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Is the heterotic superconformal three-generation model phenomenologically viable?

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Abstract

We study the phenomenology of the heterotic superconformal three-generation model and the corresponding Calabi-Yau compactification on the Schimmrigk space. We point out a potential problem with proton lifetime and indicate a possible solution.

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The Calabi-Yau (C-Y) approach [1] to the compactification of the $E_8 \times E_8$ heterotic superstring [2] has many positive features. In the field theory limit it gives rise to four-dimensional gauge models with $N = 1$ supersymmetry and chiral fermions belonging to 27 or $\overline{27}$ representations of the gauge group E_6 . For multiply-connected C-Y spaces the Wilson loop mechanism (flux breaking) offers the possibility of further breaking E_6 down to a proper subgroup at the compactification scale M_c . The net number of chiral fermion generations is a topological invariant of the C-Y manifold equal to one half the absolute value of its Euler characteristic. The global discrete symmetries of the four-dimensional model are also not arbitrary but are manifestations of the symmetries of the internal space. Extensive studies of the only known three-generation compactification of this type [3,4,5,6] has led to a phenomenology which could be characterized as surprisingly realistic [7].

Present day C-Y model building, however, although already very constrained, still has some degree of arbitrariness. The 27^3 Yukawa couplings are often computable exactly [8] but the correct normalizations of the fields involved are not. Also the number and properties of the E_6 singlets are not known in many cases, including the three-generation compactification. Finally, supersymmetry breaking is introduced by hand. All the above are important ingredients without which detailed low energy predictions are hard to obtain.

Recently Gepner [9] constructed compactified heterotic string theories by tensoring soluble $N = 2$ superconformal theories, the field theory limit of some of which seems to correspond to the field theory limit of certain C-Y compactifications. In particular, there is a three-generation superconformal construction [10] which corresponds to a fixed radius compactification on a particular realization of the three-generation C-Y manifold, namely the realization due to Schimmrigk [5].

Very recently the three-generation superconformal model has been studied in considerable detail [11]. Thus the values of all normalized three-point couplings and the content and transformation properties of the gauge singlet sector became available. The properties of the gauge singlet sector and the values of the normalized 27^3 couplings remain valid, due to non-renormalization theorems [12], in the case of the (maximally symmetric) Schimmrigk space for arbitrary compactification radius.

The above developments are certainly of great practical importance since they provide information which is usually very hard to obtain directly from the geometric C-Y construction. Given this information it certainly makes sense to ask the question whether the three-generation superconformal model or, more generally, the C-Y compactification on the Schimmrigk space are phenomenologically viable. This will be the subject of the present note.

The Schimmrigk construction of the three-generation C-Y space is defined as the complete intersection K_0 of two polynomial constraints in $CP^3 \times CP^2$, of bidegree $(3,0)$ and $(1,3)$ respectively, divided out by a non-freely-acting order 9 discrete symmetry group with the resulting fixed points resolved. Correspondence with the three-generation superconformal model requires the choice of $P_1 = \sum_{i=0}^3 z_i^3 = 0$ and $P_2 = \sum_{i=1}^3 z_i x_i^3 = 0$ for the polynomial constraints, where (z_0, z_1, z_2, z_3) and (x_1, x_2, x_3) are the homogeneous coordinates of CP^3 and CP^2 respectively. The 9 element group is the $Z_3 \times Z_3$ symmetry $G \times H$ generated by

$$g: (z_0, z_1, z_2, z_3, x_1, x_2, x_3) \rightarrow (z_0, z_2, z_3, z_1, x_2, x_3, x_1)$$

and

$$h: (z_0, z_1, z_2, z_3, x_1, x_2, x_3) \rightarrow (z_0, z_1, z_2, z_3, \alpha x_2, \alpha^2 x_3)$$

where $\alpha = \exp(2\pi i/3)$. The simply connected space K_0 has 27 generations and a discrete symmetry group which is the semi-direct product of P_3 (the permutation

embedding, which we will adopt in the following, employs the homomorphism $g \rightarrow U_g \equiv (I_3, \alpha I_3, \alpha I_3) \in SU(3)_c \times SU(3)_L \times SU(3)_R$ and the unbroken gauge group is $SU(3)_c \times SU(3)_L \times SU(3)_R$. Under it the left-handed lepton (λ), quark (Q) and antiquark (Q^c) fields in a 27 of E_6 transform as follows: $\lambda = (1, \bar{3}, 3) = \begin{pmatrix} H^1 & H^2 & l \\ e^c & \nu^c & N \end{pmatrix}$, $Q = (3, 3, 1) = \begin{pmatrix} q \\ g \end{pmatrix}$, $Q^c = (\bar{3}, 1, \bar{3}) = (u^c, d^c, g^c)$. There are 9 λ lepton fields, 7 Q (Q^c) quark (antiquark) fields, 6 $\bar{\lambda}$ mirror lepton fields and 4 $\bar{Q}$ ($\bar{Q}^c$) mirror quark (antiquark) fields which are the appropriate components of linear combinations of the fields C and $\bar{C}$. To find them we express the C 's ($\bar{C}$'s) as linear combinations of new variables λ , Q , Q^c ($\bar{\lambda}$, $\bar{Q}$, $\bar{Q}^c$) which are the 27's ($\bar{27}$'s) from which the lepton, quark, antiquark (mirror lepton, mirror quark, mirror antiquark) components respectively survive the division by G as massless fields. For the C 's we have $3^{1/2}C_{i1} = \lambda_i + Q_i + Q_i^c$, $3^{1/2}C_{i2} = \lambda_i + \alpha Q_i + \alpha^2 Q_i^c$, $3^{1/2}C_{i3} = \lambda_i + \alpha^2 Q_i + \alpha Q_i^c$ ($i = 1, \dots, 7$) and $C_8 = \lambda_8$, $C_9 = \lambda_9$. For the $\bar{C}$'s we have $3^{1/2}\bar{C}_{i1} = \bar{\lambda}_i + \bar{Q}_i + \bar{Q}_i^c$, $3^{1/2}\bar{C}_{i2} = \bar{\lambda}_i + \alpha\bar{Q}_i + \alpha^2\bar{Q}_i^c$, $3^{1/2}\bar{C}_{i3} = \bar{\lambda}_i + \alpha^2\bar{Q}_i + \alpha\bar{Q}_i^c$, $3^{1/2}\bar{C}_{21} = \bar{\lambda}_2 + \bar{Q}_2 + \bar{Q}_2^c$, $3^{1/2}\bar{C}_{22} = \bar{\lambda}_2 + \alpha\bar{Q}_2 + \alpha^2\bar{Q}_2^c$, $3^{1/2}\bar{C}_{23} = \bar{\lambda}_2 + \alpha^2\bar{Q}_2 + \alpha\bar{Q}_2^c$, $\bar{C}_3 = \bar{\lambda}_3$, $\bar{C}_4 = \bar{\lambda}_1$, $\bar{C}_5 = \bar{\lambda}_8$, $\bar{C}_6 = \bar{\lambda}_9$, $\bar{C}_7 = \bar{Q}_3$, $\bar{C}_8 = \bar{Q}_4$, $\bar{C}_9 = \bar{Q}_5$, $\bar{C}_{10} = \bar{Q}_4^c$.

The honest discrete symmetries of the 3-generation manifold K/G , for the 'standard' embedding, are the Z_2 permutation symmetry generated by p and the Z_3 R-symmetry generated by $s_1 s_2 s_3$. The scaling symmetries generated separately by s_1 , s_2 and s_3 survive only as pseudo R-symmetries. To take into account the restrictions imposed on the superpotential by the pseudosymmetries it is convenient to find the allowed couplings on the 9-generation covering space K , express the C 's ($\bar{C}$'s) as linear combinations of the new variables λ , Q , Q^c ($\bar{\lambda}$, $\bar{Q}$, $\bar{Q}^c$) and keep only the terms allowed by the gauge symmetry $SU(3)^3$. We also remind the reader of the restrictions imposed by an R-symmetry on the superpotential [13]. Assume that under the R-symmetry $\omega \rightarrow \gamma^{-1}\omega$. Also let $27 \rightarrow \beta 27$ and $\bar{27} \rightarrow \beta \bar{27}$ de-

symmetry of 3 objects) generated by g and

$$p: (z_1, z_2, x_1, x_2) \rightarrow (z_2, z_1, x_2, x_1),$$

with the $Z_3 \times Z_9$ generated by

$$s_0: z_0 \rightarrow \alpha z_0,$$

$$s_i: (z_i, x_i) \rightarrow (\alpha z_i, \beta^3 x_i) \quad (i = 1, 2, 3),$$

where $\beta = \exp(2\pi i/9)$, divided by the Z_9 generated by $s_0 s_1 s_2 s_3$. Notice that the symmetries generated by s_0 , s_1 , s_2 and s_3 are R-symmetries (they do not commute with supersymmetry) and under them the unique (3,0) harmonic form ω of the C-Y space gets multiplied by α , β^2 , β^2 and β^2 respectively. Division of K_0 by the non-freely-acting Z_3 symmetry H generated by $h = s_2^6 s_3^3$ (and subsequent resolution of the fixed point singularities) leads to a 9-generation multiply-connected space K . K has 23 (2,1) harmonic forms (27's of E_6) which we will call C_{ij} ($i = 1, \dots, 7$ and $j = 1, 2, 3$), C_8 , C_9 and 14 (1,1) harmonic forms ($\bar{27}$'s of E_6) which we will call $\bar{C}_{11}$, $\bar{C}_{12}$, $\bar{C}_{13}$, $\bar{C}_{21}$, $\bar{C}_{22}$, $\bar{C}_{23}$, $\bar{C}_3$, $\bar{C}_4$, $\bar{C}_5$, $\bar{C}_6$, $\bar{C}_7$, $\bar{C}_8$, $\bar{C}_9$, $\bar{C}_{10}$. The transformation properties of the corresponding moduli under the independent generators g , p and s_1 are given in Table 1. The action of s_0 , s_2 and s_3 can be obtained through the relations $s_0 = (s_1 s_2 s_3)^2$, $s_2 = g^2 s_1 g$ and $s_3 = g s_1 g^2$. We also list in Table 2 the transformation properties of the 24 untwisted E_6 singlets which could have couplings of the form $1 \ 27 \ \bar{27}$. Finally, the three-generation manifold is obtained by modding out the 9-generation space K by the freely-acting Z_3 symmetry G generated by g .

Flux breaking of the gauge symmetry E_6 and the massless spectrum depend on the choice of an embedding of G into E_6 defined by a homomorphism $g \rightarrow U_g$. The unbroken gauge group is the subgroup of E_6 commuting with the image of G under U_g . The possible proper subgroups are $SU(3)^3$ and $SU(6) \times U(1)$. The 'standard'

We first treat the leptons. The mass matrix of the $SU(2)_L$ and $SU(2)_R$ doublet leptons has rows (columns) labelled by the 9 H^2 's and the 6 $\bar{H}^1$'s (the 9 H^1 's and the 6 $\bar{H}^2$'s). The combinations $2^{-1/2}(H_3^2 \pm H_4^2)$ and H_5^2 pair up mainly with $2^{-1/2}(H_3^1 \pm H_4^1)$ and H_5^1 to form 3 states with mass $\sim N_8 > 10^{14}$ GeV. H_6^2 (H_7^2) pairs up mainly with H_1^1 (H_6^1) into a state of mass $\sim N_2 > 10^{14}$ GeV. $\bar{H}_2^1$ and $\bar{H}_7^2$ combine to form a state with mass $\sim f' < \bar{N}_2 > 10^6$ GeV. H_2^2 (H_8^1) pairs up with one combination out of $(\bar{H}_3^2, \bar{H}_6^2, \bar{H}_7^2)$ ($(\bar{H}_3^1, \bar{H}_6^1, \bar{H}_7^1)$) to form a state with mass $\sim S_2 > 10^{16}$ GeV. H_2^2 (H_7^1) combines with another combination out of $(\bar{H}_3^2, \bar{H}_6^2, \bar{H}_7^2)$ ($(\bar{H}_3^1, \bar{H}_6^1, \bar{H}_7^1)$) to form a state with mass $\sim S_2 > 10^{14}$ GeV ($M_e \sim 10^{16}$ GeV is the compactification scale) while H_1^2 combines with H_1^1 to form a state with mass $\sim f'^2 < S_{10} > 10^2$ GeV. $[f'$ and f'' above denote the coupling constants of the terms $(1 + g + g^2)\bar{C}_{21}^3$ and $(1 + g + g^2)C_{13}\bar{C}_{21}S_{10}$ respectively. They both vanish in the superconformal model and therefore could be tiny in the Schimmiigk space.] The combination out of $(\bar{H}_3^2, \bar{H}_6^2, \bar{H}_7^2)$ ($(\bar{H}_3^1, \bar{H}_6^1, \bar{H}_7^1)$) orthogonal to the previously mentioned ones pairs up with $\bar{H}_1^1$ ($\bar{H}_7^2$) to form a state with mass $\sim \bar{N}_2 > 10^{14}$ GeV, while $\bar{H}_4^1$ and $\bar{H}_4^2$ form a state with mass $\sim N_4' > 10^2$ GeV. Finally H_8^1 and H_8^2 form a state with mass $\sim N_4' > 10^2$ GeV. The pair H_8^1, H_8^2 (actually their supersymmetric partners) will play the role of the (naturally light) electroweak Higgs doublets.

The mass matrix of the $SU(2)_L$ doublet, $SU(2)_R$ singlet leptons is a 6×9 matrix whose rows are labelled by the 6 $\bar{H}^1$'s and its columns by the 9 H^1 's. l_6 and one combination out of $\{l_6, l_7\}$ pair up with 2 combinations out of $\{\bar{l}_3, \bar{l}_5, \bar{l}_6\}$ to form 2 states with mass $\sim S_2 > 10^{16}$ GeV and $< S_4 > 10^{12}$ GeV respectively. $\bar{l}_1$ pairs up with one combination out of $\{l_3, l_6\}$, where $l_3^2 = 2^{-1/2}(l_3 + l_4)$, to form a state with mass $\sim S_2 > 10^{14}$ GeV. The orthogonal combination pairs up with the third combination out of $\{\bar{l}_3, \bar{l}_5, \bar{l}_6\}$ to form a state with mass $\sim S_{10} > 10^{10}$

GeV. Also $\bar{l}_2$ pairs up with l_1 into a state with mass $\sim f'' < S_{10} > 10^4$ GeV. Finally $\bar{l}_4$ pairs up with the leftover combination out of $\{l_6, l_7\}$ into a state with mass $\sim < S_4 > 10^{12}$ GeV. To a good approximation the light leptons are $l_4^2 = 2^{-1/2}(l_3 - l_4)$, l_2 and l_8 . The above analysis assumes tiny ($\sim 10^{-10}$) couplings for some terms, including terms of the form $\bar{l}_i l_j (S_2)^2 / M_e$ ($i = 3, 5, 6$), in order to obtain tiny mixing, necessary for the stability of the proton, between the 'heavy' states and the mainly 'light' l_2 .

The mass matrix of the $SU(2)_L$ singlet, $SU(2)_R$ doublet leptons (ν^c, e^c) is identical to the one just discussed.

We now turn to the quark sectors. The $SU(2)_L$ and $SU(2)_R$ singlet quark mass matrix has rows labelled by the 7 g 's and the 4 $\bar{g}$'s. g_3, g_1 and $\bar{g}_1^c$ form with g_5^c, g_4^c and $\bar{g}_1$ 3 states with mass $\sim N_8 > 10^{14}$ GeV. g_6 (g_8^c) and one combination out of $(\bar{g}_3, \bar{g}_4)$ ($(\bar{g}_5, \bar{g}_1^c)$) form a state with mass $\sim S_2 > 10^{16}$ GeV. g_2 (g_2^c) and the combination out of $(\bar{g}_3, \bar{g}_4)$ ($(\bar{g}_5, \bar{g}_1^c)$) orthogonal to the one previously mentioned form a state with mass $\sim S_2 > 10^{14}$ GeV. $\bar{g}_2^c$ and $\bar{g}_2$ combine into a state with mass $\sim f' < \bar{N}_2 > 10^6$ GeV. Finally g_1 and g_1^c form a state with mass $\sim f'^2 < S_{10} > 10^2$ GeV.

In the $SU(2)_L$ doublet, $SU(2)_R$ singlet sector one combination out of $(\bar{q}_3, \bar{q}_4)$ and q_5 forms a state with mass $\sim S_2 > 10^{16}$ GeV, while the orthogonal combination and q_2 form a state with mass $\sim S_2 > 10^{14}$ GeV. Also $\bar{q}_1$ and $q_3^c = 2^{-1/2}(q_3 + \alpha^2 q_1)$ form a state again with mass $\sim S_2 > 10^{14}$ GeV. Finally $\bar{q}_2$ and q_1 form a state with mass $\sim f'' < S_{10} > 10^4$ GeV. We assume tiny ($\sim 10^{-8}$) couplings for terms of the form $q_i \bar{q}_j S_4$ ($i = 6, 7$ and $j = 3, 4$) in order to suppress the mixing between the 'heavy' q_2 and the light q_6, q_7 . With this assumption the ordinary 'light' quarks are, to a very good approximation, q_6, q_7 and $q_4^c = 2^{-1/2}(\alpha q_3 - q_1)$.

The mass matrix of the $SU(2)_L$ singlet, $SU(2)_R$ doublet antiquarks (u^c, d^c) is identical to the one just discussed (with $q_3^c = 2^{-1/2}(q_3^g + \alpha q_4^c)$ and $q_4^c = 2^{-1/2}(\alpha^2 q_3^g - q_4^c)$).

To summarize, the light leptons are contained in λ'_1, λ_2 and λ_8 . The light quarks (antiquarks) are contained in Q'_4, Q_6 and Q_7 (Q_1^c, Q_6^g and Q_7^g). The candidate electroweak Higgs doublets are H_8^1 and H_8^2 .

With the above light spectrum we can readily check that there exist Yukawa couplings leading to tree-level masses for at least the third generation of quarks and leptons. For the third-generation quark masses we use the term $1/2 f_4 \lambda_8 Q'_4 Q_4^c$, while for the third-generation lepton masses we use the term $-1/2 f_4 \lambda_8 \lambda_4'^2$. Therefore the t and b quarks (antiquarks) are contained in Q'_4 (Q_4^c) and the τ lepton in λ'_4 . Notice that the electroweak Higgs doublets giving masses to quarks and leptons are the same (and are contained in λ_8) and the coupling constant f_4 is common. Therefore the tree-level Dirac τ neutrino mass is equal to the t mass. This necessitates some sort of a see-saw mechanism.

Introduction of the non-zero $|\langle \nu_6^g \rangle| = |\langle \bar{\nu}_4^c \rangle|$ leads to a large mass term $\sim \langle S_2 \rangle < \bar{\nu}_4^c > / M_c$ for ν_4^c (τ antineutrino) through the superpotential term $\lambda'_4 S_7 \bar{\lambda}_4 S_2 / M_c$ (where $S_7 \equiv 3^{-1/2}(S_1^2 + S_2^2 + S_7^2)$). This provides the necessary see-saw mechanism. The dominant structure of the spectrum outlined earlier is not affected. The only other effect of $\langle \nu_6^g \rangle \neq 0$ is to introduce an order 10^{-4} mixing between the third-generation lepton ℓ_4 and H_4^2 and between the third-generation antiquark d_4^c and q_4^c .

Finally we arrive at the discussion of proton decay. The 'most dangerous' proton decay processes involve the exchange of a g (g') quark (antiquark) between vertices corresponding to superpotential couplings qqq ($u^c d^c g$) and $gu^c e^c$ ($g' q \ell$) (all states except g and g' are assumed to be 'light'). We first consider the (hypothetical) case in which the light leptons and quarks (antiquarks) are entirely

contained in $\lambda'_1, \lambda_2, \lambda_8$ and Q'_4, Q_6, Q_7 (Q_1^c, Q_6^g, Q_7^g) respectively and the mixing between 'heavy' and 'light' states vanishes. In this zero mixing limit it is readily seen (from the structure of the allowed 27^3 couplings) that none of the 'most dangerous' processes occurs, because in at least one of the vertices there are more than one 'heavy' state involved. With tiny $\sim 10^{-10}$ contributions of mainly 'heavy' fields to the 'light' combinations and given the $g - g'$ mass spectrum discussed earlier, one can check that proton decay at tree-level through dimension-five operators is adequately suppressed. The same is true for processes occurring through loop diagrams. It is important for these processes that, whenever all external lines correspond to states which are mainly light, there is always involved in the loop a state with mass $\sim S_2 > 10^{16}$ GeV and that there is always a suppression from the mixing angle between third- and first- (or second-) generation antiquarks which could be very small. It thus turns out that proton decay could be adequately suppressed, if mixings are small enough, in spite of the fact that all 27^3 couplings are of order unity.

We conclude by summarizing our results. We studied the phenomenology of the realization of Yau's three-generation manifold due to Schimmrigk in the light of recent computations of the properly normalized Yukawa couplings in the corresponding superconformal Gepner model. We pointed out a potential problem with proton lifetime and constructed a scenario in which, in spite of the large value of all 27^3 baryon-number-violating couplings, adequate suppression is possible if some other couplings (depending on the compactification radius) are small enough. This is certainly a feature which has never been displayed before by any other model of this type. An essential role in this scenario is played by the good knowledge of the gauge singlet sector which is usually hard to obtain in other realizations of Yau's three-generation manifold.

	g	p	s_1		g	p	s_1
C_{11}	C_{12}	C_{12}	β^0	$\bar{C}_{11}$	$\bar{C}_{13}$	$\bar{C}_{11}$	β^0
C_{12}	C_{13}	C_{11}	β^6	$\bar{C}_{12}$	$\bar{C}_{11}$	$\bar{C}_{13}$	β^6
C_{13}	C_{11}	C_{13}	β^3	$\bar{C}_{13}$	$\bar{C}_{12}$	$\bar{C}_{11}$	β^3
C_{21}	C_{22}	C_{21}	β^3	$\bar{C}_{21}$	$\bar{C}_{22}$	$\bar{C}_{21}$	β^3
C_{22}	C_{23}	C_{23}	β^3	$\bar{C}_{22}$	$\bar{C}_{23}$	$\bar{C}_{22}$	β^3
C_{23}	C_{21}	C_{22}	β^6	$\bar{C}_{23}$	$\bar{C}_{21}$	$\bar{C}_{23}$	β^6
C_{31}	C_{32}	C_{33}	β^3	$\bar{C}_{31}$	$\bar{C}_{33}$	$\bar{C}_{32}$	β^3
C_{32}	C_{33}	C_{31}	β^3	$\bar{C}_{32}$	$\bar{C}_{31}$	$\bar{C}_{33}$	β^3
C_{33}	C_{31}	C_{32}	β^3	$\bar{C}_{33}$	$\bar{C}_{32}$	$\bar{C}_{31}$	β^3
C_{41}	C_{42}	C_{43}	β^6	$\bar{C}_{41}$	$\bar{C}_{43}$	$\bar{C}_{42}$	β^6
C_{42}	C_{43}	C_{41}	β^3	$\bar{C}_{42}$	$\bar{C}_{41}$	$\bar{C}_{43}$	β^3
C_{43}	C_{41}	C_{42}	β^3	$\bar{C}_{43}$	$\bar{C}_{42}$	$\bar{C}_{41}$	β^3
C_{51}	C_{52}	C_{53}	β^6	$\bar{C}_{51}$	$\bar{C}_{53}$	$\bar{C}_{52}$	β^6
C_{52}	C_{53}	C_{51}	β^6	$\bar{C}_{52}$	$\bar{C}_{51}$	$\bar{C}_{53}$	β^6
C_{53}	C_{51}	C_{52}	β^7	$\bar{C}_{53}$	$\bar{C}_{52}$	$\bar{C}_{51}$	β^7
C_{61}	C_{62}	C_{63}	β^3	$\bar{C}_{61}$	$\bar{C}_{63}$	$\bar{C}_{62}$	β^3
C_{62}	C_{63}	C_{61}	β^3	$\bar{C}_{62}$	$\bar{C}_{61}$	$\bar{C}_{63}$	β^3
C_{63}	C_{61}	C_{62}	β^7	$\bar{C}_{63}$	$\bar{C}_{62}$	$\bar{C}_{61}$	β^7
C_{71}	C_{72}	C_{73}	β^3	$\bar{C}_{71}$	$\bar{C}_{73}$	$\bar{C}_{72}$	β^3
C_{72}	C_{73}	C_{71}	β^3	$\bar{C}_{72}$	$\bar{C}_{71}$	$\bar{C}_{73}$	β^3
C_{73}	C_{71}	C_{72}	β^3	$\bar{C}_{73}$	$\bar{C}_{72}$	$\bar{C}_{71}$	β^3
C_8	C_8	C_8	β^3	$\bar{C}_8$	$\bar{C}_8$	$\bar{C}_8$	β^3
C_9	C_9	C_9	β^3	$\bar{C}_9$	$\bar{C}_9$	$\bar{C}_9$	β^3

Table 1: Transformation properties, for the 9-generation covering space K , of the ‘massless’ E_6 non-singlet superfields under the symmetries generated by g , p , and s_1 .

	g	p	s_1
S_1	S_1^7	S_1^7	β^7
S_2	S_1^7	S_1^7	β
S_3	S_1^7	S_1^7	β
S_4	S_2^7	S_2^7	β^6
S_5	S_2^7	S_2^7	β^6
S_6	S_2^7	S_2^7	β^6
S_7	S_3^7	S_3^7	β^2
S_8	S_3^7	S_3^7	β^2
S_9	S_3^7	S_3^7	β^2
S_{10}	S_4^7	S_4^7	β^6
S_{11}	S_4^7	S_4^7	β^6
S_{12}	S_4^7	S_4^7	β^6
S_{13}	S_5^7	S_5^7	β^3
S_{14}	S_5^7	S_5^7	β^3
S_{15}	S_5^7	S_5^7	β^3
S_{16}	S_6^7	S_6^7	β^4
S_{17}	S_6^7	S_6^7	β^4
S_{18}	S_6^7	S_6^7	β^4
S_{19}	S_7^7	S_7^7	β^2
S_{20}	S_7^7	S_7^7	β^2
S_{21}	S_7^7	S_7^7	β^2
S_{22}	S_8^7	S_8^7	β^2
S_{23}	S_8^7	S_8^7	β^2
S_{24}	S_9^7	S_9^7	β^2
S_{25}	S_9^7	S_9^7	β^2
S_{26}	S_{10}^7	S_{10}^7	β^4

Table 2: Transformation properties, for the 9-generation covering space K , of the ‘massless’ untwisted E_6 singlet superfields under the symmetries generated by g , p , and s_1 .

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