) \quad \text{from muon precession ,}$$

$$\lambda = 3.183\,346\,(11) \quad \text{from muonium .}$$

The mean adjusted value adopted by Cohen and Taylor [4] is

$$\lambda = 3.183\,345\,47\,(47) .$$

From Eq. (3.1) the g -factor of the muon could be deduced to a similar accuracy if the mass m_μ of the muon was known. However the only independent measurement of m_μ from X-ray transitions in μ -phosphorus is accurate only to one part in 10^4 [15]. In 1958 this gave

$$g = 2 \left(1.0010^{+0.0001}_{-0.0002} \right)$$

confirming the theoretical value of a_μ to about 15%.

By studying the spin motion of muons in flight it proves possible to measure a frequency that is proportional not to g , but to $(g - 2)$. This result can then be combined with the magnetic moment measurement, and a precise value of g as well as of the muon mass can then be obtained.

4. MUON PRECESSION IN FLIGHT

4.1 Magnetic Field Only

To calculate the spin motion for a particle rotating at relativistic speeds, it is necessary to consider the influence of the transverse acceleration, which gives rise to an effect known as the Thomas precession. To explain this effect, we follow an argument developed by one of us in the Cargèse Lectures in Physics, 1968 [16].

Consider a gyroscope moving parallel to the x-axis OA, as shown in Fig. 4, and pointing initially along OA. It is assumed that the gyroscope is non-magnetic and is not subject to a

Fig. 4 Thomas precession. Initial motion along OA becomes motion along OB because of transverse acceleration. The arrow represents a gyroscope pointing in a fixed direction. At low speeds we expect $\phi = \theta$, but this is modified at velocities near c .

couple: it just has the property of pointing always in the same direction. Suppose, further, that the gyroscope is accelerated transversely so that it now moves along OB at an angle θ to the original direction. The angle between the gyroscope axis and its new direction of motion is called ϕ . As the gyroscope keeps its direction, the result expected is $\phi = \theta$. But unfortunately this is not true at relativistic speeds; the direction is modified by the Thomas precession. The result can be derived in a simple way by considering rotations in Minkowski space. This involves some spherical trigonometry, so we first consider a simple geometrical problem in which no velocities are involved.

Figure 5 shows a section of the globe with two lines of longitude separated by the small angle θ . There is an arrow near the pole, pointing due south along the meridian. Let the arrow be displaced parallel to itself until it lies on the line of longitude θ . The angle between the arrow and the new line of longitude is called ϕ . Clearly, near the pole, $\theta = \phi$. However, if the experiment is repeated at the equator the arrow will always point south: $\phi = 0$. At an intermediate latitude, designated by the angle α shown in the figure, the result will be between these two extremes. As only sine and cosine functions are involved in spherical trigonometry, it is not surprising to find that the general law is

$$\phi = \theta \cos \alpha . \quad (4.1)$$

Returning now to Minkowski space (Fig. 6) for a gyroscope moving in the x-direction, the time-axis t is rotated to t' through the angle given by $\cos \alpha = \gamma = (1 - \beta^2)^{-1/2}$, with $\sin \alpha =$

Fig. 5 Spherical trigonometry. When an arrow near the pole is displaced parallel to itself its angle relative to the lines of longitude changes. But at the equator this angle remains fixed.

$i\beta\gamma$, where $\beta = v/c$. An acceleration in the y -direction then implies a rotation of coordinates about the axis Ox' , bringing the time-axis to t'' . Suppose that the gyroscope is now moving at an angle θ to the x -axis; the angle between the gyroscope and the new direction of motion is, according to Eq. (4.1),

$$\phi = \theta \cos \alpha = \gamma \theta . \quad (4.2)$$

The angle ϕ is greater than the classical value, because Minkowski space is not really spherical at all, but involves imaginary angles.

Fig. 6 Derivation of Thomas precession using Minkowski space.

In effect, when the direction of the motion is changed by θ , the gyroscope apparently rotates through an angle $(\gamma - 1)\theta$ in the opposite direction. This is called the Thomas precession and is usually thought of as a rotation of the rest-frame axes associated with the transverse acceleration [17].

What happens if the gyroscope is a muon and the deflection is due to a magnetic field B ? Suppose that in time t the momentum vector is deflected through an angle θ_p . Allowing for the relativistic mass $\gamma \cdot m$,

$$\theta_p = \gamma^{-1} (e/mc) Bt . \quad (4.3)$$

The rest-frame axes then turn in the opposite direction through the Thomas angle:

$$\theta_t^* = (\gamma - 1) \theta_p . \quad (4.4)$$

(Starred quantities correspond to the rest-frame, unstarred quantities to the laboratory.) In the rest frame the magnetic field $B^* = \gamma B$ acts on the spin for the time $t^* = t/\gamma$, so the spin turns relative to the axes through

$$\theta_s^* = g \left(\frac{e}{2mc} \right) B^* t^* = g \left(\frac{e}{2mc} \right) Bt = (1 + a_\mu) \left(\frac{e}{mc} \right) Bt . \quad (4.5)$$

The overall angle between the spin and the momentum vector is therefore

$$\theta = \theta_s^* - \theta_t^* - \theta_p = a_\mu \left(\frac{e}{mc} \right) Bt . \quad (4.6)$$

The frequency at which the spin turns relative to the momentum vector is therefore

$$f_a = a_\mu (eB/2\pi mc) . \quad (4.7)$$

Combining with Eq. (3.1) one derives

$$a_\mu = f_a / (f_s - f_a) \quad (4.8)$$

$$= R / (\lambda - R) . \quad (4.9)$$

When the magnetic field in which the muons are stored is calibrated in terms of the proton resonance frequency f_p , the experiment becomes a measurement of the ratio $R \equiv f_a/f_p$. The muon spin frequency f_s in the same field is then obtained from the known ratio $\lambda \equiv f_s/f_p$ (see above), and this leads to Eq. (4.9).

Thus the measurement of f_a , the spin frequency of muons turning in a magnetic field allows $a_\mu \equiv (g - 2)/2$ to be measured directly, rather than g itself; and the only auxiliary constant required is λ .

The value of λ must be corrected for the diamagnetic shielding of the proton in water (~ 26 ppm), to give the field in vacuum seen by the muons.

It must again be stressed that the result (4.7) for the anomalous precession frequency f_a does not contain the relativistic factor γ , essentially because in going to the rest frame, time is shortened by $1/\gamma$, but the magnetic field is increased by γ , so the product Bt remains the same. This is the reason why if relativistic muons are used, the decay lifetime of the particle is lengthened, but the precession frequency remains the same as before: more precession cycles can be measured, leading to increased precision.

4.2 Magnetic and Electric Field

If an electric field is used to stabilize the orbit, we must consider whether it upsets the spin motion. In general it does, and in fact stray radial electric fields have caused considerable difficulty in the electron ($g - 2$) experiments performed at the University of Michigan. It turns out that these difficulties can be avoided by using muons of a carefully chosen ‘magic’ energy, as has been pointed out in the third ($g - 2$) experiment performed at CERN (see Section 8). The radial component E_r of the electric field bends the particle orbit; it also induces a magnetic field $B_z^* = \beta\gamma E_r$ in the rest frame of the particle, and this changes the spin precession frequency. To study this in detail, we must repeat the analysis of the relativistic spin precession in the presence of a radial electric field.

The deflection of the momentum vector in time t becomes

$$\theta_p = \gamma^{-1} \left(\frac{e}{mc} \right) (B + E_r/\beta) t . \quad (4.10)$$

The Thomas angle is again $\theta_t^* = (\gamma - 1)\theta_p$ and $B^* = \gamma B + \beta\gamma E_r$, so that

$$\theta_s^* = (1 + a_\mu)(B + \beta E_r)(e/mc) t , \quad (4.11)$$

giving

$$\theta = \theta_s^* - \theta_t^* - \theta_p = a_\mu \left(\frac{e}{mc} \right) Bt [1 + (\beta - 1/a_\mu\beta\gamma^2)(E_r/B)] . \quad (4.12)$$

So the new ($g - 2$) precession frequency is

$$f'_a = f_a [1 + (\beta - 1/a_\mu\beta\gamma^2)(E_r/B)] . \quad (4.13)$$

If $\beta^2\gamma^2 = 1/a_\mu$ the second term in Eq. (4.13) is zero and electric fields do not affect the ($g - 2$) precession frequency. Advantage was taken of this discovery in the third ($g - 2$) experiment performed at CERN, which used muons of a carefully chosen ‘magic energy’, 3.096 GeV (see Section 8).

4.3 Pitch Correction

The formulae for the $(g - 2)$ precession frequency f_a derived above assume that the particle velocity is exactly perpendicular to the magnetic field $\vec{B}$, so that the orbit lies in a plane perpendicular to $\vec{B}$. If the velocity has a small angle relative to this plane, the particle will follow a spiral path with pitch angle ψ , and the $(g - 2)$ frequency is altered. In a real storage system, the pitch angle is corrected by the ‘vertical’ focusing forces, which prevent the particles being lost. The pitch angle changes periodically between positive and negative values, and the correction to the $(g - 2)$ frequency becomes more complex.

All the $(g - 2)$ experiments for electrons and muons are in principle subject to a pitch correction. Early estimates of the pitch correction by Wilkinson and Crane [18], Henry and Silver [19], and Fierz and Telegdi [20] were superseded by the analysis of Granger and Ford [21] which, for the first time, took into proper account of the changes in pitch angle. Farley [22] obtained the same results with a different approach, and extended the analysis to include both electric and magnetic focusing. (See also Field and Fiorentini [23]).

Consider a particle spiralling in the main magnetic field B_z at a small angle ψ (pitch angle) with respect to the x-y plane perpendicular to the z-axis, referred to as the ‘axial’ direction. Suppose further that the pitch angle varies harmonically according to

$$\psi = \psi_0 \sin \omega_p t \quad (4.14)$$

owing to axial focusing forces (radial component of the magnetic field or the axial electric field). Choose a right-handed Cartesian coordinate frame rotating about the z-axis at the angular frequency

$$\omega = \gamma^{-1} \left(\frac{e}{mc} \right) B_z \quad (4.15)$$

such that the momentum vector lies always in the x-z plane, making an angle ψ to the x-axis. The spin motion will be calculated relative to this frame, and from this the frequency of spin precession relative to the momentum vector, the so-called $(g - 2)$ frequency, follows immediately. We must note that B_z is not necessarily constant, and if it varies with particle position our analysis is still valid but the time average of B_z must be inserted in the final equations. Note that B_x is zero on the average over any closed path, as no current flows through the orbit.

We distinguish two components of the spin motion: i) due to the main field B_z , and ii) due to the axial focusing forces. For (i), the angular velocity of the spin relative to the rotating frame is

$$\omega_a = \gamma^{-1} a_\mu (e/mc) B^*, \quad (4.16)$$

where B^* is the magnetic field in the rest frame of the particle. Resolving B_z parallel and perpendicular to the momentum, transforming to the rest frame, and recombining, one finds the x- and z-components of ω_a :

$$\omega_x = -\omega_0 \frac{\gamma-1}{\gamma} \psi, \quad (4.17)$$

$$\omega_z = \omega_0 \left(1 - \frac{\gamma-1}{\gamma} \psi^2 \right) \quad (4.18)$$

where $\omega_0 = a_\mu(e/mc)B_z$ is the $(g-2)$ angular frequency for $\psi = 0$, and we use the approximation $\sin \psi = \psi$ for small ψ .

For the axial focusing forces, we remark that when the momentum vector is deflected through the angle ψ in the x-z plane, the spin will rotate about the y-axis through the angle $h\psi$, where

$$h = (1 + \gamma a_\mu) \quad \text{for magnetic focusing,} \quad (4.19)$$

and

$$h = (1 + \beta^2 \gamma a_\mu - \gamma^{-1}) \quad \text{for electric focusing.} \quad (4.20)$$

Thus the harmonic pitch oscillation (4.16) assumed above will induce an instantaneous angular velocity of the spin about the y-axis:

$$\omega_y = -h\omega_p \psi_0 \cos \omega_p t. \quad (4.21)$$

We wish to follow the spin motion under the combined action of the three angular velocities (4.17), (4.18), and (4.21).

If the spin direction is defined by the polar coordinates θ, ϕ such that the projection on the Cartesian axes are

$$x = \cos \theta \cos \phi, \quad (4.22a)$$

$$y = \cos \theta \sin \phi, \quad (4.22b)$$

$$z = \sin \theta, \quad (4.22c)$$

we can calculate $\dot{\theta}$ and $\dot{\phi}$ in terms of the Cartesian components of the angular velocity ω_x, ω_y , and ω_z .

Recall that for any vector $\vec{r}$, $\dot{\vec{r}} = \vec{\omega} \times \vec{r}$. Therefore, taking the x-, y-, and z-components,

$$\dot{z} = \omega_x y - \omega_y x, \text{ etc.} \quad (4.23)$$

Differentiating Eq. (4.22c) then gives

$$\dot{\theta} = (\omega_x y - \omega_y x) / \cos \theta = \omega_x \sin \phi - \omega_y \cos \phi. \quad (4.24)$$

Furthermore, from Eqs. (4.22a) and (4.22b)

$$\tan \phi = y/x \quad (4.25)$$

and

$$\sec^2 \phi = \frac{x^2 + y^2}{x^2}. \quad (4.26)$$

So differentiating Eq. (4.25) and using Eqs. (4.22a-c) and (4.23), we find, after a little manipulation,

$$\dot{\phi} = \frac{x\dot{y} - \dot{x}y}{x^2 + y^2} = \omega_z - (\omega_x \cos \phi + \omega_y \sin \phi) \tan \theta. \quad (4.27)$$

Substituting now from Eqs. (4.20) to (4.24), we find

$$\dot{\theta} = -\omega_0 \psi_0 \gamma^{-1} (\gamma - 1) \sin \phi \sin \omega_p t + \omega_p \psi_0 h \cos \phi \cos \omega_p t, \quad (4.28)$$

$$\begin{aligned} \dot{\phi} = & \omega_0 [1 - \gamma^{-1}] (\gamma - 1) \psi_0^2 \sin^2 \omega_p t \\ & + \omega_0 \psi_0 \gamma^{-1} (\gamma - 1) \tan \theta \cos \phi \sin \omega_p t + \omega_p \psi_0 h \tan \theta \sin \phi \cos \omega_p t. \end{aligned} \quad (4.29)$$

These equations are readily solved if we replace ϕ in the right-hand side with the close approximation $\phi = (\omega_0 t + \xi)$, where ξ is an arbitrary phase. We then find from Eq. (4.28)

$$\begin{aligned} \theta = & \frac{A_1}{\omega_0 + \omega_p} \sin [(\omega_0 + \omega_p) t + \xi] \\ & - \frac{A_2}{\omega_0 - \omega_p} \sin [(\omega_0 - \omega_p) t + \xi], \end{aligned} \quad (4.30)$$

where

$$A_1 = \frac{1}{2} \psi_0 [\omega_0 \gamma^{-1} (\gamma - 1) + h \omega_p] \quad (4.31)$$

and A_2 is the same but with a negative sign before the last term.

Note that in general θ oscillates about zero with amplitude $\sim \psi_0$, the spin remaining on the average in the x-y plane. However, the amplitude becomes large if $\omega_p \approx \omega_0$, and if $\omega_p = \omega_0$, $\theta = -A_2 t \cos \xi$, so the spin turns continuously out of the plane. This is an example of the familiar depolarization resonance. It follows from Eq. (4.30) that large values of θ occur only if $(\omega_0 - \omega_p)/\omega_0 \lesssim \psi_0/\gamma \lesssim 10^{-2}$ in a typical experiment.

If we are not too close to the resonance, θ remains small, and we may substitute θ as given by Eq. (4.30) for $\tan \theta$ in Eq. (4.29). This gives:

$$\dot{\phi} = \omega_0 \left(1 - \frac{\gamma-1}{2\gamma} \psi_0^2 \right) + \frac{A_1^2}{2(\omega_0 + \omega_p)} + \frac{A_2^2}{2(\omega_0 - \omega_p)} + \text{terms oscillating at frequencies } \omega_p, \omega_0, (\omega_0 - \omega_p), \text{ etc.} \quad (4.32)$$

So the observed $(g-2)$ frequency is $\omega_a = \dot{\phi} = \omega_0(1 - C)$ where, using Eqs. (4.31) and (4.32), the correction factor C is found to be

$$C = \frac{1}{4} \psi_0^2 \left[1 - \frac{\omega_0^2}{\gamma^2(\omega_0^2 - \omega_p^2)} - \frac{\omega_p^2(h-1)(h-1+2/\gamma)}{\omega_0^2 - \omega_p^2} \right]. \quad (4.33)$$

Applying this result to three cases that are of experimental interest, we find the following:

a) When the axial focusing forces are magnetic, $h-1 = \gamma a_\mu$ and

$$C = \frac{1}{4} \psi_0^2 \left[1 - \frac{(\omega_0^2 + 2a_\mu \gamma^2 \omega_p^2) + a_\mu^2 \gamma^4 \omega_p^2}{\gamma^2(\omega_0^2 - \omega_p^2)} \right], \quad (4.34)$$

in agreement with Granger and Ford [21]. Except at very high energies, the terms in a_μ will be negligible in view of the uncertainties in determining ψ_0 . As ω_p is varied, C changes according to a dispersion curve. When $\omega_p \ll \omega_0$ (the limit of slow pitch changes), $C = 1/4 \beta^2 \phi^2$. As ω_p increase, C falls, eventually becoming negative, but after the resonance at $\omega_p = \omega_0$, C becomes large and positive, finally approaching $1/4 \psi^2 (1 + 2a_\mu)$ when $\omega_p \gg \omega_0$ (the limit of rapid pitch changes). Near resonance our approximation $\theta \approx \tan \theta$ is not valid: a more detailed analysis shows that infinitely large corrections suggested by Eq. (4.33) do not, in fact, occur.

b) With axial focusing by electric fields, $h-1 = \beta^2 \gamma a_\mu - \gamma^{-1}$, so

$$C = \frac{1}{4} \psi_0^2 \left(\beta^2 - \frac{a_\mu^2 \beta^4 \gamma^2 \omega_p^2}{\omega_0^2 - \omega_p^2} \right). \quad (4.35)$$

At low energy the second term is negligible and the correction is virtually independent of the pitch frequency.

c) With an electric field, using the 'magic' particle energy given by $\beta^2 \gamma^2 = a_\mu^{-1}$ so that the electric fields do not affect the $(g-2)$ precession, then $h = 1$ and the final term in Eq. (4.33) is zero:

$$C = \frac{1}{4} \psi_0^2 \left[1 - \frac{\omega_0^2}{\gamma^2(\omega_0^2 - \omega_p^2)} \right]. \quad (4.36)$$

The pitch correction now shows the resonant behaviour discussed above, but it contains no terms in a_μ . Far from resonance, $C = 0.25 \psi^2$ at both limits.

Fig. 7 Axes xyz rotating about z so that the momentum vector p lies always in the x - z plane. Owing to pitch oscillations the pitch angle ψ between p and the x -axis is varying. The spin angle is determined in the plane P (inclined at angle $h\psi$ to the x -axis) because a spin in this plane moves into the xy -plane when ψ passes through zero.

These results may be derived directly by means of a simple physical argument based on Eqs. (4.17) and (4.18). We are interested in the average rotation of the spin about the z -axis, i.e. the projection of the spin on the x - y plane. Let us consider the spin rotation in this plane at instants when the pitch angle is zero, thus neglecting for the average motion the small non-cumulative mutations that occur during the pitch oscillations. The essential step is to recognize that when the pitch angle is ψ , we must determine the progress of the spin in a plane making an angle $h\psi$ with the x -axis, indicated in Fig. 7 as the plane P . This is because any spin direction lying in this plane will be turned into the x - y plane when the pitch angle again becomes zero.

We therefore resolve the angular velocities (or magnetic fields) parallel ($\omega_{\parallel}$) or perpendicular ($\omega_{\perp}$) to the plane P . From Eqs. (4.17) and (4.18),

$$\omega_{\perp} = \omega_z \cos(h\psi) - \omega_x \sin(h\psi) = \omega_0 \left\{ 1 - \frac{1}{2} \psi^2 [1 + (h-1)(h-1+2/\gamma)] \right\}. \quad (4.37)$$

When the pitch oscillation is rapid ($\omega_p \gg \omega_0$), then $\omega_{\parallel}$ changes rapidly in sign and contributes nothing to the net spin precession. So, in this case, the observed spin motion will be determined by $\omega_{\perp}$ alone. Recalling that $\psi^2 = 1/2 \psi_0^2$, we see that Eqs. (4.33) and (4.37) agree exactly in the limit $\omega_p \gg \omega_0$.

In the case of slow pitching ($\omega_p \ll \omega_0$) however, $\omega_{\parallel}$ remains almost constant over many cycles of $(g-2)$ precession, and then $\omega_{\perp}$ and $\omega_{\parallel}$ must be added vectorially to give the resultant angular velocity. This is of course equal to the resultant of ω_x and ω_z , so from Eqs. (4.17) and (4.18),

$$\omega_a = \omega_0 \left(1 - \frac{1}{2} \beta^2 \psi^2 \right), \quad (4.38)$$

agreeing with Eq. (4.33) in the limit $\omega_p \rightarrow 0$.

Thus the fast-pitching and slow-pitching limits of Eq. (4.33) are confirmed by an independent argument.

4.4 Experimental Principles

To summarize the principles of the $(g-2)$ experiment, the essential requirements are a polarized source, an almost uniform magnetic field, and a polarimeter to measure the spin direction as a function of time.

An effective polarimeter would be one that measured $\vec{s} \cdot \vec{\beta}$ as a function of the storage time t spent in the magnetic field. This quantity oscillates with the frequency f_a , and in Fig. 8 we illustrate its measurement, showing the main components of the $(g-2)$ experiment in block diagram form.

Fig. 8 Relative motion of the spin and momentum in the laboratory frame and the block diagram of an ideal $(g-2)$ experiment.

The muons are produced by the decay of pions in flight, and are born longitudinally polarized in the pion rest frame. Furthermore, the muon waves a flag to show which way its spin is pointing at the moment of decay; in fact, in the process $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$, the electron angular distribution has its maximum in the direction of the muon spin. By observing a large number of decays from an ensemble of muons, one can measure the average spin direction to any desired accuracy. Thus, the pion-muon decay and the muon-electron decay provide a very simple polarized source and an efficient polarimeter. The ideal $(g-2)$ experiment consists, then, of trapping longitudinally polarized muons in a uniform magnetic field and measuring the precession frequency f_a of the spin relative to the velocity vector through the decay electron asymmetry.

The observed phenomenon is oscillatory in nature, and so the more oscillation periods there are, the more precise will be the determination of the frequency f_a . This can easily be seen by supposing, for example, that we count n cycles of a wave train, extending over a measured time t , with an uncertainty of one cycle. Then

$$f_a = \frac{n}{t} \pm \frac{1}{t}, \quad (4.39)$$

the error in the frequency decreases as the duration of t increases. It is therefore advantageous to store the charged particle in the uniform-field region for as long as possible. For the electron, this time is limited only by the quality of the trap, and may extend to months. But for the muon, its natural lifetime ($2.2 \mu\text{s}$) limits the useful storage time, and it is of advantage to dilate the lifetime by going to high energies. In doing this a real gain is made since, as we have seen, the frequency f_a does not depend on the particle energy: through the relativistic increase in the laboratory lifetime of the muon, more $(g - 2)$ modulation cycles can be measured.

In the foregoing we have outlined the general features of the precession experiment under idealized conditions. In real experiments it is not possible to inject particles of a given momentum into a fixed orbit in a static magnetic field. The open curve of the injection trajectory cannot lead to the closed curve of the fixed orbit without some localized change in the orbit parameters, such as the particle velocity or direction. The particles can, of course, be contained within a reasonable volume by allowing quasi-circular orbits to drift slowly through the storage region, but it should be remembered that the less confined the orbits are, the more difficult is the task of obtaining a precise value of the mean magnetic field $\bar{B}$ experienced by the muon sample. Different methods of injection will be discussed when briefly describing the different $(g - 2)$ experiments performed at CERN. A more detailed discussion of some experimental problems encountered in the muon $(g - 2)$ experiments is given in Ref. [24].

5. INITIAL MUON POLARIZATION AND MEASUREMENT OF THE FINAL POLARIZATION

5.1 Generalities

Muons are most readily obtained from the pion decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$, and parity non-conservation in this process leads to their polarization. The relevant decay configuration for the positive pions is shown in Fig. 9. The muons that decay close to the forward direction of the pion momentum have helicity approximately equal to -1 , and also laboratory momentum

Fig. 9 Pion-muon decay in the laboratory frame.

near the top of the available range. Selection of these maximum-momentum muons will thus yield a sample with high initial longitudinal polarization. By taking a 1% momentum bite around the maximum value of the pion momentum, the longitudinal polarization is about 97%. Thus the muons that have been produced by decays within a momentum-selected pion beam, and have momenta within the acceptance of the beam, will be strongly polarized longitudinally. The degree of polarization will depend on the momentum acceptance of the pion beam and on the decay channel.

The analysis of the longitudinal polarization of this sample as a function of time is effected by making use of the asymmetry in the decay electron angular distribution that arises from parity non-conservation. For the process $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$, for example, the decay electron angular distribution in the muon rest frame can be expressed as the differential probability

$$dP(y, \theta) = n(y)[1 + A(y) \cos \theta] dy d\Omega, \quad (5.1)$$

where $y = p_e/(p_e)_{\max}$, $d\Omega$ is a solid angle, θ is the angle between the electron momentum p_e and the muon spin, and the distribution function and the asymmetry are given by

$$n(y) = 2y^2(3 - 2y), \quad A(y) = \frac{2y - 1}{3 - 2y}. \quad (5.2)$$

The asymmetry has a maximum value of +1 at $y = 1$, changes sign at $y = 1/2$, and reaches a value of $-1/3$ at $y = 0$ (see Fig. 10).

Thus if muons that have emerged from the storage region are stopped in a non-depolarizing target, and the subsequent decay electrons are detected in a counter that is placed, for example, 'forward' with respect to the muon momentum prior to stopping, then the count rate as a function of storage time will be modulated with the $(g - 2)$ frequency f_a . The amplitude of this modulation will depend on the polarization of the muon sample, and on the

Fig. 10 The decay electron spectrum and asymmetry in the muon rest frame. The quantity y is the electron momentum expressed as a fraction of its maximum value.

Fig. 11 The decay electron momentum in the muon rest frame. The parabolas illustrate the effect in this frame of an energy cut in the laboratory.

detection efficiency of the counter system as a function of electron angle and momentum. This is essentially the technique employed in the muon experiment carried out at the CERN Synchro-cyclotron (SC), as described in more detail below.

If the muons are allowed to decay within the storage region, then we no longer have direct access to the muon rest frame, and the manner of measuring the final polarization clearly has to be modified. This measurement can be effected by selecting a subset of decay electrons above a given energy threshold in the laboratory. Such a cut can be transformed into the muon rest frame, as illustrated in Fig. 11. This figure shows the decay electron momentum space in the muon rest frame and represents a section, through the axis of symmetry, that coincides with the direction of the muon velocity β_μ . The parabolas illustrate the effect of a laboratory energy threshold E_l in this space, expressed in units of $0.5mc^2$ (~ 53 MeV). Thus the shaded region represents the decay electron momenta and directions that would be accepted for an energy cut equal to just one half the muon energy in the laboratory ($E_\mu = \gamma_\mu m_\mu c^2$).

This is a general result, which can be derived as follows. Let a moving system have velocity $\beta_c \cdot c$ in the laboratory, and in the moving system suppose particles are emitted with constant momentum p^* , energy E^* , and angular distribution $dN/d\Omega^* = f(\theta^*)$, θ^* being the angle in the moving system relative to the direction of motion. The energy of the particle in the laboratory will be given by

$$E_{lab} = \gamma_e \{E^* + \beta_e p^* \cos \theta^*\}$$

where $\gamma_e = (1 - \beta_e^2)^{-1/2}$.

As

$$\Omega^* = 2\pi (1 - \cos \theta^*)$$

$$d\Omega^* = -2\pi d(\cos \theta^*)$$

$$dE_{lab} = (\beta_e \gamma_e p^* / 2\pi) d\Omega^*,$$

so

$$dN/dE_{lab} = (2\pi/\beta_e\gamma_e p^*)(dN/d\Omega^*)$$

$$= (2\pi/\beta_e\gamma_e p^*) f(\theta^*) .$$

As $\beta_e\gamma_e$ and p^* are all independent of θ it is clear that the energy distribution in the laboratory has the same functional form as the angular distribution in the moving system. Hence selecting energy in the laboratory is equivalent to selecting angle in the moving system.

In the case of muon decay in flight to electrons this conclusion has to be modified because there is a spectrum of momenta p_e , and this will smear the one-to-one correlation of E_{lab} with θ_e . Now to have high energy in the laboratory the decay electron must a) come from near the top of the spectrum in the moving system (this means also a high asymmetry parameter A) [Eqs. (5.2)] and b) be emitted forwards.

From this simple argument we can see how the energy cut in the laboratory distinguishes between decays that are forward as opposed to backwards with respect to the muon momentum, and how decays at quite large angles in the muon rest frame may be included in the process. If we now envisage that the asymmetric probability distribution given by Eq. (5.1) is rotating in the muon rest frame with angular frequency f_a , we can see that the number of decay electrons within this selected subset will be modulated with the $(g-2)$ frequency and have the form

$$N(t) = N_0 \exp[-t/\tau] [1 - A \cos(2\pi f_a t + \phi)] , \quad (5.3)$$

where τ is the dilated muon lifetime ($\tau = \gamma\tau_0$), A is the overall asymmetry, and ϕ is the initial phase of the polarized muon sample, that is the initial phase of the spin with respect to the muon velocity vector. Elementary error analysis of the $(g-2)$ signal contained in an observed time spectrum of this form shows that the error in f_a is proportional to $(\tau A \sqrt{N})^{-1}$, where N is the total number of decay electrons detected. This expression indicates that the energy threshold should be set so as to maximize NA^2 , and it also demonstrates the importance of the dilated muon lifetime $\tau = \gamma\tau_0$. By integrating over the shaded region shown in Fig. 11 for different values of E_t , we obtain the curves of N_e , A , and $N_e A^2$ shown in Fig. 12. As can be seen, the

Fig. 12 Dependence of the number of decay electrons N_e , the asymmetry coefficient A and $N_e A^2$ versus $b = E_t/\gamma_\mu$.

required maximum in NA^2 occurs at a value of E_t that is equivalent to about 0.65 times the highest electron energy. Having established the manner of the final polarization measurement, we will return to the optimization of the $(g-2)$ signal in the discussion of each actual experimental configuration.

5.2 From an Ideal $(g-2)$ Experiment to a Real One

In a real $(g-2)$ experiment, the quality of the storage region (trap) plays an important role because the uncertainty in a_μ is in part inversely proportional to the storage time t , and it is a great advantage to make this time as long as possible.

The first requirement for the quality of the trap is therefore that the particle be trapped or stored for a long period of time. This means the elimination of the characteristics that increase the amplitude of the particle orbits to such an extent that they collide with the material limits of the region and are consequently lost. The most obvious source of such beam blow-up is collisions with gas atoms, so the storage volume must be evacuated. However, since the storage times are relatively short, the vacuum requirements are not as stringent as those of storage rings, in which particles are contained for several hours. Another source of particle loss can be the magnetic field itself. If this field contains local inhomogeneities through which the particles pass at intervals that are simply related to the period of oscillation of the particle about its equilibrium orbit, then the amplitude of these oscillations may grow in time. This distortion of the particle orbit leads to a shift in frequency of the oscillations, so that there is a phase slippage, and the perturbation then decreases the amplitude again. The resulting modulation in amplitude may well take the particle outside the physical limits of the storage volume. This phenomenon is similar to the case of a simple harmonic oscillator driven by a periodic force, and is well known to the designers of particle accelerators and storage rings. So the successful realization of a precession experiment requires a careful study of the stability of the orbits if premature loss of particles is to be avoided.

The second requirement for the quality of the trap concerns the need to measure, with great precision, the average magnetic field seen by the particles. This quantity, $\bar{B}$, must be known to a greater precision than that desired for a_μ itself, since it is just one of the contributions to the overall uncertainty. Any real beam of particles occupies a finite volume of phase space, and consequently the particles do not all follow the same orbits. The precise knowledge of $\bar{B}$ involves, therefore, a precise mapping of the magnetic field and a measurement of the distribution of particle orbits, or at least the calculation of this distribution in a reliable way. The extent to which the uncertainty in this latter leads to an error of $\bar{B}$ is determined by the uniformity of the magnetic field; for example, if the field were perfectly uniform, we would only need to know that the particles were contained within the storage region, but not precisely where.

Another consequence of the finite extension of the phase space of the beam is that it will have some angular divergence, and will thus contain particles whose component of the velocity along the magnetic field lines is not zero. This motion is not restricted by a uniform field. In a precession experiment, the total path length travelled by particles may reach several kilometres, and the confinement of these orbits within a reasonable volume implies some non-uniformity of

the magnetic field itself, in the form of a gradient, or perhaps the application of an electric field. A magnetic gradient automatically makes it more difficult to determine $\vec{B}$. Thus there is a conflict between efficient trapping and knowing the average magnetic field, which it is the art of the experimenter to resolve.

6. 1958-1962: THE $(g - 2)$ EXPERIMENT AT THE CERN SYNCHRO-CYCLOTRON

By 1958, QED was an established theory of some 10 years standing, corroborated by accurate measurements of the Lamb shift. The g -factor of the electron was known through electron spin resonance [25] to one part per million (ppm); in 1950, Karplus and Kroll [26] had shown how to calculate the higher-order corrections to g , and a numerical error in their results had been corrected by Petermann [27], Sommerfield [28], and Suura and Wichmann [29], bringing theory into line with experiment at the level of $(\alpha/\pi)^2$.

For the free electron, a direct determination of the anomalous magnetic moment $a_e \equiv (g - 2)/2$ was in progress at the University of Michigan [30] using the recently discovered principle of $(g - 2)$ spin motion explained above. Equation (4.7) had been proved to hold for relativistic velocities.

Turning to the muon, the bremsstrahlung cross-section at high energies had been measured with cosmic rays and was shown to agree with a spin assignment of $1/2$ rather than $3/2$ [31]. A similar conclusion followed from data on neutron production by cosmic-ray muons [32]. The angular distribution of electrons from the decay of polarized muons agreed with spin $1/2$ [33] and was inconsistent with spin $3/2$ [34]. Experiments with cosmic-ray and accelerator-generated muons were in progress to compare the electromagnetic scattering of muons and electrons by nuclei.

Thus evidence was accumulating that the muon behaves as a heavy electron of spin $1/2$. Berestetskii et al. [5] had emphasized that QED theory implied an anomalous magnetic moment a_μ for the muon, of the same order as for the electron, but as the typical invariant momentum transfer involved was $q^2 \approx m^2$, an experiment for the muon would test the theory at much shorter distances. Feynman [35] felt that the divergences in QED could be limited by a real energy-momentum cut-off Λ , and it seemed reasonable to expect Λ to be of the order of the nucleon mass. This would imply a 0.5% effect in a_μ . On the other hand, it was thought [36] that the muon should have an extra interaction that would distinguish it from the electron and give it its higher mass. This could be a coupling to a new massive field, or some specially mediated coupling to the nucleon. Whatever the source, the new field should have its own quantum fluctuations, and therefore give rise to an extra contribution to the anomalous moment a_μ . The $(g - 2)$ experiment was recognized as a very sensitive test of the existence of such fields, and potentially a crucial signpost to the μ - e problem.

At this stage there was no prospect of such an experiment, but in 1957 parity violation was discovered [37], muon beams were found to be highly polarized, and, better still, it was found that the angular distribution of the decay electrons could indicate the spin direction of the muon as a function of time [38]. A wide variety of muon precession and spin-resonance experiments were carried out in the next few years [39]. The $(g - 2)$ principle was invoked in the first paper on muon precession by Garwin et al. [38], who pointed out that g must be within 10% of 2.00,

because although the muon trajectory had been deflected through 100° by the cyclotron magnetic field, the muon polarization was still longitudinal.

The possibility of a $(g - 2)$ experiment for muons was envisaged, and groups at Berkeley, Chicago, Columbia, and Dubna started to study the problem [40]. If the muon had a structure that gave a form factor less than one for photon interactions, the value of a_μ should be less than predicted. Compared with the measurement for the electron, the muon $(g - 2)$ experiment was much more difficult because of the low intensity, diffuse nature, and high momentum of available muon sources. This implied large volumes of magnetic field; the lower value of (e/mc) made all precession frequencies 200 times smaller, but the time available for an experiment was limited by the decay lifetime, $2.2 \mu\text{s}$. Hence large magnetic fields would be needed to give a reasonable number of precession cycles.

One solution was to scale up the method used at Ann Arbor for the electrons, using a large solenoid and injecting the muons spirally at one end [41]. This was pursued at Berkeley and finally led to a 10% precision measurement [42].

At CERN, the work centred on the belief that it should be possible to store muons in a conventional bending magnet with a more or less uniform vertical field between roughly rectangular pole pieces. In a typical field of 1.5 T, the muon orbit would make 440 turns during the lifetime of $2.2 \mu\text{s}$. As $a_\mu \approx \alpha/2\pi \approx 1/800$, the angle between the spin and the momentum vector would develop 800 times more slowly, giving a change in beam polarization of about 180° to be studied.

The polarized muon beam from the CERN Synchro-cyclotron could fairly easily be trapped inside a magnet. The particles were aimed at an absorber in the field; they lost energy and therefore turned more sharply and remained inside the magnet. To prevent them re-entering the absorber after one turn, a small transverse (y-direction) gradient of the magnetic field was introduced, causing the orbits to drift sideways perpendicular to the gradient (x-direction). Vertical focusing was added by means of a parabolic term in the field.

If the field is of the form

$$B_z = B_0 (1 + ay + by^2), \quad (6.1)$$

where a and b are small, an orbit of radius q moves in the x-direction over a distance $s = a\pi q^2$ per turn (called the step size). On the average, the wavelength of the vertical oscillations is $2\pi/b^{1/2}$. Figure 13 is of historical interest. It shows the first evidence of particles turning several times inside a small experimental magnet. These results gave the laboratory sufficient confidence to order a very long magnet for the experiment.

An overall view of the final storage system [43] is shown in Fig. 14. The magnet pole was 6 m long and 52 cm wide, and the gap was 14 cm. Muons entered on the left through a magnetically shielded iron channel and hit a beryllium absorber in the injection part of the field. Here the step size s was 1.2 cm. Then there was a transition to the long 'storage region', where $s = 0.4 \text{ cm}$ with the field gradient $a = (1/B)(dB/dy) = 3.9 \times 10^{-4} \text{ cm}^{-1}$. Finally, a smooth transition was made to the ejection gradient, where $s = 11 \text{ cm}$ per turn. After ejection, the muons fell onto the 'polarization analyser' (Fig. 15), where they were stopped and decayed to

Fig. 13 First evidence of muons making several turns in an experimental magnet. The time of arrival of the particles at a scintillator fixed inside the magnet is plotted horizontally (time increases to the left). The first (right-hand) peak coincides with the moment of injection. The equally spaced later peaks correspond to successive turns. Owing to the spread in orbit diameters and injection angles, some muons hit the counter after nine turns (lower right), while others take 18 turns to reach the same point (Charpak et al., unpublished).

e^+ . The time t spent by a muon in the field was determined by recording the coincidences in counters 123 at the input, and counters 4566'7 at the output. The interval was measured with respect to a 10 MHz crystal.

The shimming of this large magnet to produce the correct gradients was a tour de force. This was assisted by the theorem that in weak gradients the flux through a wandering orbit is an invariant of the motion. Therefore, if the field along the centre line of the magnet was constant, unwanted sideways excursions would be avoided, and this could be checked more exactly by moving a flux coil, of the same diameter as the orbit, all along the magnet.

Fig. 14 Storage of muons for up to 2000 turns in a 6 m bending magnet. The field gradient makes the orbit walk to the right. At the end a very large gradient is used to eject the muons so that they are stopped in the polarization analyser. Coincidences 123 and 466'57, signal an injected and ejected muon, respectively. The coordinates used in the text are x (the long axis of the magnet), y (the transverse axis in the plane of the paper), and z (the axis perpendicular to the paper).

Fig. 15 Polarization analyser. When a muon stops in the liquid methylene iodide (E) a pulse of current in coil G is used to flip the spin through $\pm 90^\circ$. Backward or forward decay electrons are detected in counter telescopes 66' and 77'. The static magnetic field is kept small by the double iron shield and the mumetal shield A.

However, the constant flux theorem implied that once the particle was trapped inside the magnet it would never emerge. This was seen as a major difficulty, because the final spin direction could only be measured in a weak or zero magnetic field: otherwise, one would lose track of the spin direction while waiting for the muon to decay. For weak gradients and slowly walking orbits, calculations of the orbit confirmed these doubts, and some participants lost faith in the project. Fortunately, it was found that in large gradients, of order $\pm 12\%$ over the orbit diameter, the particles were ejected successfully.

The muons were trapped in the magnet for a time ranging from 2 to 8 μs depending on the location of the orbit centre on the varying gradient given by Eq. (6.1). About one muon per second was stopped finally in the polarization analyser, and the decay electron counting rate was 0.25 per second.

The spin direction can, in principle, be obtained from the ratio of two counting rates measured in different directions. But if two counter telescopes are used (say one forward and one backward relative to the direction of the arriving muons), it is not easy to ensure that they have equal efficiencies and solid angles. Therefore it is more reliable to use only one set of counters, but to move the muon spin direction after it has stopped. This can be done with a small constant magnetic field, but it is more efficient to turn the spin rapidly to a new position by applying a short, sharp magnetic pulse, created by applying a pulse of current to a solenoid wound round the absorber in which the muon is stopped. This flipping was accomplished within 1 μs , before the gate that selected the decay electrons was opened.

In the apparatus shown in Fig. 15, the electron counts c_+ and c_- in the forward telescope 77' were recorded in separate runs with the spin flipped through $+90^\circ$ and -90° , respectively. The asymmetry A of these counts, defined as $(c_+ - c_-)/(c_+ + c_-)$, was then related to the initial direction θ_s of the muon spin (before flipping) relative to the mean electron direction subtended by telescope 77':

$$A \equiv \frac{(c_+ - c_-)}{(c_+ + c_-)} = A_0 \sin \theta_s. \quad (6.2)$$

By flipping instead through 180° and 0° , another ratio proportional to $A_0 \cos \theta_s$ was measured; so θ_s could be determined completely. Similar, but independent, calculations were made for the telescope 66', which detected decay electrons emitted backwards.

This polarization analyser was first used to study the muon beam that was available for injection. For muons that had been through the magnet, the analyser recorded the asymmetry A as a function of the time t the particle had spent in the field. This showed a sinusoidal variation due to the $(g - 2)$ precession in the magnet. Using Eqs. (4.7) and (6.2), it follows that

$$A = A_0 \sin \theta_s = A_0 \sin [a_\mu (e/mc) Bt + \phi], \quad (6.3)$$

where ϕ is an initial phase determined by measuring the initial polarization direction and the orientation of the analyser relative to the muon beam.

Fig. 16 Asymmetry A of observed decay electron counts as a function of the storage time t . The sinusoidal variation results from the $(g-2)$ precession; the frequency is measured to $\pm 0.4\%$.

The experimental data are given in Fig. 16, together with the fitted line obtained by varying A_0 and a_μ in Eq. (6.3). Full discussion of the precautions that are necessary to determine the mean field B seen by the muons, and to avoid systematic errors in the initial phase ϕ , are given in [43].

The results of this experiment are given in Table 1 (see subsection 8.8). Preliminary runs gave $\pm 2\%$ accuracy in a_μ , and this was later improved to $\pm 0.4\%$. The figures agreed with theory within experimental errors. The corresponding 95% confidence limit for the photon propagator cut-off [see Eq. (2.11)] was $\Lambda_\gamma > 1.0$ GeV, and for the muon vertex function [Eq. (2.10)] $\Lambda_\mu > 1.3$ GeV.

This was the first real evidence that the muon behaved so precisely like a heavy electron. The result was a surprise to many, because it was confidently expected that g would be perturbed by an extra interaction associated with the muon to account for its larger mass [36, 44]. When nothing was observed at the 0.4% level, the muon became accepted as a structureless point-like QED particle, and the possibility of finding a clue to the μ - e mass difference seemed suddenly much more remote.

7. 1962-1968: MUON STORAGE RING I

7.1 A General View

By now the CERN Proton Synchrotron (PS) and Brookhaven Alternating Gradient Synchrotron (AGS) were operating, and the distinct properties of the two neutrinos ν_e and ν_μ had been established, further emphasizing the parallel but dual behaviour of the muon and the electron [45].

Muon-pair production by 1 GeV gamma rays on carbon was measured by Alberigi-Quaranta et al. [46] in agreement with theory. With this and the $(g-2)$ data, the evidence for point-like behaviour was now much better for the muon than for the electron. The

scattering of muons by lead and carbon [47] agreed with the form factors deduced for electron scattering. Logically this was the best evidence for the point-like behaviour of the electron, but was generally seen as another contribution to our knowledge of the muon. Knock-on electrons from 8 GeV muons confirmed the picture [48]. Muonium formation in high-pressure argon had been observed by Hughes et al. [49], and the hyperfine splitting of the ground state confirmed the theoretical picture to one part in 2000 [50]. For this and subsequent muonium experiments the $(g - 2)$ result was an essential input, not only for the g -factor, but also for deducing the muon mass from the precession frequency at rest, now determined to 16 ppm by Hutchinson et al. [51], see Eqs. (3.1) and (4.7).

The muon $(g - 2)$ experiment was now the best test of QED at short distances. For this reason, and to search again for a new interaction, it was desirable to press the accuracy of the experiment to new levels. It would be essential to increase the number of $(g - 2)$ cycles observed, either by increasing the field B or by lengthening the storage time. With the CERN PS available, it was attractive to see what could be done by using high-energy muons with relativistically dilated lifetimes. As there is no factor γ in Eq. (4.7), the $(g - 2)$ precession frequency would not be reduced, and more cycles would be available before the muons decayed. But to store muons of GeV energy in a magnetic field and measure their polarization required totally new techniques. Farley [52] proposed to measure the anomalous moment using a muon storage ring. As in the cyclotron, if the primary target was placed inside the magnet, muons would be produced by $\pi - \mu$ decay in flight and some of them would remain trapped in the field. With a pulsed accelerator, such as the PS, there should be no continuous background following the injection. Transfer of a pulse of protons from the PS to the muon ring could be achieved with the fast ejected beam already developed for the neutrino experiment. Estimates of the stored muon intensity and polarization looked favourable.

To determine the muon spin direction, decays in flight would need to be observed. The decay electrons would emerge on the inside of the ring and the detectors would respond only to the high-energy particles emitted more or less forward in the muon rest frame. Thus as the spin rotated, the electron counting rate would be modulated at the $(g - 2)$ frequency.

It was later realized that at injection the muons would be localized in azimuth (injection time 10 ns, rotation time about 50 ns), so the counting rate would also be modulated at the mean rotation frequency. This would enable the mean radius of the stored muons to be calculated, leading to a precise knowledge of the corresponding magnetic field.

On the evening of 21 October 1963, a random coincidence of time and place influenced the development of the project. The present authors, having first met that morning, found themselves filling a vacant evening in the same bar of the Hawthorne Hotel, Bristol. They drifted into discussing physics, and thus initiated a 26-year collaboration.

The first Muon Storage Ring (Fig. 17) [53] was a weak-focusing ring with $n = 0.13$, orbit diameter 5 m, a useful aperture of $4 \text{ cm} \times 8 \text{ cm}$ (height $\times$ width), a beam momentum of 1.28 GeV/c corresponding to $\gamma = 12$, and a dilated muon lifetime of 27 μs . The mean field at the central orbit was $\bar{B} = 72.8527(53)$ proton MHz (1.711 T).

The injection of polarized muons was accomplished by the forward decay of pions produced when a target inside the magnetic field was struck by 10.5 GeV/c protons from the

Fig. 17 Muon Storage Ring I: diameter 5 m, muon momentum 1.3 GeV/c, time dilation factor 12. The injected pulse of 10 GeV protons produces pions at the target, which decay in flight to give muons.

CERN PS. The proton beam consisted of either two or three radio-frequency bunches (fast ejection), each ~ 10 ns wide and spaced at ~ 105 ns. As the rotation time in the ring was 52.5 ns, these bunches overlapped exactly inside the ring. Approximately 70% of the protons interacted, creating, among other things, pions of 1.3 GeV/c that started to turn around the ring. The pions made, on an average, four turns before hitting the target again, and in one turn about 20% of the pions decayed. The muons created in the exactly forward decay, together with undecayed pions and stable particles from the target, eventually hit the target and were lost.

However, the decay of pions at small forward angles gave rise to muons of slightly lower momentum, and some of these fell into orbits that missed the target and remained permanently stored in the ring. Thus the perturbation, essential for inflection into any circular machine, was here achieved by the shrinking of the orbit, arising from the change of momentum in π - μ decay, and to some extent by the change in angle at the decay point, which could leave the muon with a smaller oscillation amplitude than that of its parent pion. The muons injected in this way were forward polarized, because they came from the forward decay of pions in flight. About 200 muons were stored per PS cycle. The muon injection was accomplished in a time much shorter than both the dilated muon lifetime ($27 \mu\text{s}$) and the precession period of the anomalous moment ($3.7 \mu\text{s}$).

7.2 The Injection System For Muon Storage Rings

The muon capture yield per available proton of given energy is a function of the method of injection and of the ring parameters, that is i) the aperture of the storage ring (b in the horizontal plane, a in the vertical one); ii) the radius r of the ring; iii) the magnetic field; iv) the field index n. It is difficult to express all these parameters in a general formula. However, for a given injection system and inside a limited range of the parameters, the following expression can be accepted:

$$\begin{aligned}
N_{\mu} &= F \frac{ab^3}{r^4} n^{1/2} (1-n)^{5/2} \cdot p_{\pi} Y \\
&= F_1 \frac{ab^3}{r^3} n^{1/2} (1-n)^{5/2} B \cdot Y,
\end{aligned} \tag{7.1}$$

where N_{μ} is the number of muons stored, Y is the yield of pion production (per GeV/c and steradian), which is a function of the proton and the pion momenta, and F and F_1 are factors depending on the chosen injection system. The formula can be explained simply as the product of the angular acceptance in the vertical plane $(a/r)n^{1/2}$, the angular acceptance in the horizontal plane $(b/r)(1-n)^{1/2}$, the momentum bite of the pions $(b/r)p_{\pi}(1-n)$, and finally the acceptable fraction of the muon momentum spectrum $\sim (b/r)(1-n)$.

In F , F_1 the decay probability of the pions must be included. This probability is very high in the case of the injection of protons in the ring onto an internal target; it is lower in the case of pion injection through an inflector; and it is very low in the case of backward decay trapping. In F , F_1 the losses due to the decay angles must also be included. There are three methods available for trapping muons in the storage region of the ring:

- i) by injecting protons into the ring and having them interact with a target located at the limit of the storage region;
- ii) by injecting pions of chosen momentum so that their chance to produce trapped muons is high;
- iii) by directly injecting muons of the desired momentum and putting them in stable orbits by means of a very fast inflector.

The first method has been used in this (g-2) experiment. The pions of the right momentum, produced in the internal target, travel around the ring for about four revolutions before hitting the target again; most of them will decay before, so that the muon capture yield from the circulating pions is rather high. Pions with wrong momenta travel a short distance round the ring, but their much greater number means that a considerable number of extra muons are also trapped. These muons are emitted at large angles in the pion rest frame, so the average longitudinal polarization is only 26% compared with the 95% expected. Furthermore, by injecting protons, the general background is very high around the target, and the electron counters can be located only at the opposite side of the ring (see Fig. 17).

The method of injection used in this first muon storage ring had the advantage of being very simple, but it had the following disadvantages:

- i) low muon polarization due to muons from a wide range of pion momenta;
- ii) high general background;
- iii) contamination with electrons at early times;
- iv) low average trapping efficiency.

For some time a magnetic horn was used around the target to concentrate pions of the correct energy in the forward direction. This gave a good muon polarization, but because of increased background it was not finally adopted.

The other methods of injecting muons are discussed below.

7.3 Some General Considerations for

Calculating the Number of Muons Stored in a Storage Ring

In order to calculate the number of captured muons in a muon storage ring, the regions of existence of the pions and the muons of different momenta must be traced and compared in phase space, both in the horizontal and in the vertical planes [24].

In a weak-focusing field, the trajectory of any particle can be represented in phase space (with an appropriate choice of scale factors) as a circle (Fig. 18). In the horizontal plane, the centre of the circle is the radial position of the equilibrium orbit, and in the vertical plane it is the vertical position of the median plane. One complete revolution in one of these paths corresponds to a complete betatron oscillation. Taking into account the dependence of the equilibrium orbits on the momentum and the geometrical limits of the chambers, the region of existence of the captured muons is simply limited by circles that are tangential to the given limits. The probability for a pion to produce a captured muon of a given momentum is then evaluated by following its trajectory, which is again a circle centred around the pion equilibrium orbit, inside or outside the chamber, and measuring which fraction of it crosses the region of existence of the chosen muons. When this angle is not small in comparison with the angular acceptance of the ring, a correction must be made to take into account the decay angle between muon and pion.

a) At a given time, the pions are distributed in vertical phase space along the line OT. The circle, which represents the vertical acceptance region of the stored muon, is displaced vertically by θ_v (vertical decay angle).

b) In horizontal phase space a pion trajectory originates from a line-source distribution, it is defined by the pion momentum and by the angle of injection. A pion can give a stored muon of momentum p_μ only when it is within the displaced circle, which represents the horizontal acceptance region of the stored muons displaced vertically by θ_h (horizontal decay angle).

Fig. 18

This method gives reliable results when a pion beam with a well-defined phase space, that is, momentum spectrum, and angular and spatial distribution, is injected into the ring through an inflector, in such a way that all pions will accomplish nearly one revolution around the ring before being lost by hitting the injection channel. The ratio between the length of the ring and the decay length of pions then gives a first probability factor; then in the phase space, the fraction of the area which is covered, during their revolution, by pions of given momentum and angular spread, and which is common to the existence area of muons of any chosen momentum, is easily estimated. This operation is repeated for several muon momenta, and then repeated again for several pion momenta and different pion injection angles.

A program based essentially on this method has been developed to compute the number of stored muons versus momentum, as well as the polarization, both longitudinally and radially, as a function of the momenta and of the angle of the pion injection. The programme also gives the number of muons produced per pion (trapping efficiency) as a function of the pion momentum and injection angle.

The program is somewhat less reliable when the protons hit an internal target, since the momentum range and the angular spread of the pions producing captured muons are very large, and the length of the pion trajectory inside the ring is dependent on the initial conditions. Nevertheless, the results gave a stored muon intensity and a muon distribution versus equilibrium radii, in good agreement with the measured intensity and with the reconstructed muon distribution.

It is worth while to point out here that this program was developed primarily to calculate the muon intensity in the second Muon Storage Ring.

7.4 The Count Rate $N(t)$ versus f_a

The polarization direction of the muons as a function of time is followed by recording the electrons emitted by muon decay in flight. As we have discussed above (see Section 5), these decay electrons are emitted preferentially parallel to the muon spin. Because of the combined effects of the precession of the spin and the electron decay asymmetry, the number of electrons emitted parallel to β_μ varies as $(1 + AP \cos 2\pi f_a t)$, where A is the asymmetry parameter described in Section 5 and P is the polarization of the muon sample. In Section 4 it has been shown that the kinematics of the muon-electron decay provides correlation between the angle $(\beta_e \cdot \beta_\mu)$ and the electron energy in the laboratory frame. If the electrons with energy E greater than a given value E_t are counted, they are emitted from forward decays of muons and the count rate is modulated at the $(g - 2)$ frequency

$$N_e(E > E_t, t) = N_0 e^{-t/\tau} [1 - A(E_t) \cos (2\pi f_a t + \phi)] . \quad (7.2)$$

It was pointed out in Section 4 that in order to minimize the statistical error in the angular frequency f_a , the optimum of $N_e A^2$ must be found. The maximum value of $N_e A^2$, and therefore the minimum uncertainty in f_a , is obtained at $E_t \approx 0.65 E_{\max}$, which corresponds to the values $E_t = 780$ MeV, $A = 0.42$, and $AP = 0.11$. The exact value of E_t is not critical because the curve of $N_e A^2$ has a broad maximum. The full asymmetry data obtained in this experiment are shown

in Fig. 19. The initial attempt to fit these data using Eq. (7.2) revealed a dependence of f_a and τ on the particular portion of data chosen, due to the presence of a residual uniform background and to muon losses caused by orbit perturbations. The data were therefore fitted with an eight-parameter fit of the following form:

$$N_e(E > E_t, t) = N_0 e^{-t/\tau} [1 - A(E_t) \cos(2\pi f_a t + \phi)](1 + A_L e^{-t/\tau}) + W. \quad (7.3)$$

The data fitted with this modulated function provide a value of f_a with no statistically significant dependence on the portion of data used in the least-squares fit. The frequency f_a varied by less than ± 0.2 standard deviations as a function of the starting time of the fit.

To calculate a_μ from the data using Eq. (7.3), the value of $(e/mc)\bar{B}$ is required. The quantity $\bar{B}$ is the mean magnetic field experienced by the muon sample. The value of $(e/mc)\bar{B}$ is obtained from the magnetic field measurement in terms of the proton resonance frequency f_p and the known ratio $\lambda = f_s/f_p$ for muon and proton spin precession in the same field [4] [see Eq. (4.9)].

The magnetic field was surveyed in terms of the corresponding proton spin-resonance frequency f_p . The measurement of the magnetic field B versus radius was an integrated value of 288 positions in the azimuth round the ring. At each azimuthal position, the magnetic field was measured for ten different values of radii. During the experiment, the magnetic field was surveyed as a function of both radius and azimuth.

The frequency obtained from fitting the data with the function (7.3) is actually $[\langle f_a \rangle]$, where the time average $[]$ reduces to an azimuthal average, and the ensemble average $\langle \rangle$ requires the evaluation of the average radius of the stored muons.

As the radial magnetic gradient necessary for vertical focusing implies a field variation of $\pm 0.2\%$ over the full horizontal aperture of the storage ring (8 cm), a major problem is to determine the mean radius of the ensemble of muons that contribute to the data. Since

$$\frac{\Delta B}{B_0} = n \frac{\Delta r}{r_0}, \quad (7.4)$$

where B_0 is the magnetic field at the central orbit of radius r_0 , and since $n = 0.13$, a knowledge of the average radius $\langle r \rangle$ to an accuracy of about 1000 ppm is sufficient to determine $\bar{B} = \langle B \rangle$ to 100 ppm.

Since the proton injection pulse is only 5–10 ns long, and the rotation period of the central momentum of the muons, $T = 2\pi r_0/\beta c$ is 52.5 ns, the muons are initially bunched and the counting rate in the decay electron counters is modulated with a period $T = 52.5$ ns. The low-momentum muons have a shorter rotation period than the high-momentum ones because of the spread in radius, so the bunch begins to overlap itself after about 1.5 μs and the rotational structure of the muon beam disappears after $\sim 6 \mu s$. This structure forms the basis of the primary method for finding the mean radius, since the rotation period of a given muon is determined essentially by its equilibrium radius. The analysis of the modulated records yields the mean radius to ± 2.7 mm, as shown by tests on Monte Carlo generated data (the mean frequency $f = \beta c/2\pi r$ is determined in the time interval between 1.8 μs and 5.5 μs). The

Fig. 19 Muon Storage Ring I: decay electron counts as a function of time after the injected pulse. The lower curve from 2 to 4.75 μs (lower time scale) shows 19 MHz modulation due to the rotation of the bunch of muons around the ring. As it spreads out the modulation dies away. This is used to determine the radial distribution of muon orbits. Curves A, B, and C are defined by the legend (upper time scale); they show various sections of the experimental decay (lifetime 27 μs !) modulated by the $(g-2)$ precession. The frequency is determined to 215 ppm, $\bar{B}$ to 160 ppm leading to 270 ppm in a_μ .

Fig. 20 The distribution of muons in radius (horizontal axis, cm) derived from the analysis of the decay electron events at early time. The muon rotation frequency has been analysed from $1.8 \mu\text{s}$ to $5.5 \mu\text{s}$.

reconstructed muon number versus equilibrium radius may be compared with the radial distribution of muons predicted by the injection calculations. From Fig. 20 we see that the agreement is good. The reconstructed mean radius is:

$$\langle r \rangle \equiv \bar{r} = 2494.3 \pm 2.7 \text{ mm} . \quad (7.5)$$

In the time interval in which the mean radius of the muon population was determined, there was an excess of counts; this was due partly to the fact that muons were lost, and partly to the fact that a non-rotating background was produced in the ring as consequence of the injection system (see Fig. 21). Numerous checks [53] indicate that the method of analysis by which the mean radius is determined is valid. First a direct measurement of the rotation frequency at early times was made with reduced intensity, using $1/5$ of one-bunch intensity, see Fig. 22. This determination of the mean radius gave

$$\bar{r} = 2492.5 \pm 2.1 \text{ mm} . \quad (7.6)$$

and corresponds to the time interval between $0.6\text{--}1.6 \mu\text{s}$ (before the muon bunch overlapped itself) in which the excess count factor was 3.0, compared with an excess count factor of 1.25 at $3 \mu\text{s}$. However, the mean radius was in good agreement, in spite of the different excess counting rate. Secondly a loss of particles ($\sim 30\%$) occurred between the time interval in which the mean radius was measured ($1.8\text{--}5.5 \mu\text{s}$) and the time interval in which f_a was determined. This raises the possibility of a change of radius between, let us say, $3 \mu\text{s}$ and $50 \mu\text{s}$. To study this,

a) Excess of counts at early times. The fitted curve from $20.5 \mu\text{s}$ to $189 \mu\text{s}$ has been extrapolated back to compare the corresponding counting rate at early times with the experimental data.

b) Decay electron counts modulated by the $(g - 2)$ frequency of the muons.

Fig. 21

Fig. 22 Reduced intensity experiment, clearly showing the rotation frequency of the muon bunch.

information was obtained by restricting the horizontal aperture from both the inside and the outside of the ring. These studies indicated that the mean radius did not change with time by more than $\pm 1.1 \text{ mm}$, and a conservative overall error of $\pm 3 \text{ mm}$ in radius was assigned, implying an error of 160 ppm in the value of a_μ .

The frequency f_a varied by less than ± 0.2 standard deviations as a function of the starting time of the fit. The statistical error was $\pm 23 \times 10^{-8}$, and the fluctuations of the results of eight different runs about the mean gave $\chi^2 = 7.84$, compared with 6.35 expected. The random error has been increased to $\pm 25 \times 10^{-8}$ by the square root of this ratio, the error in the magnetic field corresponding to ± 3 mm uncertainty in radius was $\pm 19 \times 10^{-8}$ in a. The two errors were combined quadratically to give the overall error in a of $\pm 31 \times 10^{-8}$.

The data-fitting program, applied to the late-time part of the data, yields

$$f_a = 0.270009 \pm 0.000054 \text{ MHz} . \quad (7.7)$$

The radius-reconstruction program, applied to the early-time part of the same data, when combined with the magnetic field map of NMR proton frequency as function of radius, gives

$$\bar{f}_p = 72.817 \pm 0.012 \text{ MHz} . \quad (7.8)$$

These two frequencies were measured with the same standard crystal, so that even if this standard was in error, it did not affect the ratio that was the final result of the Muon Storage Ring experiment:

$$R = f_a/f_p = (370805 \pm 99) \times 10^{-8} . \quad (7.9)$$

This is proportional to the extra-Dirac part of the muon magnetic moment, and the use of the somewhat unfamiliar dimensionless unit MHz/(proton MHz), merely reflects the experimental fact that all precise magnetic field measurements are made in terms of the proton NMR frequency.

We are, of course, most interested in the dimensionless number a_μ , and in order to obtain this we must combine the result of the ratio f_a/f_p with an experimental measurement of the muon magnetic moment. At the time of the second (g-2) experiment, the best such measurement came from the experiment of Hutchinson et al. [51], who stopped μ^+ in water and obtained

$$\lambda = 3.18338 \pm 0.00004 , \quad (7.10)$$

combined with the calculation of Ruderman [54], who found the diamagnetic correction for muons in water to be

$$(1 - \epsilon)^{-1} = 1.000010 \pm 0.000005 . \quad (7.11)$$

Inserting Eqs. (7.9) and (7.10) in Eq. (4.9) gave

$$a_\mu^{\text{exp}} = (116616 \pm 31) \times 10^{-8} (270 \text{ ppm}) . \quad (7.12)$$

Initially, this was nearly two standard deviations higher than the theoretical value, a sign that there was more to discover about the muon. In fact the discrepancy resulted from a defect in the theory. Theorists had originally speculated that the contribution of the photon-photon scattering diagrams to the $(\alpha/\pi)^3$ term in a_μ might be small, or perhaps cancel exactly. The experimental result stimulated Aldins et al. [55] to examine this more carefully, obtaining a coefficient of 18.4! The situation then was

$$a_\mu^{\text{exp}} - a_\mu^{\text{th}} = 28(31) \times 10^{-8} = 240 \pm 270 \text{ ppm} . \quad (7.13)$$

The difference of ~ 0.9 standard deviation cannot be called a discrepancy; rather, it confirms that the ' μ looks like a heavy electron', but it tests the theory to new levels of precision. For the photon propagator cut-off, this implied $\Lambda_\gamma > 5 \text{ GeV}$, and for the muon vertex $\Lambda_\mu > 7 \text{ GeV}$.

One of the by-products of the Muon Storage Ring was the determination of the lifetime of a muon beam decaying in flight [56]. In this experiment the Einstein time dilation in a circular orbit was confirmed to 1%. The muon lifetime was dilated from its value at rest $(2.198 \pm 0.001) \mu\text{s}$ to $(26.37 \pm 0.05) \mu\text{s}$, compared with the expected value of $26.69 \mu\text{s}$. The statistical error in the fitted lifetime was $\sim 0.2\%$, and the agreement between theory and experiment was at the level of only 1.2%. The expected lifetime corresponds to the assumption that the muon beam was oscillating around the mean equilibrium radius. The shorter measured lifetime was ascribed to a slow loss of muons due to imperfections in the magnetic field. In Section 11 we will describe a more precise confirmation of the Einstein time dilation.

8. 1969–1976: MUON STORAGE RING II

8.1 The Second ($g - 2$) Experiment at the Proton Synchrotron

By 1969 an electron-electron colliding-beam experiment had demonstrated the point-like nature of the electron ($\Lambda_\gamma > 4 \text{ GeV}$, $\Delta_e > 6 \text{ GeV}$) [57], and e^+e^- storage rings were giving useful data on vector-meson production [58]. Experiments on e^+e^- and $\mu^+\mu^-$ pair production, on wide-angle bremsstrahlung, and on muon tridents, and a comparison of e - p and μ - p scattering were all in accord with theory. (For reviews, see Ref. [59].) The pure quantum effects were less satisfactory. The Lamb-shift data [60] were consistently higher than theory, but this was resolved by a recalculation of a small theoretical term by Appelquist and Brodsky [61]. The electron ($g - 2$) data of Wilkinson and Crane [18] had been rediscussed by Farley [16], by Henry and Silver [19], and by Rich [62], who concluded that $a_e^{\text{exp}} - a_e^{\text{th}} = -(79 \pm 26) \text{ ppm}$. This discrepancy was to be resolved in a new measurement by Wesley and Rich [63]. Thus QED was doing well, but in early 1969, a_μ , a_e , and the Lamb shift all showed uncomfortably large departures from theory. It could have been the beginning of something new.

The major motivations for carrying out a third measurement were therefore as follows:

- i) to look for departures from standard QED;
- ii) to detect the contribution of strong interactions to a_μ through hadron loops in the vacuum polarization;
- iii) to search for new interactions of the muon.

The third experiment [64] had its origins very firmly rooted in the previous one, and the main challenge to a new design lay in the systematic troubles of the last experiment. Its design was an attempt to overcome the major sources of uncertainty arising in that experiment; these sources can be summarized as follows:

- i) The radial magnetic gradient required to provide the vertical focusing implied a magnetic field variation of $\pm 0.2\%$ over the aperture in which the muons were stored, and a corresponding radial dependence of f_a . Even if the mean radius was determined precisely after injection, uncertainties in radius could arise from uncontrolled muon losses.
- ii) The burst of particles created in the ring at injection upset the counting system and also produced a non-rotating background at early times, which made the interpretation of the data collected in the first few microseconds difficult.
- iii) In spite of careful shaping of the magnetic field, there were small muon losses up to at least $100 \mu s$ after injection.

The new project overcame the previous systematic troubles by introducing the following improvements:

- i) Using a ring magnet with a uniform magnetic field and focusing vertically with an electric quadrupole field. This field defocuses, of course, horizontally, slightly reducing the natural horizontal focusing of the uniform magnetic field.
- ii) Injecting a momentum-selected beam of pions, instead of protons, into the ring in order to reduce the background.
- iii) Reducing the loss of muons; this can be achieved by a combination of a more precise field shaping and by using an 'electric scraper'.
- iv) Increasing the intensity of the stored muons to improve the statistical accuracy in f_a .
- v) Increasing the product $B \cdot \gamma$, thus increasing the number of $(g - 2)$ cycles per lifetime, and therefore increasing the precision in f_a .

The previous experiment showed that even if the determination of the radial distribution from fast-rotation analysis was very precise, uncertainties about muon losses from the ring would give a non-negligible error in the mean radius. Since we do not know the relative tendency of muons at different radii to be lost, the fundamental problem was to achieve focusing but remove the dependence of f_a on r . *This can be achieved in principle because the forces that hold the muon in its orbit and give focusing for small deviations from equilibrium arise from what appears, in the muon rest frame, as an electric field; but the spin precession is determined by what appears there as a magnetic field. These two fields may therefore be varied independently by applying suitable magnetic and electric fields in the laboratory frame.*

To appreciate the advantages of this method, we rewrite the classical relativistic equations of motion of a charged particle with an anomalous magnetic moment, in the laboratory fields $\vec{B}$ and $\vec{E}$ (using cgs units), already derived above in another form [65]:

$$\frac{d\vec{\beta}}{dt} = \vec{\omega}_c \times \vec{\beta}, \quad \frac{d\vec{\sigma}}{dt} = \vec{\omega}_s \times \vec{\sigma}, \quad (8.1)$$

with

$$\vec{\omega}_c = \left(\frac{e}{mc}\right) \left[\frac{\vec{B}}{\gamma} - \left(\frac{\gamma}{\gamma^2 - 1}\right) \vec{\beta} \times \vec{E} \right] \quad (8.2)$$

$$\vec{\omega}_s = \left(\frac{e}{mc}\right) \left[\frac{\vec{B}}{\gamma} - \left(\frac{1}{\gamma + 1}\right) \vec{\beta} \times \vec{E} + a_\mu (\vec{B} - \vec{\beta} \times \vec{E}) \right] . \quad (8.3)$$

Here we have assumed that $\vec{\beta} \cdot \vec{E} = \vec{\beta} \cdot \vec{B} = 0$ (the muon charge is $-e$), and we have assumed that the electric dipole moment in the muon rest frame is zero. The precession of the spin relative to the velocity vector is given by

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = \left(\frac{e}{mc}\right) \left[a_\mu \vec{B} + \left(\frac{1}{\gamma^2 - 1} - a_\mu\right) \vec{\beta} \times \vec{E} \right] . \quad (8.4)$$

From this equation it can be seen that the final term giving a dependence of ω_a on the electric field can be made zero if γ is correctly chosen. That is, if one operates at a value of $\gamma = (1 + 1/a_\mu)^{-1}$ called the magic value $\gamma_{\text{magic}} = 29.304$ corresponding to momentum $p = 3.094 \text{ GeV}/c$. In this case the muons can be stored in a perfectly uniform magnetic field which determines the spin motion, with focusing provided by an electric field which has no influence on the spin. This new principle is the essential basis of the final CERN experiment.

The value of the electric field was chosen to give appropriate focusing, but it was not needed for calculating a_μ . Focusing in the simultaneous fields is no problem [24]. If we define $n_B = -(r/B) dB/dr$ and

$$n = n_B + \frac{r}{\beta B} \frac{dE_r}{dr} \quad (8.5)$$

then the equations for axial and radial displacements from equilibrium, $z'' + nz = 0$ and $x'' + (1-n)x = 0$, and the focusing condition $0 < n < 1$, are formally unchanged. In the last experiment, we proposed the following, particularly simple solution: $n_B = [E_r]_0 = 0$ and therefore Eq. (8.5) becomes simply

$$n = \frac{r}{\beta B} \frac{dE_r}{dr} . \quad (8.6)$$

The voltage to be applied to electric quadrupoles disposed continuously round the ring with half aperture a (vertical) $\times$ b (horizontal) then becomes

$$V = n \frac{\beta pc}{2e} \frac{a^2 + b^2}{r^2} \sim 22 \text{ kV} \quad (8.7)$$

in our case. In practice the electrodes were not continuous in azimuth and required somewhat higher voltages.

The cancellation of the effect of the electric field on the spin motion is exact only for the central momentum of the muon distribution; for other momenta a correction is required. If the muons uniformly fill the available phase space, the average correction is

$$\Delta\omega_a/\omega_a = 0.2 n(1 - n) (b/r)^2 . \quad (8.8)$$

This is 1.7 ppm for $n = 0.14$, $b = 6$ cm, $r = 700$ cm.

The system for injecting muons into the ring was designed to give maximum muon polarization, minimum background, and as large an intensity as possible. A high value of the longitudinal polarization can be achieved by starting with a momentum-selected pion beam, and accepting only decay muons whose momenta lie in a narrow band close to that of the pion beam.

It was therefore decided to locate the primary target outside the Muon Storage Ring, and to prepare a momentum-selected pion beam to be guided into the ring by a pulsed inflector. Because of the size of the inflector structure, the pions would make only one turn in the ring, and the useful aperture of the inflector would be very small. The loss of intensity due to these factors could, however, be compensated by using special beam optics, which collected pions over a large solid angle and matched them to the acceptance of the storage ring.

The injector was in the form of a coaxial line in which a $10 \mu\text{s}$ current pulse of peak value 300 kA produced the required field of about 1.5 T between the inner and outer conductors. The great technical difficulty of this method of injection was outweighed by the increased pion flux and the high longitudinal polarization of the stored muons (95%). This was borne out by the large observed modulation of the decay electron counts. The calculated polarization direction was independent of the muon equilibrium radius; and consequently any possible asymmetric muon losses could cause no significant shift in the measured spin precession frequency f_a . Finally, the background was considerably reduced with respect to the previous experiment (in which the copper target was located in the ring), and therefore the electron detectors could be located all around the ring. Other improvements can be mentioned only briefly in this review.

8.2 Electric Field and Vacuum

The electric quadrupole system has been described by Flegel and Krienien [66]; here we point out only the general features. The quadrupole electrodes were constructed in eight sectors, each sector covering about one tenth of the circumference of the ring. The sectors were arranged in two groups of four with two equal spaces between them. One of these spaces was needed for the pion injection region while the other was required to balance this omission and cancel the first harmonic closed-orbit distortion. The cross-section of the quadrupole and vacuum chamber is shown in Fig. 23. The vacuum was normally better than 10^{-7} Torr and it was obtained by ion getter pumps to avoid any deterioration of the high-voltage behaviour of the electrodes by oil vapour.

The shape of the electrodes approximated to the ideal case of a pair of hyperbolae which would be required to give a perfect quadrupole field, and was calculated by successive approximation to minimize the variations in electric field gradient throughout the storage region. The presence of the vacuum chamber affects the electric field, and so for a given geometry the uniformity of the field gradient depends upon the chosen ratio of potentials applied to the two pairs of electrodes. The ratio which minimized the contribution of other multipoles, in particular the octupole, was found by calculation and later adjusted by measuring the muon losses.

Fig. 23 Muon Storage Ring II, which consists of 40 contiguous magnet blocks. The open side of the C-shaped yoke (upper right) faces the centre of the ring. The cross-section of the vacuum chamber and electric quadrupole is shown at the bottom right. The decay electrons are detected by 20 counters. Dimensions are in mm.

It had been demonstrated in the preparation of the experiment that it was not possible to use steady electric fields for both muon polarities, because trapped low-energy electrons produced too much ionization in the residual gas. These low-energy electrons were trapped, vertically, by the electric potential, and while spiralling up and down the magnetic field lines they slowly drifted along a side electrode in the direction $\vec{E} \times \vec{B}$. At the end of the focusing system the field shape was such that these low-energy electrons passed around the electrode and returned along the reverse side; they were therefore trapped inside the electrode system, with mean energy of 1 keV, and they were able to ionize the residual gas and to cause a voltage breakdown even if the vacuum was better than 10^{-6} Torr. However this process took some milliseconds, so that the danger could be greatly reduced by operating in a pulsed mode. Voltages in the form of flat-topped pulses, about 1 ms long, were applied to the electrodes by an arrangement of spark gaps. The electric field gradient corresponding to $n = 0.185$ was $dE_r/dr = 1.17 \times 10^7 \text{ V} \cdot \text{m}^{-2}$ for $B = 1.472 \text{ T}$, implying about 40 kV applied to the horizontal electrodes.

At the beginning of the experiment the low-voltage pulser used for the vertical electrodes was made with spark gaps which could not be regulated below 5 kV. This was too high to minimize the higher multipole field components, but the resulting muon losses were too low to affect the $(g - 2)$ frequency. For the lifetime measurement, however, this mode of operation was inadequate, and the spark gaps were replaced by cascaded thyristors, which allowed the true optimum level to be found and the muon losses to be minimized.

The operation of the full electrode system of the storage ring brought with it some problems which were not present in the test set-up. For the μ^+ polarity, both high and low voltages could be switched on together before the injection of pions and switched off together some 700 μs later, while for the μ^- polarity breakdown ensued unless the low-voltage crowbar was delayed by 16–32 μs after the high-voltage pulse had been terminated.

8.3 Electric Scraping

For the measurement of the muon lifetime it was essential to reduce the late-time muon losses to a minimum. This was done by shifting the muon orbits at early times both vertically and horizontally in order to ‘scrape off’ those muons which passed near the edge of the aperture and were most likely to be lost. The orbits were shifted by applying voltages asymmetrically to each pair of electrodes, and then gradually bringing them back to normal. While one member of the pair was pulsed normally with 1 μs rise time, the other was fed with a pulse which had a 10 μs rise time. For the side electrodes the asymmetry of the voltages was reversed between the two groups of focusing units on either side of the ring. Thus the muon orbits were shifted radially outwards on one side of the ring and radially inwards on the other; to first order they were just shifted sideways with their shape remaining the same. The bottom electrodes, all the way round the ring, received the longer pulse, and so the median plane of the muon orbits was initially low and returned to its normal position with a 10 μs time constant. The pulses were switched on about 6 μs before the arrival of the pion bunch. The whole question of particle orbits, muon losses, and the effect of scraping is discussed in detail in Section 4 of the final report of the CERN Muon Storage Ring [64]. One minor disadvantage of scraping was that the horizontal perturbation allowed some protons accompanying the π^+ beam to be injected onto stored orbits, and these could hit the vacuum chamber at later times producing background in the counters.

8.4 The Storage Ring Magnet

For a given momentum the maximum accuracy in f_a is obtained by working at the highest magnetic field. The actual field of 1.47 T was chosen to meet this requirement without jeopardizing good field uniformity.

In the preparation of the magnets, the aim was to provide a field as nearly uniform as possible so that the f_a of all the muons would be the same. Because the muons sample the field all the way round the ring this requirement essentially meant that, after averaging in azimuth, the field should be independent of radius. A further aim in the construction of the storage ring was that the field should be stable and reproducible to a few parts per million (ppm). The novel procedure to achieve this spectacular result has been described elsewhere [67]. The success of this procedure is indicated by the vertical field map shown in Fig. 24. This map is a contour line plot for the storage aperture and was obtained by averaging the three-dimensional map in azimuth. The interval between the contours of equal field strength is 2 ppm or 3 μT .

As can be seen from Fig. 23, the Muon Storage Ring consisted of 40 C-shaped bending magnets, each about 1 m long, fitted together to form a regular polygon with open sides facing the centre of the ring. The pole gap was 14 cm high and 38 cm wide. Each magnet was

Fig. 24 A contour line plot of the magnetic field strength in the muon storage aperture. This map is obtained by averaging a three-dimensional map in azimuth. The interval between the contours of equal field strength is 2 ppm or $3 \mu\text{T}$.

individually supported on a ring-shaped concrete foundation. The pole pieces were cut at an angle of 4.5° at each end so that they fitted together to form a closed polygon. However the yoke was not fitted together and consequently the field at each junction was reduced; this drop in the field, after various adjustments in the geometry of the iron of the magnet, was about 400 ppm. The magnets were energized in series by four large circular coils providing continuous current all the way round the ring although for cooling purposes each coil was divided into five sectors. The field in every sector was monitored with 40 NMR probes, the signal from which was used to set the field at the correct level and to provide its stability via the control of the current through 40 compensating coils wound around the yoke close to each pole. Thus the field at 40 points in the gap just outside the muon storage region was stabilized.

It is worth emphasizing here the extreme insensitivity of the average value of the magnetic field B computed for different assumed radial distributions of muons. Even in extreme cases the average magnetic field was the same within less than 2 ppm, compared with the 160 ppm uncertainty in B in the previous experiment. Thus the uniformity of the magnetic field in the storage ring was such that the $(g-2)$ frequency was essentially independent of the distribution of muons within the storage region. However, a precise value of the mean radius of the muon population was still needed in order to derive the muon lifetime at rest from the measurement made in flight.

8.5 Stabilization and Reproducibility of the Magnetic Field

To ensure the stability and reproducibility of the magnetic field, each of the 40 magnet blocks was stabilized separately by a control system which had an NMR probe and a pick-up coil as its sensors [68]. The signals from these devices were used to determine the current automatically through additional compensating coils which were wound around the yoke of individual magnets close to each pole tip. The NMR probe contained a flat coil for modulating the field in a small plastic tube, shrunk around a cylindrical RF coil of 2 mm diameter and 4 mm length, and containing a 0.1 molar solution of NiSO_4 . The applied field modulation, B_{mod} , was a symmetric 30 Hz triangular waveform with a peak-to-peak amplitude of 0.8 mT, and the

system operation was such that the proton frequency resonance for $B_{\text{mod}} = 0$ equalled the frequency of a crystal oscillator. The signal from the pick-up coil dominated the response to changes in the magnetic field at frequencies above 1 Hz, while the NMR probe counteracted the slower drifts. By using eight different crystal frequencies for the NMR control probes, making local special shimming at the probe position and adjusting the iron shunts, the mean fields in all blocks were as closely equalized as possible. This was done in order to minimize all compensating currents. During the operation of the ring, the currents through the main coils were adjusted automatically so as to keep the average of the 40 compensating currents close to zero.

To bring the magnet into the same operating condition, a special switching-on procedure was adopted; without it the field shape would not have been reproducible, and although the field values at 40 locations of the fixed frequency stabilizer probes were the same, temporary eddy currents or hysteresis in the yoke could so modify the distribution of magnetization in the iron that the overall average field in the storage region could change by as much as 50 ppm. The procedure consisted of three rapid (50 A/s) up and down cycles of the main current to a value some 10% higher than the operating point ($I_0 \sim 4.5$ kA), followed by slow (1 A/s) oscillations about I_0 of a few per cent in amplitude and gradually decreasing. This cycling of the storage ring magnets was controlled automatically and took about half an hour to complete, at which point the stabilization system automatically switched-on.

During data-taking runs the magnetic fields were monitored in 37 of the magnets. This was done with small NMR probes which could be driven into the muon storage ring along a radial line in the median plane without breaking the vacuum. The 37 probes were connected, via multiplexers, to the same eight magnetometers used in the field mapping. The whole procedure was controlled by a computer program and only took 10 minutes. The 400 points at which these plunging probes measurements were made were located within the coordinate system of the full field map by survey and the relationship between the two sets of readings determined. This allowed the drift in the mean field to be followed throughout the periods between full-scale maps.

Between groups of data-taking runs, the whole vacuum chamber was removed and a full map made of the magnetic field with a system of eight NMR probes mounted on a measuring machine driven around the ring under computer control. The probes were connected to automatic magnetometers, and the measurements could be made and recorded whilst the machine was moving at the speed of up to 6 cm/s without significantly distorting the map. About 250 000 points were measured throughout the storage volume in steps of 1 cm vertical by 1 cm radial by 2 cm azimuthal. Careful tests had to be carried out on the reproducibility of the field. As a result of the cycling procedure, the field in the magnetic gap followed a predictable pattern; the average value rose by some 5 ppm over the first two days after switching on and then remained constant to within ± 1 ppm. The full maps of the magnetic field truly reflected the field during the data taking, subject to some small systematic effects such as the influence of the vacuum chamber and the electrodes. The effects of different parts of the chamber and electrodes were measured and the overall contribution to the average field value was less than one part per million [68].

The probes used in mapping, monitoring, and stabilizing the magnetic field were calibrated with respect to a special probe of well-defined geometry and susceptibility correction [68]. The crystal clock which was used to measure the proton resonance frequencies was also used to time the muon decays, and as the experimental value of the anomaly depends upon the ratio of these two measurements, the effect of any drift in the clock frequency was largely cancelled out. The radial component of the magnetic field was measured with a Hall-plate pendulum and found to be at the level of 10^{-4} T, sufficient to cause the median plane of the muon orbits to undulate with respect to the geometrically central plane of the storage region over a range of about 2 mm. This shift was confirmed by measurements on the muon distribution.

8.6 Radial Distribution of Muons and Magnetic Field Analysis

Information on the radial distribution was obtained as before by analysis of the bunch structure of the muons in the ring at early times. This fast rotation pattern is shown in Fig. 25. The pion bunch had a width of 10 ns, a time structure which was inherited by the decay muons. Shortly after injection, as the bunch length is much shorter than the rotation period in the ring (147 ns), the decay electrons detected by a particular counter will appear in discrete bunches. The delays of the various counters were adjusted so that the bunched data from different counters could be coherently added, simply by combining single-counter histograms suitably shifted by an integer number of bin widths (10 ns).

Two methods of analysis were available. In the first, which was applicable only to non-overlapping bunches, the centroid in time of each bunch was calculated: by fitting a

Fig. 25 The fast rotation pattern. This is the count rate at early time which clearly shows the muon bunch rotating around the ring with a period of 147 ns. Horizontal scale in μ s.

straight line to these values, the mean rotation frequency f_{rot} was obtained. The muon energy was then deduced from the average γ -parameter $\bar{\gamma}$, which was given by

$$\bar{\gamma} = 2\lambda\bar{f}_p/gf_{\text{rot}}, \quad (8.9)$$

where $\lambda = f_s/f_p$ as above; $\bar{f}_p$ is the mean proton magnetic resonance frequency, corrected to vacuum, over the muon orbits; and g is the g -factor of the muon. The mean muon radius is

$$r = \beta c / 2\pi f_{\text{rot}}. \quad (8.10)$$

In the second method more detailed information on the radial distribution was obtained by making a least-squares fit to the shape of the bunches as a function of storage time. The program to perform this analysis was also used in the previous CERN ($g-2$) experiment where knowledge of the radial distribution was crucial. Although all the muons in the storage ring had essentially the same velocity $\sim c$, the high-energy ones, because of their longer path length, gradually lagged behind those of lower energy. This resulted in spreading out the bunch as time progressed, so that eventually all bunches overlapped. The rate at which this overlap occurred was sensitive to the radial distribution of the muon orbits. The latter was found by least-squares fitting to the observed time development of the bunches. With one pion bunch injected, the bunches of detected electrons remain resolved until $\sim 10 \mu\text{s}$, and the analysis was normally carried out in the time range 5 to 20 μs or about 100 revolutions. However, the bunch structure did not disappear completely until about 40 μs after injection. The analysis was also carried out when two bunches, separated by 105 ns, were injected into the ring. In this case the fit was performed entirely on overlapping bunches, and the simple centroid-fitting procedure described above could not be used. A detailed description of the second method used to determine the mean radius is given in the two papers which describe the second and third ($g-2$) experiments [53, 64]. The statistical error on the mean radius given by the fit is typically 0.1–0.2 mm. A number of systematic checks of the fitting program were carried out, both by using Monte Carlo data as input and by varying input assumptions and the parameters in the fits to real data. Particular attention was given to systematic errors such as:

- i) rate and dead-time effects;
- ii) the effect of a flat background between bunches;
- iii) the effect of muon losses;
- iv) the width of the input pion pulses;
- v) the shape of the input pulses.

The fast rotation pattern and the radial muon distribution are shown in Figs. 25 and 26.

The determination of the average magnetic field in terms of the proton magnetic resonance frequency f_p , for each set of data which was analysed separately for f_a , involved three distinct steps:

- i) Using the parameters found in the analysis of the radial distribution of the stored muons an ensemble average of the full field map was made. The effect of the electric field for muons not exactly on the nominal radius gave a correction of the order of 2 ppm and was included in this average.

Fig. 26 The reconstructed radial distribution of muons for scraped and unscraped data compared with a Monte Carlo simulation of the latter.

- i) Using the parameters found in the analysis of the radial distribution of the stored muons an ensemble average of the full field map was made. The effect of the electric field for muons not exactly on the nominal radius gave a correction of the order of 2 ppm and was included in this average.
- ii) Using information from the plunging probe measurements a correction was applied to allow for any changes in the magnetic field between the data collection and the making of the full map.
- iii) The measured proton magnetic resonance frequency was corrected to the value in vacuum. These steps are described in Section 5.3 of Ref. [64].
- iv) The pitch correction was applied using Eq. (4.36). Assuming that the muons populate the available vertical phase space uniformly the average value of ψ_0^2 is $1/2 n(a/r)^2$. For $n = 0.135$, $a = 4$ cm, $r = 700$ cm, the pitch correction is then 0.5 ppm.

To achieve the accuracy reached in this last experiment many technical problems had to be solved. The ability to find these solutions constituted part of the beauty of the experiment and we invite the reader to consult in particular the final report on the CERN Muon Storage Ring for all details [64]. Figure 27 shows the combined decay electron counts versus storage time for the whole experiment.

Fig. 27 Muon Storage Ring II: decay electron counts versus time (in microseconds) after injection. Range of time for each line is shown on the right (in microseconds).

8.7 Data Analysis

The raw data for each detected decay electron consisted of a recorded time in bins of 10 ns within the range 0 to 655.35 μ s, and labels for counter and pulse height. The time spectrum of the decay electrons was fitted using the function minimization program MINUIT. If we denote the observed spectrum by $H(t_i)$ and the fitted function by $N(t_i)$, the likelihood function L is given, assuming Poissonian errors for $H(t_i)$, by:

$$L = \prod_i \exp [-N(t_i)] \frac{[N(t_i)]^{H(t_i)}}{H(t_i)!} . \quad (8.11)$$

The function actually minimized by the program MINUIT is

$$\ln L = \sum_i [H(t_i) \ln N(t_i) - N(t_i)] , \quad (8.12)$$

where the constant term $\sum_i \ln H(t_i)$ is dropped and $N(t_i)$ is taken as

$$N(t_i) = N_0 \{L(t_i) \exp (-t_i/\tau)[1 - A \cos (2\pi f_a t + \phi)] + W\} . \quad (8.13)$$

The function $L(t_i)$ is an empirical correction factor to allow for the effect of muon losses and gain changes at early times. The 'ideal' distribution would have $L(t_i) = 1$ and the background $W = 0$. The loss function $L(t_i)$ is parametrized in the form:

$$L(t_i) = 1 + A_L \exp (-t_i/\tau_L) . \quad (8.14)$$

The total number of parameters used in the fits is then eight: five (N_0 , τ , A , f_a , ϕ) from the 'ideal' distribution, plus three (W , A_L , τ_L). As many of the parameters in the fit are correlated with each other, some care is needed in extracting the frequency by such global fits. The only parameter strongly correlated with f_a is ϕ . However, attempting to fit data with early-time loss or gain distortions to a simple six-parameter (N_0 , τ , A , f_a , ϕ , W) function leads to an incorrect value of f_a . This effect was noted in the previous ($g-2$) experiment. For six parameters the fitted value of f_a varies with the starting time of the fit, oscillating about the correct value as a function of time with a frequency of approximately f_a . Including A_L and τ_L in the fit removes these oscillations. The mechanism of this frequency shift is easily understood in terms of 'phase pulling' of the early ($g-2$) cycles by the excess counts at early times when the simple six-parameter function is used. The final results, quoted in this experiment, were obtained with the starting time chosen for the fits at the zero crossing of the oscillation (21 μ s), so that they will be unchanged within statistical error whether the six- or the eight-parameter fit is used.

For μ^+ data with scraping, the background due to protons was at the level of approximately 10^{-3} at early times but had a negligible effect on f_a . For all the μ^- data and those for μ^+ without scraping, W was at the very low level of 2×10^{-5} .

Particular attention was given to possible systematic corrections to the ($g-2$) frequency, and checks were made with artificial light pulses during the data-taking runs. Among them the more important were:

- i) Time slewing effect due to the large counting rates experienced by the counters and the electronics after injection. The effect varied considerably from counter to counter, depending on the position relative to the injected beam. These effects on the frequency shift are very small: 0.9 ppm for a starting time of 15 μ s, and < 0.5 ppm for a starting time of 21 μ s.
- ii) Gain effects. These are included in the empirical function $L(t)$ for the $(g-2)$ frequency analysis.
- iii) Digitron calibration and linearity. Both the linearity and the absolute calibration of all digitrons used in the experiment were found to be good to a small fraction of ppm.
- iv) Queuing and dead-time losses. Experimental checks were done with great care and these experiments are discussed in Ref. [64]. The raw frequency queuing-loss correction in the worst case was 1.3 ppm.

8.8 Experimental Results

Nine separate runs were made over a period of two years to measure the $(g-2)$ precession frequency f_a , the field being determined in terms of the proton resonance frequency f_p . The ratio $R = f_a/f_p$ showed good consistency ($\chi^2 = 7.3$ for eight degrees of freedom) (see Fig. 28). The overall mean value is the principal result of the experiment:

$$R = f_a/f_p = 3.707213(27) \times 10^{-3} \text{ (7 ppm)} . \quad (8.15)$$

Fig. 28 Individual values of the ratio $R = f_a/\bar{f}_p$ for the nine experimental periods of the third experiment together with the weighted averages.

This error is made up of a 7.0 ppm statistical contribution from f_a and a 1.5 ppm systematic contribution from f_p . This number is the essential result of this experiment. Equation (4.9) allows us to calculate the anomaly if $\lambda = f_s/f_p$ is known.

The magnitude of λ has now been determined to about 1 ppm, directly from measurements of muon precession at rest [13] and indirectly from the hyperfine splitting in muonium [14]. The weighted average value of these measurements [4] is

$$\lambda = 3.18334547(47) .$$

This leads to the following results for the anomalous moment, slightly different from those published in Ref. [64] because the value of λ has changed:

$$a_{\mu^+} = 1\,165\,910\,(11) \times 10^{-9}\,(10\,\text{ppm}) \quad (8.16)$$

$$a_{\mu^-} = 1\,165\,936\,(12) \times 10^{-9}\,(10\,\text{ppm}) \quad (8.17)$$

and for μ^+ and μ^- combined

$$a_{\mu} = 1\,165\,923\,(8.5) \times 10^{-9}\,(7\,\text{ppm}) . \quad (8.18)$$

Compared with (2.9)

$$a_{\mu}^{\text{th}} = 1\,165\,918\,(2) \times 10^{-9}$$

the result is

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{th}} = (5 \pm 8.5) \times 10^{-9} . \quad (8.19)$$

Table 1 gives a summary of the measurements of a_{μ} . It includes an experiment by Henry, Shrank and Swanson [42] using a long solenoid magnet in analogy with the electron ($g-2$) experiment of Schupp, Pidd and Crane [41], which we have not described in detail.

Table 1

Experimental results for a_{μ}

		Ref.	Dates
μ^+	0.001 145 (22)	43	1961
μ^+	0.001 162 (5)	43	1962, 1965
μ^-	0.001 165 (3)	53	1966
$\mu^{\pm}$	0.001 166 16 (31)	53	1968, 1972
μ^+	0.001 060 (67)	42	1969
μ^+	0.001 165 895 (27)	64	1975
$\mu^{\pm}$	0.001 165 923 (8.5)	64	1977, 1979

8.9 Comparison Between Theory and Experiment

The main six conclusions that can be drawn from this last measurement of the anomalous magnetic moment of the muon are the following:

1. The QED calculations of the muon anomaly are verified up to the sixth order, the experimental uncertainty being equivalent to 1.2×10^{-6} in A, 3.5×10^{-3} in B, or 4.7% in C [see Eq. (2.1)].

2. The hadronic contribution to the anomaly is confirmed to an accuracy of 20%. The existence of hadronic vacuum polarization has thus been established at the level of five standard deviations.
3. There is no evidence for a special coupling of the muon. The experimental range of possible values of an extra contribution to the moment is

$$-12 \times 10^{-9} < \Delta a_\mu < 22 \times 10^{-9} \quad (8.20)$$

to 95% confidence. The limits implied for unknown boson fields then depend on the nature of the coupling and are given in Fig. 29 [64].

Fig. 29 The upper limits on the coupling constant f of the muon to a heavy neutral boson of mass M^0 , for vector (V), axial vector (A), scalar (S) and pseudoscalar (PS).

4. With the advent of renormalizable gauge theories unifying the weak and the electromagnetic interactions, the calculation of the weak interaction contribution to the muon anomaly has become reliable. In general the weak contribution depends upon the parameters of the theory, such as the masses of the Higgs and intermediate vector bosons. To the extent that we do not yet know the correct form of the weak interaction Hamiltonian, the above results for a_μ (and also the result for a_e) can be used to restrict the range of possible models. Only in the simplest of such theories, that of Weinberg [9] and Salam [9], are the parameters sufficiently well determined experimentally to give a firm prediction of the expected value of the weak anomaly. Kinoshita [3] has recently reviewed this subject. If the arguments of Weinberg on the Higgs boson are accepted, and the current limits on $\sin^2 \theta_w$ are taken into account, we obtain:

$$1.9 \times 10^{-9} \leq a_\mu(\text{weak}) \leq 2.3 \times 10^{-9}. \quad (8.21)$$

Clearly the precision of even the latest experiment is inadequate for testing this prediction.

5. The range defined by the inequalities Eq. (8.20) may be used to set limits on single contributions to the muon anomaly from other sources, including various models for breaking QED as discussed in the theoretical section. The following limits apply to 95% confidence:
- (a) The muon may not behave like a point charge, but instead have a finite size, in analogy with the proton. This would show up as a form factor, see Eq. (2.10). The limit imposed on Λ_μ would be $\Lambda_\mu > 36 \text{ GeV}$.
 - (b) The modification in the photon propagator of Eq. (2.11) leads to $\Lambda_\mu > 25 \text{ GeV}$.
 - (c) From the latest experiment it is possible to set a limit to the modification of the muon propagator [69] by a factor $(1 - q^4/\Lambda_{\text{prop}}^4)$. The value is $\Lambda_{\text{prop}} > 1.5 \text{ GeV}$.
 - (d) A possible new, undiscovered lepton of mass M_L would contribute to the vacuum polarization through a mechanism such as diagrammed in Fig. 1d. The value of the anomaly would depend on the ratio M_L/m_μ . The 95% confidence limit sets the limit $M_L > 210 \text{ MeV}/c^2$, which is not very interesting. In passing, the recently discovered heavy lepton τ [70], with mass $1.8 \text{ GeV}/c$, gives a contribution, $\Delta a_\mu(\tau) \approx 0.4 \times 10^{-9}$, well below the present sensitivity.
6. Recently Kadyshevsky [71] has given a new gauge formulation of the electromagnetic interaction theory, containing a 'fundamental length' ℓ as a universal scale constant as important as $\hbar/2\pi$ and c . This new hypothetical constant ℓ , together with $\hbar/2\pi$ and c , is expected to regulate all microscopic phenomena. The quantity $M = \hbar/2\pi\ell c$ plays the role of a fundamental mass. In the new approach the electromagnetic potential becomes a 5-vector associated with the de Sitter group $O(4,1)$. Among the various predictions given is the value of the anomalous moment for a lepton of mass m_ℓ

$$a_{\text{lepton}} \approx m_\ell^2/2M^2$$

and the electric dipole moment (EDM)

$$d_{\text{lepton}} \approx e\ell/2.$$

From the present experimental result one then obtains an upper bound for the fundamental length: $\ell < 3.9 \times 10^{-17} \text{ cm}$.

8.10 Comparison between the Muon and Electron g-Factor Experiments

It is interesting to consider the following comparison between the recent experiments to measure the anomalous parts of the muon and electron g-factors by the precession and spin resonance methods, respectively [72]. These two experiments are of vastly different scales; the electrons have non-relativistic energies in the region of 1 meV , while the muons are highly relativistic with energy of about 3.1 GeV . The electrons are trapped on cyclotron orbits of radii $\sim 1 \text{ }\mu\text{m}$ and oscillate vertically over a distance of about $200 \text{ }\mu\text{m}$, while the muons, travelling virtually at the velocity of light, follow near-circular orbits of 7 m radius and execute vertical oscillations over a range of up to 80 mm . There is little doubt about the macroscopic nature of

the motion of the many millions of muons which contribute to the measured signal, but in the mono-electron oscillator a single electron spends a sizeable part of its time in the lowest quantum level of the system.

These differences are largely dictated by the need to use the relativistic time dilation of the muon lifetime on the one hand and to minimize the relativistic frequency shift due to the electron trapping potential on the other: in spite of them there are strong similarities between the two experiments; in particular, the use of a type of Penning trap is common to both. The trap consists of an electric quadrupole superimposed upon a uniform magnetic field. Both the configurations used in these two experiments have axial symmetry, and a general form for the electric potential in cylindrical coordinates is

$$V(r,z) = V_0/b^2 \{ r^2 - 2r_0^2 [\ln(r) - \ln(r_0)] - r_0^2 - 2z^2 \} , \quad (8.21)$$

where r_0 is the radius of the circle at which $\partial V/\partial r = 0$. The potential is singular along the symmetry axis except in the case when $r_0 = 0$. At this limit the form of the potential becomes

$$V(r,z) = V_0/b^2 (r^2 - 2z^2) , \quad (8.22)$$

which is just the potential of the electron trap. For the muon experiment, r_0 is 7 m and the particle orbits are restricted by the aperture limits to radii within ± 60 mm of this value and also to within ± 40 mm of the horizontal plane $z = 0$. Thus by writing $r = r_0 + x$ the potential can be approximately reduced to the two-dimensional form

$$V(x,z) = (2V_0/b^2)(x^2 - z^2) , \quad (8.23)$$

which completely neglects the effects of curvature. Thus in cross-section the shape of the four electrodes, required to provide the quadrupole field, closely resembles that which satisfies the simple two-dimensional case, but perpendicular to this section the electrodes are curved to follow the circumference of the 7 m radius ring.

In both the electron and muon traps the classical motion of the particles can be described in the same terms as a combination of three frequencies. The first is the relatively fast cyclotron frequency due to the magnetic field, but slightly modified by the trapping potential so that the orbits are not quite circular. The centre of the cyclotron orbits can be considered to slowly drift around the axis of symmetry such that the particle executes an epitrochoidal motion, the frequency of this drift being the so-called magnetron motion. In the case of the muon storage ring the size of the horizontal aperture ensures that the centre of the 7 m radius cyclotron orbit is always within 60 mm of the symmetry axis or centre of the ring. The third frequency is that of the vertical oscillations in the direction of the magnetic field. This latter frequency and the magnetron frequency both go to zero in the absence of a trapping potential.

It should be remembered, however, that strictly speaking this classical approximation is not applicable to the quantum states of the electron in the mono-electron oscillator.

9. ELECTRIC DIPOLE MOMENT

An upper limit for the electric dipole moment (EDM) of the muon has been measured directly in the CERN Muon Storage Ring [73]. For a particle with both magnetic and electric dipole moments the electromagnetic interaction Hamiltonian contains a term $(\vec{\mu}_m \cdot \vec{B} + \vec{d} \cdot \vec{E})$, where $\vec{B}$ and $\vec{E}$ are the magnetic and electric field strengths and $\vec{\mu}_m$ and $\vec{d}$ are the magnetic and electric dipole moment operators. See Eqs. (1.3)–(1.6).

It is well known that the expectation value of the electric dipole moment $\vec{d}$ must be zero for a particle described by a state of well-defined parity. However, Purcell and Ramsey [74] stressed that the existence of an EDM for particles should be treated as a purely experimental question, and they suggested possible physical mechanisms that could lead to a non-vanishing EDM. After the discovery of parity violation in the weak interactions, it was pointed out by Landau [75] that even if P is violated, the existence of an EDM is still forbidden by T invariance, i.e. the existence of a non-vanishing EDM for a particle implies that both P and T are violated. See Field et al. [72] and Jackson [1] for comprehensive reviews of the subject.

The technique used to measure the muon electric dipole moment follows from a suggestion originally made by Garwin and Lederman [76], and the present experiment was carried out at the CERN Muon Storage Ring simultaneously with the measurement of the muon g-factor anomaly $a \equiv 1/2 (g - 2)$. The precession frequency of the muon spin relative to its velocity vector $\vec{\beta}$, which is perpendicular to the magnetic and electric fields $\vec{B}$ and $\vec{E}$, is:

$$\vec{f} = \frac{e}{2\pi mc} \left[a\vec{B} + \left(\frac{1}{\gamma^2 - 1} - a \right) \vec{\beta} \times \vec{E} + \frac{1}{2}\eta (\vec{E} + \vec{\beta} \times \vec{B}) \right], \quad (9.1)$$

where we have included the effect of an electric dipole moment defined by $|\vec{d}| \equiv (\eta/2)(eh/4\pi mc)$ in analogy with the magnetic moment.

The muon momentum was chosen such that the second term inside the square brackets of Eq. (9.1) vanishes: $\lambda = [1 + (1/a)]^{1/2} = 29.3$. The third term displays the effect of the EDM on the spin motion and, since the laboratory electric field is negligible compared with the magnetic field ($|\vec{E}| < 10^{-3}$ B), this term reduced to a precession frequency $f_{\text{EDM}} = (e/2\pi mc)(1/2 \cdot \eta \vec{\beta} \times \vec{B})$ about an axis radial to the orbit. The origin of this motion is the torque acting on the EDM from the apparent electric field in the muon rest frame. Equation (9.1) is therefore simply the vector sum of f_{EDM} with the normal $(g - 2)$ frequency f_a :

$$\vec{f} = \vec{f}_a + \vec{f}_{\text{EDM}} = -\frac{e}{mc} \left(a\vec{B} + \frac{1}{2}\eta \vec{\beta} \times \vec{B} \right). \quad (9.2)$$

The effect of the EDM is illustrated in Fig. 30. The plane of the spin precession is tilted such that its normal is at an angle $\delta = f_{\text{EDM}}/f_a = \eta\beta/2a$ to the magnetic field. This leads to a vertically oscillating component of the muon polarization with the same frequency as the precession of the horizontal polarization. The observation of this vertical component constitutes the basis of the direct measurement of an EDM. The $(g - 2)$ frequency f_a is increased to $f = f_a(1 + \delta^2)^{1/2}$, making an EDM a possible candidate for a discrepancy between the measurement of the anomaly and the theoretical prediction.

Fig. 30 Precession of the spin relative to the momentum resulting from the combination of an anomalous magnetic moment and an electric dipole moment. The plane of the precession is tilted through the angle $\delta = \beta\eta/2a_\mu$.

The evolution of the muon polarization as a function of storage time is observed through the asymmetry in the angular distribution of the decay electrons with respect to the direction of the muon spin. To be sensitive to the vertical component of the polarization, one has to record whether the decay electron is upward or downward-going. The numbers of decay electrons in these two categories are given by

$$\begin{aligned} N_{\text{up}} &= \frac{1}{2} N e^{-t/\tau} [1 - A_\mu \cos(2\pi f_{\text{at}} + \psi) + A_e \sin(2\pi f_{\text{at}} + \psi)] \\ N_{\text{down}} &= \frac{1}{2} N e^{-t/\tau} [1 - A_\mu \cos(2\pi f_{\text{at}} + \psi) - A_e \sin(2\pi f_{\text{at}} + \psi)] . \end{aligned} \quad (9.3)$$

The asymmetry A_e is proportional to the magnitude of the EDM and is a function of the energy threshold of the detectors. For a decay-electron energy threshold of 800 MeV, $A_e = (0.164 \pm 0.019)\delta$, for the counter system used in the EDM experiment [73]. Separate measurements on μ^+ and μ^- gave:

$$d_{\mu^+} = (8.6 \pm 4.5) \times 10^{-19} \text{ e} \cdot \text{cm} , \quad (9.4)$$

$$d_{\mu^-} = (0.8 \pm 4.3) \times 10^{-19} \text{ e} \cdot \text{cm} . \quad (9.5)$$

Assuming opposite EDMs for the particle and antiparticle, the combined result was

$$d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e} \cdot \text{cm} . \quad (9.6)$$

For comparison the current upper limits for the electron, proton, and neutron in units of $e \cdot \text{cm}$ [77] are electron $\leq 3 \times 10^{-24}$, proton $\leq 2 \times 10^{-20}$, and neutron $\leq 1 \times 10^{-23}$. That these limits are much lower than the limit of the muon largely reflects the fact that, unlike the muons, they are studied in neutral systems. The fundamental length of Kadyshevsky [71] can therefore not be greater than $2 \times 10^{-18} \text{ cm}$ (muon evidence) or 10^{-23} cm (electron evidence).

10. MUON LIFETIME IN FLIGHT

Accurate measurements of the muon lifetime in a circular orbit provide a stringent test of Einstein's theory of special relativity. As a bonus it sheds light on the so-called twin paradox, gives an upper limit to the granularity of space-time, and tests the CPT invariance of the weak interaction.

The muon is an unstable particle, and can therefore be regarded as a clock and used to measure the time dilation predicted by special relativity. The existence of cosmic-ray muons at ground level supports the idea of time dilation, for, if the muons lifetime was not lengthened in flight, they would all decay in the upper atmosphere [78]. Experiments verifying the time dilation in a straight path have also been made with high-energy accelerators [79]. Time dilation in a circular path has always seemed more controversial.

Hafele and Keating [80] loaded caesium atomic clocks onto a commercial aircraft on an around-the-world trip and verified the time dilation at low velocity with an accuracy of about 10%.

In the CERN Muon Storage Ring, the muon performs a round trip and so when compared with a muon at rest the experiment mimics closely the twin paradox already discussed in Einstein's first paper [81]. The circulating muons, although they return again and again to the same place, should remain younger than their stay-at-home brothers. It is indeed observed that the moving muons live longer, in agreement to one part in a thousand with the predictions of special relativity. The stationary twin's time scale is given by the muon decay rate at rest determined in a separate experiment.

An accurate measurement of the muon lifetime in a circular orbit at $\gamma = 29.3$ requires high orbit stability in a short time interval (a few hundred microseconds), for any loss of muons will set a limit to the accuracy of the measurement. The reported stability was achieved by using a scraping system that shifted the muon orbits at early times in order to 'scrape off' those muons most likely to be lost.

The experiment consisted of measuring the decay electron counting rate $N(t)$ [see Eq. (8.13)] and the fitting procedure gave the value of $\tau = \gamma\tau_0$. The rotation frequency f_{rot} of the muons obtained from the counting record at early times (see Fig. 25) with the aid of Eq. (8.9) gave

$$\gamma = 29.327(4). \quad (10.1)$$

The best value for the lifetime at rest is $2.19711(8) \mu\text{s}$ [82], which then gives $\tau_{\text{th}} = 64.435(9) \mu\text{s}$, compared with the experimental result $\tau_{\text{exp}} = 64.378(26)$. Thus the transformation of time is validated to an accuracy of $-(0.9 \pm 0.4) \times 10^{-3}$ [79].

In the actual experiment, corrections were made for a residual small loss of stored muons, for variations of the photomultiplier gain accompanying the recovery from the initial flash, and for background counts due to stored protons (in the case of μ^+). In order to measure the muon lifetime with an accuracy of 0.1% it was necessary to study carefully these three effects, which could systematically distort the recorded time spectrum.

Another check on relativity theory can be obtained by comparing $(g-2)$ measurements carried out at different values of γ . For the electron this has been argued by Newman et al. [83], and discussed by Combley et al. [84] for both e and μ . Inevitably the conclusions are model-dependent, but one can make a plausible case that these results confirm the relativistic transformation laws for magnetic field and mass, as well as for time.

11. VERIFICATION OF THE CPT THEOREM

From CPT it follows that $g_{\mu^+} = g_{\mu^-}$. The measurements in the CERN Muon Storage Ring gave to 95% confidence

$$7 \times 10^{-9} > \frac{g_{\mu^+} - g_{\mu^-}}{g_{\mu}} > -58 \times 10^{-9}. \quad (11.1)$$

From CPT it follows also that $\tau_{\mu^+} = \tau_{\mu^-}$. The experimental data on the μ^+ and μ^- lifetime in flight give the best test of this equality (as τ_{μ^-} cannot be measured at rest because of muon capture). In this connection it should be noted that the Lorentz γ -factor is the same for μ^+ and μ^- to a much higher precision than the quoted lifetime errors. The limits are

$$3.0 \times 10^{-3} > \frac{\tau_{\mu^+} - \tau_{\mu^-}}{\tau_{\mu}} > -1.4 \times 10^{-3}. \quad (11.2)$$

Thus the theorem is validated for muons to very high accuracy for the electromagnetic interaction, and rather less accurately for the weak interaction.

12. THE SITUATION TOMORROW: PLANS FOR A NEW MEASUREMENT

The main change since the last $(g-2)$ experiment has been the firm establishment of the unified electroweak gauge theory [9] which seems to fit all known facts about the electromagnetic and weak interactions. The discovery of the W and Z bosons as free particles [8] has confirmed the major premise of the theory. All present tests of the weak interactions, however, involve only first-order processes, that is the direct exchange of W and/or Z particles. Higher-order processes involving virtual loops of W and Z are an essential part of the theory, but have not so far been detected experimentally. It is important to establish their presence.

The renormalization procedures inherent in the new theory now give definite predictions for the diagrams in Fig. 3b,c,d through which the virtual weak bosons contribute to the muon g -factor. As discussed above the result is a positive contribution from the virtual W, half cancelled by the negative effect of the virtual Z, leaving a resultant of 1.95×10^{-9} that is 1.75 ppm in a_{μ} . If this could be confirmed it would be another triumph for the theory.

The above is the result obtained from the standard Weinberg-Salam-Glashow electroweak theory [9]. But a wide variety of alternative gauge theories have been proposed, all compatible with existing data, but giving [10, 85] as it happens different predictions for $\Delta a_{\mu}^{\text{weak}}$. For most

models the $(Z + W)$ effect is in the range $(1.5-3.3) \times 10^{-9}$. Including the Higgs bosons however can lead to results spreading through $(-12 \text{ to } +17) \times 10^{-9}$ depending on the Higgs mass. Thus a_μ can provide an excellent laboratory for testing electroweak gauge theories. A new measurement of a_μ at the 0.3 ppm level would provide an important guide to the Higgs masses.

Other current speculations that would alter the value of a_μ include the following:

- Composite lepton models, leading to finite-size effects or form factors, which would cut off the integrals at high q^2 .
- Lepton substructure, or excited leptons involving new diagrams such as Fig. 31, from Méry et al. in Ref. [86]. A new $g-2$ experiment would limit the branching ratio for $Z^0 \rightarrow \mu + \mu^*$ to less than 2% [10].
- Supersymmetry with many new diagrams and contributions [87].
- Axions: the standard axion model [10, 88] contributes 10 ppm to a_μ .
- Theories with multiple W and Z bosons have been proposed and would give their own values of a_μ .

In many of these cases the limits on the exotic effects from a new $(g-2)$ measurement would be better than those expected from the new generation of electron colliders such as LEP, PEP, and the SLC.

Thus there is now a strong motivation for measuring a_μ with an experimental error much smaller than 1 ppm. But independent of the experiment it will only be possible to determine

Fig. 31 Excited lepton contributions to a_μ .

$\Delta a_\mu^{\text{weak}}$ if the comparatively large hadronic contribution to a_μ can be calculated accurately. The present best result is given in Eq. (2.7), the error being dominated by the experimental uncertainty in the ratio $R(s)$ of pion and muon yields from low-energy electron-positron colliders.

To reduce the error in $\Delta a_\mu^{\text{had}}$ to 0.3 ppm, the value of $R(s)$ will need to be known to 0.6%, about 3–4 times better than the present accuracy. New measurements are in preparation at the VEPP collider in Novosibirsk, and Barkov et al. [89] have hopes of reaching the desired accuracy. The measurement of $R(s)$ using high-energy (300 GeV) positron beams annihilating on electrons at rest [90] could very probably also be improved, and is particularly valuable for the lowest values of s just above the pion threshold.

Thus the stage is set for an attempt to measure a_μ to order 0.3 ppm with a view to obtaining the contribution of the weak interaction, and hopefully finding out which of the many alternatives is to be preferred.

12.1 A Fourth ($g - 2$) Experiment

In 1984 Vernon Hughes of Yale organized a discussion meeting at Brookhaven to see whether a new ($g - 2$) measurement could be initiated, and to work out the general parameters of the experiment. Many of the collaborators in the CERN Muon Storage Ring experiments participated*). Further discussions led to a formal proposal [91]. We describe below the main features of the new experiment as we understand them at present.

The starting point was a muon storage ring working at the ‘magic energy’, 3.1 GeV, which would as before allow a uniform magnetic field to be used with vertical focusing by means of electric quadrupoles. An improvement in accuracy by a factor of 20 was taken as the target, implying a 400-fold increase in the counting statistics. A large part of this would come from the increased intensity of the Brookhaven AGS, compared to the CERN PS in 1975. (The CERN machine has also been upgraded, but is fully committed as an injector to the SPS and LEP.) The parameters of the storage ring would be re-optimized. In particular one might reduce the radius of the ring and work at a much higher magnetic field B , thus increasing the precession frequency and improving the accuracy. As before the first thought was to inject a prepared beam of pions, and allow them to decay to muons during the first turn of the ring.

The statistical error in finding the frequency f_a from the modulated exponential record of muon decays, Eq. (8.13), is

$$\delta a_\mu / a_\mu = \delta f_a / f_a = \frac{\sqrt{2}}{2\pi f_a A \gamma \tau_0 N_e^{1/2}}, \quad (12.1)$$

where N_e is the total number of decay electrons, A the amplitude of the modulation and τ the lifetime at rest; γ is fixed at the magic value 29.3.

*) The main participants were J. Bailey, H.N. Brown, F. Combley, G. Danby, S. Dhawan, F.J.M. Farley, J.H. Field, P. Franzini, M. Giorgi, V.W. Hughes, J.W. Jackson, D. Joyce, F. Krienen, D. Lowenstein, W. Lysenko, W. Mariciano, M. May, M. Month, P. Nath, S. Parke, E. Picasso and R. Siegel.

The primary way to reduce the error is to increase the observed number of decays N_e ; but increasing f_a by increasing the magnetic field B would clearly be an advantage. As the momentum is to be fixed at the magic value this means

$$\delta a_\mu \propto r/N_e^{1/2}, \quad (12.2)$$

where r is the radius of the orbit, so we must maximize N_e/r^2 .

The number N_e is proportional to the number of muons stored N_μ , given by Eq. (7.1). One option for increasing N_μ is to increase the aperture of the storage ring a (vertical) $\times$ b (horizontal). However the pitch correction of Eq. (4.36) is proportional to the square of the vertical angular acceptance, that is to $(a/r)^2$; this should not be increased. Likewise the electric field correction of Eq. (8.8) is proportional to $(b/r)^2$.

If a/r and b/r are held constant to keep these two corrections constant, then Eq. (7.1) implies that N_e is independent of r . However some radial dependent factors are concealed in the parameter F : the number of pions which decay in the first turn of the ring is proportional to r . Also the cross-sectional area of the inflection channel can be larger for larger rings. Therefore one expects

$$N_e \propto N_\mu \propto r^2 \text{ or perhaps } r^3. \quad (12.3)$$

Substituting in Eq. (12.2) it then appears that a high-field small-radius ring will not after all reduce the statistical error; it might even increase it. So bearing in mind the greater technical difficulties of a high-field ring (field uniformity, inflector design), it was decided to keep the radius at 7 m with $B = 1.47$ T.

Increasing the decay electron counting rate N_e by a factor 400 turned out not to be trivial. At CERN about 200 electrons, spread over 5 digitrons, were counted per cycle of the PS. With a muon lifetime of $64 \mu\text{s}$ this implied an initial counting rate of 0.6 per microsecond in each digitron. At rates appreciably higher than this the random overlapping of signals would lead to timing errors which would be increasingly serious as one tried to improve the absolute accuracy of the experiment. One solution is to increase the number of digitrons to, say, 80 (thus reducing the peak counting rate in each), and to repeat the experiment more often per hour. This could be achieved by filling the muon storage ring several times in each cycle of the AGS by ejecting a bunch of protons from the accelerator at intervals of, say, 10 ms during the 'flat top' of the synchrotron. The measurement could thus be repeated several times per AGS cycle. With 80 digitrons and 5 fills per cycle one could handle a factor of 80 in overall counting rate, with no increase in the peak rate per channel. Another factor of 5 could probably come from detailed electronic improvements for detecting and rejecting multiple events which might have been mistimed.

A limiting factor in the CERN experiment was a large perturbing pulse in the decay electron detectors, coinciding with the injection of pions into the ring. This was caused by a strong flash of light in the glass of the photomultipliers, light pipes and scintillators. Light continues to be produced at a lower level for many microseconds after injection, as epithermal

neutrons are gradually captured and gamma rays from (n,γ) reactions give rise to Cherenkov light. This phenomenon (also seen in other experiments) is known for convenience as the 'initial flash'. At high injection intensities it can paralyse the detector electronics for excessive periods, leading to counting and timing errors. In the CERN experiment these were controlled to less than 1 ppm in a_μ , negligible compared with the final accuracy of 7 ppm. But in a new measurement at the 0.3 ppm level, the much higher injected intensities could clearly give problems. Unfortunately these effects are hard to study quantitatively without actually building a storage ring!

An innovation which will increase the number of stored muons, while at the same time reducing the initial flash, is to inject muons instead of pions. To do this one sends the prepared beam of pions down a 'muon channel', that is a long (100 m) decay path with focusing elements to collect the muons produced in $\pi\text{-}\mu$ decay. At 3 GeV/c the pion decay distance is about 170 m, so about 50% of them decay in the channel, in contrast to only 10% of useful decays if the pions are injected directly into the ring. The muons from almost forward decay, having momentum 1-2% less than the pion momentum, can be separated out at the end of the channel by a bending magnet, giving an almost pure muon beam which is then injected into the muon storage ring.

In this case, however, one must have a full aperture kicker in the ring, to kick the muons onto permanently stored orbits: and this magnetic kick must be switched off within 140 ns, before the bunch of muons makes one turn. There is no difficulty in principle in designing such a kicker, but it must be free of iron or ferrite (which would modify the carefully prepared magnetic field), and the residual field 20 μ s later when the muon precession is being recorded must be small and measurable. As the muons continue to pass through the kicker at every turn, its residual field (including that due to any eddy currents induced in the surrounding metal vacuum chamber) must be known to a level corresponding to 0.1 ppm in the average orbit field. This corresponds to about 1:20,000 of the size of the kick, and presents some technical challenge!

In the first Muon Storage Ring the CERN team adopted the technically simple strategy of injecting high-momentum protons onto a target in the ring. The pions produced made a few turns before decaying to muons, some of which fell onto permanently stored orbits. The initial flash effects were severe, and the large number of off-momentum pions decaying at unplanned angles gave stored muons of the wrong polarization, reducing the amplitude of the precession signal. In the second CERN storage ring these difficulties were alleviated by preparing a beam of pions outside the ring and injecting them with a pulsed inflector. Following this a third storage ring in which muons are injected directly is a logical progression. Each step is better for the physics, but presents new technical difficulties.

An overall comparison of pion and muon injection is given in Table 2. With pions, 5000 particles must be injected for every muon stored, so the initial flash will be severe. With muon injection the injected beam is much lower and the flash will be about ten times smaller than in the last CERN experiment. On the other hand, because of the larger decay path, many more muons will be stored. The disadvantage is that the full aperture kicker can perturb the magnetic field in the storage region.

Table 2
Comparison of pion and muon injection

	Pion injection	Muon injection
Particles injected	6×10^7	5×10^4
Muons stored	12,000	30,000
Inflector	Yes	Yes
Full aperture kicker	No	Yes
Initial flash	Severe	Negligible

12.2 The Muon Distribution and the Average Field

Referring to Eq. (4.9) it is clear that an accurate knowledge of the mean magnetic field $\bar{B}$ seen by the muons is necessary. If the field of the storage ring is strictly uniform this is not a problem; but if, as is more likely, the field varies by some parts per million over the storage volume, it is necessary to calculate the average field weighted over the muon distribution, and corrected for the fact that muons on the inside of the ring contribute more decay electron counts than those on the outside. Ideally we should have a detailed knowledge of the distribution of muons in the available aperture, and know how this distribution affects the calculated mean field. Some useful theorems have been developed to simplify the analysis [92].

It is convenient to use a cylindrical-coordinate system, tied to the centre of the storage aperture. Let ϕ be the azimuth angle around the ring, z the height above the median plane and $(x + r_0)$ the radius from the centre of the ring: r_0 is the radius at the centre of the storage aperture, so x measures the radial distance from the centre line of the aperture.

The magnetic field B satisfies Laplace's equation

$$\frac{\delta^2 B}{\delta x^2} + \frac{\delta^2 B}{\delta z^2} + \frac{1}{x + r_0} \frac{\delta B}{\delta x} + \frac{1}{(x + r_0)^2} \frac{\delta^2 B}{\delta \phi^2} = 0 \quad (12.4)$$

For a given (x, z) let us calculate the field B averaged over all azimuths; this is the field seen by a muon circulating round the ring at the given position (x, z) in the aperture:

$$\bar{B}(x, z) = \frac{1}{2\pi} \int_0^{2\pi} B(x, z, \phi) d\phi ; \quad (12.5)$$

$\bar{B}$ satisfies Eq. (12.4) with the last term omitted, as in carrying out the integral as in Eq. (12.5) the last term cancels out.

Because the curvature of the ring is small it turns out that the third term in Eq. (12.4) is less than 1% of the first two terms, so to a good approximation the Cartesian form of Laplace's equation in two dimensions applies,

$$\frac{\delta^2 \bar{B}}{\delta x^2} + \frac{\delta^2 \bar{B}}{\delta z^2} = 0. \quad (12.6)$$

It is well known that the field can then be expressed as a series of multipoles, which each satisfy Eq. (12.6). These multipoles are most easily represented by changing to new coordinates (r, θ) in the plane of the aperture and with origin at the centre of the aperture, such that

$$x = r \cos \theta, \quad \text{and} \quad z = r \sin \theta.$$

Then the multipoles of the field are of the form $r^n \cos(n\theta)$ and $r^n \sin(n\theta)$ and in general

$$\bar{B} = \sum_n r^n [c_n \cos(n\theta) + s_n \sin(n\theta)], \quad (12.7)$$

the c_n and s_n being the multipole coefficients whose values can represent any arbitrary distribution of $\bar{B}$ in the aperture plane. The overall average field is $\bar{B} = c_0$. If the other coefficients are all zero, the field is uniform and no problems arise. How large can they be before they give an appreciable error, and how does one calculate the contribution of each multipole component to $\bar{B}$?

Let the number of muons at point (r, θ) in the aperture be $M(r, \theta)$, the total number being $N = \iint M(r, \theta) r dr d\theta$. Then the average field contributed by the n^{th} cosine multipole will be

$$\begin{aligned} \bar{B}_n &= \frac{1}{N} \iint c_n r^n M(r, \theta) \cos(n\theta) r dr d\theta \\ &= c_n I_n, \end{aligned} \quad (12.8)$$

where

$$I_n = \frac{1}{N} \iint r^n M(r, \theta) \cos(n\theta) r dr d\theta; \quad (12.9)$$

I_n may be called the n^{th} moment of the muon distribution; it has no quantities associated with the magnetic field.

Including all multipoles of the field, the overall average field will be expressed as

$$\bar{B} = \sum_n (c_n I_n + s_n J_n). \quad (12.10)$$

where the skew moments of the muon distribution are given by

$$J_n = \frac{1}{N} \iint r^n M(r, \theta) \sin(n\theta) r dr d\theta. \quad (12.11)$$

As this is a complete specification of the average field it is clear that for this purpose the n^{th} moment I_n only interacts with the n^{th} multipole coefficient c_n ; there are no cross terms. This result greatly simplifies the task of analysing the magnetic field.

It was found in the previous storage ring experiments that the muons filled the available phase space in the ring more or less uniformly. While there was no exact experimental check, this can be used as a reasonable basis for planning. With this assumption the moments I_n of the muon distribution have been calculated (a) for a rectangular aperture $10 \times 8 \text{ cm}^2$, (b) for a square aperture $9.14 \times 9.14 \text{ cm}^2$, and (c) for a circular aperture of diameter 11 cm. All three apertures have the same acceptance. (Note that the J_n are theoretically zero because of up-down symmetry.) From the moments, using Eq. (12.10) one can calculate the maximum values of c_n allowable if the mean field is to be affected by less than 0.1 ppm. The corresponding allowable field at the horizontal edge of the aperture is given in Table 3 for each multipole.

Table 3
Allowable field in ppm at the edge of the aperture
for 0.1 ppm effect on $\bar{B}$

n	Rectangular	Square	Circular
2	1.12	–	2.8
4	–1.56	0.80	–50
6	–2.00	–	286
8	5.40	0.92	–1014
10	2.06	–	2768
12	25.0	0.62	6446

From the table we see that the rectangular apertures, as used in the CERN experiments, give the muon population significant values of I_8 and I_{10} and therefore require the corresponding field multipole to be small. This is because the corners of the aperture have some muon intensity which is represented by terms in $\cos(4\theta)$, $\cos(8\theta)$, etc. These couple to the 4th, 8th and 12th multipoles which the table shows must be maintained at a very low level. The square aperture is even more sensitive to certain higher multipoles!

With a circular aperture, however, the smoothness of the boundary means that the higher cosine terms in the muon distribution are small, and we can tolerate much larger multipoles in the field. For example the $n = 8$ multipole must be known only to 1014 ppm at the edge of the aperture; this should not be difficult. Therefore it was resolved to put a circular limiting stop in the storage region to define the muon population.

12.3 Pick-Up Electrodes

The above results are based on the assumption that the muons in the ring populate the available phase space uniformly. It will be important at the new level of accuracy to obtain an independent measurement of the actual muon distribution, so that the true average field can be calculated.

The momentum of the muon determines the radius of its closed orbit in the ring, and therefore its orbital rotation frequency. As explained above the muons are bunched at injection, and therefore the rotation frequency is visible in the electron counting data. By analysing the counts one can find the distribution of muons with respect to equilibrium radius. This is an important measurement which has been helpful in finding the average field.

However, the rotation frequency does not determine the muon distribution completely, because a muon will normally oscillate both horizontally and vertically around its theoretical closed orbit. To calculate the average field one needs the distribution of muons including these orbit oscillations. Up to now no direct information on the oscillations has been available, but checks of stored intensity versus aperture size have agreed with predictions based on phase space.

In the new experiment some 30,000 muons will be circulating at 7 MHz implying a muon current of 30 nA, and it is proposed [92] to measure the distribution inside the circular aperture by means of pick-up electrodes. An interesting theorem relates the moments of the muon distribution to the Fourier components of the charge $\sigma(\psi)$ induced on a circular cylinder surrounding the storage aperture, see Fig. 32. The relation between the distribution of charges inside a long cylinder $q(r, \theta)$ and the corresponding distribution of the surface charge $\sigma(\psi)$ can be calculated in static approximation using the method of images. (See Fig. 32 for the definition of the variables.)

Fig. 32 Definition of the variables needed to calculate the distribution of charge inside a long cylinder.

Consider a long cylindrical pick-up electrode of unit radius. The centre line of the cylinder is taken as the origin for coordinates (r, θ) in the plane transverse to the axis. A distribution of line charges $q(r, \theta)$ per unit volume is placed inside the cylinder. We calculate the corresponding induced charge on the inside surface of the cylinder $\sigma(\psi)$ as a function of azimuth ψ around the cylinder.

In general $\sigma(\psi)$ may be Fourier analysed

$$\sigma(\psi) = \sum_0^{\infty} \{u_n \cos(n\psi) + v_n \sin(n\psi)\} . \quad (12.12)$$

It is then found that the amplitude c_n of the n^{th} Fourier component is proportional to the corresponding moment I_n of the charge distribution

$$u_n \propto I_n = \int \int q(r, \theta) r^n \cos(n\theta) r dr d\theta, \quad (12.13)$$

with a corresponding result for the sine components

$$v_n \propto J_n = \int \int q(r, \theta) r^n \sin(n\theta) r dr d\theta. \quad (12.14)$$

Thus the Fourier coefficients u_n and v_n of the charge induced on the pick-up cylinder are proportional to the moments I_n , J_n of the muon distribution. This is just the information needed for calculating the average field with the aid of Eq. (12.10).

In translating this theorem into practical reality there are a number of experimental difficulties. The cylinder would be divided into many longitudinal strips, each one connected to a separate amplifier. With only about 30,000 muons inside the cylinder at any one time, the induced signal on each strip will be small (of order $1 \mu\text{V}$) and this will be below the noise level in the amplifiers. This is not an absolute impediment, however, because the $g - 2$ precession signal for a single cycle of the synchrotron is also swamped in noise: by repeating the measurement many times and averaging, significant information is accumulated. In principle the same process can apply to the pick-up electrodes: by digitizing the information with appropriate signal processing, and accumulating over the duration of the experiment, one should end up with accurate position information.

Another method of finding the muon distribution is to measure the decay electron tracks carefully with wire chambers. If the track is extrapolated backwards until it is tangential to the ring this gives a fair estimate of the position of the parent muon. The μ -e decay angle creates some uncertainty, but this turns out to be small.

12.4 Magnet

The general parameters of the new experiment are summarized in Table 4. The radius of the storage ring is slightly larger than in the last CERN experiment, namely 7.11 m instead of 7.00 m. The field is reduced correspondingly to keep the same storage momentum. To have a more uniform field in the storage region the magnet gap is increased from 14 to 18 cm, and the pole width is increased in proportion from 38 to 49 cm. The yoke will be continuous, and the magnet will be energized by superconducting coils. It is expected that the uniform and steady temperature of the coils and the absence of ohmic heating will give a more stable magnet. In addition the iron of the magnet will be lagged and its temperature will be stabilized. By these measures it is hoped that the field shape will be stable to better than 1 ppm.

To increase the field uniformity there will be an air gap of 10 mm between the pole pieces and the yoke. This is a well known technique for decoupling the field in the main gap from irregularities in the iron of the yoke. In addition the field can be adjusted locally by inserting iron sheets into this air gap, instead of shimming or grinding on the pole face itself. Comparatively thick sheets can be used behind the poles to make small changes in the field. Pole face windings will be used to make final corrections to the field.

Table 4
Parameters of the New Experiment

Storage ring radius	7.112 m
Field	1.45 T
Storage momentum	3.094 GeV/c
Muon lifetime	64 μ s
Magnet gap	18 cm
Pole width at root	56 cm
Useful aperture inside quadrupole electrodes	10 $\times$ 10 cm ²
Aperture used for muon storage (diameter)	9 cm
No. of AGS protons in each RF bunch (with booster)	4 $\times$ 10 ¹²
Length of muon channel	100 m
No. of injected muons per fill	5 $\times$ 10 ⁴
Fraction of muons stored	30%
No. of stored muons per fill	1.5 $\times$ 10 ⁴
Electrons above 1.6 GeV counted per fill (20%)	3000
Fills per AGS cycle	3
Fills per hour	7714
Electron counts per hour	2.3 $\times$ 10 ⁷
Total counts required for 0.3 ppm statistical error	2.85 $\times$ 10 ¹⁰
Running time for 0.3 ppm statistics	1240 hours
Pitch correction	0.40 ppm
Electric field correction	0.69 ppm

As before the field will be surveyed and monitored with proton resonance magnetometers. By scanning the resonance slowly it is expected to achieve an accuracy of 0.1 ppm or better. A remote-controlled trolley which can automatically survey the field inside the vacuum chamber is being developed. This will enable the field to be measured without removing the vacuum chamber. (Removing the chamber normally implies switching off the field, and it may not return to exactly the same value afterwards.) If the trolley can actually move and measure under vacuum that would be an additional bonus because the field could be checked more frequently during the runs.

Another way of calibrating the magnetic field is to inject a narrow (1 ns) pulse of singly charged gold ions [93] with a kinetic energy of 23 MeV. These are available from the tandem Van de Graaff at Brookhaven, which is already linked to the AGS. They would have the same momentum as the stored muons, but would be non-relativistic ($\beta = 0.015$) so the orbit frequency becomes almost independent of radius and is a measure of the magnetic field B averaged over the orbit. The orbit frequency can be measured by kicking the gold ions onto a scintillation counter after many turns. As the orbit frequency of the gold is altered by the radial electric field of the electric quadrupoles, they must be switched off (or operated at very low

voltage), but even without any vertical focusing it seems possible to get sufficient turns to make an accurate measurement.

The gold method has the advantage of automatically averaging the field in azimuth. It may also be possible to direct the beam to different parts of the storage aperture to map the field in the x - z plane. At its lowest, the gold method could give an independent check of the NMR measurements. At its best, it might be interleaved with the muon pulses to monitor the field at frequent intervals without losing any running time with the muons.

In summary, the knowledge of the average field seen by the muons will be improved by having a more stable magnet, more frequent surveys and a cross check with the gold ions. The circular defining aperture for the storage volume will reduce the sensitivity to higher multipoles in the field, and the pick-up electrodes and wire chambers will hopefully give precise information about the distribution of muons within the aperture.

12.5 Electronics

With 3000 decay electrons to be detected per fill and timed to an average accuracy of 0.1 ppm, the electronics must be very carefully designed. As the muon lifetime will be $64\ \mu\text{s}$ the average counting rate at the beginning of the fill will be about 50 per microsecond: on the average the pulses are only 20 ns apart.

The forward decays, which have the largest asymmetry parameter, have been selected in the past by requiring a large pulse height in the electron shower calorimeters, implying high electron energy. This will not work properly if the pulses overlap and low-energy electrons are mixed into the high-energy group. The phase of the $(g - 2)$ modulation is in fact different for different electron energies, so an admixture of pulses of the wrong height due to overlapping would lead to an error in phase (at early times but not at late times when the counting rate is low). This would create an error in frequency.

Another error arises if two pulses overlap and are processed as a single pulse, with the time taken from the first pulse. This leads to a systematic bias to early times at the beginning of the fill (but not at the end), thus again creating an error in the $(g - 2)$ frequency.

The main defence proposed against pulse overlap is spatial separation of the detectors and spatial segmentation within each detector. If, say, 100 independent counters and timing electronics are used, the initial counting rate per channel drops to 0.5 per microsecond. If in addition some information is provided as to where the electron hits the counter, double hits can be detected and rejected. It is better to reject a double event and to lose counts, than to accept it with an error in time. With this strategy it looks as if the necessary timing accuracy can be achieved.

The effective span of time available for measuring the $(g - 2)$ frequency superposed on the exponential muon decay is about 3 lifetimes, that is $200\ \mu\text{s}$. This means that, on average over many events, the timing accuracy must be 20 ps. It is surprising to realise that this can be achieved with a digital timing system that sorts the signals into time bins as much as 10 ns wide. This is discussed in detail in Refs. [53] and [94].

The essential point is that the clock pulses of frequency f_{clock} which determine the channel boundaries must *not* be synchronized with the start pulse which signals the moment of muon

injection. This means that for a fixed time interval between 'start' and 'stop' the result of the measurement is statistically distributed between two neighbouring channels. Then from the counts c_n and $c_{(n+1)}$ in the two channels, the exact time interval t can be determined

$$t = f_{\text{clock}}^{-1} \left\{ n + c_{(n+1)} / [c_n + c_{(n+1)}] \right\} \quad (12.15)$$

subject to the usual statistical uncertainties.

In a test with crystal-controlled pulses with 32,000 counts per channel a timing accuracy of 25 ps was achieved with bins 5 ns wide, a ratio of 1/200 which agrees roughly with what is expected from the statistical factor $[c_n + c_{(n+1)}]^{1/2}$.

In the new experiment one expects 5×10^6 counts per channel at early times, dropping to 2.5×10^5 after 200 μs , so a statistical accuracy of 20 ps will be achieved in each channel. Because many channels contribute to the final frequency determination, this 'binning error' turns out to be entirely negligible.

12.6 Inflector

A new superconducting inflector to guide the incoming particles through the fringing field and into the storage region has been designed [95]. With a carefully structured pattern of parallel conductors carrying d.c. currents, the field of the ring magnet can be cancelled inside the inflector without perturbing the field in the storage region; the return flux is completely contained inside the inflector structure. Model tests have demonstrated the feasibility, and the next step is to test a superconducting model.

12.7 Summary

A measurement of a_μ to about 0.3 ppm has been planned in detail and looks feasible; preparations are now under way in Brookhaven and it is hoped that the experiment will be running in 1992. To reach this level of accuracy will require great patience and meticulous attention to the many detailed problems that will present themselves en route. In anticipation of this result, considerable theoretical and experimental activity has already been generated.

13. FINAL REMARKS

At its present level of accuracy the muon ($g-2$) measurement, combined with the theoretical calculation, has made an important contribution to validating QED.

The absence of a discrepancy continues to provide a constraint on the fantasy of theorists, because many new models for fundamental particles turn out at some level to change the theoretical value of a_μ . The experimental value of a_e is now known [96] 2000 times more accurately than a_μ . But new couplings common to the electron and muon usually imply a greater perturbation to a_μ by the factor $(m_\mu/m_e)^2 \sim 42,000$. Therefore the muon measurement still, in general, provides a more sensitive limit to exotic speculations.

Another consequence is that, except for possible couplings peculiar to the electron, the muon result ensures that a_e is a 'pure QED quantity' to order 2×10^{-10} in a_e , thus validating a_e as a good measurement of the fine structure constant α . Another good route to α is via the

hyperfine splitting in muonium. Here again the experiments on a_μ and a_e ensure that muonium is a 'pure QED system' [97].

We would like to conclude by drawing the reader's attention to the many previous review articles on the muon ($g - 2$) experiments [98], many of which cover particular topics in further detail.

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We dedicate this article to all those who participated directly or indirectly in the muon ($g - 2$) programme. In reviewing the work we inevitably recall again the spirit of constructive interaction which enlivened our debates. It has been our privilege and our pleasure to have a share in this endeavour.

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