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ON THE FINE-STRUCTURE SPLITTING IN $\bar{p}$ ATOMS

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ABSTRACT

Recent measurements of fine-structure splitting in $\bar{p}$ atoms of ^{174}Yb are analysed. Effects of nuclear deformation are calculated. The strength of nuclear spin-orbit coupling is determined and its implications on the $N\bar{N}$ potential are discussed.

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1. INTRODUCTION

In recent CERN measurements [1] of the X-ray transitions in antiprotonic atoms of ^{174}Yb , the level shifts and widths are determined, and contributions to the fine structure, which are due to nuclear interactions of $\bar{p}$, are detected. The results are puzzling in two respects: i) the level shifts, which arise mainly from the strong interactions, are attractive, although this is contrary to both the theoretical expectations [2–4] and the results of measurements in other atoms [5]; ii) the fine-structure splitting arising from the electromagnetic interactions is enhanced by strong interactions, contrary to theoretical expectations [2–4]. As shown in ref. [6], the first difficulty is related to the large deformation of the ^{174}Yb nucleus, giving rise to coupling of the atomic motion to the nuclear rotation. Calculations of this process for the atomic 9–8 transitions in ^{174}Yb give an attractive shift of about 600 eV [6]. The lower of the two states, i.e. $n = 8$, is split in two, with angular momentum $j = 13/2$ (lower) and $j = 15/2$ (upper). Comparing the experimental results [1] and the calculated 600 eV just mentioned, there are residual repulsive shifts of -255 eV and -315 eV for the $j = 13/2$ and $j = 15/2$ states, respectively. In ref. [6] it is suggested that these residual shifts may arise from strong interactions.

The fine-structure splitting of the $j = 13/2$ and $j = 15/2$ levels arises mainly from the electromagnetic interaction (2350 eV), but the experimental value [1] is greater by 58 ± 26 eV. Calculations of this enhancement arising from strong interactions have given [2–4] values in the range -71 to -16 eV, whilst the nuclear rotation effect gives [6] a contribution of only $+7$ eV. The purpose of this letter is to understand this discrepancy between the experimental and theoretical values of this enhancement of the electromagnetic splitting.

Two effects have been either overlooked or simplified in the calculations done so far. The first effect is the nuclear deformation which, apart from rotational coupling, induces new terms in the $\vec{L} \vec{S}$ potential. The second effect is due to specific kinematics in the atomic states. The $N\bar{N}$ interactions occur at the extreme nuclear surface and, because of the nucleon binding, the c.m. energy in the $N\bar{N}$ system is negative. The $N\bar{N}$ scattering matrices may be irregular in this region.

The results obtained, as described in this article, are the following:

- i) Nuclear deformation induces a term in the antiproton spin-orbit potential V_{so} related to the tangential gradient of the nuclear density. However, this effect is too small to change the sign of V_{so} as required by the data.
- ii) Overlaps of atomic and nuclear densities differ by $\sim 10\%$ in the upper and lower fine-structure components. This has two consequences: firstly in the lower state the width becomes larger and secondly the lower shift becomes more repulsive. The first effect explains about 50% of the experimental difference of $\Gamma_U = 1021$ (41) eV for the upper level and $\Gamma_L = 1216$ (41) eV for the lower level. The second effect is that the fine-structure splitting becomes smaller than the electromagnetic one, whilst the measured one is larger.
- iii) To counterbalance the second overlap effect of (ii), strongly repulsive $\text{Re } V_{so}$ and a weak $\text{Im } V_{so}$ are needed. The potential models of $N\bar{N}$ interactions generate repulsive $\text{Re } V_{so}$ in the subthreshold region. This is too weak, however, and the condition for $\text{Im } V_{so}$ cannot be met in the same models. Nuclear-medium effects on $N\bar{N}$ scattering are unlikely to change this situation. The atomic data require a new description of the absorption in

either the $^{33}\text{P}_1$ or the $^{13}\text{P}_0$ NN states, which may be achieved by the introduction of repulsive short-ranged potentials.

- iv) Quantitatively but not qualitatively, these results depend on the uncertainty in the nuclear deformation for the large radii involved in such processes.

2. INTERACTIONS WITH A DEFORMED NUCLEUS

The nucleus in question has no spin, and the optical potentials have to be averaged over nuclear orientations. The nuclear density profile is known from muonic atoms [7] in the form $f^a [r - c(1 + \beta' P_2)]$ or $f^c [r(1 + \beta P_2) - c]$, where f is the Woods–Saxon profile, c its half-density radius, and P_2 the Legendre polynomial. The parameters β are determined essentially by the quadrupole moment, and deformations at the large radii involved in the antiprotonic atoms are not well established. Such uncertainty is of some meaning in high angular momentum atomic states. Figure 1 shows the atomic-nuclear overlaps in the orbital angular momentum $\ell = 7$ states for the angular averaged density. No significant differences between profiles a and c are found.

The deformation changes the $\vec{L} \vec{S}$ potential, as is known [8] from inelastic proton–nucleus scattering. There, it is possible to expand f in terms of the deformation parameter β . This is not applicable now, and one has to generalize the formulas. The standard expression [9, 10] for the $\vec{L} \vec{S}$ optical potential,

$$V_{\text{so}}(r) = \frac{-2\pi}{\mu_{\text{NN}}} \sum \frac{3}{4} a_{\text{so}} \vec{\partial} \varrho(r, x) x \vec{p} \vec{\sigma} , \quad (1)$$

contains derivatives over r and $x = \hat{e} \cdot \vec{r}$, where $\hat{e}$ is a unit vector along the nuclear symmetry axis. The derivative with respect to x averaged over directions of $\hat{e}$ will again reduce V_{so} to the $\vec{L} \vec{S}$ form, but will contribute some additional terms. We have

$$V_{\text{so}} = \frac{-2\pi}{\mu_{\text{NN}}} \sum \frac{3}{4} a_{\text{so}} \frac{1}{r} [\varrho'_0(r)\alpha + \varrho'_2(r)\gamma] \vec{L} \vec{\sigma} , \quad (2)$$

where

$$\varrho'_0(r) = \int_0^1 dx \frac{\partial \varrho}{\partial r}(r, x) ,$$

$$\varrho'_2(r) = \int_0^1 dx x^2 \frac{\partial \varrho}{\partial r}(r, x) .$$

For the profile f^a , we obtain a trivial result $\alpha = 1$, $\gamma = 0$, since the deformation is r -independent. For the profile f^c , $\alpha = 1 - \beta/2$ and $\gamma = 3/2 \beta$. These contributions to V_{so} are essentially of a higher order in β . Nevertheless, in the large r limit the expression in square brackets in eq. (2) reduces to $\varrho'_0(1 + \beta)$. In the case of Yb, this makes a 17% reduction in strength in comparison with the profile f^a . The sign of V_{so} is not affected, however.

The summation in eq. (1) extends over protons and neutrons, whilst a_{so} is a special [10–14] combination of NN scattering volumes $a(P)$:

$$a_{so} = [5a(^3P_2) - 3a(^3P_1) - 2a(^3P_0)]/12. \quad (3)$$

Higher partial waves are negligible for the close-to-threshold energies. In a potential model in this energy region, $a(P)$ may be defined by

$$a^I(P) = (2\mu_{NN}/9) \left[\int dx x^4 V_{NN}(x) \Psi_{NN}(k, x) / j_1(kx) \right],$$

where V_{NN} is the NN potential, and Ψ_{NN} is the NN P-wave function of a given spin and isospin I ; j_1 is a Bessel function.

Table 1 lists the NN scattering volumes at threshold for all the spin-isospin states. The a_{so}^I of eq. (3) are dominated by the $p\bar{p}$ interactions in the $I = 0$ state. Keeping this as an assumption, we determine in fig. 2 the region of a_{so}^0 allowed by the Yb data. There is no overlap with theoretical calculations, and possible implications for the NN interactions are discussed in section 3. We will now consider the nuclear content of this ansatz.

The nuclear interaction region is peaked in the low-density nuclear tail. Overlaps of the atomic density with the absorptive $\bar{p}$ optical potential of ref. [13] are given in fig. 1. The nuclear deformation is not well known for large radii, as the muonic atoms determine essentially the quadrupole electric moment and the $\bar{p}$ atoms involve the $\langle r^{14} \rangle$ density moment (see fig. 1). An overlap with the angular averaged charge density f^a of ref. [7] differs from the overlap with a spherical density having the same r.m.s. radius as that used in ref. [2]. To minimize the uncertainty, single-particle model wave functions are used here. These are determined by fitting the separation energies of the valence neutrons and protons and the r.m.s. densities.

This calculation gives a neutron/proton ratio of about 3 in the maximum overlap region. Uncertainties of the neutron distribution may thus cover other shortcomings of the antiproton potential. The central potential we use reproduces sufficiently well the average shift and width in Yb (table 2) and also in some light nuclei [13]. In the neutron-dominant situation of ^{95}Mo , with the single-particle wave functions, we obtain 400 eV and 1840 eV, in comparison with the experimental 630 (110) eV and 2300 (400) eV [5]. An improvement of this fit is possible with small changes in the V_N strength or range (e.g. a 10% increase of the absorption). However, this is of secondary importance in understanding the $\vec{L}\vec{S}$ splitting, since the magnitude of the latter depends rather weakly on the central part of V_N . This dependence arises because the atomic-nuclear overlap is larger, by about 10%, in the lower fine-structure state, thus enhancing the lower width. Numerically, we find $\Delta\Gamma/\Gamma$ of about 7% for a broad range of antiproton potentials. About half of the experimental $\Gamma_L - \Gamma_U$ difference is explained in this way. On the other hand, the enhanced absorption enlarges the repulsive shift of the lower level. The latter reduces the electromagnetic $\vec{L}\vec{S}$ splitting opposite to the observations. Numerical studies show $\Delta E/E \approx 7\%$ for ΔE_s and Γ_s around the average experimental results (see table 2). We find this repulsion to be a strong factor that limits the allowed regions of $\text{Im } a_{so}^0$ and $\text{Re } a_{so}^0$ in fig. 2. In order to explain the data,

changes would be needed in the short-range $N\bar{N}$ interactions, presumably in the absorptive part. Some indications are given in section 3.

The nuclear rotation admixes lower atomic states. The mixing coefficients of ref. [13] yield corrections of 10 eV and 20 eV to the shifts and widths, respectively. These are already included in the results. The fine-structure splitting introduced in this way is minute.

3. A RELATION TO $N\bar{N}$ INTERACTIONS

Results of three calculations are shown in table 1. Models of refs. [11] and [12] are compared with a simple calculation consisting of pion exchange forces cut at 0.7 fm and the annihilation potential V_{ann} of ref. [11]. It is seen that the $\text{Re } a_{\text{so}}$ are determined by higher-order tensor forces that are due to pions. The absorptive parts, $\text{Im } a_{\text{so}}$, are generated by V_{ann} in a highly non-linear way, as the Born terms in eq. (4) yield only $-i 0.34 \text{ fm}^3$ for each spin-isospin state. It follows also that the strong pion-exchange potential in the $^{13}\text{P}_0$ state, and the complex phase of $\Psi_{N\bar{N}}$, produce large $\text{Im } a_{\text{so}}^0$, dominating the $\text{Im } V_{\text{so}}$. Strong reduction of $\text{Im } a_{\text{so}}^0$ may be achieved if the attractive $\text{Re } V_{\text{ann}}$ of ref. [11] is turned into repulsion in this state. The physical origin of this might be related to the vacuum quantum numbers of this state, which create specific conditions for the annihilation. Increasing the repulsion changes a_{so}^0 along the trajectory shown in fig. 2. The end-point corresponds to $\text{Re } V_{\text{ann}}(^{13}\text{P}_0, r = 0) = 1500 \text{ MeV}$ instead of -500 MeV , and is absorption range-reduced to 0.7 fm. The result comes close to the experimental limits, although the extent of required changes depends on the nuclear density distribution. The new $a^0(^{13}\text{P}_0) = -12.6 - i 0.90 \text{ fm}^3$ would reduce the 2P level widths of $p\bar{p}$ atoms by 20%.

Another option for understanding the Yb data is a change of the annihilation model in the $^{33}\text{P}_1$ state. A 1 GeV repulsive $\text{Re } V_{\text{ann}}$ makes a new $a^1(^{33}\text{P}_0) = -1.46 - i 0.17 \text{ fm}^3$ at the $N\bar{N}$ threshold. This has a stronger effect on V_{so} because $\bar{p}$ -neutron interactions are involved, and the n/p density ratio is about 3. The new a_{so}^1 is indicated in fig. 2. It would change the shift and the width of the $p\bar{p}$ atomic 2P state by $\sim 10\%$.

The extent of the modifications required to fit the atomic data depends on the nuclear density tail. As shown in fig. 2, it may be smaller with a larger $\langle r^{14} \rangle$ moment. An enhancement of the latter might arise from an uncertainty at large radii deformation. On the other hand, there seems to be no obvious justification for such an enhancement.

The sign of $\text{Re } V_{\text{so}}$ obtained here differs from that obtained with the same $N\bar{N}$ interactions at higher energies [3, 14], because the atomic kinematics requires [13] an average of a_{so}^1 over energies extending below the $N\bar{N}$ threshold. The behaviour of a_{so}^0 in this region is shown in fig. 3. Although the sign of $\text{Re } V_{\text{so}}$ is plausible, the strength is not sufficient. Nuclear effects and external fields cannot change this picture in the low-density region of interest. This statement is borne out by calculations done at higher energies (ref. [14]). The same conclusion is obtained also in the subthreshold region with our pion-exchange model. We summarize the result: the atomic data require changes in the description of $N\bar{N}$ annihilation. Indications are that these should involve the $^{13}\text{P}_0$ and/or $^{33}\text{P}_1$ states. Scattering experiments with polarized $\bar{p}$ would be useful for clarifying the situation.

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Table 1
The $N\bar{N}$ scattering volumes (in fm³)

State	Ref. [11]	Ref. [12]	Ref. [11] Pion exchange + annihilation
$^{13}\text{P}_0$	-9.552 -i 5.196	-8.356 -i 5.036	-8.217 -i 6.208
$^{13}\text{P}_1$	5.137 -i 0.077	4.131 -i 0.049	4.860 -i 0.079
$^{13}\text{P}_2$	-0.315 -i 0.811	-0.077 -i 0.709	0.234 -i 0.557
a_{so}^0	0.176 +i 0.547	0.327 +i 0.556	0.252 +i 0.822
$^{33}\text{P}_0$	2.686 -i 0.127	2.291 -i 0.000	2.536 -i 0.157
$^{33}\text{P}_1$	-2.076 -i 0.862	-1.594 -i 0.774	-1.682 -i 0.765
$^{33}\text{P}_2$	-0.128 -i 0.502	-0.184 -i 0.459	0.044 -i 0.468
a_{so}^1	0.018 +i 0.028	-0.058 -i 0.001	0.016 +i 0.022

Table 2

The upper and lower shifts and widths in the $n = 8$ state of ^{174}Yb .

Row 2 includes corrections due to the deformation [6]. All calculations are done with $V_{\bar{N}}^{\text{central}}$ of ref. [13] (separable version); the V_{so} is based on a_{so} of ref. [11].

	Γ_U	Γ_L	ΔE_U	ΔE_L	$\Delta \Gamma_s$	ΔE_s
Experiment	1021 (41)	1216 (41)	283 (36)	341 (43)	195 (59)	58 (26)
Strong int.	1021 (41)	1216 (41)	-326 (36)	-272 (43)	195 (59)	51 (26)
Theory $V_{\bar{N}}^{\text{cent}}$	1088	1174	-210	-227	86	-17
$V_{\bar{N}}^{\text{cent}} + \text{Im } V_{\bar{N}}^{\text{LS}}$	1074	1191	-203	-236	117	-33
$V_{\bar{N}}^{\text{cent}} + V_{\bar{N}}^{\text{LS}}$	1070	1196	-205	-234	126	-29

Figure captions

- Fig. 1** Overlaps of atomic and nuclear densities ($r^2 \Psi_N^2 \rho$, pure Coulomb wave function) in the $n = 8$ circular level of ^{174}Yb . Continuous line: angular averaged charge density, profile f^a for ref. [7], proton radius removed; dashed line: spherical charge density of the same r.m.s. [2]; dot-dashed line: single-particle density of protons. O_0 : overlap with $\text{Im } V_N$ of ref. [13]; O_I : the same but with Ψ_N deformed by V_N ; R : the ratio of overlaps in the two split states.
- Fig. 2** Regions of a_{so} allowed by the Yb data. Continuous lines: a_{so}^0 allowed with single-particle nuclear densities, and the theoretical value [11] $a_{so}^1 = 0.013 + i 0.028 \text{ fm}^3$, marked with a cross. The theoretical value [11] $a_{so}^0 = 0.10 + i 0.32 \text{ fm}^3$ is marked with a circle. Other models yield similar results. The trajectory indicates the effects of increasing repulsion in the $^{13}\text{P}_0 \text{ NN} \bar{N}$ state. Dot-dashed lines: the region allowed when extended [2] proton density is used. Dashed lines: the region of allowed a_{so}^1 with theoretical [11] a_{so}^0 . The trajectory starting with the cross indicates increasing repulsion in the $^{33}\text{P}_1$ state. The short arrows indicate nuclear-medium effects.
- Fig. 3** The isospin-0 spin-orbit volumes a_{so}^0 in the threshold region of the $\text{NN} \bar{N}$ model of ref. [11], continued above threshold with a separable approximation of ref. [13].

Fig. 1

Fig. 2

Fig. 3