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Multiple string flips in ultrarelativistic hadronic collisions from the strong coupling QCD expansion

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ABSTRACT

We apply the strong coupling expansion of lattice QCD to derive a multiple-string-flip expansion for hadronic collisions. This expansion is exactly of the type used in the phenomenological string model VENUS. This formalism also justifies the use of strings rather than gluons as basic objects in such a model.

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1. Introduction

Phenomenological classical string models [1-3] very successfully describe high energy hadron-hadron collisions, as well as hadron-nucleus and nucleus-nucleus scattering. One type of model (VENUS [1] and Dual Parton Models [2]), use the “string-flip” (or color exchange) mechanism as the basic principle to form strings. The constituents of the models are quarks q , antiquarks $\bar{q}$ and strings (rather than gluons!). A typical proton-proton collision would be of the type (brackets () indicate color singlets):

$$(q_p q_p q_p) + (q_t q_t q_t) \rightarrow (q_p q_p q_t) + (q_t q_t q_p) \quad (1)$$

where a string flip between the projectile proton ($q_p q_p q_p$) and the target proton ($q_t q_t q_t$) (left-hand side of eq. (1)) results in two diquark quark strings (right-hand side of eq. (1)) with a large relative momentum between the quark q and the diquark qq . We are, however, also referring to objects with little relative momentum, like the initial nucleons in eq. (1), as strings. Since a nucleon may have a more complicated structure, like $(qqq)(q\bar{q})$, in VENUS also “diffractive scattering” [4]

$$(q_p q_p q_p)(q_p \bar{q}_p) + (q_t q_t q_t)(q_t \bar{q}_t) \rightarrow (q_p q_p q_p) + (q_t \bar{q}_t) + (q_t q_t q_p) \quad (2)$$

contributes, where the “surviving” nucleon ($q_p q_p q_p$) keeps a large fraction of the initial momentum, thus giving rise to a diffractive peak at large momenta. Once considering diffraction eq. (2), consistency requires us to take into account also multiple-string-flips. In VENUS, for example, the following double-string-flip occurs:

$$(q_p q_p q_p)(q_p \bar{q}_p) + (q_t q_t q_t)(q_t \bar{q}_t) \rightarrow (q_p q_p q_t) + (q_p \bar{q}_t) + (q_t q_t q_p), \quad (3)$$

leading to four strings rather than the two strings as in the lowest order contribution eq. (1). As shown in [1] for VENUS, one can set up a consistent model by expanding the whole contribution σ as

$$\sigma = \sum_n w_n \sigma_n \quad (4)$$

where σ_n are contributions with n string flips, w_n the corresponding weight. These weights w_n are assumed to be of the form

$$w_n \sim c^n \quad (5)$$

with some constant c .

It is the purpose of this paper to derive the “string-flip” principle, and in particular the string-flip expansion eq. (4) from the basic properties of the Hamiltonian formulation of QCD on the lattice. We are going to derive the exponential form eq. (5) for w_n , and we shall be able to express the constant c in terms of the strong coupling constant g . This means: the multiple-string-flip expansion eq. (4) is just a strong coupling QCD expansion.

2. Strong coupling lattice QCD

Let us first discuss some elements of Hamiltonian lattice QCD, introduced by Kogut and Susskind [5] (see also [6]). A space point on the lattice is denoted by a triplet $n = (n_x, n_y, n_z)$ of integers. A triplet integer $e = (e_x, e_y, e_z)$ denotes a unit lattice vector $((\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1))$. Clearly [5] a discrete Hamiltonian of the form

$$H_q = \frac{1}{ia} \sum_{n,e} q^\dagger(n) \alpha_j e_j q(n+e) + m \sum_{n,e} \beta q^\dagger(n) q(n) \quad (6)$$

with Dirac spinors $q^\dagger, q$ and the Dirac matrices α_j yields the Dirac equations

$$i\dot{q} = (-i\alpha_j \partial_j + \beta m) q \quad (7)$$

in the continuum limit $a \rightarrow 0$. The Hamiltonian eq. (6) is, however, not invariant under local SU(3) gauge transformations

$$q(n) \rightarrow V(n) q(n) \quad (8)$$

with

$$V(n) = \exp\left\{i \frac{\lambda_j}{2} w_j(n)\right\} \quad (9)$$

with λ_j being the generators of SU(3) and $w_j(n)$ being arbitrary coefficients. In order to achieve gauge invariance, quark creation and annihilation operators acting at different points n and m have to be linked by string operators $U(n, m)$ transforming as

$$U(n, m) \rightarrow V(n) U(n, m) V^\dagger(m). \quad (10)$$

So in order to achieve gauge invariance, the Hamiltonian eq. (6) is replaced by

$$H_q = \frac{1}{ia} \sum_{n,e} q^\dagger(n) \alpha_j e_j U(n, n+e) q(n+e) + m \sum_n \beta q^\dagger(n) q(n). \quad (11)$$

Choosing the string operator U as

$$U(n, n+e) = \exp\left\{iag \frac{\lambda^\alpha}{2} A^\alpha(n, n+e)\right\}$$

with A^α being the gluon field, we obtain for H_q in the continuum limit $a \rightarrow 0$

$$H_q \xrightarrow{a \rightarrow 0} \int d^3r \bar{q}(r) \left\{ -i\gamma_j \partial_j + g\gamma_j \frac{\lambda^\alpha}{2} A_j^\alpha(r) + m \right\} q(r), \quad (12)$$

representing continuum QCD. The gluon Hamiltonian of continuum QCD can be written as

$$\int d^3r \text{Tr} \{ E_i E_i + B_i B_i \} \quad (13)$$

with the electric field $E_i = \frac{1}{2} \partial_0 A_i^a$ ($A_0 = 0$ gauge) and a magnetic field $B_i = \frac{1}{2} \epsilon_{ijk} F_{jk}$. A lattice Hamiltonian which reproduces eq. (13) in the continuum limit $a \rightarrow 0$ is given as

$$H_g = H_E + H_M; \quad (14)$$

the magnetic part is

$$H_M = \frac{1}{ag^2} \sum_{\text{plaquettes } p} \text{Tr}[2 - U(p) - U(p)^\dagger] \quad (15)$$

where the plaquette operator $U(p)$ is a product of four string (link) operators

$$U(p) = U(n, n + e_1)U(n + e_1, n + e_1 + e_2)U(n + e_2, n + e_2 + e_3)U(n + e_3, n) \quad (16)$$

representing an elementary string around a plaquette. The electric part of the gluon Hamiltonian eq. (14) is given as

$$H_E = \frac{g^2}{2a} \sum_{\text{links } l} E(l)^2 \quad (17)$$

with

$$E(l)^2 \equiv E(n, n + e)^2 = E^\alpha(n, n + e)E^\alpha(n, n + e) \quad (18)$$

where α are the 8 color indices. The complete lattice Hamiltonian is a sum of the quark Hamiltonian eq. (11) and the electric and magnetic gluon contribution eqs. (15,17):

$$H = H_q + H_E + H_M. \quad (19)$$

For a perturbation expansion it is useful to rewrite H as

$$H = H_0 + H_1 \quad (20)$$

with the "strong coupling Hamiltonian" H_0 containing the electric gluon contribution as well as the quark mass contribution:

$$H_0 = \frac{g^2}{2a} \sum_{\text{links } l} E(l)^2 + m \sum_n \beta q^\dagger(n)q(n) \quad (21)$$

and with H_1 containing the magnetic gluon part and the kinetic quark part:

$$H_1 = \frac{1}{ag^2} \sum_{\text{plaquettes } p} \text{Tr}[2 - U(p) - U(p)^\dagger] + \frac{1}{ia} \sum_{n, e} q^\dagger(n) \alpha_j e_j U(n, n + e) q(n + e). \quad (22)$$

In the large distance, strong coupling limit $a, g \rightarrow \infty$ the term H_1 vanishes, so in a certain regime of large g the Hamiltonian H_1 can be considered as perturbation, acting on eigenstates of the unperturbed Hamiltonian H_0 . It is of course not a priori clear that a strong coupling expansion has anything to do with continuum QCD, since $a \rightarrow 0$ implies $g \rightarrow 0$ and thus any strong coupling expansion breaks down. However, numerical results indicate that there seems to be a smooth transition from the strong to the weak coupling regime, justifying strong coupling lattice QCD as a useful tool to investigate large distance phenomena. And this is just what we are interested in: we want to study soft hadronic collisions, involving transverse momentum transfer of few hundred MeV, corresponding to a scale of ~ 1 fm, which is large enough as we will see later.

3. The string-flip expansion

We are now going to discuss eigenstates of the strong coupling Hamiltonian H_0 [7]. First of all there is the vacuum $|0\rangle$, where neither sites nor links are occupied, with zero energy. Closed strings, a sequence of neighbouring links

$$\text{Tr} [U(l)|0\rangle] = \text{Tr} U(n, n + e_1)U(n + e_1, n + e_2) \dots U(n + \sum e_i, n)|0\rangle \quad (23)$$

with the same initial and end point (see fig. 1(a)), are gauge-invariant eigenstates with energy $\frac{2}{3} \frac{g^2}{a^2} L$, if L is the length of the string. The simplest eigenstate containing quarks is a quark and an antiquark, connected by a string (see fig. 1(b)), with energy

$$m_q + m_{\bar{q}} + \frac{2}{3} \frac{g^2}{a^2} L. \quad (24)$$

Eq. (24) leads to a linear potential σL , with the string tension σ given as $\sigma = \frac{2}{3} \frac{g^2}{a^2}$, which demonstrates that for $\sigma = 1 \text{ GeV}/\text{fm}$ and a typical distance $a = 1 \text{ fm}$ the strong coupling condition $g \gg 1$ is indeed fulfilled. More complicated eigenstates of H_0 can be constructed, like three quarks connected by strings (T-shape, for example) which represent baryons, or even more complex multi-quark states.

In the strong coupling limit ($H_1 = 0$), for a given quark and antiquark position, the lowest eigenstate corresponds to the shortest string between the two partons. The operator $U(p)$ in H_1 has the effect of deforming a given string path, the deformation having the size of one plaquette (see fig. 2). So for finite but small perturbation H_1 the shortest path still provides the dominant contribution, however also deformed paths contribute. If the area of a deformation is ν times the plaquette area, the weight of this contribution is of order ν in smallness of H_1 , so large deformations are unlikely. We obtain a flux tube structure [7].

As an asymptotic state for meson-meson scattering one might consider two fluxtubes, with a large distance Na between them. Let us for simplicity consider two elementary strings, with each string being just one link (see fig. 3(a)). An interaction between the two mesons (a coupling between the state in fig 3(a) with something like the one in fig.

3(b)) is at least of order N , so for large distances Na the two independent mesons shown in fig. 3(a) are indeed approximately eigenstates of $H_0 + H_1$.

For small distances the situation is completely different. Let us consider two elementary strings, being parallel to each other and just one link apart (see fig. 4(a), left-hand side). We investigate the effect of H_1 on this two-meson state. The term $q^\dagger \alpha U q$ can for example annihilate a quark of one meson as well as an antiquark of another meson and at the same time provide a link between the two points (see fig 4(a)). So this operator provides fusion (meson + meson $\rightarrow$ meson). The plaquette operator U , the other contribution to H_1 , can provide "string flips" as shown in fig. 4(b), by annihilating two links and creating two new ones.

In an adiabatic approximation we might consider the two-meson state as depending on the distance N as parameter. The distance N on the other hand be time dependent: $N = N(t)$. In this way H_1 acts as a time dependent perturbation, giving a first order contribution only when the mesons are just one link apart. Let us discuss such first order contributions more in detail. We only consider the "string flip" operator U for the following, assuming that this is the dominant part of H_1 for high energy collisions (fusion due to the operator $q^\dagger \alpha U q$ should be negligible). Using $|(q_a \bar{q}_b)(q_c \bar{q}_d)\rangle$ for a two meson state, the string flip process shown in fig 4(b) corresponds to

$$-\frac{1}{ag^2} U | (q_p \bar{q}_p)(q_t \bar{q}_t) \rangle = | (q_p \bar{q}_t)(q_t \bar{q}_p) \rangle \quad (25)$$

where we used the indices p and t for projectile and target to differentiate between the partons. We also used the proper factor $\frac{1}{ag^2}$ in front of U , see eq. (22). If $|q_p \bar{q}_p\rangle$ and $|q_t \bar{q}_t\rangle$ are two mesons moving very fast relative to each other, the operator U has the effect of rearranging color flux lines such that we obtain two (high mass!) strings, consisting of partons from different mesons. Such a string-flip interaction could also be called "color exchange". Of course a similar kind of string-flip also occurs for meson-baryon and baryon-baryon scattering; the latter one may look like:

$$-\frac{1}{ag^2} U | (q_p q_p q_p)(q_t q_t q_t) \rangle = | (q_p q_p q_t)(q_t q_t q_p) \rangle, \quad (26)$$

(see fig. 4(c)) where the interaction results in two diquark-quark strings, consisting of a diquark and a quark from different baryons. This corresponds exactly to the non-diffractive one-string-flip (or one-color-exchange) contribution eq. (1) in the phenomenological model VENUS!

Realizing that sea quarks and antiquarks contribute to a hadron, we have to consider more complicated states than shown in eq. (26), like for example the state $|(qqq)(q\bar{q})\rangle$ containing one quark-antiquark pair from the sea. Therefore the following string-flip may occur:

$$-\frac{1}{ag^2} U | (q_p q_p q_p)(q_p \bar{q}_p)(q_t \bar{q}_t) \rangle = | (q_p q_p q_p)(q_t \bar{q}_p)(q_t q_t q_p) \rangle, \quad (27)$$

which is just the diffractive VENUS contribution eq. (2). In addition to eqs. (26,27) there are obviously two more cases. So acting on the initial two-baryon state

$$|i\rangle = | (q_p q_p q_p)(q_p \bar{q}_p)(q_p \bar{q}_p) \cdots (q_t q_t q_t)(q_t \bar{q}_t)(q_t \bar{q}_t) \cdots \rangle \quad (28)$$

we have the following four one-string flip contributions (only "active" $q-\bar{q}$ pairs are written down):

$$\begin{aligned} (a) \quad & -\frac{1}{ag^2} U |i\rangle = | (q_p q_p q_t)(q_t q_t q_p) \rangle \\ (b) \quad & -\frac{1}{ag^2} U |i\rangle = | (q_p q_p q_p)(q_t \bar{q}_p)(q_t q_t q_p) \rangle \\ (c) \quad & -\frac{1}{ag^2} U |i\rangle = | (q_p q_p q_t)(q_p \bar{q}_t)(q_t q_t q_t) \rangle \\ (d) \quad & -\frac{1}{ag^2} U |i\rangle = | (q_p q_p q_p)(q_p \bar{q}_t)(q_t \bar{q}_p)(q_t q_t q_t) \rangle \end{aligned} \quad (29)$$

These are exactly the four basic one-string-flip (or one-color-exchange) contributions used in VENUS [1] referred to as (a) non-diffractive (b,c) single diffractive and (d) double Pomeron exchange (contribution (a) having by far the largest weight).

We are now going to discuss multiple string flips. The string flip operator $\frac{1}{ag^2} U$ applied twice to the initial state $|i\rangle$ can provide a "double-string-flip":

$$-\frac{1}{ag^2} U -\frac{1}{ag^2} U |i\rangle = | (q_p q_p q_t)(q_p \bar{q}_t)(q_t q_t q_p)(q_t \bar{q}_p) \rangle \quad (30)$$

resulting in two diquark-quark strings and two quark-antiquark strings. Eq. (30) corresponds to the double-string-flip contribution eq. (3) for VENUS. In a similar fashion an arbitrary n -string-flip in VENUS corresponds to an operator $(\frac{1}{ag^2} U)^n$ acting on the initial state $|i\rangle$.

Let $|f\rangle$ be a two-hadron state differing from $|i\rangle$ (eq. (28)) by exactly n string flips. The transition amplitude is then, to lowest order in $\frac{1}{ag^2}$:

$$A_{if} = \frac{1}{i^n} \left(\frac{-1}{ag^2} \right)^n \sum_m \int dt_{n-1} \cdots \int dt_1 \langle f | U | m_{n-1} \rangle \cdots \langle m_1 | U | i \rangle. \quad (31)$$

In order to provide a non-zero contribution the intermediate states $|m_i\rangle$ have to be such that two consecutive states $|m_i\rangle$ and $|m_{i+1}\rangle$ differ by one of the n flips, which make the difference between $|m_0\rangle = |i\rangle$ and $|m_n\rangle = |f\rangle$. There are $n!$ possibilities, on the other hand we get a factor $1/n!$ by removing the time ordering in eq. (31), so we get

$$A_{if} = \left(\frac{i}{ag^2} \right)^n (A_1)^n \quad (32)$$

with some average 1-string-flip matrix element

$$A_1 = \int dt \langle m_{i+1} | U | m_i \rangle, \quad (33)$$

so the probability for n string flips is

$$w_n = \left(\frac{A_1^2}{a^2 g^4} \right)^n \equiv c^n. \quad (34)$$

This means: by applying strong coupling QCD perturbation theory we recover the exponential distribution e^n (eq. (5)) for the probability w_n to have n string flips, which has been used in VENUS!

4. Summary

We demonstrated that the prescription of string formation in hadron-hadron scattering, as used by the phenomenological string model VENUS [1], can be derived using the strong-coupling QCD expansion.

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Fig. 1. Gauge invariant eigenfunctions of the strong coupling Hamiltonian H_0 : (a) a closed string and (b) a quark and antiquark linked by a string.

Fig. 2. The plaquette operator $U(p)$ deforming a string path.

Fig. 3. (a) Two basic strings (one link long) at a large distance Na and (b) the two strings after string flip. The mixing between the two configurations is very small.

Fig. 4. The effect of the perturbation Hamiltonian H_1 on elementary hadronic states, just one link apart: the operator $q^+ \alpha U q$ provides fusion (a), the operator U provides string-flips between mesons (b) as well as baryons (c).