Electromagnetic Properties of Nucleons from Lattice QCD

Terrence Draper* and R.M. Woloshyn
TRIUMF, 4004 Wesbrook Mall,
Vancouver, B.C. V6T 2A3, Canada

Keh-Fei Liu
Department of Physics and Astronomy,
University of Kentucky, Lexington, KY 40506, USA

21 OCT 1989

Abstract

A numerical simulation of QCD on a quenched $24 \times 12 \times 12 \times 24$ lattice is used to calculate, at small momentum transfer, the electric and magnetic form factors of a variety of hadrons.

(submitted to Physics Letters B)

* Present address: Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA

CM-P00068263

1181-11 03-01
C

If lattice QCD Monte Carlo calculations are to provide an attractive alternative to phenomenological models it is important that they be able to explain such basic hadron properties as electromagnetic structure. Early lattice computations with $SU(2)$ color investigated the electric form factor of the pion [1]. This was later extended to $SU(3)$ [2,3]. More recently, the electric form factor of the proton was studied [4]. Here we report on the techniques and results of a calculation of both electric and magnetic form factors for proton and neutron. For completeness, some results for the pion and rho-meson are also included. The computation was done using Wilson fermions with $SU(3)$ color on a quenched $24 \times 12 \times 12 \times 24$ lattice at $\beta = 5.9$.

The nucleon form factors F_1 and F_2 are defined in terms of the matrix element of the electromagnetic current j_μ .

$$\langle p, s | j_\mu(0) | p', s' \rangle = N \bar{u}(p, s) \left(i \gamma_\mu F_1(q^2) - i \sigma_{\mu\nu} \frac{q^\nu}{2m} F_2(q^2) \right) u(p', s') \quad (1)$$

where $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2i$. The Dirac gamma matrices γ_μ are Hermitian and satisfy $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. We'll follow the notation of [5], using an Euclidean metric so that $q^2 > 0$ for space-like momentum transfers. The factor N depends on the normalization conventions for states and spinors, and is such that $F_1(0)$ equals the total charge of the nucleon. One can determine this matrix element (and thus the form factors) by computing the ensemble average (denoted $\langle \dots \rangle$) of three-point Green functions

$$\begin{aligned} & \langle G_{Nj_\mu N}(t_2, t_1; \vec{p}, \vec{p}', \Gamma) \rangle \\ & \stackrel{\text{def}}{=} \sum_{\vec{x}_2, \vec{x}_1} e^{-i\vec{p}'\vec{x}_2} e^{+i(\vec{p}-\vec{p}')\vec{x}_1} \Gamma^{\beta\alpha} \langle \text{vac} | T \left(\chi^\alpha(x_2) (-i) j_\mu(x_1) \bar{\chi}^\beta(0) \right) | \text{vac} \rangle \end{aligned} \quad (2)$$

(The factor of $-i$ is inserted for convenience so that the simplest Green functions are real.) Upon inserting complete sets of states this correlation function takes the limit

$$\sum_{s, s'} N' e^{-E_p(t_2-t_1)} e^{-E_{p'} t_1} \Gamma^{\beta\alpha} \langle \text{vac} | \chi^\alpha | p, s \rangle \langle p, s | (-i) j_\mu | p', s' \rangle \langle p', s' | \bar{\chi}^\beta | \text{vac} \rangle \quad (3)$$

for large Euclidean time separations $t_2 - t_1 \gg 1$ and $t_1 \gg 1$, as contributions from excited states are exponentially suppressed. χ is a suitable nucleon interpolating field, and Γ is a Hermitian 4×4 Dirac matrix whose entries can be chosen to select some desired linear combination (in initial and final Dirac indices) of correlation functions. We'll consider only those components of the nucleon correlation functions which are

non-relativistically large, by setting the entries of Γ to be zero except for the upper left 2×2 submatrix.

The factors $\langle \text{vac} | \chi | p, s \rangle$ (with momentum p and polarization s) can be determined from the two-point function $\langle G_{NN}(t; \vec{p}, \Gamma) \rangle \stackrel{\text{def}}{=} \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \Gamma^{\beta\alpha} \langle \text{vac} | T \left(\chi^\alpha(x) \bar{\chi}^\beta(0) \right) | \text{vac} \rangle$, and cancel, for large time separations, in the following ratio [1] of correlation functions:

$$\begin{aligned} R(t_2, t_1, \vec{p}, \vec{p}', \Gamma, \Gamma') &\stackrel{\text{def}}{=} \sqrt{\frac{\langle G_{N_{j\mu}N}(t_2, t_1; \vec{p}, \vec{p}', \Gamma) \rangle \langle G_{N_{j\mu}N}(t_2, t_1; -\vec{p}', -\vec{p}, \Gamma') \rangle}{\langle G_{NN}(t_2; \vec{p}, \Gamma') \rangle \langle G_{NN}(t_2; -\vec{p}', \Gamma') \rangle}} \\ &\rightarrow \sqrt{\left(\frac{E_p + m}{2E_p} \right) \left(\frac{E_{p'} + m}{2E_{p'}} \right)} \hat{R}(\vec{p}, \vec{p}', \Gamma, \Gamma') \end{aligned} \quad (4)$$

Kinematical normalizations have been factored out of the time-independent asymptotic limit leaving a reduced ratio $\hat{R}(\vec{p}, \vec{p}', \Gamma, \Gamma')$ which is directly proportional to the form factors. With a judicious choice of Γ 's one can extract the Sachs combinations for the electric, $\mathcal{G}_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2} F_2(q^2)$, and magnetic, $\mathcal{G}_M(q^2) = F_1(q^2) + F_2(q^2)$, form factors. For example, one can show (without making any non-relativistic approximations) that

$$\hat{R}(\vec{q}, \vec{0}, \Gamma_4, \Gamma_4, 4) = \mathcal{G}_E(q^2) \quad (5a)$$

$$\hat{R}(\vec{q}, \vec{0}, \Gamma_j, \Gamma_4, k) = \mathcal{G}_M(q^2) |\epsilon_{ijk} q_i| / (E_q + m) \quad (5b)$$

$$\hat{R}(\vec{q}, \vec{0}, \Gamma_4, \Gamma_4, k) = \mathcal{G}_E(q^2) |q_k| / (E_q + m) \quad (5c)$$

with

$$\Gamma_j = \frac{1}{2} \begin{pmatrix} \sigma_j & 0 \\ 0 & 0 \end{pmatrix} ; \quad \Gamma_4 = \frac{1}{2} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

Thus the nucleon electric and magnetic form factors can be extracted from a lattice computation of two and three-point functions. We use Wilson's formulation for both the gauge action $\mathcal{S}_G$ and the fermionic action $\mathcal{S}_F$:

$$\begin{aligned} S_F(\{U\}) &\stackrel{\text{def}}{=} \bar{\psi} M(\{U\}) \psi \stackrel{\text{def}}{=} \sum_{x, j} \bar{\psi}^{(j)}(x) \psi^{(j)}(x) \\ &\quad - \sum_{x, \mu, j} \kappa_f \left(\bar{\psi}^{(j)}(x) (1 - \gamma_\mu) U_\mu(x) \psi^{(j)}(x + \hat{\mu}) \right. \\ &\quad \left. + \bar{\psi}^{(j)}(x + \hat{\mu}) (1 + \gamma_\mu) U_\mu(x)^\dagger \psi^{(j)}(x) \right) \end{aligned} \quad (6)$$

An $SU(2)$ flavor symmetry is enforced by equating the Wilson hopping parameters $\kappa_u \stackrel{\text{def}}{=} \kappa_d \stackrel{\text{def}}{=} \kappa$. For the proton interpolating field we'll use $\chi^\alpha = \epsilon^{abc} (\psi_a^{(u)\beta} \tilde{C}_{\beta s} \psi_b^{(d)\delta}) \psi_c^{(u)\alpha}$

where a, b, c are color indices and α, β, δ are Dirac indices. (For the neutron, simply exchange u and d .) The 4×4 Dirac matrix $\tilde{C} = C\gamma_5$, with C the charge conjugation matrix, satisfies $\tilde{C}\gamma_\mu \tilde{C}^{-1} = +\gamma_\mu^T = +\gamma_\mu^*$. Standard lattice techniques give the two-point Green function as the configuration average of a sum of products of quark propagators $S \stackrel{\text{def}}{=} M^{-1}$

$$\begin{aligned} G_{NN}(t; \vec{p}) &= \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \epsilon^{abc} \epsilon^{a'b'c'} \left(\text{tr} \left[\Gamma S_{aa'}^{(uu)}(x, 0) \tilde{S}_{bb'}^{(dd)}(x, 0) S_{cc'}^{(uu)}(x, 0) \right] \right. \\ &\quad \left. + \text{tr} \left[\Gamma S_{aa'}^{(uu)}(x, 0) \right] \text{tr} \left[\tilde{S}_{bb'}^{(dd)}(x, 0) S_{cc'}^{(uu)}(x, 0) \right] \right) \end{aligned} \quad (7)$$

The trace is over Dirac indices only, and $\tilde{S}_{bb'}^{\alpha\beta}(x, 0) \stackrel{\text{def}}{=} (\tilde{C} S_{bb'}(x, 0) \tilde{C}^{-1})^{\beta\alpha}$ for compactness of notation. The corresponding connected Wick contractions of the three-point function

$$\begin{aligned} G_{N_{j\mu}N}(t_2, t_1; \vec{p}, \vec{p}', \Gamma) &= \sum_{\vec{x}_2} e^{-i\vec{p} \cdot \vec{x}_2} \epsilon^{abc} \epsilon^{a'b'c'} \\ &\cdot \left(\text{tr} \left[\Gamma q_u \tilde{S}_{aa'}^{(uu)}(x_2, 0; t_1, \vec{q}, \mu) \tilde{S}_{bb'}^{(dd)}(x_2, 0) S_{cc'}^{(uu)}(x_2, 0) \right] \right. \\ &\quad + \text{tr} \left[\Gamma S_{aa'}^{(uu)}(x_2, 0) q_d \tilde{S}_{bb'}^{(dd)}(x_2, 0; t_1, \vec{q}, \mu) S_{cc'}^{(uu)}(x_2, 0) \right] \\ &\quad \left. + \text{tr} \left[\Gamma S_{aa'}^{(uu)}(x_2, 0) \tilde{S}_{bb'}^{(dd)}(x_2, 0) q_u \tilde{S}_{cc'}^{(uu)}(x_2, 0; t_1, \vec{q}, \mu) \right] + \dots \right) \end{aligned} \quad (8)$$

are similar to those of the two-point function, except that each quark propagator S has been replaced, in turn, by $q_f \hat{S}$ where q_f is the quark charge, and

$$\begin{aligned} \hat{S}(x_2, 0; t_1, \vec{q}, \mu) &= \sum_{\vec{x}_1} e^{+i\vec{q} \cdot \vec{x}_1} \kappa \left(S(x_2, x_1 + \hat{\mu}) (1 + \gamma_\mu) U_\mu(x_1)^\dagger S(x_1, 0) \right. \\ &\quad \left. - S(x_2, x_1) (1 - \gamma_\mu) U_\mu(x_1) S(x_1 + \hat{\mu}, 0) \right) \end{aligned} \quad (9)$$

describes the quark propagator coupled, with momentum $\vec{q}$, to $(-i)j_\mu$ with j_μ the electromagnetic current

$$\begin{aligned} j_\mu(x) &= i q_u \kappa_u \left(\bar{\psi}^{(u)}(x + \hat{\mu}) (1 + \gamma_\mu) U_\mu(x)^\dagger \psi^{(u)}(x) \right. \\ &\quad \left. - \bar{\psi}^{(u)}(x) (1 - \gamma_\mu) U_\mu(x) \psi^{(u)}(x + \hat{\mu}) \right) + (u \rightarrow d) \end{aligned} \quad (10)$$

This current, derived via the Noether procedure from the fermionic action S_F , has been normalized to reduce to the continuum current in the (classical) $a \rightarrow 0$ limit, and is exactly conserved on the lattice [6]. Thus *no renormalization is required* to relate its matrix elements on the lattice to those in the continuum. The lattice Ward

identity responsible for charge conservation $\hat{S}(x_2, 0, t_1, \vec{0}, 4) = S(x_2, 0)$ (if $t_2 > t_1 \geq 0$; $\hat{S} = 0$ otherwise) ensures that $G_{N,N}(t_2, t_1; \vec{p}, \vec{p}, \Gamma) = Q_N G_{N,N}(t_2; \vec{p}, \Gamma)$ (if $t_2 > t_1 \geq 0$; $G_{N,N} = 0$ otherwise) for a nucleon of charge Q_N . Thus, for the ratio of correlation functions defined in eqn. 4, one is guaranteed to measure $F_1(0) = Q_N$ identically, up to convergence errors, configuration by configuration. Therefore, there is no overall uncertainty in the normalization of the form factors, as would be the case if one used (as in references [3,4]) a nonconserved current which is subject to renormalization and greater statistical fluctuations.

The quark propagators S and $\hat{S}$ are the only ingredients necessary for a calculation of nucleon form factors. In fact they can be used to construct the form factors of *any* hadron, since given a hadronic two-point function defined as a sum of products of quark propagators, the corresponding three-point function is obtained by replacing each S , in turn, by $q/\hat{S}$. Thus, for example, the ratio of three-point to two-point pion correlation functions, defined analogously to eqn. 4, has the asymptotic limit $\frac{1}{2}(\sqrt{\frac{E}{E}} + \sqrt{\frac{E}{E}})F(q^2)$ where F is the pion form factor defined by $\langle p | j_\mu | p \rangle = N''(p + p')_\mu F(q^2)$.

A sequential source technique [7] is used to compute S and $\hat{S}$ for each configuration. First, S is computed by solving $MS = b$, with b a delta-function source at the origin, using a sparse matrix solver. Secondly $\hat{S}$ is used to construct the secondary source

$$\begin{aligned} \hat{b}(x; t_1, \vec{q}, \mu) = & e^{+i\vec{q}\cdot\vec{x}} \kappa \left((1 + \gamma_\mu) U_\mu(x - \hat{\mu})^\dagger S(x - \hat{\mu}, 0) \delta(t_2 - \hat{\mu}, t_1) \right. \\ & \left. - (1 - \gamma_\mu) U_\mu(x) S(x + \hat{\mu}, 0) \delta(t, t_1) \right) \end{aligned} \quad (11)$$

and $\hat{S}$ is computed by using the *same* solver on $M\hat{S} = \hat{b}$. This step is repeated for each choice of t_1 , $\vec{q}$ and μ .

The correlation functions are defined in principle as averages over an infinite ensemble of equilibrium gauge field configurations, but they are approximated by an average over a *finite* sample of configurations. Therefore it is important to construct an unbiased estimator with minimal noise for these. The discrete symmetries of the Green functions are a useful guide in doing so. We have found [8] that it is crucial to take advantage of the parity of the Green functions by averaging correlation functions with their $\vec{p}, \vec{p}', \vec{q} \rightarrow -\vec{p}, -\vec{p}', -\vec{q}$ partners. Since there are two independent momenta, it would be incorrect simply to construct a secondary source symmetric under $\vec{q} \rightarrow -\vec{q}$, since this would contribute unwanted cross terms which mix different values of q^2 .

Rather, two separate inversions must be done, once each for $\pm\vec{q}$. The extra expense is well worthwhile. The quality of the signal is dramatically improved (much more so than a simple increase in the number of configuration could accomplish) as the procedure removes contributions which are zero in the infinite ensemble average, but which are noise for a finite number of configurations.

Similarly, the symmetry $S(x, 0; \{U\}) = (\tilde{C}S(x, 0; \{U\})\tilde{C}^{-1})^*$ (which is a combination of $S(x, 0; \{U\}) = (\gamma_5 S(0, x; \{U\})\gamma_5)^\dagger$ and charge conjugation $S(x, 0; \{U^*\}) = (CS(0, x; \{U\})C^{-1})^\dagger$) instructs us [2,8] to discard the imaginary parts (or, if appropriate, the real parts) of correlation functions, since these are noise and must vanish in the infinite ensemble average. In constructing two and three-point correlation functions, one must be careful to choose only those Γ 's which satisfy $(\tilde{C}\Gamma\tilde{C}^{-1})^* = \pm\Gamma$.

From eqns. 5b and 5c, notice that a single $\hat{S}$ obtained using a secondary source constructed from a spatial component j_k of the current, at a three-momentum transfer $\vec{q}$ which has non-zero components both parallel and perpendicular to the direction $\hat{k}$, is sufficient to determine both the electric and magnetic form factors at that momentum transfer. However, we are interested in calculating form factors at the minimum possible momentum transfer available on the lattice and have lengthened the lattice in one spatial direction accordingly. Thus we chose to set $\vec{q} = (2\pi/24, 0, 0)$ and to determine $\mathcal{G}_E$ and $\mathcal{G}_M$ from eqns. 5a and 5b respectively. (This requires that $M\hat{S} = \hat{b}$ be solved for two choices of $\hat{b}$.) An advantage of doing so is that the signal to noise ratio for $\mathcal{G}_E$ as calculated from eqn. 5a is an order of magnitude better than that of eqn. 5c.

Twenty-eight quenched configurations were generated by the Cabibbo-Marinari [9] pseudo-heatbath method on a $24 \times 12 \times 12 \times 24$ periodic lattice, with $\beta = 5.9$. After 5000 thermalization sweeps from a cold start, configurations were selected every 1000 sweeps. We used periodic boundary conditions in the spatial directions for the quark fields, and Dirichlet or "fixed" boundary conditions in the time direction. Time slices are labelled from -3 to 20 , with the primary delta function source (and hence the hadronic interpolating fields) at the origin $t = 0$. The current j_μ (more precisely, the symmetric combination $\frac{1}{2}(j_\mu(x_1 - \hat{\mu}) + j_\mu(x_1))$) was centered at time slice $t_1 = 8$. For the data presented subsequently, $|\vec{p}'|a = 0$ and $|\vec{p}|a = |\vec{q}|a = 2\pi/24 = 0.26$, the minimum nonzero discrete three-momentum possible on the lattice. Errors were

calculated with a single elimination jackknife [10].

Figure 1 displays plots of ratios R of Green functions whose asymptotic limits are proportional to the electric form factor of the proton $\mathcal{G}_{Ep}$, the magnetic form factor of the proton $\mathcal{G}_{Mp}$, the magnetic form factor of the neutron $\mathcal{G}_{Mn}$, and $\mathcal{G}_{Mn}/\mathcal{G}_{Mp}$. This graph includes points for which $t_2 < t_1$. For the electric form factor at zero momentum transfer, the jump at $t_1 = 8$ would be (with fixed temporal boundary conditions) a step function from 0 to Q_N (Physically, the charge density operator j_4 measures zero charge if not between the interpolating fields.) It is satisfying that the ratios R do indeed approach a constant for large times, so that form factors can be extracted using eqns. 4 and 5. Table 1 presents our results for the electric and magnetic form factors so extracted, as a function of κ .

The monopole form $(1 + q^2/\lambda^2)^{-1}$ is an accurate empirical fit for low q^2 to the experimental data for the pion form factor [11]. To high precision $\lambda = m_\rho$ as suggested by vector meson dominance (VMD). It is satisfying that the calculated pion form factor agrees very well (to within $\approx 1\%$) with the monopole form, provided that one uses the lattice rho-meson mass.

The lattice rho-meson form factor is also consistent with the VMD prediction. Parametrizing the form factor to the monopole form, and extracting a radius via $< r^2 > = -6dF/dq^2$, gives the radius of the mesons as a function of κ , as shown in table 1. The radii increase with decreasing quark mass, as is physically reasonable. The rho-meson is larger than the pion, with the difference increasing with decreasing mass to $\approx 10\%$ at our largest κ value. We also note that the calculated radii for the proton are essentially the same as for the mesons.

To calculate results that can be compared with experimental numbers, it is necessary to extrapolate to the chiral limit. However, it would be misleading to extrapolate the pion form factor data linearly in κ^{-1} to κ_c^{-1} for fixed $\vec{q}$. For although the variation in $q^2 = 2m_\pi(\sqrt{m_\pi^2 + |\vec{q}|^2} - m_\pi)$ versus κ^{-1} is small (4% over the range of masses investigated, for which $m_\pi > |\vec{q}|$), it differs by a factor of 2 from its value at κ_c where $m_\pi < |\vec{q}|$.

For the nucleon, q^2 varies slowly for fixed $\vec{q}$ as κ is varied from the values at which the lattice measurements are made right through to κ_c . Thus an extrapolation in κ^{-1} of

nucleon form factors is reliable. For the proton charge radius we get $r = 0.64 \pm 0.08$ fm which is 2.5σ smaller than the physical size of 0.84 fm. Finite size effects are not expected to be significant since the measured radii ($r/a \approx 4 - 5$) are smaller than the minimum spatial size of the lattice, and much smaller than the size of the lattice ($L_x = 24$) in the direction in which the measurement is made. A comparison with reference [4] of the proton's electric charge radius cannot be made, since their data was too noisy to allow for an extrapolation to the chiral limit. However, they find, as do we, that the proton's electric form factor is close to the VMD value.

The electric form factor of the neutron is small (it is $Q_n = 0$ at $q^2 = 0$). Near $q^2 = .16\text{GeV}^2$ (where our calculations were performed) dispersion fits [12] to the experimental data (which have errors of 50%) give $\mathcal{G}_{En} = 0.04$. The simplest quark models predict zero. But a more sophisticated one [13] suggests that the quark-spin-dependent interaction that breaks the mass degeneracy of the ground state baryons is more repulsive for quarks with parallel than with anti-parallel spins, so that the induced charge radius $< r_n^2 >$ is negative. This mechanism is available on the lattice, even in the quenched approximation. Our data (table 1) shows a hint of a non-zero signal of the correct sign and magnitude. But since this is at the 1σ level, we cannot draw a stronger quantitative conclusion.

Next we turn to a consideration of magnetic properties. Our method allows the calculation of magnetic form factors only at non-zero values of momentum transfer. Ideally, one would want to extrapolate to $q^2 = 0$ to compare with magnetic moments. However, this is not practical as an extrapolation in q^2 would suffer from large statistical errors that would tend to obscure systematic effects. We chose, therefore, to use only the minimum three-momentum transfer where the statistical errors are smallest. To make contact with empirical results, we use the fact that the experimentally measured form factors obey, to a very good approximation at low q^2 [12], the "scaling law"

$$\frac{\mathcal{G}_{Ep}(q^2)}{\mathcal{G}_{Ep}(0)} \approx \frac{\mathcal{G}_{Mp}(q^2)}{\mathcal{G}_{Mp}(0)} \approx \frac{\mathcal{G}_{Mn}(q^2)}{\mathcal{G}_{Mn}(0)} \quad (12)$$

where $\mathcal{G}_{Ep}(0) = Q_p = 1$ is the electric charge, and $\mathcal{G}_{Mp}(0) = g_p \stackrel{\text{def}}{=} \frac{\mu_p}{(e/2m_p)} = 2.793$ the magnetic moment (in nuclear magnetons) of the proton. This suggests that we compare ratios of lattice form factors (which have smaller statistical errors than the individual factors) to the corresponding experimental quantities.

From table 1 and figure 2 we see that ratios of form factors have a gentle dependence on κ (for a pure Dirac particle, g is independent of mass) so that a linear extrapolation in κ^{-1} is reliable. The calculated ratio G_{Mn}/G_{Mp} is $\approx 15\%$ less (in magnitude) than the experimentally observed value $G_{Mn}(q^2)/G_{Mp}(q^2) \approx G_{Mn}(0)/G_{Mp}(0) = g_n/g_p = -0.68$, and G_{Mp}/G_{Ep} is $\approx 15\%$ less than $g_p = 2.79$. If one assumes that lattice form factors obey a scaling law analogous to eqn. 12, then the proton's magnetic moment is underestimated by about 15%. We note that this is consistent with an early lattice calculation [14] of nucleon magnetic moments which used a completely different method on a much smaller lattice with fewer configurations. They found $\mu_p = 2.3^{+0.8}_{-0.3}$ nuclear magnetons, and $\mu_p/\mu_n = -1.7 \pm 0.1$.

A sequential source technique can evaluate the connected Wick contractions of any n -point Green function with at most $(n-1)$ matrix inversions; however, it cannot calculate disconnected contractions. Our 3-point function has such a disconnected graph wherein the external current communicates with the hadron propagation only indirectly via gluons and not directly to the valence quarks. Fortunately, for meson propagation, this contribution can be shown to vanish identically because of charge conjugation symmetry [2]. The argument fails to generalize to baryons and so, in this case, we must drop the contribution in the spirit of the quenched approximation. The contribution is equal for the proton and neutron (since, for equally massive up and down quarks, the spectator nucleon propagation is the same), and so does not contribute for the isovector combination. Somewhat surprisingly, however, it is the lattice calculation for the isoscalar, G_{Mn}/G_{En} , which agrees more closely with experiment (a factor of 1.0 ± 0.2 different), than does the corresponding ratio for the isovector (a factor of 0.8 ± 0.2 different). So it appears that the effect of dropping the disconnected graph is no stronger than that of making the quenched approximation.

We have presented the formalism for and results of a method for calculating electric and magnetic form factors for *any* hadron. The advantage of using the conserved vector lattice current was stressed — a Ward identity guarantees that $F(0)$ equals the electric charge identically; thus there is no uncertainty in overall normalization, and furthermore, statistical errors are reduced. The choice of the current [1], rather than an interpolating field [2,3,4], as the source for the secondary matrix inversion [7] allows for a

calculation for any hadron with any momentum, and permits the construction of ratios of Green functions which are free of both time dependences and vacuum-field-state amplitudes and which are directly proportional to the form factors. It was emphasized that fluctuations can be greatly reduced by ensuring that the averages of correlation functions over a finite set of configurations have the exact discrete symmetries of the full ensemble averages [8]. In agreement with earlier simulations [1,2,3], the lattice pion form factor is consistent with the monopole form at low momentum-transfer, even away from the chiral limit. The rho-meson form factor is also consistent with the monopole form with the rho-meson having a slightly larger radius than the pion. There is a 1σ signal of the correct sign and magnitude for the electric form factor of the neutron. The proton's electric charge radius is underestimated on the lattice. We find $r_{Ep} = 0.64 \pm 0.08$ fm compared to the experimental value of 0.84 ± 0.01 fm. The ratio of the proton's magnetic to electric form factors is calculated to be about 15% smaller than in experiment, with better agreement for the isoscalar component. The ratio of the neutron to proton magnetic form factors is about 15% smaller in magnitude than the experimental value. It is encouraging that the calculation from *first principles* of these quantities can reproduce experimental results to the accuracy that quenched lattice QCD can reproduce the hadronic spectrum.

We thank J. Connolly and his staff at the Center for Computational Sciences and D. Hurley and his staff at the Computing Science Department of the University of Kentucky for their generous help in providing us with time on their IBM 3090-300E supercomputer. T.D. thanks D. Leinweber for useful conversations. This work is supported in part by the National Sciences and Engineering Research Council of Canada, the U.S. Department of Energy under grant no. DE-FG05-84ER40154, and an IBM ACIS grant.

References

- 1) W. Wilcox and R.M. Woloshyn, *Phys. Rev. Lett.* **54** (1985) 2653; R.M. Woloshyn, *Phys. Rev.* **D34** (1986) 605
- 2) T. Draper, R.M. Woloshyn, W. Wilcox and K.F. Liu, *Nucl. Phys.* **B318** (1989) 319; T. Draper, R.M. Woloshyn, W. Wilcox and K.F. Liu, *Nucl. Phys.* **B** (Proc. Suppl.) **4** (1988) 524
- 3) G. Martinelli and C.T. Sachrajda, *Nucl. Phys.* **B306** (1988) 865
- 4) G. Martinelli and C.T. Sachrajda, *Nucl. Phys.* **B316** (1989) 355
- 5) J. Sakurai, *Advanced Quantum Mechanics*, (Addison-Wesley, Reading, 1967)
- 6) L.H. Karsten and J. Smit, *Nucl. Phys.* **B183** (1981) 103
- 7) C. Bernard, in *Gauge Theory on a Lattice*, eds. C. Zachos, W. Celmaster, E. Kovacs, and D. Sivers (National Technical Information Service, Springfield, 1984); C. Bernard, T. Draper, G. Hockney, and A. Soni, in *Lattice Gauge Theory: A Challenge in Large-Scale Computing*, eds. B. Bunk, K.H. Mutter, and K. Schilling (Plenum, New York, 1986)
- 8) T. Draper, R.M. Woloshyn, W. Wilcox and K.F. Liu, in *Field Theory on the Lattice*, eds. P. Mackenzie and A. Kronfeld *Nucl. Phys.* **B** (Proc. Suppl.) (1989) (in press)
- 9) N. Cabibbo and E. Marinari, *Phys. Rev. Lett.* **119B** (1982) 387
- 10) B. Efron, *SIAM Rev.* **21** (1979) 460
- 11) S.R. Amendolia et al., *Nucl. Phys.* **B277** (1986) 168
- 12) G. Höhler et al., *Nucl. Phys.* **B114** (1976) 505, and references therein.
- 13) R.D. Carlitz, S.D. Ellis, and R. Savit, *Phys. Lett.* **68B** (1977) 443
- 14) C. Bernard, T. Draper, K. Olynyk, and M. Rushton, *Nucl. Phys.* **B220**[FS8] (1983) 508

Table 1: Masses m_π (m_ρ , m_p) of the pion (respectively rho-meson, proton), electric form factors F_π (F_ρ) of the pion (rho-meson) and a comparison to vector meson dominance (VMD) predictions, the electric form factors $\mathcal{G}_{E\rho}$ ($\mathcal{G}_{En}$) and the magnetic form factors $\mathcal{G}_{M\rho}$ ($\mathcal{G}_{Mn}$) of the proton (neutron), various ratios thereof, and the electric charge radii r_π (r_ρ , r_p) of the pion (rho-meson, proton). $\mathcal{G}_*$ ($\mathcal{G}_w$) is the isoscalar (isovector) combination of the proton and neutron.

	$\kappa = 0.152$	$\kappa = 0.154$	$\kappa = 0.156$	$\kappa_c = 0.1599(3)$
m_π/a	0.62(1)	0.53(1)	0.43(1)	0
m_ρ/a	0.68(1)	0.60(1)	0.53(2)	0.38(4)
m_p/a	1.08(4)	0.95(4)	0.83(3)	0.59(6)
F_π	0.880(4)	0.858(6)	0.831(14)	
F_ρ	0.87(1)	0.84(1)	0.80(2)	
F_π/F_π^{VMD}	1.005(5)	1.01(1)	1.02(2)	
$F_\rho/F_\rho^{\text{VMD}}$	0.99(1)	0.99(1)	0.99(2)	
$\mathcal{G}_{E\rho}$	0.86(1)	0.83(1)	0.81(3)	0.78(6)
$\mathcal{G}_{En}$	0.001(2)	0.006(7)	0.02(2)	0.02(2)
$\mathcal{G}_{M\rho}$	1.9(1)	1.8(2)	1.8(2)	1.8(4)
$\mathcal{G}_{Mn}$	-1.2(1)	-1.1(1)	-1.1(2)	-1.0(3)
$\mathcal{G}_{M\rho}/\mathcal{G}_{E\rho}$	2.2(1)	2.2(2)	2.3(2)	2.3(4)
$\mathcal{G}_{Mn}/\mathcal{G}_{En}$	0.79(6)	0.81(8)	0.85(12)	0.88(19)
$\mathcal{G}_{Mw}/\mathcal{G}_{Ew}$	3.5(2)	3.6(3)	3.7(4)	3.8(7)
$\mathcal{G}_{Mn}/\mathcal{G}_{M\rho}$	-0.63(1)	-0.62(2)	-0.61(4)	-0.60(6)
r_π/a	3.5(1)	3.9(1)	4.4(2)	
r_ρ/a	3.7(1)	4.2(1)	4.8(2)	
r_p/a	3.9(1)	4.2(2)	4.5(4)	5.2(8)

Figure Captions

Fig. 1: Time dependence of correlation function ratios R (at $\kappa = 0.154$) which are asymptotically proportional to $\mathcal{G}_{Ep}$ ($\bullet$), $\mathcal{G}_{Mp}$ ($\circ$), $-\mathcal{G}_{Mn}$ ($\diamond$) and $-\mathcal{G}_{Mn}/\mathcal{G}_{Mp}$ ($\ast$). Data are similar for $\kappa = 0.152$ (0.156), except that the statistical errors are smaller (larger).

Fig. 2a: The lattice ratio $\mathcal{G}_{Mp}/\mathcal{G}_{Ep}$ of magnetic to electric form factors of the proton at $q^2 \approx 0.07$, divided by the experimental value $\mathcal{G}_{Mp}(0)/\mathcal{G}_{Ep}(0) \approx \mathcal{G}_{Mp}(q^2)/\mathcal{G}_{Ep}(q^2)$. If the 'scaling law' (eqn. 12) of nucleon form factors were exactly true on the lattice, then this quotient would equal 1, which is drawn for comparison. The quotient was calculated at $\kappa = 0.152$, 0.154, and 0.156 ($\circ$), and extrapolated in κ^{-1} to $\kappa_c = 0.159(3)$ ($\bullet$).

Fig. 2b: Same as for figure 2a, but for $-\mathcal{G}_{Mn}/\mathcal{G}_{Mp}$.

Fig. 1

12

a)

b)

Fig. 2