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ASYMPTOTIC BEHAVIOUR OF MULTIPLICITIES IN MODELS
WITH INTERACTING POMERONS

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ABSTRACT

Using renormalization group techniques, the asymptotic behaviour of $\langle n^p \rangle$ in models with strong coupling Pomeron interactions is found to be $\ln s^{p(1+\gamma)}$. In the Gribov Reggeon calculus $\gamma = \eta$ ($\eta = \lim_{s \rightarrow \infty} \ln \sigma_T(s) / \ln \ln s$), in an absorptive model $\gamma = -\eta$. The values of $\langle n^p \rangle / \langle n \rangle^p$ are calculated perturbatively. The Gribov model results are shown to be consistent with positivity requirements, and in surprisingly good agreement with experimental values at present energies.

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1. PROBLEM AND METHOD

It is attractive, and also suggested by high-energy phenomenology, to assume that the Pomeron (P) singularity has intercept exactly one at $t = 0$. If this is the case, the cuts corresponding to multiple P exchange have all a branch point at $J = 1$ for $t = 0$, and the problem of the nature of the P singularity is formally similar to the problem of the infrared behaviour of a theory containing massless particles.

The renormalization group techniques (or alternatively the Wilson method) have recently been exploited¹⁻³⁾ for the study of the asymptotic properties of the elastic amplitude. Some properties of inelastic processes (behaviour of $\langle n \rangle$ and of σ_n) have also been considered in the framework of the Gribov Reggeon calculus (GRC), in Ref. 1.

It is our purpose here to investigate systematically the high-energy behaviour of the multiplicity moments in a variety of models for which the asymptotic limit of A_{el} is determined by a Pomeron interaction of the form

$$L_I = \frac{i}{2} g \psi \psi^\dagger (\psi + \psi^\dagger) \quad (1)$$

where ψ is the Pomeron field. This form of interaction is the relevant one in the ϵ -expansion³⁾, and from the results of Refs. 1 and 2 we know that a fixed point exists at $g^* = f(\epsilon)$ [$\epsilon = 4-D$, D being the dimensionality of the space of the transverse momentum, and $f(0) = 0$]. The high-energy limit of the total cross-section is consequently $\sigma_T(s) \approx (\ln s)^\eta$, $\eta = (\epsilon/12) + O(\epsilon^2)$. Since we want the optical theorem to hold, it must be possible to express the contribution of each Reggeon graph to $\text{Im } A_{el}(s, 0)$ summing its possible s -channel discontinuities. It turns out that these cutting rules are not uniquely determined by this requirement. Indeed we will explicitly consider two schemes, the former inspired by the rules obtained by Gribov and co-workers⁴⁾ from the study of classes of Feynman diagrams, the latter suggested by absorptive model considerations⁵⁾.

In both schemes we will assume the validity of the following property: the possible ways of cutting that give a contribution to the leading term in $\text{Im } A_{el}$ have each P-line completely cut or completely uncut (for a discussion of this point, see Ref. 4).

Let us now construct the generating function

$$S(z, s) = \sum z^n \sigma_n(s) \quad (2)$$

If a graph contains k cut lines, that produce $n, \dots, n_k$ particles ($\sum n_k = n$) we write $z^n = \prod z^{n_i}$, and attribute each factor z^{n_i} to the corresponding i^{th} cut line.

We will now assume that the n -particle discontinuity of a P -line behaves as it does in a ladder model, i.e. that upon multiplying by z^n and summing over n , one obtains a shift in the intercept of the Regge pole under consideration. We can therefore compute $S(z,s)$ by formally considering a theory of two different Pomerons in interaction with each other: a cut Pomeron (CP), of intercept $\alpha_c(z)$, and an uncut Pomeron (UCP) of intercept $\alpha_p = 1$.

For the purpose of computing the leading behaviour of the multiplicity moments, it is sufficient to expand $\alpha_c(z)$ to first order around $z = 1$:

$\alpha_c(z) = 1 + c(z - 1)$. The value of c is irrelevant for our analysis, hence $c = 1$ from now on^{*}).

In order to be able to apply the renormalization group method, we need that $\alpha_c(z)$ stays around 1: therefore we expand $S(z,s)$ in terms of the multiplicity moments $n_p = \langle n(n-1) \dots (n-p) \rangle$

$$S(z,s) = \sum \frac{1}{p!} (z-1)^p n_p(s) \sigma_T(s) \quad (3)$$

Hence the values of n_p can be obtained by taking the p^{th} derivative of the total cross-section with respect to α_c at $\alpha_c = 1$.

In the language of field theory the derivatives with respect to α_c (which has to be considered as the bare mass of the field describing the cut lines) correspond to the mass insertions. Therefore our problem is to find the anomalous dimensions of a mass operator. This is strongly reminiscent of the techniques used in the theory of critical phenomena to find the dependence on the temperature T around $T = T_c$.

2. THE RENORMALIZATION GROUP EQUATION

Generally it is not possible to construct the allowed s -channel discontinuities introducing a Lagrangian formalism to describe the interaction of the CP and the UCP, since it happens that diagrams constructed with allowed vertices do not correspond to allowed discontinuities. Nevertheless, the allowed discontinuities satisfy (at least in the models considered here) a Dyson equation,

^{*}) To be precise, the quantity which is related to z is the bare value of $\alpha_c(z)$, i.e. its value without taking into account interactions among Pomerons. In general, $\alpha_c(z) = 1 + \delta + c(z - 1) + \dots$. Obviously the value of δ is immaterial for us. The analogy with the ladder would suggest that also α'_c depends (approximately linearly) on z , and in principle we should consider also the dependence on z of $S(z,s)$ which comes in through the α'_c dependence. However, it is easy to see that in the spirit of the ϵ -expansion these contributions are negligible, since they have only anomalous dimensions and no "normal" one [the contribution to n_p coming from $p \alpha'_c$ derivatives would be of the form $\ln s^{p\gamma_5}$, with $\gamma_5 = O(\epsilon)$, to be compared with the $\ln s^{p(1+\gamma_4)}$ that comes from through the z -dependence of α_c].

which is all we need to apply the renormalization group methods. (A detailed discussion of this point can be found in the appendix.)

Let us define $\Gamma^{(n,m;k,\ell)}$ to be the amputated Green functions for m incoming and n outgoing UCP, k incoming and ℓ outgoing CP. We work as usual^{1,2)} in the space of t -channel angular momentum J , and define $E = 1-J$, in such a way that the free P propagator is $i(E - \alpha' k^2 + i\epsilon)^{-1}$, and k is a D -dimensional transverse momentum vector. The renormalization group analysis is carried out at $D = 4-\epsilon$, and ϵ is treated as a small parameter. To study the asymptotic behaviour of the theory, we have therefore to determine its $E \rightarrow 0$ limit.

We show in the appendix that the equations for all $\Gamma^{(m,n;k,\ell)}(E_i, k_i)$ reduce in the $z \rightarrow 1$ limit to the equation satisfied by $\tilde{\Gamma}^{(m+k;n+\ell)}(E_i, k_i)$ of the usual theory containing uncut Pomerons only. We are therefore guaranteed that the asymptotic behaviour of our two-component theory is controlled by a fixed point in the space of the various coupling constants in which they all approach the common value g^* obtained in Refs. 1 and 2.

Therefore the anomalous dimension that controls the $E \rightarrow 0$ limit of $\Gamma^{(0,0;1,1)}$ is the same η that determines $\tilde{\Gamma}^{(1,1)}$. As long as the anomalous dimensions represent a perturbation with respect to the normal ones, the leading contribution to the multiplicity moments is determined by the n -fold mass insertions in the CP propagator:

$$A_n^B = \frac{\partial^n}{\partial d_c^n} \Gamma^{(0,0;1,1)}(E, k^2=0) \Big|_{d_c=1} \quad (4)$$

Since A_1^B is logarithmically divergent (for $D = 4$) we introduce a renormalized quantity A_1 by the condition ($E_0 < 0$):

$$A_1(E_0, 0) = -i \quad (5)$$

The renormalized A_n are now completely specified. Formally this renormalization defines a renormalized mass insertion operator $(\phi^2)^R = Z_4^{-1}(\phi^2)^B$. Hence $A_n^B = Z_4^n A_n$. The dimensions of A_n are E^{1-n} . We can now combine the renormalization group equation and the dimensional analysis as usual^{1,2)} to obtain at the fixed point

$$\left(E \frac{\partial}{\partial E} + (n-1) + n \gamma_4 - \eta \right) A_n(E, 0) = 0 \quad (6)$$

The normalization condition (4) determines $\gamma_4 = -E_0(\partial/\partial E_0) \ln Z_4$ to be

$$\gamma_4 = \eta - i E_0 \frac{\partial}{\partial E} A_1(E, 0) \Big|_{E=E_0} \quad (7)$$

The behaviour of A_p when $E \rightarrow 0$ is therefore:

$$A_p \rightarrow E^{1-p-p\gamma_4+\eta} \quad (8)$$

The Mellin transform of $\sigma_T(s) n_p(s)$, given by the p^{th} derivative with respect to α_c of the complete propagator of the cut Pomeron $[\Gamma^{(0,0;1,1)}(E, k^2 = 0)]^{-1}$, has therefore the following behaviour for $E \rightarrow 0$:

$$M(\sigma_T(s) n_p(s)) = c E^{-1-p-\gamma_4-\gamma} F_p(E_0) \quad (9)$$

and

$$M(\sigma_T(s)) = c [\Gamma^{(0,0;1,1)}(E, 0)]^{-1} = c E^{-1-\gamma} F_0(E_0) \quad (10)$$

Remembering that

$$\Gamma(x+1) \int dE s^{-E} E^{-x-1} = (\ln s)^x \quad (11)$$

we obtain

$$m_p = \frac{F_p(E_0) \Gamma(1+\gamma)}{F_0(E_0) \Gamma(1+\gamma+p(1+\gamma_4))} \ln s^{p(1+\gamma_4)} \quad (12)$$

3. CALCULATION OF THE ANOMALOUS DIMENSIONS

In this section we want to compute the value of γ_4 perturbatively in g^* , i.e. in ϵ . To first order we have to consider the one-loop insertion in the CP propagator. Three possible loops have to be considered, according to whether zero-, one-, or two-loop lines are CP (Fig. 1). Putting α' equal to 1 from now on, we have:

$$A_i = \frac{\partial}{\partial \alpha_c} \left[-i (E + \alpha_c - 1) - \frac{g^{*2}}{2} \left(\sum (\alpha_c, E) - \sum (\alpha_c, E_0) \right) \right]_{\alpha_c=1} \quad (13)$$

$$\begin{aligned} \sum (\alpha_c, E) = \int dE' d^2k & \left[\frac{A}{(E-E'-k^2)(E'k^2)} + \frac{B}{(E-E'-k^2)(E'+(\alpha_c-1)-k^2)} \right. \\ & \left. + \frac{C}{(E-E'+\alpha_c-1-k^2)(E'+\alpha_c-1-k^2)} \right] \quad (14) \end{aligned}$$

The relative weights A, B, C of the three contributions depend on the model that we want to consider. The first graph (Fig. 1a) represents the contribution of the large-gap events, the second (Fig. 1b) the effect of absorption (i.e. elastic rescattering) on normal (ladder) production, whereas the third graph (Fig. 1c), in which both P-lines are cut, represents the contribution of double multiplicity inelastic rescattering events.

In the Gribov Reggeon calculus (GRC) all three are allowed, with weights⁴⁾ $A = 1$, $B = -4$, $C = 2$. Obviously $A + B + C = -1$, corresponding to the usual negative two-Pomeron cut. An alternative way of flipping the sign of the Amati-Fubini-Stanghellini cut is provided by the s-channel-unitarity motivated absorptive model⁵⁾ (AM), in which the double multiplicity events are not considered,

and the weights are $A = 1$, $B = -2$, $C = 0$ *) . We will also consider for reference a simplified absorptive model (SAM) in which the contribution of large-gap events is not taken into account, and the weights are $A = 0$, $B = -1$, $C = 0$ **).

It is simple to check that in the GRC $(\partial/\partial\alpha)\Sigma = 0$, and therefore $\gamma_4 = \eta$. This happens because the contribution from double multiplicity events exactly makes up for the depletion caused by absorption, and the result $\gamma_4 = \eta$ actually holds to all orders in ϵ , as a result of the properties of the cutting rules in the GRC, as discussed at more length in the appendix.

For the AM one has

$$\frac{\partial}{\partial\alpha} \Sigma(\alpha, E) \Big|_{\alpha=1} = 2 \frac{\partial}{\partial E} \Sigma(1, E) \quad (15)$$

Since η is determined at order ϵ by²⁾:

$$\eta = -\frac{i}{2} g^2 E_0 \frac{\partial}{\partial E_0} \frac{\partial}{\partial E} \Sigma(1, E) \Big|_{E=E_0} \quad (16)$$

we obtain from (6) $\gamma_4 = -\eta$. In the SAM, instead, $\gamma_4 = 0$, since:

$$\frac{\partial}{\partial\alpha} \Sigma(\alpha, E) \Big|_{\alpha=1} = \frac{\partial}{\partial E} \Sigma(1, E) \quad (17)$$

We thus obtain

$$\begin{aligned} \text{GRC: } n_p &\approx \ln s^{P(1+\eta)}; & \text{SAM: } n_p &\approx \ln s^P \\ \text{AM } n_p &\approx \ln s^{P(1-\eta+O(\epsilon^2))} \end{aligned} \quad (18)$$

These behaviours are simply interpreted physically. In the SAM there is only one cut line, with no large rapidity gaps, and therefore the multiplicity distribution is unaffected by the Pomeron interactions. In the AM the presence of large gaps decreases the multiplicity and its moments by powers of $\ln s$. This effect is overcompensated in the GRC by the inelastic rescattering events, and the net result in this model could be obtained by naively considering a Muller diagram and inserting p times the behaviour of σ_T (i.e. the two-Pomeron/two-particle vertex carries no anomalous dimensions). In particular, the presence of inelastic rescattering destroys the boundedness of the invariant momentum transfer limitation that presumably holds for the production from one P-line, and therefore the theorem⁸⁾ that forbids n_p to grow faster than $(\ln s)^P$ no longer applies.

*) Recently this count has been obtained⁶⁾ also for the Mandelstam diagram, at variance with the so far accepted GRC one.

**) We reproduce in this way the results of Ref. 7.

4. CALCULATION OF THE GENERATING FUNCTION

In the previous paragraph we have computed the asymptotic behaviour of n_p , and it turned out that in all models the coefficients $c_p = n_p / \langle n \rangle^p$ are asymptotically constant. We now turn our attention to a perturbative calculation of C_p (obviously $c_p = 1$ in the $\epsilon \rightarrow 0$ limit).

The Mellin transforms of σ_T and of $\sigma_T n_p$ are given in our scheme by:

$$M[\sigma_T] = c \left(E/\epsilon_0 \right)^{-1-\gamma} 1/\epsilon_0 \quad (19)$$

$$M[\sigma_T n_p] = c \left(\frac{E}{\epsilon_0} \right)^{-1-\gamma-P(1+\gamma_4)} \frac{\partial^p}{\partial \epsilon^p} \left[P(\epsilon_0)^{(q_0; 1,1)} \right]^{-1} \Big|_{\epsilon=\epsilon_0} \quad (20)$$

The Mellin transform $\psi(z, E)$ of the generating function $S(z, s)$ is therefore given by

$$\begin{aligned} \psi(z, E) &= \sum \frac{1}{p!} (z-1)^p M[\sigma_T n_p] = \\ &= c \left(\frac{E}{\epsilon_0} \right)^{-1-\gamma} \left[\epsilon_0 + (z-1) \left(\frac{E}{\epsilon_0} \right)^{-1-\gamma_4} + P(\epsilon_0, (z-1) \left(\frac{E}{\epsilon_0} \right)^{-1-\gamma_4}) \right]^{-1} \end{aligned} \quad (21)$$

The function P is given to first order in ϵ in the various models by (from now on $\epsilon_0 = -1$ is no longer indicated):

$$P^{GR}(x) = -\frac{\epsilon}{6} \left[\ln(1-x) - (1-2x) \ln \left(\frac{1-2x}{1-x} \right) \right] \quad (22)$$

$$P^{AM}(x) = 2 P^{SAM}(x) = \frac{\epsilon}{6} \left[x \ln(1-x) - x - \ln(1-x) \right] \quad (23)$$

The structure of the SAM is particularly simple: in fact to first order in ϵ , and defining $x = (z-1)[1 - (\epsilon/12)]$ we obtain

$$\psi(z, E) = (-E-z)^{-1-\gamma}; \quad \frac{S(z, s)}{\sigma_T(s)} = s^{(z-1)(1-\frac{\epsilon}{12})}$$

Hence all c_p are 1 and the correlations vanish asymptotically.

For the other two models we will have to take the Mellin transform of each n_p separately. In the AM we obtain

$$m_p = \frac{(\ln s)^{p(1-\frac{\epsilon}{12})} \Gamma(1+\gamma) p! \left(1 - \frac{\epsilon}{6} K_p\right)}{\Gamma(1+\gamma + p(1-\gamma))} \quad (24)$$

$$K_p = \sum_{l=2}^p \frac{(p-l+1)}{l(l-1)}$$

and

$$c_p = p! \left(1 - \frac{\epsilon}{6} K_p\right) [\Gamma(1+\gamma)]^{1-p} \Gamma^{-1}(1+\gamma+p(1-\gamma)) \quad (25)$$

The first few values are $c_2 = 1 + (\epsilon/24)$, $c_3 = 1 + (\epsilon/9)$. To first order in ϵ

$$c_p \simeq 1 + \frac{\epsilon}{12} \left[p \sum_{l=3}^p \left(\frac{1}{l} - \frac{2}{l(l-1)} \right) + \frac{p}{2} - 1 + \sum_{l=2}^p \frac{1}{l} \right] \quad (26)$$

from which it is obvious that $c_{p+1} > c_p > 1$ for all p , and presumably $c_p \simeq e^{c\epsilon p}$ for large p . In the SAM one would obtain an analogous result

$$c_p = \left[\Gamma\left(2 + \frac{\epsilon}{6}\right) \right]^p p! \left(1 - \frac{\epsilon}{6} K_p\right) [\Gamma(1+\frac{\epsilon}{6})]^{1-p} \Gamma^{-1}\left(p+1+\frac{\epsilon}{6}\right) \simeq 1 + O(\epsilon^2) \quad (27)$$

in agreement with the results of Ref. 7.

For the GRC model it is harder to obtain the c_p in a compact form. We have computed a few:

$$c_2 = p^2 \left(2 + \frac{\epsilon}{6}\right) \left(2 + \frac{\epsilon}{3}\right) \Gamma^{-1}\left(1 + \frac{\epsilon}{12}\right) \Gamma^{-1}\left(3 + \frac{\epsilon}{4}\right) \simeq 1 + \frac{\epsilon}{8} \quad (28)$$

$$c_3 = (6 + 3\epsilon) \Gamma\left(1 + \frac{\epsilon}{12}\right) p^3 \left(2 + \frac{\epsilon}{6}\right) \Gamma^{-1}\left(4 + \frac{\epsilon}{3}\right) \Gamma^{-3}\left(1 + \frac{\epsilon}{12}\right) \simeq 1 + \frac{7\epsilon}{18} \quad (29)$$

For large p the behaviour of c_p is readily obtained using the fact that the coefficients F_p defined in Eq. (9) are the p -derivative with respect to α_c of $[\Gamma(0,0;1,1)]^{-1}$ [see Eq. (19)]. Since the leftmost singularity of $[\Gamma(0,0;1,1)]^{-1}$ as a function of α_c is the logarithmic branch cut at $\alpha = \frac{1}{2}$ [the discontinuity of which vanishes at the tip, see Eq. (22)], the behaviour of F_p for large p is $F_p \simeq p! [1 + (c\epsilon/p^2)e^{p \ln 2}]$. This leads for c_p to an exponential behaviour of the form

$$c_p \simeq \left(1 + \frac{c\epsilon}{p^2} e^{p \ln 2}\right) \left(1 - \frac{\epsilon}{12} p \ln p\right) \quad (30)$$

for sufficiently large p , and ϵ small enough to guarantee a meaningful expansion.

5. CONCLUSIONS

We have investigated the asymptotic behaviour of the multiplicity moments in theories of interacting Pomerons. We found quite generally that in these models $\langle n_p \rangle \simeq \ln s^{p(1+\gamma_4)}$, where γ_4 depends on the model and is of order ϵ . We interpret this result as follows: a naive Muller expansion of the p -particle inclusive distribution yields the leading behaviour of n_p provided that one introduces an

anomalous dimension $\gamma_4 = \eta$ for the two-Pomeron/two-particle vertex. Specializing to particular models we found in the Gribov Reggeon calculus $\gamma_4 = \eta$ to all orders in ϵ , in the absorptive model $\gamma_4 = -\eta$ at first order in ϵ , in the simplified absorptive model $\gamma_4 = 0$, when in all cases $\sigma_T \approx (\ln s)^\eta$.

The ratios $c_p = n_p / \langle n \rangle^p$ are asymptotically constant in all models, and equal to 1 for $\epsilon = 0$. The departure from 1 of c_p shows the lack of factorization which is expected in the Muller diagram that gives the leading contribution to n_p whenever the exchanged Pomerons are not single poles. Let us remark that the positivity of σ_n forces the relation $c_{p+1} \geq c_p \geq 1$ for all p . Furthermore, in a scaling theory with long-range correlations one expects $c_p \approx e^{cp}$ for large p . Our results confirm the validity of these properties for the AM. This is not surprising, since the absorptive model is constructed in s-channel terms and from positive cross-sections to start with. The validity of these relations for the Gribov model is more interesting, since on the one hand the model is constructed to satisfy t-channel properties, and has no s-channel unitarity or positivity built in *a priori*, and on the other hand a naive Muller analysis with multiple exchange of factorized Pomerons more singular than a single pole would lead to $c_p < 1$ for all p , and $c_p \approx p^{-p}$ for large p ⁹⁾. Obviously as soon as ϵ is larger than 0, and the Pomeron becomes more singular than a pole, the factorization property breaks down, and we have found that this happens in a way that makes the model consistent with s-channel requirements. The numerical values of the first-order calculation for this model at $\epsilon = 2$ turn out to be $c_2 = 1.25$ and $c_3 \approx 1.8$. The excellent agreement with the experimental values should by all means be considered fortuitous, but it might turn out not to be so after all.

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CUTTING RULES AND DEFINITION OF THE MODELS

As explained in the introduction, we want to study the possible s-channel discontinuities of a diagram of interacting Reggeons in terms of the interactions of two formally different fields, corresponding to the uncut (UCP) and cut (CP) Pomeron lines. The vertices that we need to consider are in general four (Fig. A1), since an incoming UCP line never turns into CP lines. When a CP line is incoming we can have in general two outgoing UCP lines (with the cut passing between them: this corresponds to diffractive dissociation, Fig. A1b), one CP line and a UCP line (absorptive corrections, Fig. A1c), and two CP lines (inelastic rescattering, Fig. A1d). Let us immediately remark that the structure of our theory cannot possibly be described by a Lagrangian. In fact the combination of two allowed vertices (Figs. A1b and c) introduces an off-diagonal cut-uncut propagator (Fig. A2) that never occurs in the s-channel discontinuity diagrams. We can, however, write consistent Dyson equations that automatically generate all (and only) the allowed discontinuities. Before examining the different models in detail, let us remark that, quite generally, having forbidden the cut to uncut propagator and all the vertices $\text{uncut} \rightarrow \text{cut} + X$, the Dyson equations for uncut propagator and vertices completely decouple from the cut field, and reduce to the familiar ones (Figs. A3 and A4). Here and in the following we do not consider explicitly the contributions to the Dyson equation for the vertex like the one in Fig. A5, which anyhow would give higher-order corrections in the ϵ -expansion.

Let us now consider the rules for the Gribov Reggeon calculus. The Dyson equations for the CP propagator and the vertices with a CP incoming line are given in Figs. A6-A9. The relative weights of each diagram are indicated in the figures and are obtained with the following rules^{*)}.

- 1) The CP propagator carries a factor of 2.
- 2) Calling ig the vertex with three UCP lines, a vertex with at least one CP and an odd number of UCP lines is $ig/2$, with an even number of UCP lines is $g/2$.
- 3) For each diagram there is a plane through which the cutting takes place: this plane cannot intersect UCP lines, which in our diagrams can be thought of as rubber strings that can be pulled above or below the aforementioned plane. Whenever the vertex of Fig. A1b occurs, the cutting plane must pass

^{*)} A set of equivalent rules has been obtained by G. Wimbow, private communication.

between the two uncut lines. All possible positions of the UCP lines with respect to the cutting plane have to be counted as independent.

- 4) The vertices referring to UCP lines that stay below the cutting plane are the complex conjugate of the vertices of the UCP lines which stay above.

The reader can check that in the limit in which $\alpha_c \rightarrow 1$, and therefore analytically there is no distinction between cut and uncut lines, the equation of Fig. A6 reduces to the equation of Fig. A3 and the equations of Figs. A7-A9 to the equation of Fig. A4.

For more complicated diagrams the aforementioned cutting rules satisfy a basic property: the relative weight of the possible combinations of cut and uncut configurations of a certain number of parallel lines in a general diagram does not depend on the structure of the diagram, and therefore it is equal in particular to their relative weight when the P-lines under consideration are directly attached to the external ones. This guarantees that the contribution to the multiplicity of each line of every graph sums up to zero, and therefore that the result $\gamma_4 = \eta$ holds to all orders. As an example, the reader can consider the graph of Fig. A10. The allowed discontinuities and their weight are shown in Fig. A11, and it is easy to see that they sum up to twice the value of the contribution of this graph to $\text{Im } A_{e1}$, computed according to the usual rules of the Reggeon calculus. One can also check that the sum of the diagrams in which a given line is cut is always zero.

The rules for the other model are comparatively much simpler. In the simplified absorptive model only the vertices a and c of Fig. A1 are allowed, and the Dyson equations are straightforward.

In the complete absorptive model the vertices a, b, and c of Fig. A1 are allowed and the Dyson equations for the propagator and vertices are shown in Figs. A12 and A13.

----- UNCUT POMERON
 _____ CUT POMERON
 .-.-.-.-. EXTERNAL LINE

FIG.1

FIG. A1

FIG. A2

FIG. A3

FIG. A4

FIG. A5

$$\begin{aligned}
 & \text{Diagram with a solid line and a black dot} \quad \text{value } 2 = \text{Diagram with a solid line} \quad \text{value } 2 + \text{Diagram with a solid line and a loop with two black dots} \quad \text{value } 4 \\
 & + \text{Diagram with a solid line and a dashed loop with two black dots} \quad \text{value } +2 + \text{Diagram with a solid line and a solid loop with two black dots} \quad \text{value } -4 + \text{Diagram with a solid line and a dashed loop with two black dots} \quad \text{value } -4
 \end{aligned}$$

FIG. A6

$$\begin{aligned}
 & \text{Diagram with a solid line and a vertex} \quad \text{value } 1/2 = \text{Diagram with a solid line and a vertex} \quad \text{value } 1/2 + \text{Diagram with a solid line and a loop with two black dots} \quad \text{value } -1/2 + \text{Diagram with a solid line and a loop with two black dots} \quad \text{value } -1/2
 \end{aligned}$$

FIG. A7

FIG. A8

FIG. A9

FIG.A10

FIG.A11

FIG. A12

FIG. A13

FIG. A14