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Study of the Strongly Interacting Higgs Sector

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ABSTRACT

The most general structure of the strongly interacting Higgs sector has two unknown parameters and corresponds, in this unitarized one loop calculation, to the possible existence of a $I = 0, J = 0$ Higgs scalar boson and a $I = 1, J = 1$ vector boson. The vector boson width is shown to satisfy the KSFR relation while the Higgs boson width is reduced by a factor of $2/3$ from the standard tree value. As the calculation is completely unitary, no bound can be obtained for the Higgs boson and the vector boson masses.

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The Minimal Standard Model (MSM) $SU(2)_L \times U(1)_Y$ which unifies electromagnetic and weak interactions is known to be consistent with the experimental evidence obtained until now. In this model the spontaneous symmetry breaking is realized by the introduction of a complex scalar doublet. Three of these degrees of freedom are the Nambu-Goldstone bosons which are absorbed by the $W^\pm$ and Z to make them massive and the remaining one becomes the physical Higgs boson whose mass is undetermined in the MSM. If the Higgs mass is sufficiently large, say in the TeV region, the longitudinal components of the $W^\pm$ and Z become strongly interacting at this energy scale [1,2,3]. More precisely if the energy of the $W^\pm W^\pm$, $W^\pm Z$ or ZZ scattering is much larger than the boson masses, their mutual interactions can be described by the linear σ -model of Gell-Mann and Levy [4] where the longitudinal components of the $W^\pm$ and the Z play the role of the Nambu-Goldstone pions and the Higgs field the role of the σ scalar meson.

In the MSM the Higgs Lagrangian can be written as

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}m_H^2 H^2 - \frac{1}{2}m_H^2(\Phi^2 + H^2)H - \frac{m_H^2}{8v^2}(\Phi^2 + H^2)^2 \quad (1)$$

with $v = (\sqrt{2}G_F)^{-1/2} = 246\text{GeV}$ and $m_H^2 = 2\lambda v^2$ with λ being the selfcoupling of the scalars. $\Phi = (w_1, w_2, w_3)$ are the Nambu-Goldstone bosons. The calculation of the T -matrix for the $W_i \rightarrow W_i$ scattering, $T(W_i W_i \rightarrow W_i W_i)$, is simplified to the study of $T(w_i w_j \rightarrow w_k w_l)$ (this equivalence is accurate up to corrections of order $m_w/\sqrt{s}$ where $\sqrt{s}$ is the centre-of-mass energy) [2,5]. By denoting the T -matrix for $w_i w_j \rightarrow w_k w_l$ as

$$T_{ij,kl} = A(s, t, u)\delta_{ij}\delta_{kl} + A(t, s, v)\delta_{ik}\delta_{jl} + A(u, t, s)\delta_{il}\delta_{jk}$$

then

$$A(s, t, u) = \frac{m_H^2}{v^2} \left(\frac{s}{m_H^2} - s \right). \quad (2)$$

Nevertheless, the Lagrangian in eq.(1) and hence eq.(2) is not sufficiently general. It is possible for example to couple the Φ fields to a heavy vector field $I = 1, J = 1$ in a chiral invariant manner. This can be done à la Weinberg [6], as previously shown in the context of $\pi - \pi$ scattering [7], to yield a more general expression than eq.(2)

$$A(s, t, u) = \frac{s}{v^2} \frac{m_H^2}{m_H^2 - s} + \frac{g_{V\Phi\Phi}^2}{m_V^4} \left(\frac{s-u}{m_V^2 - t} + \frac{u(s-t)}{m_V^2 - u} \right) \quad (3)$$

where $g_{V\Phi\Phi}$ is the strongly interacting vector boson coupling to the Φ fields. Eq.(3) is a tree level amplitude and cannot be directly compared to the experiment since the particles involved in it appear as zero width particles.

The most general description of the dynamics of the Nambu-Goldstone bosons considered here is provided by a phenomenological chiral Lagrangian [8] in the same spirit of [9] which can be organized in an expansion in terms of the energy, or equivalently, in terms of the number of derivatives. The simplest choice of this Lagrangian that is compatible with the experimental constraint $\rho \simeq 1$ is the one associated to the $SU(2)_L \times SU(2)_R$ non-linear σ -model. We will consider here the effective chiral Lagrangian at order s^2 that, at tree level, is given by:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \quad (4)$$

where

$$\mathcal{L}_0 = \frac{v^2}{4} \text{tr}(\partial_\mu U \partial^\mu U^\dagger)$$

$$\mathcal{L}_1 = \frac{E}{32} \text{tr}([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2) + \frac{G}{8} (\text{tr}(\partial_\mu U \partial_\mu U^\dagger))^2$$

with $U(x) = \exp(iw_a(x)\sigma_a/v)$ and σ_a being the Pauli matrices. The expression for $\mathcal{L}_1$ is simply the Skyrme and the non-Skyrme terms with $E = 1/e^2$ and $G = \gamma/e^2$ in the standard notation of Skyrme physics. The tree level amplitudes obtained from $\mathcal{L}_1$ can be interpreted as the low energy expansion of eq.(3) through the identification $E = 2v^2/m_V^2$ and $G = v^2/m_H^2$ (here the KSRF relation is assumed). At the one-loop level the quadratic terms in $\mathcal{L}_1$ provide the needed counter-terms making the model renormalizable at order s^2 . In the following, we will regard $\mathcal{L}_1$ as the Lagrangian which simulates the low energy contribution to the scattering amplitudes to order s^2 of any heavy fields (say, the Higgs particle, vector bosons, fermion loops effects, etc.) that could couple to the Nambu-Goldstone bosons. The coefficients E and G are the only two free parameters in this model. For appropriate choice of these parameters, Higgs and vector bosons can appear as resonances in the scattering amplitudes. Because there are only two parameters once the particle masses are fixed, their widths are predicted in our calculation.

The tree level amplitude constructed from the effective Lagrangian $\mathcal{L}$ is given by:

$$A^{tree}(s, t, u) = \frac{s}{v^2} + \frac{E}{4v^4}(-2s^2 + t^2 + u^2) + \frac{G}{v^4}s^2. \quad (5)$$

The corresponding amplitude to one loop level is given by:

$$A^{1-loop}(s, t, u) = \frac{s}{v^2} + \frac{E_R(\nu)}{4v^4}(-2s^2 + t^2 + u^2) + \frac{G_R(\nu)}{v^4}s^2$$

$$+ \frac{1}{(4\pi)^2 v^4} \left(-\frac{1}{12}(3t^2 + u^2 - s^2) \log\left(\frac{-t}{\nu^2}\right) - \frac{1}{12}(3u^2 + t^2 - s^2) \log\left(\frac{-u}{\nu^2}\right) - \frac{1}{2}s^2 \log\left(\frac{-s}{\nu^2}\right) \right) \quad (6)$$

where $E_R(\nu)$ and $G_R(\nu)$ are the renormalized parameters that now depend on an arbitrary scale ν . The amplitude $A^{1-loop}(s, t, u)$ is independent of this scale provided the evolution equation of the parameters is given by:

$$E_R(\nu) = E_R(\nu_0) - \frac{1}{6\pi^2} \log \frac{\nu}{\nu_0}$$

$$G_R(\nu) = G_R(\nu_0) - \frac{1}{8\pi^2} \log \frac{\nu}{\nu_0} \quad (7)$$

The whole effect of the one-loop correction is changing the E and G bare parameters by the renormalized ones $E_R(\nu)$ and $G_R(\nu)$, and the addition of the logarithmic terms in s, t and u appearing in eq.(6), to the tree level result [9].

Now it is possible to compute the partial wave amplitudes from eq.(6). Let us write these amplitudes in the form $a_{IJ}(s) = a_{IJ}^{(0)}(s) + a_{IJ}^{(1)}(s)$, where $a_{IJ}^{(0)}(s)$ contains the contribution from $\mathcal{L}_0$ at tree level and $a_{IJ}^{(1)}(s)$ the remaining terms. As it was previously shown [10] the partial wave amplitudes $a_{IJ}(s)$ violate unitarity and cannot be compared with the

experiment unless $|a^{(1)}| < |a^{(0)}|$. Notice that they verify $Ima^{(1)} = |a^{(0)}|^2$ instead of $Ima = |a|^2$ as elastic unitarity requires. In order to avoid this problem let us use here the Padé approximant method. It is straightforward to show that the first diagonal $[1, 1]$ Padé approximant partial wave amplitude $a_{IJ}^{[1,1]}(s)$ defined as

$$a_{IJ}^{[1,1]}(s) = \frac{a_{IJ}^{(0)}(s)}{1 - \frac{a_{IJ}^{(1)}(s)}{a_{IJ}^{(0)}(s)}} \quad (8)$$

satisfies exactly the elastic unitary condition. On the other hand, the appearance of the denominator in eq.(8) makes it possible that the Padé improved partial wave amplitudes develop a resonant behaviour for some range of the parameters E_R and G_R . That last property is of course very important and is not present in the original perturbation series, eq.(6). This is so because eq.(8) is a resummation of an infinite series generated by the one loop Feynman diagrams. Notice that a similar study of the partial wave amplitudes but in the context of the $\pi - \pi$ scattering was already carried out by Lehmann in [9] (in a slightly different form) and by Jhung and Willey in [9].

The Padé partial wave amplitudes corresponding to the lowest I and J values are given by:

$$\begin{aligned} a_{00}^{[1,1]}(s) &= \frac{s}{16\pi v^2} \frac{1}{1 - \frac{s}{4v^2}(-\frac{1}{3}E_R(\nu) + \frac{22}{3}G_R(\nu) + \frac{1}{(4\pi)^2}(\frac{11}{27} - \frac{50}{9}\log\frac{s}{v^2} + 4i\pi))} \\ a_{11}^{[1,1]}(s) &= \frac{s}{96\pi v^2} \frac{1}{1 - \frac{s}{v^2}(\frac{3}{4}E_R(\nu) - G_R(\nu) + \frac{1}{(4\pi)^2}(-\frac{1}{18} + \frac{i\pi}{6}))} \\ a_{20}^{[1,1]}(s) &= \frac{s}{32\pi v^2} \frac{1}{1 + \frac{s}{2v^2}(\frac{4}{3}(G_R(\nu) + \frac{E_R(\nu)}{2}) + \frac{1}{(4\pi)^2}(\frac{25}{54} - \frac{20}{9}\log\frac{s}{v^2} + i\pi))} \end{aligned} \quad (9)$$

The above amplitudes have been applied very recently to the $\pi - \pi$ scattering problem and the results were found in excellent agreement with the experimental data in terms of only two parameters that in the hadronic context take the values $E_R(785MeV) = 0.029$ and $G_R(785MeV) = 0.0073$ [11]. Of course, the value of the corresponding parameters in the strong interaction Higgs sector case remains unknown but they could be measured in the next generation of colliders [12]. In the following, we will use eq.(9) in order to get some simple but important relations between the widths and the masses of the resonances which may be present in the spectrum of the strongly interacting Higgs sector.

Let us consider first the P wave channel. From eq.(7) it is very easy to show that the combination $3E_R(\nu)/4 - G_R(\nu)$ does not depend on the scale v . Fixing the physical vector boson mass m_{VP} at the vanishing of the real part of the denominator, we have:

$$m_{1P}^2 = \frac{v^2}{\frac{3}{4}E_R(\nu) - G_R(\nu) - \frac{1}{18(4\pi)^2}} \quad (10)$$

We will therefore have a $I = 1, J = 1$ resonance for those values of the renormalized parameters that make positive the value of m_{1P}^2 in eq.(10). In this case, the width can be read off from eq.(9):

$$\Gamma_V = \frac{m_{1P}^2}{96\pi v^2} \quad (11).$$

This relation was first given by Lehmann [9] for $\pi - \pi$ scattering and is nothing but the KSRF relation [13] in the chiral limit. Unlike the demonstration of this relation by Brown and Goble [14] which requires the assumption that the left hand cut of the P wave amplitude does not contribute, we show here in the chiral limit that it is indeed the case (see also [11]). We effectively demonstrate $g_{1P}^2 = m_{1P}^2/2v^2$ of eq. (3).

In the case of the $I = 0, J = 0$ partial wave we obtain the mass of the corresponding resonance (the Higgs particle) in a similar way, i.e., at the point where the real part of the denominator of the S -partial wave amplitude vanishes. As the physical quantities calculated from eq.(9) are scale independent (e.g. masses, widths, etc.), it is convenient to take the scale $v = m_{HP}$. Then the equation for the physical mass of the Higgs particle becomes

$$m_{HP}^2 = \frac{4}{-\frac{4}{3}E_R(m_{HP}) + \frac{22}{3}G_R(m_{HP}) + \frac{11}{27(4\pi)^2}} 4v^2 \quad (12)$$

If this equation has a solution for m_{HP} then the Higgs resonance exists and its width can be read off from eq.(9)

$$\Gamma_H = \frac{m_{HP}^2}{16\pi v^2} \quad (13)$$

This expression has to be compared with the standard formula obtained at the tree level from eq.(1)

$$\Gamma_H^{tree} = \frac{3m_H^2}{32\pi v^2} \quad (14)$$

Therefore the width of the physical Higgs is reduced by a factor of $2/3$ from the tree level value. Our result of eq.(13) is important because a reduction of the Higgs width value could improve the possibilities of its experimental detection in the next generation colliders. It is interesting to note that if we had used the Brown and Goble assumption that the left hand cut does not contribute in the $I = 0, S$ wave amplitude, the Higgs width would have been larger by a factor of two than that given in eq.(13). This demonstrates that the assumption of Brown and Goble is not always correct.

Finally some comments are in order. Firstly we would like to stress that as the S -wave $I = 0$ amplitude satisfies exactly the elastic unitarity condition, no upper bound can be put on the Higgs mass. The frequent statements on the upper bound of the Higgs mass based on the violation of the unitarity of the perturbation series are therefore incorrect from our point of view. Secondly, the general treatment followed here for the strongly interacting symmetry breaking sector of the SM depends on two unknown and independent parameters which are fixed by the also unknown underlying dynamics. For a wide range of these parameters, resonances like the physical Higgs particle or vector bosons can appear in the corresponding channel but the ratio between their masses cannot be known a priori [15].

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