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Lattice QCD, Quark-Gluon Plasma and Heavy-Ion Collisions[†]

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Abstract

These lectures give a strongly biased introduction into the physics of the quark gluon plasma. We will discuss the present status of Monte Carlo simulations of lattice regularized QCD at finite temperature. In particular we look at results for the order of the finite temperature phase transition and the quantitative determination of the transition temperature as well as the equation of state and screening lengths in the QCD plasma phase. We also discuss briefly first experimental attempts to study dense nuclear matter in heavy ion experiments.

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1. INTRODUCTION

Quantum Chromodynamics (QCD) is well established as fundamental theory of strong interaction physics. In particular its successes in describing perturbative features of high energy hadron physics led to its general acceptance as the correct theory of strong interactions. However, QCD is also expected to account for all the non-perturbative features in low energy hadron physics, many of which are not at all well understood theoretically; until now we are, for instance, not able to prove that QCD indeed describes confinement or predicts the correct hadron mass spectrum. The most systematic quantitative approach to deal with these non-perturbative questions is based on the numerical simulations of lattice regularized QCD¹⁾.

Probably one of the most exciting and at the same time experimentally least accessible predictions of QCD is the existence of a phase transition at finite temperature and/or finite baryon number density, which separates the non-perturbative low energy regime, characterized by confinement and chiral symmetry breaking, from a perturbative, chiral symmetric high density phase - the quark-gluon plasma. Also the existence of a phase transition, which is an entirely non-perturbative feature of QCD, is not proofed rigorously. However, it is strongly suggestive that the low and high energy regimes of hadronic matter which show such widely different properties are separated by a phase transition²⁾. A first quantitative study of the finite temperature phase structure of QCD became possible with the introduction of the lattice regularized formulation

of QCD^{3,4)}. First Monte Carlo simulations of lattice regularized QCD⁵⁾ suggested that indeed there exists a phase transition to a nearly free (quark-) gluon gas at a temperature which is of the order of 200 MeV⁶⁻⁸⁾. This transition temperature is close to what had been estimated earlier in approximate statistical models⁹⁾. Equally important was, however, the observation that beyond the phase transition the energy density approaches quickly that of an ideal gas of quarks and gluons at the same temperature⁷⁾. This suggests that energy densities of the order of 3 GeV/fm³ should be sufficient to create a quark-gluon plasma.

Almost certainly a quark-gluon plasma existed in the early universe¹⁰⁾. Is it possible to recreate this state of matter also in the laboratory? In order to reach a situation where thermodynamics is applicable one certainly has to achieve that a dense hadronic system is created in a reasonably large volume (a few fm³) for a sufficiently long time (a few fm). Such conditions may indeed be reached in heavy ion experiments. First crude estimates that it might be possible to obtain energy densities of several GeV/fm³ in ultra-relativistic nucleus-nucleus collisions¹¹⁾. These estimates, however, were subject to uncertainties such as the transparency of nuclei at high energies was completely unknown. Would the stopping power be large enough to create a dense hadronic system in the interaction region? First fixed-target nucleus-nucleus experiments with relatively light ions (oxygen, sulphur) have recently been performed at CERN to study the conditions that can be reached in such experiments and to look for possible signals of a

phase transition to a quark-gluon plasma¹²⁾.

In the following three lectures we want to discuss attempts to understand the physics of the quark-gluon plasma both theoretically through Monte Carlo simulations of lattice QCD as well as experimentally in heavy ion experiments. In section 2 we start with a discussion of the lattice approach to finite temperature QCD. We will then discuss the current knowledge about the QCD finite temperature phase transition in section 3. In particular we will look at recent attempts to determine the order of the deconfinement phase transition in pure SU(3) gauge theory from finite size scaling analyses. In section 4 we present recent results for the equation of state, the heavy quark potential and other thermodynamic quantities. In section 5 we will discuss basic parameters of heavy ion experiments and give a brief discussion of some observables which may shed some light on the thermal conditions reached in these experiments. Finally we give some conclusions in section 6.

2. FINITE TEMPERATURE QCD ON THE LATTICE

In this section we will discuss the formulation of finite temperature QCD on the lattice. We start in section 2.1 with a brief introduction into the lattice formulation of QCD, which mainly serves to fix our basic notations. For a more detailed discussion of basic aspects of lattice QCD and results obtained from simulations at zero temperature we refer to recent reviews on these topics^{13,14)}. In section 2.2 we concentrate on aspects more specific to the finite temperature formulation of QCD on the lattice.

2.1. QCD on the lattice

The partition function of a $SU(N)$ gauge theory coupled to n_f flavours of fermions with masses m_i , $i = 1, \dots, n_f$, is given by

$$Z(T, V) = \int D A D \bar{\psi} D \psi \exp[-S(T, V)] \quad (2.1)$$

Here we have suppressed the explicit dependence of the partition function on the quark masses (m_i), as well as colour (N) and flavour (n_f) degrees of freedom. The dependence on temperature T and volume V enters in the action $S(T, V)$ through the integration limits for the four-dimensional space-time integral over the QCD Lagrangian

$$S(T, V) = \int_0^{1/T} dx_0 \int_V d^3x \mathcal{L}(\mathcal{A}(x), \bar{\psi}(x), \psi(x)) \quad (2.2)$$

with

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{i=1}^{n_f} \bar{\psi}_i (D(\mathcal{A}) + m_i) \psi_i \quad (2.3)$$

Here $D(\mathcal{A}) = \gamma_\mu [i\partial_\mu + \frac{1}{2}g_s \mathcal{A}_\mu^a \lambda^a]$ denotes the covariant derivative and $F_{\mu\nu}^a$ is the gluonic field strength tensor, $F_{\mu\nu}^a = \partial_\mu \mathcal{A}_\nu^a - \partial_\nu \mathcal{A}_\mu^a + g f^{abc} \mathcal{A}_\mu^b \mathcal{A}_\nu^c$. The fact that in the Hamiltonian formulation the partition function is a trace over all possible states of the system is in the Euclidean formulation reflected in the temporal boundary conditions for the fields. While the bosonic fields $\mathcal{A}_\mu(x_0, \vec{x})$ obey periodic boundary conditions in Euclidean time, $\mathcal{A}_\mu(1/T, \vec{x}) = \mathcal{A}_\mu(0, \vec{x})$, the fermionic fields $\bar{\psi}(x)$, $\psi(x)$ obey anti-periodic boundary conditions in this direction of space-time.

Let us start our discussion of lattice QCD by defining the basic degrees of freedom of QCD on a four-dimensional discrete space-time lattice. First of all we replace the integration over the four-volume $T^{-1}V$, appearing in the definition of the action in eq.(2.2), by a sum over sites of a four-dimensional hypercubic lattice. On this lattice we define bosonic (fermionic) fields $U_{n,\mu}$ ($\bar{\chi}_n$, χ_n) which replace the continuum fields $\mathcal{A}_\mu(x)$ ($\bar{\psi}(x)$, $\psi(x)$). Here $n = (n_0, n_1, n_2, n_3)$ labels a site on the four-dimensional hypercubic lattice, which is related to the continuous space-time coordinate x through $x = n \cdot a$, with a denoting the lattice spacing. Gluonic fields are represented by elements of the colour gauge group $SU(N)$, $U_{n,\mu} \in SU(N)$. They are defined on links (n, μ) of the lattice that originate at a site n and end at the neighbouring site, $n + \hat{\mu}$, in μ -direction, $\mu = 0, 1, 2$ or 3 . The link variables $U_{n,\mu}$ are related to the gauge fields $\mathcal{A}_\mu(x)$ through

$$U_{n,\mu} = \exp \left[-iga \sum_{i=1}^{N^2-1} \lambda^i \mathcal{A}_\mu^i(x) \right] \quad (2.4)$$

where g is the bare coupling constant of QCD. Using this relation one can easily verify that the single plaquette action proposed by Wilson¹⁾

$$S_G = \frac{2N}{g^2} \sum_{n, 0 \leq \mu < \nu \leq 3} P_{n,\mu\nu} \quad (2.5)$$

with

$$P_{n,\mu\nu} = 1 - \frac{1}{N} \text{ReTr} U_{n,\mu} U_{n+\hat{\mu},\nu} U_{n+\hat{\mu}+\hat{\nu},\mu}^\dagger U_{n,\nu}^\dagger \quad (2.6)$$

approximates the gluonic part of the continuum action up to terms of $O(a^6)$, $P_{n,\mu\nu} = \frac{1}{4}a^4 F_{\mu\nu}^b F_{\mu\nu}^b + O(a^6)$. In the continuum limit, $a \rightarrow 0$, these higher order corrections become irrelevant.

The discretization of the fermionic part of the action leads to some difficulties. The fermion action contains only first derivatives of the fields. As a consequence of this one finds that a naive discretization of the action leads to additional poles in the lattice fermion propagator. In the continuum limit these additional poles would give rise to 16 additional, unwanted fermion species rather than only the one we started with. By distributing the four components of the continuum spinor over different sites of the lattice it is possible to reduce somewhat the number of additional species¹⁵⁾. This problem of species doubling can be avoided entirely by giving a large mass to the additional species¹⁾. However, this is possible only for the price of explicitly breaking the chiral invariance of the continuum action. In fact, if we insist in introducing only short range interactions among the fermionic fields on the lattice it is in general impossible to avoid species doubling and at the same time preserve chiral symmetry¹⁶⁾.

For the purpose of finite temperature calculations on the lattice it is convenient to work with a lattice action which preserves at least a part of the $SU(n_f) \times SU(n_f)$ chiral symmetry of the continuum action as phase transitions at finite temperature are generally characterized by the spontaneous breaking/restoration of global symmetries of the QCD Lagrangian. In the following we thus will restrict our discussion to the staggered fermion formulation¹⁵⁾. If we introduce $\bar{f}$ different fermion species on the lattice the staggered lattice action will lead to $n_f = 4\bar{f}$ species of fermions in the continuum limit. It preserves, however, a global $U(\bar{f}) \times U(\bar{f})$ chiral symmetry, i.e. an abelian subgroup of the continuum chiral symmetry.

The staggered fermion action is given by

$$S_F = \sum_{i=1}^{\bar{f}} \bar{\chi}_{n,i} Q_{n,m}^i \chi_{m,i} \quad (2.7)$$

Here there fermion fields, χ ($\bar{\chi}$), are anticommuting Grassmann variables defined on the sites of the lattice and the fermion matrix $Q_{n,m}^i$ is defined as

$$Q_{n,m}^i = \sum_{\mu=0}^3 D_{n,m;\mu} + m_i a \delta_{n,m} \quad (2.8)$$

The hopping matrices $D_{n,m;\mu}$ have non-zero elements only for $m = n \pm \hat{\mu}$. They are given by

$$D_{n,m;\mu} = \frac{1}{2} \eta_\mu(n) [U_{n,\mu} \delta_{n,m-\hat{\mu}} - U_{m,\mu}^\dagger \delta_{n,m+\hat{\mu}}] \quad (2.9)$$

with phase factors $\eta_\mu(n) = (-1)^{n_0 + \dots + n_{\mu-1}}$ for $\mu > 0$ and $\eta_0(n) = 1$. Finally the partition function takes on the form

$$Z = \int \prod_{n,\mu} dU_{n,\mu} \prod_{n,i} d\chi_{n,i} d\bar{\chi}_{n,i} e^{[-S_G - S_F]} \quad (2.10)$$

As the fermionic part of the action is bilinear in the fields $\bar{\chi}_{n,i}$, $\chi_{n,i}$, these can be integrated out easily and the partition function can be represented in terms of gluonic degrees of freedom only,

$$Z = \int \prod_{n,\mu} dU_{n,\mu} \prod_{i=1}^{\bar{f}} \det Q^i e^{-S_G} \quad (2.11)$$

Using this partition function expectation values of any observable X can be calculated in the usual way,

$$\langle X \rangle = Z^{-1} \int \prod_{n,\mu} dU_{n,\mu} X \prod_{i=1}^{\bar{f}} \det Q^i e^{-S_G} \quad (2.12)$$

Finally we have to discuss how results of a lattice calculation can be converted into physical units and how the lattice cut-off can be removed in order to get results relevant for the continuum limit. Physical quantities are calculated on the lattice in units of the lattice spacing a ; a mass will be given in units of a^{-1} , the energy density in units of a^{-4} , etc.. The lattice action, however, does not contain any dimensionful parameters. The lattice cut-off a^{-1} enters through the renormalization of the bare coupling g^2 (and the quark masses $m_i a$). In the continuum limit ($g^2 \rightarrow 0$) g^2 and a are related through the renormalization group equation

$$a\Lambda_L = (b_0 g^2)^{-b_1/2b_0^2} e^{-1/2b_0 g^2} \quad (2.13)$$

with b_0 , b_1 given by

$$\begin{aligned} b_0 &= \frac{1}{16\pi^2} \left[11 \frac{N}{3} - \frac{2}{3} n_f \right] \\ b_1 &= \left(\frac{1}{16\pi^2} \right)^2 \left[\frac{31}{3} N^2 - \left(\frac{10}{3} N + \frac{N^2 - 1}{N} \right) n_f \right] \end{aligned} \quad (2.14)$$

This can then be used to eliminate the cut-off from measured quantities in favour of the scale parameter Λ_L . If the lattice simulations have been performed in

the continuum regime, results should be cut-off independent, i.e. physical quantities expressed in units of Λ_L should be independent of the coupling g^2 used in the actual calculation. Numerical studies of the QCD β -function using Monte Carlo renormalization group techniques gave evidence that the asymptotic scaling behaviour, described by eq.(2.13), is approximately valid for $g^2 < 1$ ¹⁷⁾. For $g^2 \geq 1$, however, strong violations of asymptotic scaling have been observed.

2.2. Finite temperature QCD on the lattice

For simplicity lattice simulations are generally performed on isotropic lattices, i.e. lattices with identical lattice spacings a_τ and a in temporal and spatial directions. However, in order to discuss the finite temperature formalism on the lattice it is convenient to work on an anisotropic lattice ($a_\tau \neq a$) and introduce the anisotropy parameter¹⁸⁾

$$\xi = \frac{a}{a_\tau} . \quad (2.15)$$

This allows to keep track of the temperature dependence, which enters in a complicated way the lattice action. While T appears in the continuum formulation explicitly only as an integration limit for the integration over the time components of the fields, on the lattice it appears through the finite number of lattice sites in the time direction of the lattice as well as through the lattice cut-off ($1/a_\tau$) in that direction. The continuum four-volume $T^{-1}V$ is replaced by a lattice of size $N_\tau \times N_\sigma^3$ with lattice spacings a_τ (a) in the temporal (spatial) directions such that

$$\begin{aligned} 1/T &= N_\tau a_\tau , \\ V^{1/3} &= N_\sigma a . \end{aligned} \quad (2.16)$$

As we have seen in the previous section, the lattice spacings (a_τ , a) do not enter the action explicitly. They can only be tuned through the bare couplings. An anisotropy can be introduced on the lattice by choosing different couplings for space-space and space-time plaquettes in the gluonic action and similarly by introducing a new coupling in the 0-th component of the Dirac operator. The gluonic action, eq.(2.5), then becomes¹⁸⁾

$$S_G = \beta_\sigma \left[\sum_{n; 0 < \mu < \nu \leq 3} P_{n,\mu\nu} + \gamma_G^2 \sum_{n; 0 < \mu \leq 3} P_{n,0\mu} \right] \quad (2.17)$$

and the fermion matrix, eq.(2.8), now reads

$$Q_{n,m}^i = \sum_{j=1}^3 D_{n,m;j} + \gamma_F D_{n,m;0} + m_i a \delta_{n,m} . \quad (2.18)$$

Let us look at these expressions in somewhat more detail. The QCD action on anisotropic lattices depends on three dimensionless couplings β_σ , γ_G and γ_F , as well as the bare quark masses $m_i a$. The lattice spacings a, a_τ enter only through the renormalization of these couplings in the continuum limit. All three couplings may then be considered as functions of g^2 , defined on isotropic lattices ($\xi = 1$), and the anisotropy parameter ξ . The dependence on the latter can be established perturbatively by demanding rotational invariance in the continuum limit, even though this limit is performed on an anisotropic lattice¹⁹⁻²¹⁾. In the naive classical continuum limit rotational invariance is guaranteed if we choose $\gamma_G = \gamma_F = \xi$ and $\beta_\sigma = \xi^{-1} \beta$, with $\beta = 2N/g^2$ denoting the bare coupling on an isotropic Euclidean lattice. The latter is related to the lattice cut-off $1/a$ through the QCD renormalization group equation, eq.(2.13). Quantum corrections to this leading order relations have been calculated for β_σ and γ_G in the pure gauge theory^{19,20)} as well as for QCD²¹⁾ by demanding rotational invariance of the effective action. Similarly $O(g^2)$ corrections have been calculated for γ_F by demanding rotational invariance for the fermion propagator²¹⁾. One finds then

$$\begin{aligned} \beta_\sigma &= \xi^{-1} \beta [1 + c_\sigma(\xi) g^2 + O(g^4)] , \\ \gamma_G &= \xi [1 + c_G(\xi) g^2 + O(g^4)] , \\ \gamma_F &= \xi [1 + c_F(\xi) g^2 + O(g^4)] . \end{aligned} \quad (2.19)$$

In the definition of the gluonic part of the action, eq.(2.17), we have introduced the couplings β_σ and γ_G in order to stress the similarity of the modifications in the gluonic and fermionic part of the action required on anisotropic lattices. The couplings γ_G and γ_F determine the relative weights of space- and time-like components in the gluonic and fermionic action. Quite often one uses instead the pair β_σ and $\beta_\tau \equiv \beta_\sigma \gamma_G^2$, where β_τ now denotes the coupling in front of the temporal plaquettes. Its weak coupling expansion may be written as $\beta_\tau = \xi \beta (1 + c_\tau(\xi) g^2 + O(g^4))$, with c_τ being related to the functions c_σ and c_G through

$$c_G(\xi) = \frac{1}{2} (c_\tau(\xi) - c_\sigma(\xi)) . \quad (2.20)$$

The functions $c_\sigma(\xi)$, $c_\tau(\xi)$ and $c_F(\xi)$ depend on the number of colours and flavours. Their explicit form^{19-21,21)} is not of importance here, as we are finally interested in the isotropic limit $\xi \equiv 1$ where these functions vanish. The derivatives of c_σ , c_τ and c_F with respect to ξ , however, do not vanish for $\xi \equiv 1$. This is of importance as thermodynamic observables are obtained from the partition function by taking suitable derivatives with respect to V and/or T . On a lattice of fixed size $N_\tau \times N_\sigma^3$ the temperature derivative can be written as

$$\frac{\partial}{\partial(1/T)} = -\frac{\xi^2}{N_\tau a} \frac{\partial}{\partial \xi} \quad (2.21)$$

Derivatives of c_σ , c_τ and c_F thus will enter thermodynamic expressions even in the isotropic limit. These derivatives for $\xi \equiv 1$ are given by^{19-21,21)}

$$\begin{aligned} c'_\sigma &= \frac{dc_\sigma}{d\xi} \Big|_{\xi=1} = 4N \left[\frac{N^2-1}{32N^2} 0.58644 + 0.000499 \right] \\ &\quad - \frac{1}{2} n_f 0.00062 \\ c'_\tau &= \frac{dc_\tau}{d\xi} \Big|_{\xi=1} = 4N \left[-\frac{N^2-1}{32N^2} 0.58644 + 0.005306 \right] \\ &\quad - \frac{1}{2} n_f 0.00782 \\ c'_F &= \frac{dc_F}{d\xi} \Big|_{\xi=1} = -\frac{N^2-1}{2N} 0.1599 \end{aligned} \quad (2.22)$$

Using these expressions one obtains for instance for the energy density

$$\frac{\epsilon}{T^4} = \frac{(\epsilon_G + \epsilon_F)}{T^4} = \left(\frac{N_\tau}{N_\sigma} \right)^3 \frac{\partial}{\partial \xi} \ln Z \Big|_{\xi=1} \quad (2.23)$$

with the gluonic contribution ϵ_G given by

$$\begin{aligned} \frac{\epsilon_G}{T^4} &= -3N_\tau^4 \frac{d\beta_\sigma}{d\xi} \Big|_{\xi=1} [\langle P_\sigma \rangle + \langle P_\tau \rangle] \\ &\quad - 6\beta N_\tau^4 \frac{d\gamma_G}{d\xi} \Big|_{\xi=1} \langle P_\tau \rangle \end{aligned} \quad (2.24)$$

and the corresponding fermionic part ϵ_F

$$\begin{aligned} \frac{\epsilon_F}{T^4} &= N_\tau^4 \frac{n_f}{4} \left(\frac{d\gamma_F}{d\xi} \Big|_{\xi=1} \langle \text{Tr } D_0 Q^{-1} \rangle \right. \\ &\quad \left. + \frac{dma}{d\xi} \Big|_{\xi=1} \langle \text{Tr } Q^{-1} \rangle \right) \end{aligned} \quad (2.25)$$

Unless explicitly stated we will assume equal masses for all fermionic species and suppress the flavour index i

in the following. Furthermore we have used the abbreviation P_τ for space-time plaquettes $P_{n,\mu}$, $\mu = 1, 2, 3$ and P_σ for space-space plaquettes $P_{n,\mu\nu}$, $\mu, \nu = 1, 2, 3$. In eqs.(2.24) and (2.25) we have introduced the normalized expectation values $\langle \bar{X} \rangle$, which are defined by

$$\langle \bar{X} \rangle = \langle X \rangle_{N_\tau \times N_\sigma} - \langle X \rangle_{N_\tau \times N_\sigma} \quad (2.26)$$

This takes care of the subtraction of vacuum contributions to thermodynamic quantities and thus normalizes the gluonic and fermionic parts of the energy density to zero at $T = 0$.

The pressure can be obtained from $\ln Z$ by taking a derivative with respect to the volume, $V = N_\sigma a$,

$$\begin{aligned} \frac{p}{T^4} &= \frac{\epsilon}{3T^4} - \frac{1}{3} N_\tau^4 \frac{d\beta}{d \ln a} (\langle \bar{P}_\sigma \rangle + \langle \bar{P}_\tau \rangle) \\ &\quad + N_\tau^4 \frac{n_f}{12} \frac{dma}{d\xi} \Big|_{\xi=1} \langle \text{Tr } Q^{-1} \rangle \end{aligned} \quad (2.27)$$

Note that deviations from the ideal gas equation of state, $\epsilon = 3p$, are proportional to the QCD renormalization group equations for the bare couplings g^2 and ma . This is similar to continuum perturbative results, where deviation from the ideal gas relations come only through the temperature dependence of the running coupling constant²⁵⁾. Note also that for massless QCD, $ma \equiv 0$, we find that $\epsilon - 3p$ is given entirely in terms of gluonic operators.

Using eqs.(2.23) and (2.27) we obtain for the entropy density

$$\begin{aligned} \frac{S}{T^3} &= \frac{\epsilon + p}{T^4} \\ &= \frac{4}{3} \frac{\epsilon}{T^4} - \frac{1}{3} N_\tau^4 \left[\frac{d\beta}{d \ln a} (\langle \bar{P}_\sigma \rangle + \langle \bar{P}_\tau \rangle) \right. \\ &\quad \left. - \frac{n_f}{4} \frac{dma}{d\xi} \Big|_{\xi=1} \langle \text{Tr } Q^{-1} \rangle \right] \end{aligned} \quad (2.28)$$

We note that the analysis of the thermodynamic observables discussed above involves the calculation of two conceptually different quantities. First of all one has to evaluate expectation values of certain gluonic and fermionic operators. This can be done in a Monte Carlo simulation. In addition one also needs to know the various derivatives of couplings entering the expressions given in eqs.(2.24-2.28). Using eqs. (2.19) and (2.23) the derivatives of β_σ , γ_G and γ_F with respect

to ξ can be replaced by the leading order perturbative results. However, in particular in the fermionic sector these relations may not be accurate enough to get a correct description of all non-perturbative aspects of the thermodynamics of the high temperature phase²¹⁾ and higher order corrections may be needed.

An entirely non-perturbative treatment of the thermodynamics of QCD on the lattice would also require a non-perturbative calculation of the derivatives of the couplings discussed above. This can be achieved in principle²²⁾; in practice it is, however, rather time consuming to obtain accurate results for these quantities. At present one thus usually uses the perturbative relations given in eq.(2.22).

A detailed discussion of the numerical analysis of the thermodynamic quantities defined above will be given in section 4. However, before coming to that we start in the next section our discussion of Monte Carlo simulations of finite temperature QCD with a detailed analysis of the finite temperature phase structure of QCD as it emerges from these simulations.

3. DECONFINEMENT AND CHIRAL SYMMETRY RESTORATION

The low temperature phase of QCD is characterized by the confinement of quarks and gluons as well as the spontaneous breaking of chiral symmetry. The asymptotic freedom of QCD at high temperatures suggests that these non-perturbative features of QCD are lost in the high temperature phase. The linearly rising $q\bar{q}$ potential is expected to be replaced by a Debye screened Coulomb potential and chiral symmetry is expected to be restored in the plasma phase.

The mechanisms leading to confinement and chiral symmetry breaking are seemingly unrelated and it thus has been speculated that QCD may undergo two separate phase transitions²⁶⁾. Deconfinement could set in at some critical temperature T_d giving rise to free partons which still have an effective mass. Chiral symmetry thus would still be broken and would get restored during a second phase transition at $T_{ch} \geq T_d$. As will become clear from the following discussion this scenario does not seem to be realized for QCD at finite temperature, i.e. the Monte Carlo data suggest $T_d = T_{ch}$. Two separate transitions have, however, been found in gauge

theories with fermions in higher representations²⁷⁾. It may also be possible that two separate transitions occur in a cold QCD plasma, i.e. at low temperature as a function of baryon number density.

3.1. Global symmetries of the lattice action

Phase transitions are generally characterized by the spontaneous breaking/ restoration of global symmetries of the system under consideration. The QCD Lagrangian possesses exact global symmetries only in the limiting cases of vanishing (chiral limit) or infinite (pure gauge theory limit) quark masses. In the latter case the QCD action reduces to the gluonic action, S_G , defined in eqs.(2.5) and (2.6). S_G has a global $Z(3)$ symmetry, the breaking of which can be related to the deconfinement phase transition. If we perform a global $Z(3)$ rotation of all timelike gauge fields $U_{n,0}$ originating at sites $n = (n_0, \vec{n})$ of a given temporal hyperplane (fixed n_0) of the lattice,

$$U_{(n_0, \vec{n}), 0} \rightarrow U'_{(n_0, \vec{n}), 0} = z U_{(n_0, \vec{n}), 0} , \quad z \in Z(3), \quad n_0 \text{ fixed}, \quad (3.1)$$

the action remains unchanged, i.e. $S_G(\{U'_{n,\mu}\}) = S_G(\{U_{n,\mu}\})$, while for instance the Polyakov loop,

$$L_{\vec{n}} = \prod_{n_0=1}^{N_\tau} U_{(n_0, \vec{n}), 0} , \quad (3.2)$$

transforms non-trivially under this transformation,

$$L_{\vec{n}} \rightarrow z L_{\vec{n}} , \quad z \in Z(3) . \quad (3.3)$$

The thermal expectation value of the Polyakov loop

$$\langle L \rangle = \frac{1}{N_\tau^3} \langle \sum_{\vec{n}} \text{Tr} L_{\vec{n}} \rangle , \quad (3.4)$$

will thus vanish as long as the global $Z(3)$ symmetry of the action is preserved but will acquire a non-vanishing value if this symmetry is broken spontaneously. The Polyakov loop can be thought of as representing a static fermionic test charge which probes the screening properties of the surrounding gluonic medium^{6,28)}. Its expectation value is related to the excess free energy, $F_q(T)$,

$$e^{-F_q(T)/T} \simeq \langle L \rangle , \quad (3.5)$$

induced by the presence of this source in the gluonic heat bath. In the absence of dynamical quarks

($ma \rightarrow \infty$) a single coloured charge cannot be screened in the confined phase, its free energy, F_q , is infinite and $\langle L \rangle = 0$. In the deconfined phase, however, F_q is finite and $\langle L \rangle \neq 0$. The expectation value $\langle L \rangle$ is thus an order parameter for the deconfinement phase transition in the pure $SU(3)$ gauge theory, which is related to the spontaneous breaking of a global $Z(3)$ symmetry of the gluonic action S_G .

In the massless limit, $ma \rightarrow 0$, the QCD Lagrangian has a global chiral symmetry. Using staggered fermions on the lattice the symmetry group is $U(\bar{f}) \times U(\bar{f})$, i.e. a subgroup of the $SU(n_f) \times SU(n_f)$ chiral symmetry group of the continuum theory. In the low temperature phase this symmetry is spontaneously broken and leads to a non-vanishing chiral condensate,

$$\langle \bar{\chi}\chi \rangle = \frac{1}{N_\tau N_\sigma^3} \frac{\partial}{\partial ma} \ln Z \quad (3.6)$$

This symmetry is expected to be restored at high temperature.

In fig.1 we show results for the temperature dependence of the deconfinement order parameter $\langle L \rangle$ and the chiral condensate $\langle \bar{\chi}\chi \rangle$ obtained in a simulation of QCD with 4 light quark flavours ($n_f = 4$, $\bar{f} = 4$) of mass $m/T = 0.1$ ^[29]. This shows that the restoration of the chiral symmetry is accompanied by a large change in the excess free energy of a heavy quark source.

The numerical analysis of the deconfinement/chiral phase transition concentrates around two questions - the order of the phase transition and a quantitative determination of the transition temperature. As can be seen from fig.1 the transition is rather abrupt. Whether it is continuous or whether there is a discontinuity in the order parameter which would signal the presence of a first order transition is a subtle question. To answer it a careful analysis of the transition region on various lattice sizes is needed. As we will see, the problem is not completely settled even for the simplest case - the pure $SU(3)$ gauge theory. Similarly a quantitative determination of the transition temperature requires good control over the approach to the continuum limit in order to be able to translate the critical couplings β_c , determined in lattice simulations, into physical values for the transition temperature. These two problems will be discussed in the next two subsections.

Figure 1: The chiral order parameter $\langle \bar{\chi}\chi \rangle$ (triangles) and the Polyakov loop $\langle L \rangle$ (dots) versus $\beta = 6/g^2$ for QCD with $n_f = 4$ and quarks of mass $m/T = 0.1$ on a 1×8^3 lattice. Data are taken from ref.29.

3.2. Order of the phase transition

It has been pointed out first by Svetitsky and Yaffe that the critical properties of four-dimensional $SU(N)$ gauge theories at finite temperature can be related to that of a 3-d effective spin theory. Universality arguments can then be used to classify the order of the phase transition in the pure gauge sector^[30]. A similar argument has later on been used by Pisarski and Wilczek to classify the phase transitions in the chiral limit^[31].

Svetitsky and Yaffe argue that the effective theory for the (3+1)-dimensional finite temperature $SU(N)$ gauge theory is a 3-d, $Z(N)$ symmetric spin model with short-range interactions, dominated by a ferromagnetic nearest neighbour coupling.

In the case of $SU(2)$ the validity of these arguments has been verified in great detail. A Monte Carlo renormalization group analysis has shown that the interactions in the effective theory are indeed short-ranged and ferromagnetic^[32]. One would then expect that the critical properties are the same as those of the 3-d Ising model. Indeed, excellent agreement has been found between the critical exponents of the $SU(2)$ gauge theory and those of the 3-d Ising model^[33].

In the case of $Z(3)$ symmetric models it is of par-

ticular importance to verify that the effective theory is dominated by ferromagnetic interactions as it is known that anti-ferromagnetic interactions can give rise to second order phase transitions³¹⁾ in this case. In the class of globally $Z(3)$ symmetric models with ferromagnetic short-range interactions, on the other hand, no fixed point is known, and one thus expects that these models have first order phase transitions. In the strong coupling limit one can explicitly derive the effective 3-d spin model from the QCD partition function. One can carry out the integration over all spatial degrees of freedom, $U_{n,i}$, $i = 1, 2, 3$. This leads to an effective theory in terms of the Polyakov loop operator^{30,35,36)}

$$Z_{eff} = \int \prod_{\vec{n}} dL_{\vec{n}} \exp[\beta' \sum_{\vec{n}} \sum_{i=1}^3 \text{Tr} L_{\vec{n}} \text{Tr} L_{\vec{n}+\vec{i}}^\dagger + c.c.] \quad (3.7)$$

with $\beta' = \beta^{N_\tau} + \dots$. This is a 3-d, $Z(3)$ spin model with ferromagnetic coupling, $\beta' > 0$. It can be shown that also in higher orders of the strong coupling expansion the interactions are ferromagnetic. Also at larger values of β the effective spin model characterizing the deconfinement phase transition in pure $SU(3)$ gauge theory has been constructed recently³⁷⁾. On a 4×24^3 lattice near T_c ($\beta = 5.6925$) it has been found that the effective spin model has only short ranged ferromagnetic interactions. This gives strong support for the applicability of the universality arguments given by Svetitsky and Yaffe for the $SU(3)$ gauge theory and it therefore is expected that the $SU(3)$ gauge theory indeed exhibits a first order deconfining phase transition.

The numerical analysis of the effective model³⁸⁾ defined by eq.(3.7) as well as the 3-d, 3-state Potts model^{39,40,41)} shows that these $Z(3)$ symmetric spin system with ferromagnetic couplings indeed undergo first order phase transition. Thus, at least for QCD motivated 3-d, $Z(3)$ symmetric models the first order nature of the phase transition seems to be established. The early numerical investigations of $SU(3)$ pure gauge theory, performed on rather small lattices, gave evidence that also in this case the $Z(3)$ symmetry breaking deconfinement phase transition is first order. Rather large discontinuities in the order parameter $\langle L \rangle$ and the internal energy of the system have been observed in these simulations^{12,13)}. Later systematic studies¹¹⁻¹⁷⁾ of the transition showed that at the critical point the system flipped between ordered and disordered phases giving

Figure 2: Time history of the phase, $\Theta \equiv \arg\{\langle L \rangle\}$, of the Polyakov loop operator, $L \equiv |L| \exp(i\Theta)$, for $SU(3)$ gauge theory on a 4×24^3 lattice at $\beta = 5.6925$ (fig.2b). Noisy regions indicate the time intervals during which the system is in the symmetric phase, the remaining time it spent in one of the 3 degenerate vacua with broken $Z(3)$ symmetry where $\Theta \simeq 0, \pm 2\pi/3$. Fig.2a shows the corresponding abrupt changes in the entropy density when the system flips from the broken to the symmetric phase and back. Data are taken from ref.47.

rise to a double peak structure in the probability distribution of the Polyakov loop that is characteristic for a first order phase transition. The time history of such a Monte Carlo simulation for the order parameter $\langle L \rangle$ and the entropy density is shown in fig.2.

Although these results are very suggestive of a first order phase transition, some care is in order. Strictly speaking there are no discontinuities in the order parameter or other physical quantities on finite lattices. When averages are calculated over the complete time history of a Monte Carlo simulation the frequent flips between different metastable states will only lead to a rapid, but smooth, changes in all thermodynamic observables. Discontinuities will develop only in the infinite volume limit. One thus has to study the finite size dependence of thermodynamic quantities in order to judge whether the transition is first or second order in the infinite volume limit. The finite size scaling behaviour is expected to show clear differences in these cases. For instance the critical coupling determined on finite lattices from the relative population of the peaks in the probability distribution of the order parameter, L , or the peak in the specific heat, is expected to scale like

$$\beta_c = \beta_c(V) + \frac{A}{V^{1/\nu_d}} \quad (3.8)$$

Figure 3: Finite size analysis of the critical couplings in the 3-d, 3-state Potts model (a) and the $SU(3)$ gauge theory for $N_\tau = 4$ (b).

with $\nu d = 1$ for a first order transition and $\nu d < 1$ for a second order transition. Such an analysis has been attempted for the $SU(3)$ gauge theory⁽⁴¹⁾. In fig.3 we show results for lattices with $N_\tau = 4$ and compare with a similar analysis performed for the 3-d, 3-state Potts model⁽³⁹⁾. As can be seen in the latter case the determination of ν is statistically significant and clearly supports the first order nature of the transition whereas in the case of the $SU(3)$ gauge theory at best consistency with a first order transition can be deduced from such an analysis.

Additional evidence for the first order nature of the transition comes from a finite size scaling analysis of response functions like the specific heat or susceptibility. Consider, for instance, the susceptibility of the order

parameter defined as

$$\chi = V(\langle \text{Abs}(L)^2 \rangle - \langle \text{Abs}(L) \rangle^2) \quad (3.9)$$

On a finite lattice this quantity reaches a maximum value, χ^{peak} , at β_c . With increasing lattice size χ^{peak} increases and will diverge in the infinite volume limit, irrespective of the order of the transition, i. e. $\chi^{\text{peak}} \sim V^\alpha$. For a first order phase transition $\alpha = 1$, while $\alpha < 1$ for a continuous transition. The finite size scaling behaviour of χ^{peak} has been analyzed in $Z(3)$ symmetric spin models and supports the first order nature of the transition^(39,38). A similar analysis has recently been performed for the $SU(3)$ gauge theory on lattices of size $4 \times N_\sigma^3$, $N_\sigma = 8 - 32$, which also suggests that $\chi^{\text{peak}} \sim V^{0.77}$.

The analysis of the finite size scaling behaviour of the critical coupling and the peaks in the specific heat or susceptibility allows a clear distinction between first and second order phase transitions. The situation is somewhat more subtle for correlation lengths near a first or second order phase transition. This is partly due to the fact that the correlation length cannot be obtained by a simple measurement of a lattice operator but has to be extracted from the long distance behaviour of a two-point correlation function. Moreover, one has to be careful in disentangling the physical mass gap (inverse correlation length) from the tunnelling correlation length, which diverges at β_c irrespective of the order of the transition. Unfortunately the divergence of the tunnelling correlation length itself cannot be used to classify the order of the transition. In a recent large scale analysis of the deconfinement transition the first order nature of the $SU(3)$ deconfinement transition has been questioned⁽⁴⁸⁻⁵⁰⁾ on the basis of a finite size analysis of the correlation length at the phase transition. The analysis of the Polyakov loop correlations as well as plaquette-plaquette correlations close to T_c indicated that for these quantities the correlation lengths increased at T_c with increasing spatial volume of the lattice. In fact the correlation lengths at T_c turned out to rise proportionally to the linear size of the lattice thus suggesting that they actually diverge in the infinite volume limit. This led to a detailed study of correlation lengths in the vicinity of first and second order phase transitions^(37,38,39,41,51,52). In particular the analysis of the correlation length of both the 3-d, 3-state Potts

model³⁹⁾ and the 3-d effective spin model³⁸⁾, defined by eq.(3.7), has shown that the correlation length in these models becomes large close to the critical point. However, in order to judge whether the physical correlation length (inverse mass gap) diverges at β_c one has to be careful in separating its contribution to the correlation functions from that of the tunneling correlation length³⁹⁾. The latter diverges for all $\beta > \beta_c$ in the infinite volume limit, irrespective of the order of the transition. Qualitatively the behaviour of the correlation length found in the $Z(3)$ symmetric spin models seems to be very similar to that observed in the $SU(3)$ gauge theory, although the transition clearly is first order in the former case. It thus seems that it is particularly difficult to deduce the order of the phase transition from the scaling behaviour of the correlation length alone. We will give a more detailed discussion of the behaviour of the $SU(3)$ screening length in section 4.4.

Although there seems to be strong evidence that the $SU(3)$ deconfinement transition is first order, the above discussion shows that a detailed finite size analysis of the pure gauge theory on even larger lattices is still required to establish the order of the deconfinement transition beyond any doubt. In the presence of dynamical fermions the situation is by far more complicated as it is difficult to perform high statistics analyses of full QCD on large lattices that would allow a similarly detailed finite size study as it has now been done in the pure gauge sector. The qualitative changes due to dynamical fermions are, however, theoretically well understood and reproduced by Monte Carlo simulations. The presence of dynamical fermions leads to an explicit breaking of the global $Z(3)$ symmetry of the pure gauge action. In terms of a $1/ma$ expansion of the fermion determinant it can be seen that fermions act like an additional magnetic field in the effective spin model^{35,36,53)}. This leads to a weakening of the 1st-order transition. It is thus expected that the transition seen in the pure gauge sector weakens for finite quark masses and it has been speculated that in an intermediate mass regime there would be no trace of the deconfinement transition left and only in the chiral limit would the restoration of the chiral symmetry lead to a new phase transition.

The order of the chiral transition has been analyzed by Pisarski and Wilczek³¹⁾ using universality arguments

for 3-d chiral models with the general Lagrangian

$$\begin{aligned} L_\Phi = & -\frac{1}{2}\text{Tr}(\partial_\mu \Phi^\dagger \partial_\mu \Phi) - \frac{1}{2}m^2\text{Tr}(\Phi^\dagger \Phi) \\ & + \frac{\pi^2}{3}g_1(\text{Tr}(\Phi^\dagger \Phi))^2 + \frac{\pi^2}{3}g_2\text{Tr}((\Phi^\dagger \Phi))^2 \\ & + c(\det\Phi + \det\Phi^\dagger) \quad , \end{aligned} \quad (3.10)$$

for a field Φ_{ij} , $i, j = 1, \dots, n_f$, which is invariant under the chiral symmetry group $Z(n_f) \times SU(n_f) \times SU(n_f)$. A renormalization group (RG) analysis of this Lagrangian shows that the nature of the transition is strongly dependent on the number of flavours. For $n_f \geq \sqrt{3}$ the RG-equations have no infra-red stable fixed point. The transition is thus expected to be first order in the chiral limit for $n_f = 3$ and larger. A non-zero mass term weakens this transitions and it will disappear completely for masses larger than a certain critical mass. For $n_f = 2$ the term proportional to $\det\Phi + \det\Phi^\dagger$ acts itself like a mass term and thus may change the order of the transition, depending on the magnitude of the coefficient c . The RG analysis thus does not lead to a prediction of the order of the transition for $n_f = 2$. Finally for $n_f = 1$ it predicts that there is no chiral transition at all, as $\det\Phi + \det\Phi^\dagger$ acts like an external magnetic field in this case.

The above scenario for the n_f -dependence of the chiral transition has been found to be correct in the instanton liquid approximation of QCD⁵⁴⁻⁵⁷⁾. A mean-field analysis of this model suggests that there is no chiral phase transition for $n_f = 1$. However, it predicts a second order transition for $n_f = 2$ and a first order transition for all $n_f > 2$ ^{56,57)}.

Monte Carlo simulations for full QCD with dynamical quarks have by now been performed for a variety of different quark masses and flavours (A detailed summary can be found in refs. 14 and 58.). Indeed the transition has been found to be rather sensitive to the choice of quark masses and flavours. It seems that for $n_f > 4$ the transition is first order for all quark masses m . For $n_f \leq 4$ the situation is still to some extent uncertain. Nonetheless, there are strong indications that for sufficiently small quark masses ($m/T \lesssim 0.1$) the transition is first order for all $2 \leq n_f \leq 4$. In fig.1 we have shown results of a simulation for $n_f = 4$ with light quarks of mass $m/T = 0.1$ on a 4×8^3 lattice, which strongly suggests a first order transition. At intermediate quark masses there seems to be a region where

Figure 4: Probability distribution of $|L|$ for $n_f = 2$ and $m/T = 0.4$ on a 4×8^3 lattice at $\beta = 5.38$ (a) and a 4×12^3 lattice at $\beta = 5.375$ (b). These β values are close to the critical couplings on these lattices which have been estimated to be 5.3825(50) and 5.376(3), respectively. Data are taken from ref.62.

the transition is likely to be continuous, although still accompanied by a very rapid change in the order parameters $\langle L \rangle$ and $\langle \bar{\chi} \chi \rangle$. However, one should once again stress the uncertainties in the determination of the order of the chiral transition in Monte Carlo simulations. At present most finite temperature simulations of QCD with light quarks are performed on rather small lattices, $N_\tau \simeq 4 - 12$, $N_\tau = 4, 6$, and it seems that the dependence of the chiral transition on the spatial lattice size is not fully understood. At intermediate quark masses ($0.4 \lesssim m/T \lesssim 1.0$) the volume dependence has been studied⁵⁹⁻⁶². This showed that metastabilities, which could be considered as being indicative for a first order transition, tend to disappear with increasing lattice size. In fig.4 we present results from a finite size analysis of the phase transition at $m/T = 0.4$ and $n_f = 2$. As can be seen the distribution of the Polyakov line is broad on a small lattice and gives a clear two-

state signal, which has been taken as indication of a 1st order transition. However, on the larger lattice the distribution becomes narrower for all values of β and there remains little evidence for a first order transition. If this behaviour persists at smaller quark masses and on larger lattices a second order transition, particularly for $n_f = 2$, cannot be ruled out.

For the actual case of QCD it is also important to understand the influence of an intermediately heavy strange quark ($m_s/T \sim O(1)$) in addition to a light isodoublet ($m_u = m_d \simeq 0$). Simulations which try to approximate this situation^{63,64} indicate that the influence of a strange quark with $m_s/T = 1$ on the order of the transition is small. The critical behaviour of QCD thus seems to be controlled by the $n_f = 2$ chiral theory.

3.3. The critical temperature

For all $SU(N)$ gauge groups it has been rigorously established that the global $Z(N)$ centre symmetry is spontaneously broken on the lattice for couplings β larger than a certain critical coupling $\beta_c = 2N/g_c^2$ ⁶⁵. A similar existence proof has been given for the chiral phase transition in the case of $SU(2)$ lattice gauge theory⁶⁶. However, in both cases the bounds on β_c are weak and, in particular, they do not rule out that the transition temperature shifts to infinity in the continuum limit.

A quantitative determination of the critical temperature from lattice simulations involves two steps. First of all one has to show that the transition temperature is independent of the lattice cut-off, i.e. when increasing the temporal lattice size N_τ the transition has to occur at a smaller lattice spacing, $a(g_c)$. The critical coupling, g_c , has to scale such that the product $N_\tau a(g_c) = 1/T_c$ stays constant. In the continuum limit the dependence of g_c on N_τ is given by the renormalization group equation,

$$T_c/\Lambda_L = N_\tau^{-1} (b_0 g_c^2)^{b_1/2b_0^2} e^{1/2b_0 a_c^2} \quad (3.11)$$

Monte Carlo renormalization group studies¹⁷ suggest that this asymptotic scaling relation holds approximately for $g_c^2 < 1$. In order to achieve that the critical coupling is smaller than 1 large temporal lattices are necessary. Using nonetheless the asymptotic scaling relation, eq.(2.13), also on smaller lattices to extract T_c/Λ_L leads to results which will still depend on N_τ (or g^2), indicating that the validity regime of the asymp-

Figure 5: $T_c/\Lambda_{\overline{MS}}$ for pure gauge theory, $n_f \equiv 0$, (dots) and massless QCD with $n_f = 2$ (triangles) and $n_f = 4$ (squares) flavours versus temporal extent of the lattice. The solid line gives the critical temperature for the pure gauge theory, $T_c\Lambda_{\overline{MS}} = 1.78 \pm 0.03$, estimated on the basis of the results obtained for $N_\tau = 10, 12$ and 14 .

otic scaling relation has not been reached. Results for the $SU(3)$ gauge theory are shown in fig.5. (Rather than using Λ_L to fix the scale we converted this to the continuum scale parameter $\Lambda_{\overline{MS}}$ which is less sensitive to the number of flavours.) As can be seen $T_c/\Lambda_{\overline{MS}}$ is roughly N_τ independent on lattices with $N_\tau \geq 10$, or equivalently for $g_c^2 \leq 1$.

In order to extract the critical temperature in physical units from these measurements one has to eliminate the scale parameter, Λ_L , by measuring a second physical quantity in the continuum regime. Using measurements of the string tension⁶⁷⁾ or hadron masses in the quenched approximation⁶⁸⁾ one obtains

$$T_c = \begin{cases} 0.58(4)\sqrt{\sigma} & = 254(18)\text{MeV} \\ 0.30(5)m_\rho & = 231(38)\text{MeV} \\ 0.21(2)m_N & = 197(20)\text{MeV} \end{cases} \quad (3.12)$$

Here we have used $\sigma = 0.192\text{GeV}^2$ for the string tension⁶⁹⁾ and $m_\rho = 770\text{MeV}$, $m_N = 940\text{MeV}$ for the rho and nucleon masses respectively. The difference in T_c obtained by either using m_ρ or m_N to fix the lattice scale $a\Lambda_L$ reflects the difficulties one has in reproducing the experimental mass spectrum in lattice simulations. The ratio m_N/m_ρ calculated on the lattice does not

agree with the experimental result so far, it comes out too high (see ref. 14 for a detailed discussions of hadron mass calculations on the lattice).

A similar detailed analysis of the transition temperature for QCD with light quarks does not yet exist. So far only rather small lattices with $N_\tau \leq 6$ could be analyzed. Most detailed results exist for $n_f = 2$ and 4 on lattices with $N_\tau = 4$ and 6 . Rather surprisingly it has been found that the violations of asymptotic scaling seen in the measurement of the critical temperature when going from $N_\tau = 4$ to $N_\tau = 6$ are in good quantitative agreement with those present already in the pure gauge sector. In order to convert the critical couplings into a critical temperature, measured in units of $\Lambda_{\overline{MS}}$ using the asymptotic scaling relation, eq.(2.13), an additional extrapolation to zero quark mass is required. A comparison of these critical temperatures for $n_f = 2$ and 4 with the corresponding results for $n_f = 0$ is also shown in fig.5. A further conversion of the critical temperature into units of MeV again requires the measurement of a second physical quantity, for instance m_N or m_ρ , for the same value of the lattice cut-off (same coupling β . For full QCD this leads to considerably smaller values for the critical temperature than in the pure gauge sector. For $n_f = 2$ and quark masses $m/T = 0.15$ one finds⁷⁰⁾

$$T_c = \begin{cases} 0.19(3)m_\rho & = 116(23)\text{MeV} \\ 0.12(1)m_N & = 113(9)\text{MeV} \end{cases} \quad (3.13)$$

on lattices with $N_\tau = 6$. Results obtained for $N_\tau = 4$ lead to about 10% smaller values for T_c ^{70,71)}. A similar analysis of the transition temperature for $n_f = 4$ and $N_\tau = 4$ and 6 ⁷²⁾ leads to values for T_c measured in units of hadron masses that are about 15% smaller than those obtained for $n_f = 2$. However, one should keep in mind that at present the calculations with dynamical fermions are by far not as accurate as corresponding simulations in the pure gauge sector. The finite size effects still present in the determination of both T_c as well as m_ρ or m_N have the tendency to lead to larger $T_c/m_{\rho(N)}$ ratios when the spatial lattice size is increased. On larger lattices β_c will shift to larger values and $m_{\rho(N)}$ will become smaller. Although the first effect is expected to lead only to small modifications the effect of the latter is difficult to judge. Also the changes in the $T_c/m_{\rho(N)}$ ratio with increasing N_τ are not studied systematically.

4. THERMODYNAMICS OF THE PLASMA PHASE

4.1. Equation of state

Here we want to discuss Monte Carlo results for the equation of state (e.o.s.) in the high temperature phase. We will compare these results with perturbation theory to $O(g^2)$. Deviations from this will give some indication of the importance of non-perturbative effects in the high temperature plasma phase. In order to perform this comparison we have to take into account that Monte Carlo simulations are still performed on rather small lattices. In particular the small temporal lattice size, needed to simulate finite temperature effects, leads to rather severe finite size effects. This prohibits a direct comparison of the Monte Carlo data with continuum perturbation theory. As an intermediate step one thus calculates the lowest order perturbative expressions on finite lattices⁷³⁻⁷⁵ and compares these with the Monte Carlo data.

Let us start with a discussion of the entropy density defined in eq.(2.28). This expression still involves the derivative of the couplings with respect to the anisotropy parameter. We will approximate these derivatives by their lowest order weak coupling expressions^{19,21,24}. The entropy density can then be written as

$$\begin{aligned} \frac{S}{T^3} = & \frac{8N}{g^2} N_\tau \left(1 - \frac{c'_\sigma - c'_\tau}{2} g^2\right) (\langle P_\sigma \rangle - \langle P_\tau \rangle) \\ & + \frac{n_f}{3} N_\tau \left(1 + c'_F g^2\right) \left(\langle Tr D_0 Q^{-1} \rangle - \frac{N}{4}\right) \end{aligned} \quad (4.1)$$

where $c'_{\sigma(\tau)}$ and c'_F are given by eq.(2.22) and we have ignored an additional term which is proportional to the bare quark mass (see eq.(2.28)). We will first consider the pure gauge sector only. The entropy density involves then only differences of space-space (P_σ) and space-time (P_τ) plaquettes.

In fig.6 we show results for S/T^3 obtained on lattices of size $4 \times N_\sigma^3$, $N_\sigma = 12^{76}$, 16^{17} and 24^{17} . The numerical results are compared with perturbative behaviour up to $O(g^2)$ obtained on a 4×12^3 lattice⁷⁴. It can be seen that the entropy density approaches ideal gas behaviour at temperatures $T \sim (2-3)T_c$. Close to T_c ($\beta_c \simeq 5.692$), however, the entropy drops to about 30% of the corresponding ideal gas value.

It is instructive to analyze what causes these large deviations from ideal gas behaviour close to T_c . To

Figure 6: Entropy density in units of T^3 versus β on lattices of size $4 \times N_\sigma^3$, with $N_\sigma = 12$ (dots)⁷⁶, 16 (triangles)¹⁷ and 24 (squares)¹⁷. The data on the 4×16^3 lattice have been shifted by $\Delta\beta = -0.02$ (see discussion in ref.47). The dashed and dashed-dotted curves give the perturbative results on a 4×12^3 lattice. Perturbative results for larger N_σ differ only little from this. Lines are drawn to guide the eye.

Figure 7: Energy density and pressure in units of the ideal gas values on finite lattices versus T/T_c . Data on the 4×12^3 lattice (dots and triangles) are taken from ref.76 and include a subtraction of the vacuum contributions. The data on the 4×24^3 (squares) are taken from ref.47 and do not include a subtraction of the vacuum part. Lines are drawn to guide the eye.

this extent we look separately at the contribution of the energy density and the pressure to S/T^3 . These quantities are displayed in fig.7. A comparison with the perturbative results shows that the deviations from ideal gas behaviour seen in S/T^3 close to T_c are mainly due to the non-perturbative behaviour of the pressure. The energy density on the other hand is quite close to the Stefan-Boltzmann limit already at $T \gtrsim 1.2T_c$.

At least for $T \gtrsim 1.2T_c$ the good agreement between the 4×12^3 and 4×24^3 data in fig.6 and 7 shows that finite size effects are small for this size lattices. There are, however, large discrepancies between the results for the latent heat obtained in earlier simulations^{77,78)} on smaller lattices and the recent analysis on a 4×24^3 lattice¹⁷⁾ close to T_c . Simulations on a 4×10^3 lattice gave for the latent heat of the transition $\Delta\epsilon/T_c^4 \simeq 5.53 \pm 0.37^{78)}$. This is substantially larger than what has been found from the simulation on a $N_\tau \times 24^3$ lattice¹⁷⁾,

$$\frac{\Delta\epsilon}{T_c^4} = \begin{cases} 2.51 \pm 0.12 & , \text{ for } N_\tau = 4 \\ 2.48 \pm 0.24 & , \text{ for } N_\tau = 6 \end{cases} \quad (4.2)$$

The origin of this discrepancy is not completely clarified. One would expect any discontinuity to be smoothed out on smaller lattices and becoming more pronounced on larger lattices. In view of the large finite size dependence of the latent heat measurements found for lattices with only 4 sites in temporal direction ($N_\tau = 4$) one has to be sceptical about the accuracy obtained so far for $\Delta\epsilon$ on lattices with larger N_τ ⁷⁹⁾ (the signal for $\Delta\epsilon$ decreases as N_τ^{-1} !).

The result for the latent heat, given in eq.(4.2), should be compared with the energy density of an ideal gluon gas calculated on a lattice of same size. For $N_\tau = 4$ (6) one finds $\epsilon_G/T^4 = 7.85$ (6.16), i.e. the latent heat is about 1/3 of the energy density of an ideal gas at the same temperature. Notice, however, that even for $N_\tau \rightarrow \infty$ the lattice result for ϵ_G/T^4 is larger by a factor 1.49 (1.18) than the continuum Stefan-Boltzmann value $\epsilon/T^4 = \frac{8}{15}\pi^2$ for a free gas of 8 massless gluons. Only for $N_\tau \rightarrow \infty$ does the lattice ideal gas approach the continuum value. It thus is to be expected that the present estimates for $\Delta\epsilon$ are still affected by finite lattice size effects and it seems that a more detailed finite size analysis is needed in order to establish an accurate value for the latent heat.

Qualitatively similar results for the entropy density as well as for energy density and pressure have been

obtained for the $SU(2)$ gauge theory^{18,80-82)} and also for QCD with light quarks^{83,84)}. In fig.8 we show results for the entropy density, S/T^3 , obtained for two flavour QCD ($n_f = 2$) with light quarks of mass $m/T = 0.1^{21)}$. The corresponding results for energy density and pressure are given in fig.9⁸³⁾. As can be seen also in this case ϵ/T^4 rises rapidly at T_c and stays approximately constant already close to T_c . The pressure, on the other hand, rises only slowly and the ideal gas relation $\epsilon = 3p$ is satisfied only for $T \sim 2T_c$. Also the entropy density approaches quickly the value expected for an ideal quark-gluon gas. This is similar to the pure gauge sector. However, in the case of QCD the ideal gas value seems to be approached from above, whereas in the pure gauge sector the approach is from below and even seems to be in agreement with the $O(g^2)$ perturbative corrections calculated on the lattice. Whether this overshooting of the perturbative value observed for QCD is a lattice artifact or will persist in the continuum limit is at present difficult to judge. At this point it is crucial to keep in mind that the thermodynamic observables calculated on the lattice are not exact. We have used approximate perturbative expressions for the derivatives of the couplings with respect to the anisotropy (see discussion in section 2.2). It has been demonstrated recently that it is important to include the $O(g^2)$ corrections $c'_{\sigma,(\tau,F)}$ in the expressions used on the lattice to calculate the entropy density, eq.(4.1), and other observables. This insures that their perturbative evaluation on the lattice is consistent with continuum perturbation theory. Leaving them out leads to an even larger overshooting of the perturbative limit⁸⁵⁾. It is thus conceivable that also the presently used expressions for the thermodynamic quantities are not accurate enough in the presence of light quarks and a non-perturbative determination of the derivatives appearing in the definitions of thermodynamic quantities given in eq.(2.23-2.28) might be necessary^{21,22)}.

In the pure gauge sector as well as for full QCD a large difference in behaviour is observed for the pressure and the energy density in the plasma phase close to T_c . The equation of state in this region is obviously highly non-perturbative. At least for the pure gauge sector the deviations from the ideal gas behaviour seen in the entropy density close to T_c also rules out the validity of a simple bag equation of state in this region, which

Figure 8: Entropy density in units of T^3 versus β for two flavour QCD with quarks of mass $m/T = 0.1$. Shown are results obtained on 4×8^3 (triangles) and 4×12^3 (dots) lattices⁸⁴⁾. The full line is the lowest order perturbative result on a lattice of size 8×12^3 , the dashed line is the corresponding $O(g^2)$ correction.

Figure 9: Energy density and pressure in units of T^4 versus β for two flavour QCD with quarks of mass $m/T = 0.1$ on a 4×8^3 lattice. Data are taken from ref.83, they, however, do not include contributions from $O(g^2)$ corrections to $d\gamma_F/d\xi$ given in eq.(4.1). Lines are drawn to guide the eye.

would lead to $S = 4a_{SB}T^3$, with

$$a_{SB} = \frac{\pi^2}{45}(N^2 - 1 + \frac{7}{4}Nn_f) \quad (4.3)$$

denoting the Stefan-Boltzmann constant for an ideal quark-gluon gas. The present analysis of the bulk thermodynamic quantities suggests that at least for temperatures $T \gtrsim 1.2T_c$ the plasma phase can well be described by an e.o.s. which is of the form⁸⁰⁾

$$\begin{aligned} \epsilon &= 3a_{SB}T^4 + O(g^2) \\ p &= \frac{1}{3}\epsilon - bT^3, \end{aligned} \quad (4.4)$$

with $b \simeq 1.5T_c^3$.

The different behaviour of energy density and pressure is not unexpected: in the case of a second order phase transition the singular part of the partition function is expected to be of the form $\ln Z \sim VT^3(1-t)^{2-\alpha}$, $t = T_c/T \leq 1$. Here α is the thermal exponent that governs the singular behaviour of the specific heat, $c_V = (\partial\epsilon/\partial T)_V \sim (1-t)^{-\alpha}$, at T_c . From this ansatz it follows that close to T_c the energy density ϵ and the pressure P are given by

$$\begin{aligned} \epsilon &\sim (T - T_c)^{1-\alpha}, \\ P &\sim (T - T_c)^{2-\alpha}. \end{aligned} \quad (4.5)$$

P thus will drop faster close to T_c than ϵ ⁸⁶⁾. For a weak first order phase transition a remnant of this behaviour may be visible as P is continuous through the transition and has to connect smoothly to the low pressure in the hadronic phase whereas the energy density jumps at T_c .

Independent of these general considerations, which are valid only close to T_c , the quasi ideal gas behaviour of the energy density and the non-perturbative behaviour of the pressure has been taken as indication for the importance of massive excitations^{81,87,88)} in the plasma phase. These are expected to affect the pressure more than the energy density in the plasma phase. The basic idea is the observation that in the plasma phase, heavy colour singlet excitations will contribute dominantly in the low momentum regime to the thermodynamic potential, while for large momenta a free massless gluon gas gives the dominant contribution. A simple model calculation based on this idea shows that the introduction of heavy low frequency modes in the gluon propagator affects predominantly the pressure whereas

the energy density remains basically unaffected⁽⁸⁵⁾. In the simplest version one may think of a single mass scale which defines the relevant momentum cut-off in the dispersion relation

$$E(\vec{k}) = \begin{cases} |\vec{k}| & , |\vec{k}| > m_c \\ \sqrt{\vec{k}^2 + m_c^2} & , |\vec{k}| < m_c \end{cases} \quad (4.6)$$

The partition function reads then

$$\ln Z = dV T^3 \frac{\pi^2}{90} - dV h(T, m_c) \quad (4.7)$$

where the first term is just the contribution of an ideal gas with d degrees of freedom and the second term parametrizes the deviations due to massive excitations,

$$h(T, m_c) = -\frac{T^3}{2\pi^2} \int_0^{m_c/T} dx x^2 \times \ln \frac{1 - e^{-\sqrt{x^2 + (m_c/T)^2}}}{1 - e^{-x}} \quad (4.8)$$

For the energy and pressure one obtains from this ansatz

$$\begin{aligned} p &= d \frac{\pi^2}{90} T^4 - d T h(T, m_c) \\ \epsilon &= 3d \frac{\pi^2}{90} T^4 - d T^2 \frac{\partial}{\partial T} h(T, m_c) \end{aligned} \quad (4.9)$$

For $m_c \sim T$ it turns out that $h(m_c, T)$ is only weakly dependent on T and the derivative term contributing to ϵ in eq.(4.9) is thus negligible, whereas the contribution in p coming from $h(m_c, T)$ is linear in T and thus leads to a correction term for the pressure as given in eq.(4.4). The above ansatz can be further refined to get a good phenomenological description of the e.o.s. extracted from lattice simulations of QCD. An explicit comparison of the above ideas with the high statistics data available for the $SU(2)$ thermodynamics shows that a good quantitative description of the behaviour of energy density and pressure can be obtained this way⁽⁸²⁾.

4.2. Strangeness content of the plasma

A realistic simulation of the QCD e.o.s. has to take into account also the contribution of strange quarks to the partition function. Their mass is of the order of the phase transition temperature and thermal production of strange quarks is thus expected to become relevant for $T \sim T_c$. They may influence the chiral phase transition itself^(63,64) as well as the dynamics of the high temperature phase. As the strangeness content of the low temperature phase is small it has been

Figure 10: Fermionic energy density of light u- and d-quarks and heavy s-quark in units of T^4 versus β on a 4×12^3 lattice. Data are taken from ref.64. The dashed line gives the lowest order ideal gas behaviour on a finite lattice.

argued that strangeness production in the plasma phase can yield genuine signals of plasma formation in heavy ion experiments^(89,90).

On the basis of ideal gas relations for massive fermion gases one would expect a large contribution of the strange quark sector to the energy density. The energy density in a massive fermion gas is given by

$$\epsilon_F(m/T) = d T^4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} \left[15 \left(\frac{mn}{T^4} \right)^2 K_2(mn/T) + 3 \left(\frac{mn}{T} \right)^3 K_1(mn/T) \right] \quad (4.10)$$

with d counting the number of degrees of freedom and K_i , $i = 1, 2$ denoting Bessel-functions of second kind. From eq.(4.10) one finds for the energy density of quarks with masses which are of the order of the temperature of the system

$$\frac{\epsilon_F(m/T=1)}{\epsilon_F(m/T=0)} = 0.88 \quad (4.11)$$

In a non-interacting gas of massive fermions with $m \simeq O(T)$ the energy density at a given temperature is thus only little less than the corresponding energy density in a gas of massless fermions.

The contribution of strange quarks to the energy density of QCD has been studied recently⁽⁶⁴⁾ in a simulation with a light isodoublet of mass $m_u/T = m_d/T =$

0.05 and a heavy quark of mass $m_s/T = 1.0$. Results for the fermionic part of the energy density are shown in fig.10. They indicate that only at rather high temperatures is the perturbative result, eq.(4.11), approached. Close to T_c one finds $\epsilon_F(m_s/T)/\epsilon_F(m_u/T) \simeq 0.5$.

However, looking only at the fermionic part of the energy density is not completely correct, as the presence of fermions also strongly influences the gluonic contribution, ϵ_G , to the total energy density. Furthermore, the derivative $d\gamma_F/d\xi$ appearing in the definition of the energy density, eq.(2.25), has been approximated by 1 in the above analysis. It is known, that the $O(g^2)$ corrections to this will lead to a reduction of the fermionic contribution to the total energy density and, furthermore, this reduction will also be dependant on the quark mass²¹⁾. These effects have not been studied so far. The present result for $\epsilon_F(m_s/T)/\epsilon_F(m_u/T)$ thus should only be taken as a first attempt to analyze the influence of an intermediately heavy strange quark on the e.o.s.. However, if verified by a more detailed analysis on larger lattices it suggests that the thermal production of strange quarks close to T_c is smaller than it would be in an ideal gas. In any case, as should have become clear from our discussion of the QCD e.o.s. near T_c , it is certainly not to be expected that ideal gas relations are valid for the strangeness sector in the plasma phase in this temperature regime.

4.3. The heavy quark potential

The heavy quark potential plays a central role in our attempts to understand the phase structure of QCD. A quantitative analysis of its short and intermediate distance properties is important for the understanding of the heavy quark bound state spectrum. Its temperature dependence recently found much attention and is expected to lead to characteristic signatures for plasma formation in heavy ion collisions^{91,92)}.

Besides the interest in the quantitative properties of the potential at intermediate distances an understanding of its asymptotic large distance behaviour is of fundamental importance. At low temperatures the potential rises linearly at large distances leading to confinement. In the deconfined phase, however, it is screened due to the presence of nearly free colour sources. We thus expect a characteristic change in the functional form of the potential as we cross the QCD phase tran-

sition:

$$V_{q\bar{q}}(r, T) = \begin{cases} \frac{\sigma(T)}{r} + \sigma(T)r & , T \leq T_c \\ \frac{\alpha(T)}{r} e^{-\mu(T)r} & , T > T_c \end{cases} \quad (4.12)$$

Here σ denotes the string tension and μ the screening mass in the high temperature phase. Their temperature dependence is of fundamental interest and recently led to conflicting statements on the order of the phase transition^{37,17,18)}.

On the lattice the potential for a static $q\bar{q}$ pair separated by a distance $R \equiv r/a = |\vec{n}|$ can be obtained from the correlation function of two Polyakov loops, $\langle \text{Tr} L_0 \text{Tr} L_{\vec{n}}^\dagger \rangle$. We define the heavy quark potential by

$$\exp(-V_{q\bar{q}}(r, T)/T) = \frac{\langle \text{Tr} L_0 \text{Tr} L_{\vec{n}}^\dagger \rangle}{\langle |L| \rangle^2} \quad (4.13)$$

The normalization of the loop-loop correlation function with $\langle |L| \rangle^2$ is used to eliminate divergent self-energy terms in the potential. Eq.(4.13) then provides a well-defined potential, free of self-energy divergences. The heavy quark potential defined this way actually gives a thermal average of colour singlet and octet quark-antiquark potentials²⁸⁾. In perturbation theory the singlet potential is attractive, whereas the octet potential is repulsive. Their relative strength is such that it leads to a cancellation of the leading, $O(g^2)$, contributions to the colour averaged potential defined in eq.(4.13). The leading perturbative contribution thus is $O(g^4)$,

$$V_{q\bar{q}}(r, T)/T = -\frac{1}{16} \frac{V_1(r, T)^2}{T^2} \quad (4.14)$$

with the singlet potential V_1 given by

$$V_1(r, T) = -g^2(T) \frac{N^2 - 1}{8\pi N} \frac{1}{r} e^{-m_E(T)r} \quad (4.15)$$

In leading order perturbation theory the screening mass, m_E , of course vanishes. However, including self energy terms in the bare gluon propagator, leads to eq.(4.15) with m_E denoting the electric screening mass, which to leading order perturbation theory is given by

$$m_E(T)^2 = \left(\frac{N}{3} + \frac{n_f}{6} \right) g^2(T) T^2 \quad (4.16)$$

Whether this leading order result for the heavy quark potential is of any significance at all, or gets completely invalidated due to the infra-red problems that

occur at higher orders is unclear⁹³⁻⁹⁵). A detailed non-perturbative analysis of the functional form of the heavy quark potential thus is of particular importance for understanding more about the infra-red properties of high temperature QCD. In the following we will discuss some Monte Carlo results for the heavy quark potential and compare these with leading order perturbation theory. This will give some indication about the applicability of the leading order perturbative result.

Let us first discuss the relation between the potential calculated on the lattice and the corresponding continuum potential. Due to the periodic spatial boundary conditions the lattice potential is periodic around $N_\sigma/2$ and the quark-antiquark separation can only vary between $R = 1, 2, \dots, N_\sigma/2$. On lattices with fixed asymmetry N_σ/N_τ there is thus only a limited range of distances measured in units of T accessible,

$$\frac{1}{N_\tau} \leq rT \leq \frac{1}{2} \frac{N_\sigma}{N_\tau} \quad (4.17)$$

no matter how large N_σ and N_τ are chosen. For instance, on a lattice with $N_\sigma/N_\tau = 2$ the potential will be periodic around $rT = 1$ and one may expect that the potential will be strongly distorted for $rT \sim 1$ due to the periodicity of the lattice. The behaviour of the heavy quark potential in the continuum limit can then be determined by comparing calculations of the lattice potential on different size lattices performed at the same temperature $T = 1/N_\tau a$. In fig.11 we show results for the $SU(2)$ heavy quark potential at a high temperature, $T = 250\Lambda_L$ ($T \simeq 6T_c$), obtained on lattices of various sizes and with varying asymmetry between spatial and temporal lattice extent, $N_\sigma/N_\tau = 1, 2$ and 3 . We note that results from different size lattices follow a unique curve as long as the ratio N_σ/N_τ is the same. This suggests that we are indeed calculating the continuum heavy quark potential in a finite physical volume. In addition it is obvious that the potential becomes steeper with increasing N_σ/N_τ and that the distortion of the infinite volume potential is largest around $rT \sim N_\sigma/2N_\tau$. A detailed analysis of the finite size dependence of the $SU(2)$ potential⁹⁶) shows that this can be parametrized as

$$\ln \left[\frac{V_\infty(r, T)}{T} \right] = \ln \left[\frac{V(r, T)}{T} \right] - \frac{c(r, T)}{1/T^3} \quad (4.18)$$

An extrapolation to infinite volume based on this ansatz is also shown in fig.11.

Figure 11: The heavy quark potential $-V(r, T)/T$ versus rT at fixed temperature $T = 250\Lambda_L$. The couplings used in the simulation are $\beta = 2.988$ ($N_\tau = 4$), $\beta = 3.147$ ($N_\tau = 6$) and $\beta = 3.26$ ($N_\tau = 8$). The dashed line gives the potential extrapolated to infinite volume.

In order to compare the Monte Carlo results for the heavy quark potential in the plasma phase with perturbation theory it seems to be appropriate to use a general ansatz of the form

$$\frac{V(r, T)}{T} = \frac{\alpha(T)}{(rT)^d} e^{-\mu r} \quad (4.19)$$

for fits to the numerical data. Such fits show that the perturbative prediction for the subleading $1/rT$ -term, i.e. $d = 2$, is correct only for very large temperatures. Even at temperatures as high as $6T_c$ the asymptotic behaviour is not valid. For the case shown in fig.11 the fits gave $d \simeq 1.6$. As the temperature is lowered one finds that d decreases further and it is compatible with $d = 1$ for $T \simeq T_c$ ⁹⁶).

Fig.11 also shows that the potentials are flatter on smaller lattices. This suggests that finite size effects

tend to lower the screening mass in a finite volume. In fact, fits to the potential using the ansatz given in eq.(4.19) show that even for $N_\sigma/N_\tau = 3$ the screening mass is still about 20% lower than the asymptotic infinite volume result⁹⁶⁾. This volume dependence of the screening mass is in qualitative agreement with perturbative calculations on finite lattices⁹⁷⁾.

Like in the discussion of bulk thermodynamic quantities we thus find also here that deviations from perturbative predictions are large close to T_c . Similar results have been obtained for the functional form of the $SU(3)$ potential⁹⁸⁾. Also in that case one finds indications that the perturbative form of the potential with $d = 2$ is valid only at very high temperature, whereas a potential with $d \equiv 1$ gives a better description of the Monte Carlo close to T_c .

4.4. Correlation lengths

The correlation length extracted from the asymptotic large distance behaviour of the heavy quark potential has been studied recently for pure $SU(3)$ gauge theory on rather large lattices^{37,47,48)}. In particular its behaviour close to T_c has been analyzed in detail. The mass gap (inverse correlation length) can be extracted from the large distance behaviour of the Polyakov-loop correlation function

$$\mu/T = -N_\tau \lim_{|\vec{n}| \rightarrow \infty} \ln \frac{1}{|\vec{n}|} \times \quad (4.20)$$

$$\times \left[\langle \text{Tr} L_0 \text{Tr} L_{\vec{n}}^\dagger \rangle - \langle L \rangle^2 \right].$$

A comparison with eqs. (4.12) and (4.13) shows that μ is identical to the screening mass in the heavy quark potential for $T > T_c$ and the string tension divided by the temperature, σ/T , for $T < T_c$.

In practice it is difficult to extract information about the asymptotic behaviour of the correlation functions, as it is rather difficult to measure correlations at large values of $rT = |\vec{n}|N_\tau$. In the first numerical determinations of the mass gap¹⁰¹⁻¹⁰³⁾ the potential could only be analyzed up to distances $rT \sim 1$. In this intermediate distance regime the subleading power behaviour of the potential is still very important and, as we have seen in the previous section, the exponent of the subleading $1/r$ is essentially unknown. Recent high statistics simulations on large lattices⁴⁷⁻⁴⁸⁾ allowed a determination of the mass gap μ from the heavy quark potential for $1 < rT < 5$. These calculations lead to sub-

Figure 12: Mass gap in units of the lattice spacing for the pure $SU(3)$ gauge theory obtained on lattices of size 4×24^3 [ref.37(squares), ref.47(dots)] and $4 \times 8^2 \times 32$ [ref. 48(triangles)].

stantially smaller values for μ/T than those obtained in earlier simulations^{101,102)}, where the mass gap has been extracted from the region $rT \sim 1$. Some results for μ/T for pure $SU(3)$ gauge theory are shown in fig.12. It can be seen that the mass gap decreases strongly close to T_c ($\beta_c \simeq 5.692$). In fact, it has been suggested that $\mu(T_c) \rightarrow 0$ if the spatial lattice size moves to infinity¹⁸⁻⁴⁹⁾. However, this does not seem to be supported by a recent study of the mass gap on a large, 4×32^3 , lattice³⁷⁾. There are thus still uncertainties concerning the exact behaviour of the mass gap at T_c . However, it should also be noted that the regime of couplings shown in fig.12 corresponds to a rather small temperature interval around T_c ($\Delta T/T_c \lesssim 0.2$). Already at $T \simeq 1.2T_c$ ($\beta = 5.75$) one finds $\mu/T \simeq 2.5$, and this ratio stays constant for larger temperatures^{98,104)}. We note that the above result for the screening mass in the plasma phase is already close to the lowest order perturbative expression, $\mu = 2m_E \sim 2T$, with m_E given by eq.(4.16) if we assume $g(T) \sim 1$.

Simulations for QCD with light quarks^{84,105)} are at present in a stage similar to the early $SU(3)$ calculations. The statistics collected in these simulations is not sufficient to go to large distances or analyse the potential with an arbitrary power for the $1/rT$ -

term. Nonetheless, as we have seen, eq.(4.19) with $d \equiv 1$ seems to give a satisfactory description of the intermediate-distance part of the potential close to T_c , and can be used to analyse the Monte Carlo data in this regime. One should, however, interpret the screening mass extracted from these data as an effective mass, μ_{eff} , that characterizes the behaviour of the potential for $rT \leq 1$. Present calculations for QCD give $\mu_{eff}/T \simeq 3.5$ for $T \simeq 1.1T_c$ which rises up to $\mu_{eff}/T \simeq 5.0$ for $T \gtrsim 2T_c$ ^{84,105}. The behaviour is thus similar to what has been found in the pure gauge sector. The asymptotic value at large temperatures, however, seems to be larger than for pure $SU(3)$ gauge theory, which is in accordance with the expectation that the presence of light quarks leads to a more efficient screening of colour charges.

4.5. Other observables

We have seen that the analysis of bulk thermodynamic quantities in pure $SU(3)$ gauge theory as well as QCD indicate that for $T \gtrsim 1.2T_c$ the thermodynamic behaviour is close to what one would expect for an ideal gas of quarks and gluons. This is also reflected in the behaviour of other observables. For instance the quark number susceptibility¹⁰⁶ shows that it is easy to create baryonic excitations in the plasma phase, suggesting that the relevant degrees of freedom can be associated with light quarks. Also the analysis of transport coefficients in the high temperature phase suggests that the heat conductivity is small as one would expect for a weakly interacting gas¹⁰⁷.

Close to T_c one observes significant non-perturbative effects. There are large deviations from the ideal gas equation of state, $\epsilon = 3p$. This is also reflected in a calculation of the velocity of sound, $c_s^2 = dp/d\epsilon$ in pure $SU(3)$ gauge theory^{108,109}. While c_s is compatible with the ideal gas value, $c_s = \sqrt{1/3}$, for $T \gtrsim 1.2T_c$, it drops suddenly close to T_c .

The discontinuity in the entropy density (latent heat) at T_c seems to be small. At present the numerical data are indeed not accurate enough in order to rule out completely the possibility that the transition for pure $SU(3)$ gauge theory is continuous¹⁸⁻²⁰. A careful finite size analysis will be needed to clarify this point. If the transition turns out to be first order different phases can coexist at T_c . The interface between these phases will have a non-vanishing surface

energy. The thermodynamics of such interfaces has been studied in mean-field approximation¹¹⁰ and simple 2-dimensional models^{111,112}. A first analysis of the interface energy density for the $SU(3)$ gauge theory has been performed recently on a rather small lattice with only two sites in temporal direction ($N_\tau = 2$)^{113,114}. In this case the deconfinement transition is clearly first order and a stable interface between the confined and deconfined phases can be produced. This exploratory study yields for the surface energy, α , in units of the deconfinement temperature, $\alpha/T_c^3 = 0.21 \pm 0.06$.

5. HEAVY ION COLLISIONS

We have seen in the previous sections that lattice simulations of finite temperature QCD strongly suggest the existence of a quark-gluon plasma phase at high temperatures and/or baryon number densities. The numerical simulations of QCD with light quarks yield astonishingly low values for the critical temperature of the phase transition. At present it is, however, not clear whether the large decrease relative to results found in pure gauge theories is physical (it may well be so as a larger number of partons is contributing to energy and pressure at a given temperature in this case) or whether the systematic errors in the presence of dynamical fermions due to the lattice discretization (finite lattice size effects, finite cut-off effects) are still too large in these calculations. This requires further systematic studies. For our present purpose, however, it is sufficient to use the critical temperature determined in pure gauge theory, which is much better known, as a guide line. Any decrease in the critical temperature would make the situation for plasma formation in heavy ion collisions, which we are going to discuss in the following, even more favourable.

Lattice simulations of pure $SU(3)$ gauge theory suggest a transition temperature, $T_c \sim 200\text{MeV}$, at which the energy density rises sharply and quickly reaches values which are close to those an ideal quark-gluon gas at the same temperature would have. For QCD with three light quarks one thus expects that the critical energy density for the formation of a quark-gluon plasma is well estimated as

$$\epsilon_c \simeq 3a_{SB}T_c^4 \simeq 15.6T_c^4 \simeq 3\text{GeV/fm}^3 \quad (5.1)$$

Can such large energy densities be reached in heavy ion experiments? A rough estimate of the energy density can be obtained from the multiplicities and transverse energies of the hadrons produced in a heavy ion collision⁽¹¹⁾. In ultra-relativistic collisions target and projectile nucleus are contracted to discs of transverse radius $R_A \simeq 1.2A^{1/3}$ and a longitudinal extent of about $1\text{fm}^{(11)}$. After the collision they leave behind a highly excited region of phase space, from which particles are emitted. After an initial time τ_0 the central region, in which the number of produced particles per unit rapidity is roughly independent of y , has a size $l = \tau_0 \Delta y$, with Δy denoting the central rapidity interval. The interaction volume can then be estimated as

$$V_{\text{int}} \simeq \pi R_A^2 l, \quad (5.2)$$

and the total transverse energy of the particles emitted from this region of phase space is given by

$$E_T \simeq \frac{dE_T}{dy} \Delta y \simeq \frac{3}{2} \langle E_T \rangle \frac{dN_{ch}}{dy} \Delta y, \quad (5.3)$$

where N_{ch} denotes the total number of charged particles observed in a given nucleus-nucleus collision. Energy and particle densities obtained in a collision at the initial time τ_0 may then be estimated by the Bjorken formulae⁽¹¹⁾

$$\begin{aligned} \epsilon &\simeq \frac{E_T}{V_{\text{int}}} = \frac{3}{2} \frac{\langle E_T \rangle}{1.44\pi A^{2/3} \tau_0} \frac{dN_{ch}}{dy} \text{fm}^{-2}, \\ n_H &\simeq \frac{3 N_{ch}}{2 V_{\text{int}}} = \frac{3}{2} \frac{dN_{ch}/dy}{1.44\pi A^{2/3} \tau_0} \text{fm}^{-2}. \end{aligned} \quad (5.4)$$

Initially it was not at all clear how transparent nuclei would be in very high energy collisions and the number of charged particles produced in such collisions could thus only roughly be estimated. First heavy ion experiments have been performed at CERN in 1986 and 1987 with oxygen and sulphur projectiles at $E_{\text{Lab}} = 200\text{GeV}/A$ in order to test whether large energy and particle densities can indeed be reached in collision with heavy targets (gold, uranium...). Indeed it has been found that for instance in a central O-U collision of the order of 100 charged particles per rapidity interval can easily be produced at that energy⁽¹²⁾. In the three central rapidity intervals these particles have an average transverse energy $E_T \sim 300\text{MeV}$. With a

formation time $\tau_0 \sim 1\text{fm}$, one then finds from eq.(5.4) that energy densities of the order of $(1-2)\text{GeV}/\text{fm}^3$ and number densities of about $5/\text{fm}^3$ must have existed in the initial stages of the collision (For comparison the energy density in a proton is about $0.15\text{GeV}/\text{fm}^3$ and the hadronic density in a system with densely packed pions is only $1.2/\text{fm}^3$). This dense hadronic system occupies a volume $V \sim V_{\text{int}} = 85\text{fm}^3$ (140fm^3) in the case of central oxygen (sulphur) collisions with a heavy target nucleus. The number of nucleons $\bar{B}$, participating in a central A - B collision, may be estimated from the size of the cylinder which a (smaller) projectile nucleus A cuts out of the target nucleus B in a central collision, $\bar{B} = \frac{3}{2} B^{2/3} A^{1/3}$. One thus estimates that about 140 (180) target nucleons are involved in a central O-U (S-U) collision.

The above estimates show that one may indeed expect that in a heavy ion collision a macroscopic region of very dense hadronic matter is created. Are these conditions sufficient to create a quark-gluon plasma? A crucial question is, of course, whether at least local equilibrium has been reached in these collisions, which would justify the application of thermodynamic or hydrodynamic concepts in the interpretation of the data. An even more urgent problem is, however, to find observables which can provide unique information about the initial stages of such a violent collision. The initially formed dense system will expand quickly and cool down. It may go through a phase transition but eventually the density will become so low that hadronization sets in and we are finally left with a large number of hadrons reaching the detectors. What are then the good observables to look at which can tell us about the existence of a quark-gluon plasma in the early stages of the collision? There exist various proposals for signatures of the quark-gluon plasma phase; none of these is perfect in the sense that its observation can uniquely be related to the existence or non-existence of a quark-gluon plasma in a heavy ion collision. Most likely one has to analyze several observables which hopefully can consistently be described through a hydrodynamic model that incorporates the existence of a quark-gluon plasma. In the following we want to concentrate on some observables which are closely related to the kind of thermodynamic calculations we have analyzed in the previous sections. We will discuss the possibility of measuring the equa-

Figure 13: Schematic behaviour of $\langle p_T \rangle$ as a function of energy density for nucleons, kaons and pions in a hydrodynamic model (solid curves) with a first order phase transition. The dashed curve shows the idealized case of vanishing transverse flow velocities.

tion of state, the strangeness content and the screening properties of the dense hadronic system created in a heavy ion collision.

5.1. The equation of state

Lattice Monte Carlo simulations suggest that the finite temperature phase transition in QCD is first order (see section 3). A characteristic feature of a first order phase transition is the non-vanishing latent heat at the critical point; i.e. the energy density is discontinuous and jumps at constant temperature, $T = T_c$. As we have discussed above the energy density can be related to experimentally accessible quantities like the transverse energy of emitted particles or to the number of charged particles per rapidity interval. Similarly a measure for the temperature of the hadronic system created in a collision is given by the average transverse momentum p_T of the emitted particles. The measurement of p_T as a function of E_T or dN_{ch}/dy thus is closely related to the equation of state of the system^[15]. The constant temperature in the mixed phase of a first order phase transition should then be reflected as a plateau in $\langle p_T \rangle$ as function of E_T or N_{ch} .

There are, of course, several problems which arise

Figure 14: $\langle p_T \rangle$ as a function of E_T for negative charged particles in S-W collisions^[17] (fig.14a) and $\langle p_T \rangle$ as function of all charged particles in $p\bar{p}$ collisions^[18] (fig.14b).

when one tries to apply these simple considerations to the actual situation in a heavy ion experiment: The observed hadrons are mainly pions which can be produced also at late stages of the evolution of the hadronic system. Their p_T spectrum thus is an integrated observable reflecting the entire thermal history of the evolution of the hadronic system (The p_T spectrum of heavy particles may be a better observable in this respect, as heavy particle decouple at an earlier stage from the thermal evolution of the entire system.). Moreover, the (radial) expansion of the entire hadronic system influences also the p_T spectrum of particles created at an early stage

in the collision. If this expansion can be described by ordinary fluid hydrodynamics one would expect that the produced particles drift in a medium which has a constant (transverse) flow velocity^[16]. They thus will pick up additional momentum from this collective flow. This will partly wash out the plateau in p_T versus E_T expected for a static equilibrated plasma. However, it also has the nice effect that it will disentangle the momentum distributions for different particles as the constant flow velocity leads to larger momenta for nucleons than, for instance, for kaons and pions. A schematic view of the expected $\langle p_T \rangle$ distribution is given in fig.13. The dashed curve shows the idealized situation of an equilibrated thermodynamic system without transverse flow.

Some results for $\langle p_T \rangle$ obtained in S-W collisions at CERN^[17] are shown in fig.14. Also shown here are results from a $p\bar{p}$ collider experiment at $\sqrt{s} = 1.8\text{TeV}$ performed at Fermilab^[18]. In particular the similarity of the Fermilab collider data with the schematic picture drawn in fig.13 is striking. The heavy ion data on the contrary show only little variation with E_T . Partly this may be due to the fact that the CMS-energy is much lower in this case. Applying the Bjorken formula, eq.(5.4), to the $p\bar{p}$ data indeed leads to energy densities as large as $\epsilon \simeq 1\text{GeV/fm}^3$. It is, however, questionable, whether the interaction volume in a $p\bar{p}$ collision is large enough to create a macroscopic thermal environment.

The Fermilab data can indeed be explained in the framework of a hydrodynamic model^[19]. However, there exist also more conventional interpretations of the data which relate the shape of $\langle p_T \rangle$ to an enhanced jet production in these high energy collisions^[20]. A better understanding of the situation may come from a detailed analysis of p_T -spectra for different hadrons which would give more information about the hydrodynamic conditions reached in these experiments.

5.2. Strangeness production

The strange quark mass is of the same order of magnitude as the temperature needed to form a quark gluon plasma ($m_s \simeq 150\text{MeV}$). Although only present as *sea* quarks in the target and projectile nuclei, they will be produced copiously in a thermalized plasma. This is also supported by the Monte Carlo data discussed in section 4.2. The strangeness content of the plasma is expected to be enhanced even further in a baryon rich

plasma^[21] as the large chemical potential for u and d quarks favours in this case the production of the heavier s and $\bar{s}$ quarks which have vanishing chemical potential. If ideal gas relations are applicable to this situation (the Monte Carlo data discussed in section 4.2 indicate that this may not be a good approximation close to T_c), the relative abundance of strange quarks and anti-quarks is given by

$$\frac{n_{\bar{s}}}{n_u + n_{\bar{u}}} \simeq \frac{1}{2} \left(\frac{m_s}{T} \right)^2 K_2(m_s/T) e^{\mu_B/3T}, \quad (5.5)$$

where $\mu_B/3$ denotes the chemical potential for u and d quarks and n_q denotes the number density for quarks with flavour q . The large amount of s ($\bar{s}$) quarks present in the plasma should then be reflected in a larger amount of strange hadrons produced in nucleus-nucleus collision relative to nucleon-nucleon collisions. In particular one would expect an increase in the K/π ratio. Unfortunately large K/π ratios can also be obtained in other ways and details depend very strongly on the assumptions about the equilibration of various flavours during the hydrodynamic evolution of the hadronic system^[90]. At present it is thus not clear to which extent strangeness enhancement can be considered as a good indication for plasma formation. Also here a systematic analysis of various strange particle yields will help to distinguish different models for chemical equilibrations. In particular a study of strange baryon ratios like $\Lambda/\bar{\Lambda}$ are expected to yield important information^[22].

5.3. Screening of the heavy quark potential

The QCD heavy quark potential undergoes a characteristic change during the phase transition to a quark-gluon plasma. The confining $q\bar{q}$ potential of the hadronic phase gets replaced by a screened Coulomb potential in the plasma phase. In particular the spectrum of heavy quark resonances is well understood in terms of the linearly rising confinement potential at zero temperature^[91]. In the quark-gluon plasma phase these resonances may still exist as Coulombic bound states in a Debye screened potential. For large enough temperatures, however, the Debye screening mass, $\mu(T)$, is proportional to T and there will thus inevitably exist a critical temperature beyond which the formation of heavy quark bound states will become impossible with such a potential. An analysis of heavy quark

Figure 15: Dilepton spectrum from O-U collisions as function of the invariant $\mu^+\mu^-$ mass in the region of the J/ψ resonance for two different transverse energy cuts. The curves are normalized to the same background.

bound states in a screened Coulomb potential using a non-relativistic Schrödinger equation shows that for the $c\bar{c}$ -system bound states do not exist for $\mu(T) > 700\text{MeV}^{92}$. Using results obtained for $\mu(T)$ in lattice simulations (see section 4.4) one has to conclude that already at a temperature $T_D \simeq 1.1T_c$ no $c\bar{c}$ bound states will exist. This temperature is still much smaller than the charmed quark mass itself and the thermal production of c and $\bar{c}$ quarks will be negligible. It then is likely that $c\bar{c}$ pairs formed in initial hard collisions will disintegrate in a strongly screened quark-gluon plasma due to the lack of confining forces. The small number of thermal charmed quarks (anti-quarks) makes it unlikely that a suitable charmed partner is found at a later stage of recombination into colourless hadrons. The c and $\bar{c}$ quarks are thus expected to go into open charm channels leading to a net suppression of J/ψ (or χ) states in nucleus-nucleus collisions relative to the rates expected from nucleon-nucleon collisions^{91,123}.

Indeed a suppression of J/ψ production has been observed in heavy ion experiments¹²⁴. It further has been found that the suppression of J/ψ is more efficient in events with large E_T and/or for J/ψ 's with small transverse momentum. Some results for the dilepton spectrum measured for different transverse energy cuts

are displayed in fig.15.

A quantitative description of the expected suppression pattern in the plasma model requires a specification of the initial conditions, i.e. the density or temperature profile at the initial time t_i at which the system reaches thermal equilibrium, and a model for the subsequent time evolution of the plasma phase. Let us assume that at $t_i = \tau_0$ the temperature profile is given by

$$T(r) = T_i \left(1 - \left(\frac{r}{R_A} \right)^2 \right)^{b/3}, \quad (5.6)$$

where b parametrizes the transverse density distribution in the projectile and target nucleus¹²⁵. At later times the plasma is assumed to cool rapidly due to isentropic longitudinal expansion. At time $t \geq t_i$ the temperature is then related to the one at time t_i by

$$T_i^3 t_i = T^3 t, \quad (5.7)$$

which fixes the plasma lifetime as

$$t_f = t_i \left(\frac{T_i}{T_c} \right)^3. \quad (5.8)$$

Eq. (5.7) can be used to determine the initial energy density in the plasma phase as a function of the plasma lifetime. Assuming that locally an ideal gas relation holds for the energy density,

$$\epsilon(r) = 3a_{SB}T^4(r), \quad (5.9)$$

the total energy in the interaction volume V_{int} can be related to the plasma lifetime and phase transition temperature. Using eqs.(5.2), (5.6) and (5.9) one finds¹²⁵

$$\begin{aligned} E/A^{2/3} &= \frac{9a_{SB}}{4b+3} 1.44 T_c^4 \left(\frac{t_f}{t_i} \right)^{1/3} \times \\ &\times \left[1 - \left(\frac{t_i}{t_f} \right)^{(4b+3)/3b} \right]. \end{aligned} \quad (5.10)$$

We note that E is proportional to the transverse size of the projectile nucleus, $E \sim A^{2/3}$, and the fourth power of the phase transition temperature T_c . The total energy calculated this way has to be related to the transverse energy measured in a heavy ion experiment. However, the critical temperature as well as several other parameters entering the above relation are only approximately known and the correct relation between E and E_T is thus difficult to establish. Nonetheless,

Figure 16: Ratio $S \equiv [\text{no. of } J/\psi\text{'s} / \text{no. of back-ground } \mu^+\mu^- \text{ pairs}]$ in a given E_T bin (indicated by horizontal error bars) divided by the same quantity for the lowest E_T bin versus $E_T/A^{2/3}$ for O-U (dots) and S-U (squares) collisions.

some basic features are obvious. With increasing transverse energy the plasma lifetime increases and so does the initial temperature T_i . From eq.(5.6) we see that this results in an increasing transverse size of the region with $T > T_c$, i.e. the region which initially was in the plasma phase. We thus expect an increasing amount of J/ψ suppression with increasing transverse energy. Fig.16 shows the amount of suppression predicted by the plasma model, compared to the experimental data from the NA38 collaboration at CERN⁽²⁴⁾. Here the experimental data have been normalized to the lowest E_T -bin and a linear relation has been assumed between the experimentally measured E_T and E given by eq. (5.10)⁽²⁵⁾. We see that the plasma model reproduces quite well the slope of the E_T dependence for the large E_T events.

A second feature of the experimental data is the strong p_T dependence of the suppression pattern. This too can be understood in the plasma model.⁽²⁵⁾ A $c\bar{c}$ pair formed at rest in the plasma will proceed to separate; at the hadronization point, the c and the $\bar{c}$ will in general be too far apart to bind into a J/ψ anymore. If, however, the $c\bar{c}$ pair moves fast with respect to the plasma rest frame, the c and $\bar{c}$ can still be close enough together to bind to a J/ψ when it leaves the deconfining environment. A $c\bar{c}$ pair with momentum $\vec{p}$ will form a

J/ψ at a time

$$t = \tau_{J/\psi} \sqrt{1 + (\vec{p}/m)^2} \quad , \quad (5.11)$$

where $\tau_{J/\psi} \simeq 0.9\text{fm}$ denotes the J/ψ formation time in the $c\bar{c}$ rest frame⁽²²⁾. J/ψ 's with large momentum thus form at a late stage in the plasma rest frame. At this time the region covered with a hot plasma is reduced and we thus expect less suppression of large momentum J/ψ . In particular, J/ψ 's with a momentum larger than $p_c = m\sqrt{(t_f/\tau_{J/\psi})^2 - 1}$ will form at $t > t_f$. They are thus not affected by the plasma at all and lead to normal resonances.

The plasma model discussed above is based entirely on the assumption that modifications of the J/ψ yield in different E_T bins are due to final state interactions: the disintegration is due to screening of the heavy quark potential, and the momentum dependence of the suppression pattern to time dilatation altering the "growth" of the $c\bar{c}$ system in the plasma rest frame. A short plasma lifetime then leads to a strong momentum dependence of the suppression pattern.

First attempts to explain the experimentally observed J/ψ suppression in terms of disintegration in dense hadronic matter⁽²⁶⁾ failed because the high density necessary to get a large amount of suppression also led to a long life-time of such purely hadronic systems. This in turn produced a rather flat p_T dependence. Recently it was noted that this difficulty could be overcome by taking into account that initial state interactions with other partons may strongly alter the momentum distribution of the gluons. The J/ψ 's, which are produced through gluon fusion, thus get shifted to higher p_T ; this mechanism allows a description of the observed p_T dependence of the suppression⁽²⁷⁾.

If there are indeed strong initial state effects that alter the momentum distribution of the $c\bar{c}$ pair⁽²⁷⁾, then these have to be taken into account in the plasma model as well. This would here lead to an even stronger p_T dependence. To some extent, however, this could be compensated by an increase of the plasma lifetime and a modification of the temperature profile ($b \rightarrow 1$) in the model calculations. At present it is not clear to what extent the momentum dependence is already explained by initial state interactions alone, or whether there is still room for the additional p_T dependence coming from the plasma formation. There are also attempts to ex-

plain the modified p_T spectrum of the J/ψ 's formed in nucleus-nucleus collisions by a modification of the gluon structure functions in a nucleus relative to that in a nucleon¹²⁸⁾. This is an alternative to the initial state interaction models.

It seems that a more systematic study of the rescattering effects of gluons in the initial state on the momentum distribution of produced $c\bar{c}$ pairs is needed, before a further analysis of the disintegration mechanism in the final state can be performed. J/ψ suppression still is one of the most interesting results of the first round of heavy ion experiments. However, the above discussion should have made clear that also this mechanism is not the unique signature for quark-gluon plasma formation one is looking for.

6. CONCLUSIONS

QCD predicts the existence of an entirely new state of hadronic matter at high temperature and/or density. We have tried to give an introduction into the physics of this new state of matter. We have concentrated on a discussion of quantitative results emerging from Monte Carlo simulations of lattice regularized QCD and gave a short overview of experimental results related to the thermodynamic observables studied on the lattice.

We have seen that realistic simulations with light quarks start becoming feasible. The physics of the pure gauge theory seems to be well understood and there is growing evidence that the deconfinement phase transition is indeed first order. Also the analysis of the physically most interesting case of two-flavour QCD with light quarks seems to indicate that the chiral phase transition is first order.

Bulk thermodynamic quantities agree well with the perturbative predictions for $T \gtrsim (2 - 3)T_c$. For lower temperatures strong deviations from perturbative predictions have been found, which lead to deviations from the ideal gas equation of state. Non-perturbative effects are clearly visible in the behaviour of the pressure and the entropy density for $T_c \leq T \lesssim (2 - 3)T_c$, while the energy density is in good agreement with perturbative results already for $T \gtrsim 1.2T_c$. This can be understood as the energy density is less sensitive to the short distance properties of the plasma phase.

The heavy quark potential seems to deviate clearly from perturbative predictions close to T_c . There are,

however, indications that the potential approaches the expected perturbative form for high temperatures and that the screening mass is close to the lowest order perturbative value already for $T \sim 1.2T_c$. Given the poor understanding of the screening mechanism gained from perturbative analyses so far, a further detailed numerical analysis of the large-distance part of the potential at high temperatures is certainly needed.

Present simulations of full QCD with light quarks suffer from large finite size effects. Although technically feasible simulations on large enough lattices are extremely time consuming. While, for instance, in the gluonic sector the transition temperature has been determined on lattices with temporal sizes up to $N_\tau = 11$, the corresponding results for full QCD exist only for $N_\tau = 4$ and 6. At present simulations for $N_\tau = 8$ are being performed. A determination of T_c/m_ρ on a lattice of size 8×16^3 will, however, require about 30.000 hours on a CRAY Y-MP.

First ultra-relativistic heavy ion experiments led to encouraging results. The large number of particles produced in nucleus-nucleus collisions indicates that indeed very dense hadronic systems can be produced in this way. The first experiments led to new, interesting results like, for instance, the strong E_T and p_T dependence of J/ψ production. Whether these results are indicative for the formation of a quark-gluon plasma and in how far the hadronic systems produced in nucleus-nucleus collisions can be described as a dense thermalized system hadronic gas or even as a quark-gluon plasma, requires further detailed studies. In particular experimental data with higher statistics than those obtained from the first pilot runs is needed in order to distinguish predictions from different model calculations.

Where are we going? Further heavy ion experiments will be performed at CERN in 1990. However, the big step forward is expected to occur in 1992 when a lead beam should become available for heavy ion experiments at CERN. Also lattice simulations will become more reliable with the steady improvement of computer resources. This year a special purpose computer at Columbia university has been completed, which is about ten times faster than a 4-processor CRAY X-MP. At present it is entirely devoted to simulations of finite temperature QCD. It is expected that by 1992 an upgraded version of this machine gives at least another

factor 10 in performance. This will allow to perform high statistics simulations of QCD with light quarks on large lattices and will considerably improve our knowledge about non-perturbative features of QCD.

The early 90's are thus expected to be an interesting period with major developments in quark-gluon plasma physics taking place.

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