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TRIPLE REGGE LIMIT IN THE ABSORPTIVE MULTIPERIPHERAL MODEL

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A B S T R A C T

The $x \rightarrow 1$ limit of the inclusive one-particle distribution is studied in the absorptive multiperipheral model : we find a leading behaviour of the type $(1-x)^{-1} \ln^{-3}(1-x)$, which gives an integrable contribution to the energy conservation sum rule.

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1. - MOTIVATIONS

One of the most attractive properties of the multiperipheral (MP) model in the ladder approximation, the existence of a leading Regge pole in the imaginary part of the forward elastic amplitude ¹⁾, turns out, at a close examination, to be more a liability than an asset. Apart from the fact that the intercept of this pole moves indefinitely to the right with increasing coupling constant g , thus violating the Froissart's bound for large values of g , the existence of a leading pole is hard to reconcile with the requirement that diffractive channels exist, i.e., that σ_{tot} approaches a constant (at least within logarithmic factors) asymptotically.

In fact, in order to satisfy this requirement we should have the leading pole at $\alpha(0) = 1$, but the repeated exchange of such a pole in the MP ladder leads again to a violation of the Froissart's bound ²⁾. From another point of view, the existence of a leading pole in the ladder approximation leads to inclusive short range correlations, and therefore to a probability for producing a fixed number of particles n $P_n(s) = (\sigma_n(s)/\sigma_{\text{tot}}(s))$ that decreases faster than any inverse power of $\ln s$ ³⁾, against what one would expect in the presence of diffraction (for instance, if $\sigma_{\text{tot}} \rightarrow \text{const}$, σ_{el} should behave at worst like $\ln s^{-p}$ from the optical theorem).

In the inclusive language the difficulties related to the existence of a leading factorizable pole with $\alpha(0) = 1$ manifest themselves in the non-integrable behaviour $(1-x)^{-1} \ln^{-1}(1-x)$ of the scaling function

$$f(x) = \frac{1}{\sigma_{\text{tot}}(s)} \int E \frac{d^2 \sigma}{d^2 p_{\perp} d p_L} d^2 p_{\perp}$$

in the limit $x \rightarrow 1$ ⁴⁾.

These inconsistencies can be circumvented by decoupling the leading pole at $t = 0$ from the inelastic channels but recently Brower and Weiss ⁵⁾ have given strong arguments to show that in this case the leading pole decouples also from the elastic channel (and therefore cannot be used to obtain a constant total cross-section).

In view of these difficulties the recent results of Bassetto, Sertorio and Toller ⁶⁾ become particularly interesting. These authors have shown that some properties that, in the ladder approximation, are immediately

understood as consequences of the existence of a leading pole (like the $\ln^p s$ behaviour of $\langle n^p \rangle$), actually follow from the validity of an upper bound on the modulus of the production amplitudes for large values of the momentum transfers t_i . This suggests a way of extending the definition of multi-peripheralism beyond the ladder approximation, and therefore with the possibility of including models in which the leading singularity P is not a factorizable pole.

A natural possibility is to consider the ladder approximation as a Born term, and try to enforce some s channel unitarity by taking into account initial state absorptive correction, that can be chosen to have the single form (multiplicative in impact parameter b) $S_{e\ell}^{\frac{1}{2}}(b,s)$ ⁷⁾. Since the absorbed ladder decreases rapidly in the momentum transfers t_i if the unabsorbed ladder does, this procedure generates a multiperipheral scheme [in the generalized sense of Ref. 6] that prevents itself from violating the Froissart's bound by essentially the same mechanism present in the eikonal framework ⁸⁾.

Finkelstein and Zachariasen ⁹⁾ have shown that a self-consistent leading singularity P can be found in this scheme. The solution to this bootstrap has a simple representation in impact parameter, for instance $\Theta(\ln s - b)$, and, as expected, saturates the Froissart's bound.

Even if there is little phenomenological support for this model, and also some theoretical features [like the "hard" cut $((J-1)^2 - t)^{-\frac{3}{2}}$] are unpalatable, the model offers a convenient test ground for the effects that unitarity corrections and long range correlations are likely to have on the ladder approximation. In a previous paper ¹⁰⁾ (I) we have studied the behaviour of the partial cross-sections $\sigma_n(s)$ and shown that they approach a constant asymptotically : thus in this model, due to the non-factorizable nature of P and the consequent long range correlations, diffraction can be accommodated without leading to any inconsistency. Also in (I) we have studied the inclusive distributions in the pionization region and shown that they are scaling, as expected. Here we want to study the behaviour of the one particle inclusive distribution in the "triple Regge" limit ^{*)} to see how the inconsistency found when the leading singularity is a factorizable pole is avoided in the non-factorizable absorptive scheme.

*) We keep using the notation "triple Regge" limit meaning the $x \rightarrow 1$ limit of the inclusive distribution : remember, however, that a triple Regge vertex cannot be defined in a non-factorizable scheme like the one that we are considering.

2. - CALCULATIONS

Since there is a close similarity between the mechanism that determines the "triple Regge" limit and the one that produces the asymptotic behaviour of the partial cross-sections, we briefly recall from (I) the impact parameter representation of the partial cross-sections σ_n in the absorptive multiperipheral model :

$$\sigma_n(Y, \vec{B}) = \frac{[g^{1/3}(Y^2 - B^2)^{1/2}]^{3n-5}}{(3n-5)!} \exp(-g^{1/3}(Y^2 - B^2)^{1/2}) \theta(Y^2 - B^2) \quad (1)$$

(from now on $Y = \ln s$).

The integral over d^2B that has to be performed in order to obtain $\sigma_n(s)$ can be divided in two regions : a central region that, asymptotically in Y , includes any finite B , in which the factor $\exp(-g^{1/3} \sqrt{Y^2 - B^2})$ gives a vanishing factor of the type $s^{-g^{1/3}}$, and a fringe contribution in which $Y^2 - B^2$ remains finite, that contributes the asymptotic constant term to $\sigma_n(s)$.

Following closely the procedure of (I), let us now compute the impact parameter representation of the inclusive distribution of the first particle of the multiperipheral chain. We obtain, in analogy to Eq. (34) of (I) :

$$\begin{aligned} \frac{d\sigma}{dy}(b, b') &= \frac{1}{g^{(1-x)}} \int d^2\vec{B} d^2\vec{B}' \delta^{(2)}(\vec{B} - \vec{B}' - \vec{b} + \vec{b}') \\ &\quad \theta(Y^2 - B^2) \theta(y^2 - b^2) \theta(y'^2 - b'^2) \theta((Y-y)^2 - (\vec{B} - \vec{b})^2) \theta(Y^2 - B'^2) \\ &\quad [(Y^2 - B^2)(Y'^2 - B'^2)]^{1/4} [(Y-y)^2 - (\vec{B} - \vec{b})^2]^{-1/2} \\ &\quad \exp \left\{ g^{1/3} [(Y-y)^2 - (\vec{B} - \vec{b})^2]^{1/2} - \frac{g^{1/3}}{2} (Y^2 - B^2)^{1/2} - \frac{g^{1/3}}{2} (Y'^2 - B'^2)^{1/2} \right\} \end{aligned} \quad (2)$$

Here x is the Feynman scaling variable, and y is the rapidity gap between the first and the second particle, related to x in the $x \rightarrow 1$ limit by $y \simeq -\ln(1-x)$. The impact parameters $\vec{B}, \vec{B}', \vec{b}, \vec{b}'$ are defined in the figure, and the $\delta^{(2)}(\vec{B}-\vec{b}-\vec{B}'+\vec{b}')$ factor is a consequence of the fact that in the missing mass variable $Y-y$ we are computing a total cross-section. The two exponentially decreasing factors in $\log s$ represent the effect of the absorption in the initial state whereas the exponentially increasing factor in the logarithm of the missing mass $Y-y$ is the shadow of the multiple Pomeron exchange.

From (2) the usual double differential cross-section can be computed

$$\frac{d^2\sigma}{dy dt} = \int \frac{d\sigma}{dy}(\vec{b}, \vec{b}') J_0(b\sqrt{-t}) J_0(b'\sqrt{-t}) d^2\vec{b} d^2\vec{b}'$$

but this quantity is not easy to compute. However, in a theory in which the Pomeron is a factorizable pole, the difficulties related with the triple Pomeron coupling appear in the large y (i.e., $x \rightarrow 1$) behaviour of

$$f(x) = \frac{1}{\sigma_{tot}(s)} \int \frac{d^2\sigma}{dy dt} dt$$

which, in our model, has the simpler form

$$f(x) = \frac{1}{Y^2} \frac{1}{g(1-x)} \int d^2\vec{B} d^2\vec{b} [(Y-y)^2 - (\vec{B}-\vec{b})^2]^{-1/2} \\ (Y^2 - B^2)^{1/2} \Theta(Y^2 - B^2) \Theta(y^2 - b^2) \Theta((Y-y)^2 - (\vec{B}-\vec{b})^2) \\ \exp \left\{ g^{1/3} [(Y-y)^2 - (\vec{B}-\vec{b})^2]^{1/2} - g^{1/3} [Y^2 - B^2]^{1/2} \right\}$$

In this integral, we can distinguish two regions, in analogy with the case of the partial cross-section : for fixed $\vec{B}$, the term $\exp[-Y^2 - B^2]^{\frac{1}{2}}$ gives asymptotically a factor that decreases like a power of s . The increasing term $\exp[(Y-y)^2 - (\vec{B}-\vec{b})^2]^{\frac{1}{2}}$ gives a correspondent increase like a power of M^2 , and the two combine to change the dangerous $(1-x)^{-1}$ factor into an integrable $(1-x)^{-1+g^{1/3}}$.

Again we expect the leading behaviour in the $x \rightarrow 1$ limit to be provided by the edge of the disk $|\vec{B}| \simeq Y$. To compute this contribution let us remark that, at fixed $Y, y, \vec{b}$, the maximum in the exponential factor is obtained for $\vec{B} = \vec{b}(Y/y)$: therefore we change variables from $\vec{B}$ to $\vec{A} = \vec{B} - \vec{b}(Y/y)$, and expand the exponential around $\vec{A} = 0$, setting $\vec{A} = 0$ in the other factors. We obtain :

$$\begin{aligned} & \left[(Y-y)^2 - (\vec{B}-\vec{b})^2 \right]^{\frac{1}{2}} - (Y^2 - B^2)^{\frac{1}{2}} \rightarrow -(y^2 - b^2)^{\frac{1}{2}} - \\ & - \frac{y}{2Y(Y-y)} \frac{1}{(1 - b^2/y^2)^{3/2}} \left[A^2 \left(1 - \frac{b^2}{y^2}\right) + \frac{(\vec{A} \cdot \vec{b})^2}{y^2} \right] + O(A^4) \end{aligned}$$

The $d^2\vec{A}$ integration in Eq. (3) can now be explicitly performed, and yields

$$\begin{aligned} f(x) & \simeq \frac{1}{(1-x)} \quad y^{-3} \int_0^y d^2b \quad e^{-g^{1/3} \frac{(y^2 - b^2)^{\frac{1}{2}}}{(y^2 - b^2)}} \\ & = (1-x)^{-1} \left[\ln^{-3}(1-x) + O((1-x)^{g^{1/3}}) \right] \end{aligned} \quad (4)$$

As expected, the leading contribution is provided by the region $|\vec{b}| \simeq y$, that corresponds to $|\vec{B}| \simeq Y$. The finite b region contributes terms that vanish like a power of $(1-x)$ with respect to this border contribution in the $x \rightarrow 1$ limit.

3. - CONCLUSIONS

The main result of our calculation is the appearance of the factor $\ln^{-3}(1-x)$ instead of the usual $\ln^{-1}(1-x)$ obtained after the t integration in the coupling of three moving factorizable poles with intercept equal to one *).

The two extra powers of $\ln(1-x)$ in the denominator are sufficient to ensure a convergent result for the contribution of the $x \simeq 1$ region to the energy sum rule integral : therefore the absorptive multiperipheral model does not present any problem in this respect, and the energy conservation constraint $\int_0^1 f(x)dx = 1$ can be enforced by properly choosing the details of the model.

The problem of computing the full double differential cross-section $d^2\sigma/dtdy$, rather than its integral over dt , is technically harder. We consider a safe guess that the integration over dt produces a factor $\ln^{-2}(1-x)$ through the strong shrinkage that characterizes this model : therefore, $(1/\sigma_{\text{tot}})(d^2\sigma/dtdy)$ is expected to behave like $(1-x)^{-1} \ln(1-x)$ at $t = 0$. Another feature of the t distribution that can be anticipated on the basis of our calculation is that the $x \simeq 1$ region will be unusually sharp in t , since, as we have seen, the highest partial waves are responsible for the leading term in the $x \rightarrow 1$ limit. The mechanism that produces this sharpening is the same that sharpens the forward peak of two-body reactions in the presence of absorption.

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*) Let us remark that the non-factorizable nature of the leading singularity again plays an essential rôle in obtaining these results : if our three Pomerons were factorizable, i.e., if we were to apply absorption in the missing mass channel, we would obtain the same results in Eq. (4) but without the factor $e^{-g^{1/3}(y^2-b^2)^{1/2}}$, and $f(x)$ would behave like $(1-x)^{-1} \ln(1-x)$. Upon integration over x up to $1-O(s^{-1})$ we would obtain a violation of the energy conservation sum rule of order $\ln^2 s$, as naïvely expected from the coupling of three triple poles (three times $\ln^2 s$ minus $\ln^2 s$ from the shrinkage and minus $\ln^2 s$ from the behaviour of σ_{tot}).

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FIGURE CAPTION

Definition of rapidities and impact parameters.