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OSp(2n/2m) COVARIANT COMPUTATIONS IN THE GREEN-SCHWARZ SUPERSTRING

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ABSTRACT

Using the $Osp(9,1/4)$ suggested by Green and Hull we give the correct reclassification of the infinite ghost sector discussed by one of the authors in the Lorentz covariant quantization of the Green-Schwarz superstring. We then show how to perform $Osp(2n/2m)$ covariant computations in this infinite sector, without regulators, using supergroup techniques introduced some time ago by one of the authors and his collaborators. In particular the conformal anomaly is shown to be a function of the graded dimension and quadratic index of a spinor representation which are natural $OSp(2n/2m)$ invariants. Therefore, like many other supergroup invariants these quantities are equal to the corresponding $SO(2n-2m)$ or $Sp(2m-2n)$ invariants that are functions of $n - m$.

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Recent progress in the covariant quantization of the Green-Schwarz superstring suggests that an infinite set of ghosts, ghosts for ghosts, extra fields, etc. is necessary [1,2,3]. It has been observed [3] that these fields could be described in terms of a spinor (or “metaplectic”, “singleton”) representation of the $\text{OSp}(9,1/4)$ supergroup that was mathematically described previously in [4,5]. However, the fields appearing in the constructive approach of [1] were not clearly identified as corresponding to the states of this representation. This has caused some confusion and indeed has not allowed progress by using supergroup techniques in computations. Instead, it seemed that regulators were needed in order to compute finite quantities in this infinite system. Since one could regulate in various ways and obtain various finite numbers, it seemed that the desired graded dimension (i.e. fermionic degrees of freedom minus bosonic degrees of freedom) and conformal anomaly (i.e. central charge in the Virasoro algebra) of the ghosts fields were not on firm footing.

In the present paper we first give a correct $\text{OSp}(9,1/4)$ reclassification of the original space-time fermions together with the towers of ghosts, ghost for ghosts and extra fields required in the Batalin-Vilkovisky quantization [6] as performed in [1] and then show how to use supergroup techniques to perform covariant calculations.

It was shown in [1] that all of these fields could be labelled as $\Theta_{p,q}$, $\bar{\Theta}_{p,q}$, $\Pi_{p,q}$, $\bar{\Pi}_{p,q}$ where p, q count the number of $z, \bar{z}$ conformal tensor upper or lower indices. In addition, each one of these fields, depending on the values of p, q was assigned to be in either the 16 or $\bar{16}$ spinor representation of $\text{SO}(9,1)$. The Batalin-Vilkovisky triangles that describe all the fields are given in figures 1 and 2 on which the conformal dimensions of the fields are indicated (i.e. $\lambda = p - q$ for lower indices and $\lambda = q - p$ for upper indices). Note that $\Theta_{0,0}$ is the classical 10-dimensional space-time fermion and is assigned to the 16-dimensional spinor representation of $\text{SO}(9,1)$. It was noticed that the action for these fields looked like the standard conformal ghost b-c system $(b(z)\partial_z c(z))$, with conformal weights $-1 - \lambda$ and λ for b, c respectively, for which the conformal anomaly is $c = 6\lambda^2 + 6\lambda + 1$. On this basis an infinite tower of $B - C$ assignments were made, choosing the members for C by taking “branches” from the Θ and the Π triangles in an alternating manner [1]. The conformal anomaly was then computed as an infinite sum which had no obvious convergence properties. Choosing a regulator, the desired values were obtained, but as emphasised above a different answer could be obtained with a different regulator.

In this paper we will abandon the $B - C$ assignments that were made previously and make new assignments that are consistent with representation theory of $\text{OSp}(9,1/4)$. The new assignments will differ from the old ones in renaming some of the b 's as c 's and vice versa. This is possible since, after an integration by parts and using anticommuting properties we can write $b(z)\partial_z c(z) = c(z)\partial_z b(z)$ and also we have the identity $c = 6(-1 - \lambda)^2 + 6(-1 - \lambda) + 1 = 6\lambda^2 + 6\lambda + 1$. We shall then find that the quantities of interest are related to invariants of the supergroup and they are then computed in a supergroup covariant approach that eliminates the need for regulators and gives unambiguous answers.

To obtain an assignment of the fields to a representation of the supergroup, we

begin by observing that on each horizontal line of the Θ -triangle the states fit into an $\text{SU}(2)$ multiplet $[j, m > (\text{ignoring the } 16 \text{ or } \bar{16} \text{ quantum numbers})]$. Thus, we identify $j = (p + q)/2$ and $m = \pm(p - q)/2$, where the $\pm$ signs are taken for lower/upper indices respectively. Furthermore we note that when $2j$ is even the field is a fermion assigned to the 16 of $\text{SO}(9,1)$, while when $2j$ is odd it is a boson assigned to the $\bar{16}$ which has opposite $\text{SO}(9,1)$ chirality. The pattern is similar for the Π -triangle. This is precisely the pattern of states contained in the spinor representation of $\text{OSp}(9,1/4)$ as further explained below on more general grounds. This representation would be unitary for $\text{OSp}(10/4)$ but non-unitary for $\text{OSp}(9,1/4)$ since the spinor of $\text{SO}(9,1)$ is not a unitary representation.

We will assign all of the Θ -triangle to C -fields and all of the Π -triangle to B -fields. While this differs from the previous $B - C$ assignments the patterns of $c = 6\lambda^2 + 6\lambda + 1$ on the triangle are identical to the ones obtained before thanks to the invariance of this expression under the interchange of λ with $-1 - \lambda$ as explained above. The advantage is that now we can identify the conformal dimensions $\lambda = 2m$ as the eigenvalue of the isospin operator $2J_0$ which is a generator of $\text{OSp}(9,1/4)$ and is embedded in $\text{Sp}(4)$ as described below. Thus in order to compute the total conformal anomaly for the entire representation, we simply need to compute the supertrace

$$c = 2 \text{Str}_{\text{spinor}} \left(6(2J_0)^2 + 6(2J_0) + 1 \right), \quad (1)$$

where the supertrace is taken in the spinor representation. The factor of 2 is due to the fact that we have arranged the fields in the triangles in the form of a complex spinor representation. Since J_0 has zero supertrace the linear term drops out. The constant term simply gives the graded dimension d_{spinor} since it is the supertrace of the identity operator, while the quadratic term is proportional to the quadratic index q_{spinor} times a normalization constant. For any representation R we have $\text{Str}_R(J_0^2) = q_R \times (\text{normalization})$, where the normalization of the generator does not depend on the representation. Therefore we may write

$$c = 12 \frac{q_{\text{spinor}}}{q_{\text{vector}}} \text{Str}_{\text{vector}}(2J_0)^2 + 2d_{\text{spinor}}, \quad (2)$$

where the normalization factor is accounted for by evaluating the supertrace in the finite dimensional vector representation. In the vector representation of $\text{OSp}(9,1/4)$ the isospin operator of interest to us is embedded in the fundamental representation of $\text{Sp}(4)$ and is a diagonal operator with eigenvalues $2J_0 = (0, 0, 0, 0, 0, 0, 0, 0, 1, -1, -1, -1)$ as explained below. Thus, it gives the normalization

$$\text{Str}_{\text{vector}}(2J_0)^2 = -4, \quad (3)$$

where the negative number emerges from the definition of the supertrace. It is now evident that the group theoretical knowledge of the quantities d_{spinor} , q_{spinor} , q_{vector} is sufficient to evaluate completely the desired expression. It was already observed in [7,8] that generally certain invariants of $SU(n|m)$ or $OSp(2n/2m)$ are identical to the corresponding invariants of $SU(n-m)$ or $SO(2n-2m) \sim Sp(2m-2n)$ respectively. Thus, the desired numbers are finite and are functions of $n-m$. For the present case we have $OSp(10/4) \sim SO(6)$ so that these quantities are simply equal to

$$d_{spinor}(SO(6)) = 4, \quad \frac{q_{spinor}(SO(6))}{q_{vector}} = \frac{1}{2}. \quad (4)$$

Putting all the information together we have

$$c = 12(1/2)(-4) + 2(4) = -16, \quad (5)$$

which is the correct value of the anomaly for conformal invariance of the theory. We emphasize that we have not used a regulator, but rather we first reduced the computation to invariants of $OSp(9,1/4)$ and then simply used the general theorem of [7-9] concerning the equivalence $OSp(2n/2m) \sim SO(2n-2m) \sim Sp(2m-2n)$ when computing invariants.

In the above discussion we kept the group theory aspects to a minimum in order to emphasize the physics issues. However, for the reader who has little experience with supergroups it is desirable to shed more light on the technical aspects of the computations and also convince him of the absence of a regulator when performing calculations in an $OSp(2n/2m)$ covariant way. It is also useful to provide a general expression for the anomaly for $OSp(2n/2m)$ with arbitrary n, m , which may be useful in future model building when some of the dimensions are compactified. Furthermore, the new system of classical fields and infinite tower of ghosts could provide a new point of view for the super Poincaré group independent of the Green-Schwarz string. Thus, we describe here in a more general setting the spinor representation of $OSp(2n/2m)$ in a convenient basis following the work of [4,7-9] and explain the origin of the similarity $OSp(2n/2m) \sim SO(2n-2m) \sim Sp(2m-2n)$. When applying the ideas above to a more general case involving $OSp(2n/2m)$ we should keep in mind that the operator that assigns the dimensions λ could be different from the one we identified above as $2J_0$; accordingly (1) and (2) may be modified to accommodate some new assignment of operator, but the basic technique of calculation would be the same.

As emphasized above the relevant spinor representation is a unitary representation of $OSp(10/4)$ but is non-unitary for $OSp(9,1/4)$. Since the distinguishing factor is its unitarity for $OSp(10/4)$, we will discuss more generally the unitary spinor representation of $OSp(2n/2m)$ for arbitrary n, m , in which the $SO(2n)$ subgroup is compact and is represented in the spinor representation but the $Sp(2m)$ subgroup is non-compact and requires an

infinite dimensional representation. A general harmonic oscillator technique for building large classes of unitary representations for any non-compact supergroup with a "Jordan structure" was given in [4] where these representations were first constructed explicitly. The decomposition

$$OSp(2n/2m) \supset SU(n/m) \times U(1) \quad (6a)$$

shows that this supergroup has a Jordan structure, with a compact sub-supergroup. It is useful to analyze the representations of $OSp(2n/2m)$ from the point of view of this sub-supergroup. The harmonic oscillator construction of the generators was given in [4]. The representation relevant in the current physical application is the simplest case, involving only one copy of oscillators. The harmonic oscillators are assigned to the fundamental $2n+2m$ dimensional representation as $(\alpha^{\mu\dagger}, \alpha_\mu, a^{\dagger i}, a_i)$, where $\mu = 1, 2, \dots, n$ and $i = 1, 2, \dots, m$, belong to the fundamental representation of $SU(n)$ and $SU(m)$ respectively. Thus, in the case of $OSp(10/4)$ these indices belong to the 5- and 2-dimensional representations of $SU(5)$ and $SU(2)$ respectively. This $SU(2)$ is the one that we discussed above, and the eigenvalues of its third generator $2J_0$ labels the dimensions of the fields as shown in the triangles of figs.1,2. Hence the normalization of the eigenvalues of $2J_0$ in the vector representation occupied by the oscillators is just as given in eq.(3). The α oscillators are fermions while the a oscillators are bosons. It is useful to introduce the super-oscillators $\xi^{A\dagger} = (\alpha^{\mu\dagger}, a^{\dagger i})$ and $\xi_A = (\alpha_\mu, a_i)$ that belong to the fundamental representation of $SU(n/m)$. The generators which close under super-commutations are

$$\begin{aligned} SU(n/m) &\rightarrow \xi^{A\dagger} \xi_B - (\xi^\dagger \cdot \xi) \delta_B^A / (n-m), \\ U(1) &\rightarrow 1/2 - (\xi^\dagger \cdot \xi) / (n-m), \\ coset &\rightarrow \xi^{A\dagger} \xi^{B\dagger}, \quad \xi_A \xi_B. \end{aligned} \quad (6b)$$

Note that the $U(1)$ generator is simply related to the total number operator $N = \xi^{A\dagger} \xi_A$ for all bosonic and fermionic oscillators.

In order to construct a unitary representation in Fock space, following [4] we need to find a lowest multiplet of states $|\Omega\rangle$ that is annihilated by half of the coset generators $\xi_A \xi_B$ and forms an irreducible representation of the subgroup $SU(n/m) \times U(1)$. Then, applying all powers of the remaining coset generators $\xi^{A\dagger} \xi^{B\dagger}$ we generate all the states in the unitary representation. In the present oscillator construction there are only two candidates for $|\Omega\rangle \rightarrow |\Omega\rangle > 0$ or $\xi^{A\dagger} |0\rangle > 0$, where $|0\rangle > 0$ is the Fock vacuum. Following this prescription we see that the states for the two possible representations contain either an even or an odd number of oscillators, and they take the form

$$|even\rangle \sim \xi^{A_1\dagger} \xi^{A_2\dagger} \dots \xi^{A_{2k}\dagger} |0\rangle > 0,$$

$$|odd\rangle \sim \xi^{A_1} \xi^{A_2} \dots \xi^{A_{2k+1}} |0\rangle, \quad (7)$$

with $k = 0, 1, 2, \dots$. The even and the odd states cannot be mixed with each other by applying any generator, and therefore they form irreducible bases. In terms of $SU(n/m)$ representations these correspond to a direct sum of completely super-antisymmetric tensors in an even or odd number of indices respectively and they are represented by single column super tableaux in $2k$ or $2k+1$ boxes respectively. The $U(1)$ quantum numbers of the states are given by $(1/2-2k)/(n-m)$ in the even case and by $(1/2-(2k+1))/(n-m)$ in the odd case. To see the $SU(n) \times SU(m)$ content of these states we rewrite everything in terms of the α, a oscillators. We then see that we have the following decomposition

$$|even\rangle \sim |even, even\rangle + |odd, odd\rangle, \quad |odd\rangle \sim |even, odd\rangle + |odd, even\rangle, \quad (8)$$

where the first entry refers to the α -oscillators and the second to the a -oscillators. We recall that the even powers of the fermionic α -oscillators create the states of one of the spinor representations of $SO(2n)$, while the odd powers create the other spinor representation of $SO(2n)$. Therefore, collecting all the states, we can rearrange them to display the $SO(2n)$ rather than the $SU(n)$ properties

$$\begin{aligned} |even\rangle &\sim |even\ spinor\ even\rangle + |odd\ spinor, odd\rangle, \\ |odd\rangle &\sim |even\ spinor, odd\rangle + |odd\ spinor, even\rangle. \end{aligned} \quad (9)$$

The second entry describes completely symmetric products of bosonic a oscillators and therefore corresponds to completely symmetric $SU(m)$ tensors of rank $even = 0, 2, 4, \dots$ or $odd = 1, 3, 5, \dots$, which are represented by single row Young tableaux. The representation is infinite dimensional since we can take infinite products of bosonic oscillators. Thus, (9) gives the $SO(2n) \times SU(m)$ content of the representation and corresponds to a rearrangement of the states described in (8) or (7). Note that the even/odd spinors have opposite $SO(2n)$ chirality and also opposite bose/fermi properties that are inherited from the fermionic property of the α -oscillators.

Comparing this structure of states to the triangles in figs.1,2 for the case $OSP(10/4)$, we see that we reproduce the $SO(10)$ spinor representations $16, \bar{16}$ for fields that have opposite bose/fermi properties and that they come in $SU(2)$ multiplets $|jm\rangle$, in which $2j$ is identified with the number of a -oscillators that create the appropriate state (recall that for $SU(2)$ the α -oscillators carry spin $1/2$). Now, using the equivalence of the set of states in (8,9) and (7) it is more efficient to do computations using the supergroup classification of (7).

In many computations one is interested in invariants of $OSP(2n/2m)$. All invariants in a representation R can be computed from the super-character $\chi_R(U) = Str_R(U)$ of an arbitrary group element $U = exp(H)$, where H is a supertraceless arbitrary matrix in the algebra of $OSP(2n/2m)$. Thus, if H is a $(2m+2n) \times (2n+2m)$ matrix in the fundamental vector representation $OSP(2n/2m)$, then expanding the character in powers of H defines coefficients that are simply related to all the desired invariants

$$Str_R(e^H) = d_R + 0 + q_R Str(H^2)/(2!) + c_R Str(H^3)/(3!) + \dots, \quad (10)$$

where on the left hand side the supertrace is taken in the R -representation, while on the right it is in the vector representation. Then $d_R = Str_R(1)$ is the graded dimension consisting of the number of bosons minus the number of fermions in the representation, q_R is the quadratic index, c_R is the cubic index, etc. We can always use $OSP(2n/2m)$ rotations in order to diagonalize H partially or completely. Therefore, we can take a block diagonal form for H corresponding to the subgroup $SU(n/m) \times U(1)$

$$U = \begin{pmatrix} u & 0 \\ 0 & (u^T)^{-1} \end{pmatrix}, \quad H = \begin{pmatrix} h & 0 \\ 0 & -h^T \end{pmatrix}, \quad (11)$$

where h is a graded $U(n/m)$ matrix applied on the basis of $n+m$ oscillators (α_μ, a_i) . Then from the decomposition described by (7) we can write the character in the spinor representations of $OSP(2n/2m)$ as

$$Str_{even}(U) = \sum_k Str_{2k}(u), \quad Str_{odd}(U) = \sum_k Str_{2k+1}(u), \quad (12)$$

where on the right hand side we need to sum over the $U(n/m)$ characters in the completely super-antisymmetric representations described by single column super tableaux with $2k$ or $2k+1$ boxes. Fortunately we know a closed expression for these characters [7,8]

$$Str_p(u) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-ip\phi} Sdet(1 + e^{i\phi} u), \quad (13)$$

where the superdeterminant is evaluated in the $(n+m) \times (n+m)$ dimensional fundamental representation of $U(n/m)$. As explained in [7,8] the form of this expression is identical to the character of ordinary $U(N)$ in the completely antisymmetric representation described by a single column Young tableau with p boxes, except for the fact that a superdeterminant and a supermatrix u replace the ordinary determinant and ordinary matrix. Plugging

this form into (12) we have a closed form for characters of $OSp(2n/2m)$ in the spinor representations

$$Str_{even}(U) = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{Sdet(1 + e^{i\phi}u)}{1 - e^{-i2\phi}}, \quad Str_{odd}(U) = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{e^{-i\phi} Sdet(1 + e^{i\phi}u)}{1 - e^{-i2\phi}}. \quad (14)$$

The expansion of these expressions in powers of $Str(h^n) = \frac{1}{2} Str(H^n)$ gives the desired invariants as the coefficients of expansion.

$$d_{even} = \int \frac{d\phi}{2\pi} \frac{(1 + e^{i\phi})^{n-m}}{1 - e^{-i2\phi}} = 2^{n-m-1}, \quad q_{even} = \frac{1}{2} \int \frac{d\phi}{2\pi} \frac{e^{i\phi} (1 + e^{i\phi})^{n-m}}{1 - e^{-i2\phi}} = 2^{n-m-1}, \quad (15)$$

and the same numbers for the *odd* representation. The origin of $n - m$ is the superdeterminant evaluated at $u = 1$. Note that we have normalized the coefficients so that $q_{vector} = 1$.

We see that the result in (15) is in agreement with the $OSp(2n/2m) \sim SO(2n - 2m)$ computation given above for $OSp(10/4) \sim SO(6)$. This relationship can be explained generally as follows. Consider the expansion of the $SU(n/m)$ character in (13) in powers of h . Since the structure of the expression is identical to $SU(N)$, we expect that the algebraic coefficients in the expansion are the same as $SU(N)$ except for the replacement of superdeterminant by determinant and supertrace by trace. Then, at $h = 0$ we have $Sdet(1 + e^{i\phi}) = (1 + e^{i\phi})^{n-m}$, which is the same expression as if we are computing with $SU(N)$, where $N = n - m$. This expression multiplies the integrand for all coefficients $d_p, q_p, c_p, \dots$ and determines the $N = n - m$ dependence of all these coefficients. Thus, for $p \leq N$, the coefficients d_p, q_p are the binomial coefficients

$$d_p = \binom{N}{p}, \quad q_p = \binom{N-2}{p-1}, \quad etc. \quad (16)$$

Pursuing this idea we naturally arrive at the conclusion that $d_p = 0, q_p = 0$ for $p > (n - m)$, since for $SU(N)$ we cannot antisymmetrize in more than N indices. Thus when the character in the form (12) is expanded in a $SU(n/m)$ covariant fashion in powers of h , then the d, p coefficients can get contributions only from the terms up to $2k$ or $(2k+1) \leq (n - m)$, eliminating the need for regulators. That is,

$$d_{even} = \sum_k d_{2k}, \quad q_{even} = \sum_k q_{2k}, \quad (17)$$

and similarly for the *odd* representation. But this is precisely what we get if we write the character of the spinor representation of $SO(2N)$ in a $SU(N)$ basis as in (12), for which $2k$ or $(2k+1)$ are limited to go up to N anyway for the general character. Thus, we see the origin of the equivalence $OSp(2n/2m) \sim SO(2n - 2m)$ in the spinor representations since they produce the same sums for the invariant $d, q, c, \dots$ coefficients up to powers h^{n-m} (although the general character is different). Actually this equivalence of invariants up to order $N = n - m$ applies to all representations, not just the spinor, as it follows from the general equivalence $SU(n/m) \sim SU(n - m)$ for all Young tableaux representations as demonstrated in [8]. By analysing $SO(2n/2m)$ in a $SU(n/m)$ basis and comparing the Clebsch-Gordan series it is easy to establish that the general equivalence holds for all Casimir invariants of $SO(2n-2m)$ up to Casimirs of order $N=n-m$. If we happen to have $n < m$, then the equivalence is $OSp(2n/2m) \sim Sp(2m - 2n)$ which can be seen by similar arguments. Using these equivalences can be extremely useful in computations as demonstrated above in the example of $OSp(9,1/4)$ which is of physical relevance.

We have seen the utility of using $OSp(2n/2m)$ covariant computations in the sector involving the fields described in the triangles of Figs.1,2. We may now return to the question of whether this symmetry is shared by the remainder of the Lagrangian and whether it might be useful to carry out symmetric computations with the entire Lagrangian. This appears to be gauge dependent. The conformal gauge does not seem to have an overall $OSp(9,1/4)$ symmetry, but the harmonic gauge does. It is not clear that the structure of all the fields in various gauges has been settled since the question of a nilpotent BRST operator is not yet completely understood [10]. It may be that an $OSp(9,1/4)$ covariant approach could shed light on these problems.

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FIGURE CAPTIONS

Fig.1 - Diagram for $\Theta_{p,q}, \bar{\Theta}^{p,q}$.

Fig.2 - Diagram for $\Pi_{p,q}, \bar{\Pi}^{p,q}$.

- Figure 1 -

- Figure 2 -