CERN LIBRARIES, GENEVA

CM-P00058798

Ref. TH.1297
26 February 1971SELF-CONSISTENT RISING CROSS-SECTIONSJ. Finkelstein^{*)} and F. Zachariasen^{**)}

CERN, Geneva, Switzerland.

ABSTRACT

It is shown that the multiperipheral model, when corrected for absorption, can accommodate increasing total cross-sections. In this case, a particle appears as a totally absorbing disk, with a radius which increases with energy; this property is independent both of the properties of the production amplitudes when not all of the subenergies are large, and also of the details of the absorption. If one does use the absorbed multiperipheral matrix element over all of phase space, then the resulting bootstrap has a self-consistent solution given by

$$T(s, t) = 4\pi i s R_0 \log s (-t)^{-1/2} J_1(R_0 \log s (-t)^{1/2})$$

*) A.P. Sloan Foundation Fellow, on leave from Stanford University, Stanford, Calif., USA.

**) J.S. Guggenheim Memorial Foundation Fellow, on leave from Calif. Inst. of Technology, Pasadena, Calif., USA.

The absorptive part of an elastic scattering amplitude can be determined, through s-channel unitarity, from a knowledge of the n-particle production amplitudes. The most popular contemporary model of production amplitudes is the multiperipheral model¹⁾. In this model, the production amplitudes are presumed to factor into separate pieces, correlating adjacent particles in the multiperipheral chain. The separate factors are themselves determined by the elastic scattering amplitude. The phrase "multiperipheral bootstrap" is therefore appropriate, in that self-consistency can, in principle, be required between the "input" elastic amplitude in terms of which the production amplitude is expressed, and the "output" elastic amplitude calculated through the unitarity relation. In practice, however, no one has yet succeeded in finding a self-consistent solution to this program; the best that has been achieved is the imposition of some constraints at $t = 0$.

One of these constraints is that if any solution does exist, it must be one in which the total cross-section falls to zero as the energy increases; in particular if the leading j-plane singularity at $t = 0$ is a pole, then the total cross-section falls as a power of the energy. Experimentally, this power must of course be very small. Theoretically, it is not obvious that it must be small; therefore the experimental near-constancy of total cross-sections must, in the multiperipheral bootstrap model, be regarded as a coincidence.

In this paper we show that, by including the effects of absorptive corrections to the multiperipheral matrix element, it is possible to obtain a total cross-section which does not fall as the energy increases. With this modification, the scattering target appears as a black disk, with a radius increasing indefinitely with the energy. Outside of this radius there may be an additional semi-transparent region. This first result is independent of any assumption about the production amplitudes outside that tiny region of phase space in which all of the sub-energies are large.

If we make the stronger assumption that an absorbed multiperipheral matrix element describes the production process throughout all of phase space, then one obtains a bootstrap condition, and as our second result we are able to exhibit an amplitude which, in the limit $s \rightarrow \infty$, is a

self-consistent solution to this absorbed multiperipheral bootstrap at finite as well as at zero values of t .

* * *

A fundamental principle which is not taken into account in the multiperipheral bootstrap model is that of full $(n \rightarrow m \text{ body})$ s -channel unitarity. It is likely that almost any specific form for production amplitudes $T_{2 \rightarrow n}$ can be accommodated within a fully unitary theory if arbitrarily peculiar amplitudes $T_{m \rightarrow n}$ ($m, n > 2$) are permitted; hence multibody unitarity by itself is probably not enough to tell us much about $T_{2 \rightarrow n}$. We can, however, guess with some plausibility what the primary effect of imposing full unitarity, with physically reasonable $T_{m \rightarrow n}$, would be on a multiperipheral production amplitude: it would be to introduce absorption. The assumption of factorizability made in the multi-Regge model does not allow for initial or final state interactions; in a fully unitary theory these effects should be expected to exist.

What will be the qualitative result of introducing absorption into a multiperipheral production amplitude? It will evidently be to limit the size of the inelastic contributions to the unitarity equation. Crudely speaking, we could expect the inelastic contribution to be given by the usual multiperipheral result multiplied by some function which vanishes if the multiperipheral approximation diverges. Thus, for example, a multiperipheral model with a leading input singularity at $j = 1$ should, when absorbed, not be expected to lead to a violation of the Froissart bound. It is therefore likely that, in an absorbed multiperipheral model, self-consistent cross-sections need not decrease with increasing energy.

* * *

We write the unitarity relation for the elastic amplitude $T_{22}(s, t) = D(s, t) + i A(s, t)$ as

$$A(s, t) = \int d\Phi_2 T_{22} T_{22}^* + \sum_{n=3}^{\infty} \int d\Phi_n T_{2n} T_{n2}^* \quad (1)$$

Consider, for definiteness, the model suggested by Caneschi²⁾ in which the production amplitude T_{2n} is given by the multiperipheral expression for T_{2n} , which we call T_{2n}^{MR} , multiplied by the usual³⁾ absorption factor:

$$T_{2n} = (S_{22})^{1/2} T_{2n}^{MR} \quad (2)$$

where S_{22} is the elastic S-matrix. We wish to stress that our results are independent of the exact way in which the absorption is done, and would be completely unchanged if we were to replace $(S_{22})^{1/2}$ in Eq. (2) by $f(S)$, where $f(S)$ is any function of S which vanishes only when S does. To emphasize this, we write

$$T_{2n} = f(S) T_{2n}^{MR} \quad (3)$$

From Eqs. (1) and (3), we see that, in the impact parameter representation,

$$A(s, b) = T_{22}(s, b) T_{22}^*(s, b) + f^2(S) A^{(0)}(s, b) \quad (4)$$

where we have defined

$$A^{(0)} = \sum_{n=3}^{\infty} \int d\Phi_n T_{2n}^{MR} T_{n2}^{MR*} \quad (5)$$

$A^{(0)}$ has the interpretation of an "unabsorbed" multiperipheral expression for the overlap function.

The solution to Eq. (4) evidently has the property that

$$\begin{aligned} A \rightarrow \frac{1}{2}, \quad S \rightarrow 0 \quad \text{if} \quad A^{(0)} \rightarrow \infty \\ \text{and} \quad A \rightarrow T_{22} T_{22}^* \quad \text{if} \quad A^{(0)} \rightarrow 0 \end{aligned} \quad (6)$$

The function $A^{(0)}$ can be calculated by techniques which have been developed to sum multiperipheral contributions to the overlap function. In particular, we know that if the elastic amplitude which is the input into T_{2n}^{MR} is dominated by a Regge pole with intercept one, or indeed by

any singularity which yields an integrated cross-section which disappears no faster than $(1/\log s)$, then the leading contributions to $A^{(0)}$ from the region of phase space in which all subenergies are large will be given by a moving Regge pole whose intercept is greater than one. This means that, at least in the limit $s \rightarrow \infty$, $b/(\log s)^{1/2} = \text{const.}$, the contribution to $A^{(0)}(s, b)$ from that region of phase space alone will grow like a power of s , and contributions from the rest of phase space can only make $A^{(0)}(s, b)$ even larger. We therefore see that the solution to Eq. (4) will have the property

$$\lim_{s \rightarrow \infty} S'(s, b) = 0 \quad \text{at least for} \quad b \leq \text{const.} (\log s)^{1/2} \quad (7)$$

Thus there will be full absorption at least out to a radius of $(\log s)^{1/2}$; hence a lower limit to the total cross-section is $\log s$.

* * *

We now wish to exhibit a solution to the bootstrap equations obtained from the much stronger assumption that Eq. (3) correctly represents, at least on the average, the production amplitude in the whole of phase space. Under this assumption, and with various kinematic simplifications⁴⁾, the t-channel partial waves of $A^{(0)}$ are given by

$$A^{(0)}(t, j) = \frac{C_1 [A_{AFS}(t, j)]^2}{1 - C_1 A_{AFS}(t, j)} \quad (8)$$

where C_1 is a constant, and we have defined $A_{AFS}(t, j)$ to be the t-channel partial wave of

$$A_{AFS}(s, b) \equiv T_{22}(s, b) T_{22}^*(s, b) \quad (9)$$

Note that Eq. (9) is written in terms of s-channel partial waves, while Eq. (8) is in terms of t-channel partial waves.

We shall show that an (asymptotically) exact self-consistent solution to this bootstrap is given by

$$T(s, t) = 4\pi i s \frac{R_0 \log(s/s_0)}{(-t)^{1/2}} J_1(R_0 \log(s/s_0) (-t)^{1/2}) \quad (10)$$

[Our normalization is such that $A(s,0) = s\sigma_T(s)$.] This corresponds to scattering from a fully absorbing sphere of radius $R \log(s/s_0)$. Therefore, we have $A_{AFS}(s,t) = \frac{1}{2}A(s,t)$, and it is easy to deduce that

$$A_{AFS}(t,j) = \frac{2\pi R_0^2 s_0}{[(j-1)^2 - R_0^2 t]^{3/2}} \quad (11)$$

Thus A_{AFS} contains two complex conjugate cuts of the $(j - \alpha_c)^{-3/2}$ type with branch points at $\alpha_c^\pm = 1 \pm iR_0\sqrt{-t}$. From Eqs. (8) and (11) we find

$$A^{(0)}(t,j) \propto \left\{ [(j-1)^2 - R_0^2 t]^{3/2} - 2\pi R_0^2 s_0 C_1 \right\}^{-1} \quad (12)$$

This amplitude has two j -plane poles, at $j = \alpha_+$ and $j = \alpha_-$, where

$$\alpha_\pm = 1 \pm (C_2 + R_0^2 t)^{1/2} \quad (13)$$

and where we write $C_2 = (2\pi R_0^2 s_0 C_1)^{2/3}$. Note that for $t > -C_2/R_0^2$ and in particular at $t = 0$, $\text{Re } \alpha_+ > 1$. Therefore, if we stopped here, and pretended that the multiperipheral model result were the final answer, we would find a violation of the Froissart bound. However, for us $A^{(0)}$ is only an intermediate step in the calculation; in the absorbed multiperipheral model we must proceed.

The next step is to compute $A^{(0)}(s,t)$; this of course is simply given by $\beta_+(s/s_0)^{\alpha_+} + \beta_-(s/s_0)^{\alpha_-}$, plus the cut contributions, where $\beta^\pm$ are the residues of the poles at $\alpha_\pm$. We have, to be specific

$$\beta_\pm = \pm C_3 (C_2 + R_0^2 t)^{-1/2}$$

where C_3 is another constant. Neglecting the cut contribution we next calculate $A^{(0)}(s,b)$ and obtain

$$A^{(0)}(s,b) = \frac{2C_3 s}{R_0^2} \frac{\cosh \left[(C_2)^{1/2} \left([\log(s/s_0)]^2 - b^2/R_0^2 \right)^{1/2} \right]}{\left([\log(s/s_0)]^2 - b^2/R_0^2 \right)^{1/2}} \quad (14)$$

if $b < R_0 \log s/s_0$, and zero if $b > R_0 \log s/s_0$. Thus $A^{(0)}(s,b)$ blows up as $s \rightarrow \infty$ for b fixed, but vanishes if $b \rightarrow \infty$ when s is fixed.

Finally, to complete the cycle, we use Eq. (4) to recompute $A(s,b)$ and hence $A(s,t)$. In fact we need only the property (6), from which we see that we recover precisely the result assumed in Eq. (10). The bootstrap is complete, and Eq. (10) has been shown to be fully self-consistent.

We conclude with several comments:

i) Our final result is exactly the same as that obtained in Quantum Electrodynamics, and certain other field theories, by summing diagrams⁵⁾. The detailed mathematics differs considerably in many important ways, but the conclusion is identical. We consider this to be support for the physical correctness of the final picture.

ii) It is likely that the conclusion we have obtained will continue to hold even if many of the detailed assumptions that we have made are false. For example, other methods of absorbing the multiperipheral model may be envisaged; however, we believe that nearly any of them will lead to similar results. As an illustration, one may imagine Eq. (4) replaced by an eikonal formula [this is what happens in QED⁵⁾]; nevertheless, if $A^{(0)}$ behaves the same way, we still end up with Eq. (10). Further, it may well be that the simple version of the multiperipheral model we used to calculate $A^{(0)}$, namely Eq. (8), is faulty. However, more sophisticated multiperipheral models will still yield an $A^{(0)}$ violating the Froissart bound if the input is as in Eq. (10), and this is the only result we really need.

The point is that our conclusion, because it depends essentially only on saturating the Froissart bound, is insensitive to any details as to how that saturation occurs.

iii) It is apparent that the amplitude suggested in Eq. (10) does not, by itself, satisfy t -channel unitarity. There is no reason why it should, since it is only intended to be that part of the amplitude

which, for negative t , gives the leading behaviour at large s , and we know of no constraints which t -channel unitarity places on this part^{*)}.

iv) The amplitude given in Eq. (10) is, for $t \neq 0$, a mixture of even and odd signature in the t -channel. We could obtain an amplitude which had only even signature by using Eq. (10) for the imaginary part, and then determining the real part through a dispersion relation. In this case, the proof we have given for exact self-consistency would not be valid, except at $t = 0$.

v) In spite of the wiggles which have been observed in the proton-proton elastic angular distribution⁶⁾, and in spite of some very indirect indications that some total cross-sections may be slowly increasing at high energies⁷⁾, we must concede that there is at present no compelling evidence that elastic scattering amplitudes look at all like the expressions in Eq. (10). We can only say that our results are strictly asymptotic ones, and that the asymptotic region may lie at an energy higher than is now available.

Acknowledgements

We should like to thank the Theoretical Division of CERN for its hospitality.

*) For example, it is not true that the trajectory function of a moving pole, or of a "hard" cut, must contain a singularity at the t -channel threshold t_0 . As an illustration consider the amplitude $T(t, j) = ((t_0 - t)^{\frac{1}{2}} - (t_0 - 2t + j - 1)^{\frac{1}{2}})^{-1}$, which is unitary for each fixed t when j is sufficiently large and which has a pole at $\alpha(t) = t + 1$.

References and Footnotes

- 1) See, for example, G.F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).
- 2) L. Caneschi, Phys. Rev. Letters 23, 257 (1969).
- 3) See, for example, K. Gottfried and J.D. Jackson, Nuovo Cimento 34, 735 (1964).
- 4) M.L. Goldberger, C.-I. Tan and J.-M. Wang, Phys. Rev. 184, 1920 (1969).
- 5) H. Cheng and T.T. Wu, Phys. Rev. Letters 24, 1456 (1970).
- 6) J. Allaby et al., Phys. Letters 38 B, 67 (1968).
- 7) V. Barger and R.J.N. Phillips, Phys. Letters 31 B, 643 (1970).