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CLAN STRUCTURE AS A SOURCE OF INTERMITTENCY
IN HIGH ENERGY MULTIHADRON PRODUCTION

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ABSTRACT

Recently the systematic study of charged hadron multiplicity distributions in high energy hadronic and leptonic collisions has revealed the general occurrence of negative binomial regularities in symmetric c.m. rapidity windows and of intermittency in very small windows in the central region. The negative binomial regularities imply a particular form of clustering called clan structure, which embodies all Mueller multiparticle correlations. We show that this clan structure leads to intermittency when the simplest assumptions are made on the behaviour of the clans in very small windows.

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$$f_D(u) = (1-\mu)^{-k_D}, \quad F_D(u) = \exp[k_D r_D^{-1} u \ln(1+\mu)], \quad (3)$$

$$g_D(u) = 1 - r_D [\ln(1-\mu) / \ln(1+\mu)], \quad m = \bar{n}_D / k_D$$

r_D is an "empty-clan" parameter, with the meaning that $1-r_D$ is the probability that a clan contains no particle in D. It cannot be deduced from the data which only determine $\bar{n}_D$ and k_D . Hence one usually chooses $r_D=1$.

We now consider a small window d in D. The simplest assumption for the MDs in d is to take as g.f.

$$F_d(u) = F_D(Au), \quad g_d(u) = g_D(au) \quad (4)$$

for the number of clans and the number of particles per clan, respectively. It corresponds to binomially distributed splittings of the multiplicities pertaining to D, with probabilities A and a respectively to pertain to d. This can easily be seen from Eq.(1) by writing

$$(1+Au)^n = [A(1+u) + (1-A)]^n$$

and expanding in powers of (1+u), and similarly for a. The resulting MD of particles in d is

$$f_d(u) = F_d[g_d(u)-1] = F_D[A(g_D(au)-1)]. \quad (5)$$

For $d \rightarrow 0$ we expect that both A and a $\rightarrow 0$. As we now show, this implies indefinitely growing normalized factorial moments, i.e., intermittency.

3. The factorial moments are given by the successive derivatives of the g.f. with respect to u at $u=0$. From (5) one immediately finds (we drop the subscript D in the derivatives of F_D and g_D)

$$f_d' = AaF'g', \quad f_d'' = Aa^2(F'g'' + AF''g'^2),$$

$$f_d''' = Aa^3(F'g''' + 3AF''g'g'' + A^2F'''g'^3), \dots \quad (6)$$

The first equation (6) gives the mean multiplicity $\bar{n}_d = Aa \bar{n}_D$, the others then giving the normalized factorial moments $G_q = f_d^{(q)} / [f_d']^q$:

$$G_2 = \frac{1}{Aa} \frac{g''}{g'^2} + \frac{F''}{F'^2}, \quad G_3 = \frac{1}{A^2} \frac{g'''}{g'^3} + \frac{3}{A} \frac{F''g''}{F'^2g'} + \frac{F'''}{F'^3}, \dots \quad (7)$$

They are independent of a and tend to infinity for $A \rightarrow 0$, i.e., when the small window d tends to zero; this behaviour corresponds to intermittent

1. In recent years the systematic study of charged hadron multiplicity distributions (MD) of high energy reactions in limited intervals (to be called windows) of the (pseudo)rapidity y has revealed two classes of unexpected properties, which many experiments then showed to have a high degree of generality in hadronic collisions, e^+e^- annihilation and deep inelastic semileptonic scattering: the negative binomial (NB) regularities in windows symmetric in the centre of mass (c.m.) frame [1,2], and the intermittency property related to the sporadic appearance of spikes in the MDs and characterized by the growth of the normalized factorial moments for windows decreasing to very small size (down to $d < 0.1$) in the central c.m. rapidity region [3]. The NB regularities imply a particular form of clustering which was called clan structure to distinguish it from the cluster concept commonly used to describe the two-particle correlations [1]; as explained in section 4 of [4], the clan structure embodies all multiparticle correlations as defined by A. H. Mueller [5]. The aim of this letter is to show that the clan structure implies intermittency when the simplest assumption is made on the behaviour of the clans in very small windows.

2. In mathematical terms, the clan structure is most conveniently expressed by means of generating functions (g.f.). For any MD with probabilities p_n we define the g.f. by

$$f(u) = \sum p_n (1+u)^n. \quad (1)$$

The normalization condition is $f(0)=1$ and the mean is $\bar{n} = f'(0)$. Let $f_D(u)$ be the g.f. of the MD in some finite window D. Its clan structure is expressed by

$$f_D(u) = F_D[g_D(u)-1] \quad (2)$$

where $F_D(u)$ is the g.f. for the distribution of the number of clans and $g_D(u)$ the g.f. for the distribution of the number of particles in an average clan. When $f_D(u)$ corresponds to a NB distribution of parameters $\bar{n}_D$ and k_D , F_D can be taken poissonian (independent emission of clans, all correlations in the clans) and g_D logarithmic:

tency. If we assume the derivatives $g^{(q)}(u)$ of $g_D(u)$ at $u=0$ not to vanish, which means that the multiplicity per clan is not limited to some finite value, we find that the leading term of the normalized factorial moment for general q is given by

$$G_q = (A\Gamma')^{1-q} g^{(q)} / g^{1-q} + O(A^{2-q}) . \quad (8)$$

Let us limit ourselves to the leading terms [they would be the only ones if $F_D(u)$ were linear in u], and let us assume

$$A \propto d^c, \quad a \propto d^{1-c}, \quad 0 < c < 1, \quad (9)$$

which implies $\bar{n}_d = Aa\bar{n}_D \propto d$ for the mean multiplicity in the window d . We then get for the moments the power behaviour $G_q \propto d^{(1-q)c}$ and the intermittency exponents $h_q = (q-1)c$ correspond to a single fractal dimension $(q-1)^{-1}h_q = c$. We see no reason, however, to expect such a rather special behaviour to hold in reality.

4. We now illustrate the above considerations for the realistic case where the MD in the window D is of NB type, as expressed in Eq.(3). We then get with Eq.(4) for the small window d

$$F_d(u) = \exp[Ak_D \bar{r}_D^{-1} u \ln(1+u)], \quad g_d(u) = 1 - r_D [\ln(1+u) / \ln(1+m)] . \quad (10)$$

The resulting MD in d , given by (5), is immediately found to be

$$f_d(u) = \exp\{-Ak_D \bar{r}_D^{-1} u \ln(1+u)\} = (1+amu)^{-Ak_D} . \quad (11)$$

We note that the empty-clan parameter r_D of (3) has disappeared. Eq.(11) is the g.f. of the NB distribution of parameters

$$k_d = Ak_D, \quad \bar{n}_d = amk_d = Aa\bar{n}_D . \quad (12)$$

The intermittency now results from the fact that $k_d \rightarrow 0$ for $d \rightarrow 0$. Eqs.(10) and (11) are of the form (3) with D replaced by d and r_D by

$$r_d = r_D \ln(1+m) / \ln(1+m) .$$

This expresses the fact that some of the clans which contain particles in D do not contain any falling in d . Replacing this value by r_d^{-1} then redefines the clans pertaining to the NB distribution (10), (11) by simply discounting those which contain no particle in the small window d .

When this has been done, the mean number $\bar{n}_{c,d}$ of particles per clan and the mean number $\bar{N}_d$ of clans in the window d are given by the standard equations of the clan analysis [1]

$$\bar{n}_{c,d} = \bar{n}_d / \{k_d \ln(1+\bar{n}_d/k_d)\}, \quad \bar{N}_d = \bar{n}_d / \bar{n}_{c,d} . \quad (13)$$

In the limit $d \rightarrow 0$ they are to leading order

$$\bar{n}_{c,d} = 1 + \bar{n}_d / 2k_d = 1 + a\bar{n}_D / 2k_D, \quad \bar{N}_d = \bar{n}_d = Aa\bar{n}_D . \quad (14)$$

For $d \rightarrow 0$, the quantity $\bar{n}_{c,d}^{-1}$ (which is the mean excess of particles per clan above the minimum of one) decreases only as a , whereas the mean number of clans $\bar{N}_d$ decreases as Aa , i.e., faster by the factor A . This expresses for very small windows d the strong clustering of particles responsible for intermittency.

5. We end with a few concluding remarks. Both the intermittency property of factorial moments and the negative binomial regularities with the related clan structure properties came as unexpected surprises, none of them predicted on the basis of the existing dynamical models for multihadron production. Experiment revealed their high degree of generality (both types of effects in hadronic collisions and e^+e^- annihilation, NB regularities in semileptonic reactions for which we do not yet know of intermittency studies). Much work is going on to incorporate them into dynamical models, with different approaches for the two classes of effects, mainly QCD parton shower models for the NB regularities [4,6] and selfsimilar cascade models for intermittency [7]. As shown in this paper, the two problems are related; when extended to very small rapidity windows, the clan structure naturally leads to intermittency, in the sense that it predicts a continued increase of the normalized factorial moments for decreasing windows. In general, this decrease does not follow a simple power law, in contradistinction to the most characteristic prediction of selfsimilar cascade models (the experimental situation is still inconclusive on this point). While we are aware that other sources of intermittency can exist, we suggest that a complete dynamical understanding of the NB and clan structure properties (such an understanding is not yet available, especially for the clans, see [6], last ref.) could go a long way toward explaining the very short-range multiparticle correlations revealed by the Biafas-Peschanski method of factorial moment analysis.

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