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Gravitational Equilibria of Self-interacting Charged Scalar Fields*

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Abstract

I present the static spherically symmetric gravitational equilibria of scalar fields coupled to a U(1) gauge field and with a possible $\frac{\lambda}{4}(\phi^*\phi)^2$ self-interaction. These configurations are obtained by solving numerically the coupled system of Einstein-Maxwell-Klein-Gordon equations for non-singular and asymptotically flat solutions. Static solutions only exist for values of the gauge coupling constant such that $e^2/4\pi \leq G_N m^2$, where m is the mass of the scalar particle. The maximum mass of the Bose star increases with increasing value of the gauge coupling constant. I discuss also the dynamical stability of the equilibrium configurations, for which I derive a pulsation equation, which governs the time evolution of the infinitesimal radial oscillations, as well as a variational principle for its eigenvalues. The equilibrium configurations with a central density bigger than ρ_{crit} , corresponding to the critical mass, are dynamically unstable.

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I present the static spherically symmetric gravitational equilibria of scalar fields coupled to a $U(1)$ gauge field and with a possible $\frac{1}{4}(\phi^*\phi)^2$ self-interaction. These configurations are obtained by solving numerically the coupled system of Einstein-Maxwell-Klein-Gordon equations for non-singular and asymptotically flat solutions. Static solutions only exist for values of the gauge coupling constant such that $e^2/4\pi \leq G_N m^2$, where m is the mass of the scalar particle. The maximum mass of the Bose star increases with increasing value of the gauge coupling constant. I discuss also the dynamical stability of the equilibrium configurations, for which I derive the pulsation equation, which governs the time evolution of the infinitesimal radial oscillations, as well as a variational principle for its eigenvalues. The equilibrium configurations with a central density bigger than ρ_{crit} , corresponding to the critical mass, are dynamically unstable.

The more recent developments in particle physics and cosmology suggest that scalar fields may have played an important role in the evolution of the early universe, for instance in primordial phase transitions and that they may make up part of the missing dark matter. Models for galaxy formation using cold dark matter and the inflationary scenario suggest that the ratio of baryonic (luminous) matter to dark matter can be of the order of 10 %. These facts naturally raise the question whether cold gravitational equilibrium configurations of massive scalar fields - Bose stars - may exist and whether such configurations are dynamically stable. Spherically symmetric equilibrium configurations of scalar fields have been found by solving the coupled Einstein-Klein-Gordon equations without [1-4] or with a quartic self-interaction [5]. In all these models, if one plots the mass for equilibrium configurations of Bose stars against their central density, one finds a behaviour very similar to that of neutron stars. The mass quickly rises to a maximum for $\rho = \rho_{crit}$, drops a little, oscillates and approaches an asymptotic value at large central densities. A similar behaviour is found for the particle number.

Here I discuss the static spherically symmetric solutions for a system of complex scalar fields coupled to a $U(1)$ gauge field [6]. I also allow for the possibility of a quartic self-interaction. The main effect of having scalar fields coupled to a $U(1)$ gauge field is to increase the maximum mass with increasing gauge coupling constant e . At the same time the binding energy per particle decreases. Static solutions only exist for $e^2/4\pi \leq G_N m^2 = e_{crit}^2$. I consider also the problem of the dynamical stability of the equilibrium configurations in the framework of general relativity [7]. I analyse the time evolution of infinitesimal radial oscillations, which conserve the total number of particles, following the

method developed by Chandrasekhar [8]. I find an eigenvalue equation, called also pulsation equation, which determines the normal modes of the radial oscillations and a variational principle for determining the eigenvalues [7,9]. As in the cases without charge [4], the particle number and mass have their extrema, in particular their maximum, at the same value of the central density. From this fact it follows that the pulsation equation has a zero mode, where M and N have their extrema [10]. Therefore one expects the central density ρ_{crit} , corresponding to the maximum mass, to be the boundary between stable (for ρ smaller than ρ_{crit}) and unstable equilibrium configurations. However in order to establish this fact completely one has to analyse the eigenvalues of the pulsation equation to see if they are real and positive for central densities smaller than ρ_{crit} . M and N as a function of the central density have other stationary points for ρ bigger than ρ_{crit} . We expect the behaviour of the stability not to change there, a fact which is already suggested by the shape of the particle number versus radius diagram [6], which is bent counterclockwise at the critical points. A rigorous proof, however, can be given using the variational principle; in fact one can easily find an upper bound to the lowest eigenvalue at the stationary points of M and N for ρ bigger than ρ_{crit} . It turns out that the upper bounds are negative, therefore the lowest eigenvalue is negative which establishes that the equilibrium configurations remain unstable for all ρ bigger than ρ_{crit} .

The system we consider is a complex scalar field with a $U(1)$ charge coupled to gravity, whose action is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G_N} + g^{\mu\nu} (D_\mu \phi)^* (D_\nu \phi) - m^2 |\phi|^2 - \frac{\lambda}{2} |\phi|^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right], \quad (1)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu \phi = \partial_\mu \phi + ie A_\mu \phi$. The action is invariant under a local $U(1)$ gauge transformation, thus the total particle number is conserved. We take all the fields as functions of r and t alone since we consider only spherically symmetric equilibrium configurations. We express the metric in Schwarzschild coordinates

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (2)$$

where ν and λ are functions of r and t only ($g^{00} = -e^{-\nu}$ and $g^{rr} = e^{-\lambda}$). We choose the gauge field A_μ such that we have only electric charges and no magnetic ones, therefore we set $A_\mu = (A_t = A, A_r = C = 0, A_\theta = 0, A_\varphi = 0)$. We write the complex scalar field as $\phi = (\phi_1 + i\phi_2)e^{i\omega t}$, where ϕ_1 and ϕ_2 are two real fields. For the equilibrium configuration we set $\phi_1(r, t) = \phi_0(r)$, $\phi_2(r, t) = 0$ and all the other fields and metric functions are time independent. The coupled system of equations, which can be derived from the above action by varying with respect to the various fields, is given by the two Einstein equations

$$\lambda'_0 = \frac{(1 - e^{\lambda_0})}{r} + 8\pi G_N r e^{\lambda_0} \left[((\omega + e A_0)^2 e^{-\nu_0} + m^2) \phi_0^2 + \frac{\lambda}{2} \phi_0^4 + \phi_0^2 e^{-\lambda_0} + \frac{A_0^2}{2} e^{-\nu_0 - \lambda_0} \right], \quad (3)$$

$$\nu'_0 = \frac{(e^{\lambda_0} - 1)}{r} + 8\pi G_N r e^{\lambda_0} \left[((\omega + e A_0)^2 e^{-\nu_0} - m^2) \phi_0^2 - \frac{\lambda}{2} \phi_0^4 + e^{-\lambda_0} \phi_0'^2 - \frac{A_0^2}{2} e^{-\nu_0 - \lambda_0} \right], \quad (4)$$

the Maxwell equation

$$A_0'' + \left(\frac{2}{r} - \frac{\nu'_0 + \lambda'_0}{2} \right) A_0' - 2e\phi_0^2 e^{\lambda_0} (\omega + e A_0) = 0, \quad (5)$$

and the scalar wave equation

$$\phi_0'' + \left(\frac{2}{r} + \frac{\nu_0' - \lambda_0'}{r}\right)\phi_0' + e^{\lambda_0}((\omega + eA_0)^2 e^{-\nu_0} - m^2 - \bar{\lambda}\phi_0^2)\phi_0 = 0. \quad (6)$$

As boundary conditions we impose $\phi_0(0)=\text{const}$, $\phi_0'(0)=0$ and $\phi_0(\infty)=\phi_0'(\infty)=0$ in order to have a localized particle distribution. We allow the possibility of ϕ_0 having nodes, giving rise to excited states. Furthermore we demand that the electric field be absent at the origin, giving $A_0'(0)=0$, and we normalize the potential by $A_0(\infty)=0$. The condition $e^{\nu_0(\infty)}=1$ is imposed to get asymptotically the ordinary Minkowski metric and the condition $e^{\lambda_0(0)}=1$ is a regularity condition. With these boundary conditions the equations (3-6) form an eigenvalue equation for ω . For a detailed discussion of the equilibrium solutions see Ref.[6]. The total particle number N_0 , which is related to the total charge by $Q = eN_0$, is given by

$$N_0 = 8\pi \int_0^\infty dr r^2 (\omega + eA_0) \phi_0^2 e^{(\nu_0 - \lambda_0)/2}. \quad (7)$$

Consider now the situation where the equilibrium configuration is perturbed in a way such that the spherical symmetry is still preserved. These perturbations will give rise to motions in the radial direction. The equations governing the small perturbations are obtained by expanding all functions to first order and by linearizing the equations. By suitably combining the perturbation equations [7,9] and supposing a time-dependence of the form $e^{i\omega t}$, one gets the following eigenvalue equation for f_1 and f_2 , which are related to the radial part of $\delta\phi_1(r, t), \delta\phi_2(r, t)$ and $\delta C(r, t)$ [7]

$$L_{ij}f_j = \sigma^2 M_{ij}f_j \quad i, j = 1, 2 \quad (8)$$

where

$$M_{ij} = e^{\lambda_0 - \nu_0} \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix}$$

with $G_1 = r^2 e^{(\nu_0 - \lambda_0)/2}$ and $G_2 = r^2 \phi_0^2 e^{(\nu_0 - \lambda_0)/2}$,

$$L_{ij} = \begin{pmatrix} -\frac{\partial}{\partial r} G_1 \frac{\partial}{\partial r} + G_1 C_1 & -2\frac{\partial}{\partial r}(\bar{\omega} G_1 \phi_0) + G_1 C_3 \\ 2\bar{\omega} G_1 \phi_0 \frac{\partial}{\partial r} + G_1 C_3 & -\frac{\partial}{\partial r} G_2 \frac{\partial}{\partial r} + G_2 C_2 \end{pmatrix}$$

($\bar{\omega} = \omega + eA_0$), C_i ($i=1,2,3$) are complicated expressions involving the equilibrium solutions (see Ref.[7]). Eq.(8) is the required "pulsation equation". The appropriate boundary conditions are for $r \rightarrow \infty$: $f_1 \rightarrow 0$, $r^2 \phi_0^2 f_2 \rightarrow 0$ and for $r \rightarrow 0$: $f_1 = \text{const}$, $r^2 f_2 \rightarrow 0$. With these boundary conditions both the operators L_{ij} and M_{ij} are symmetric, this also in the case where ϕ_0 has nodes [7,11], and the total particle number is automatically a conserved quantity [7]. The eigenvalues are real and dynamical instability will occur whenever $\sigma_0^2 \leq 0$.

Eq.(8) can also be obtained from the following variational principle

$$\sigma^2 \int_0^\infty \frac{1}{2} e^{\lambda_0 - \nu_0} (G_1 f_1'^2 + G_2 f_2'^2) dr = \int_0^\infty \left[\frac{1}{2} G_1 f_1'^2 + \frac{1}{2} G_1 f_1^2 C_1 + \frac{1}{2} G_2 f_2'^2 + \frac{1}{2} G_2 f_2^2 C_2 + f_2 f_1' \frac{\partial}{\partial r} \phi_0 (\omega + eA_0) G_1 + f_2 f_1 G_1 C_3 \right] dr. \quad (9)$$

A sufficient condition for dynamical instability to occur is that the right-hand side of eq.(9) vanishes (or becomes negative) for some chosen pair of trial functions f_1 and f_2 which satisfy the above boundary conditions. It turns out that for central densities bigger than ρ_{crit} the equilibrium configurations are unstable.

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Figure Captions

Fig. 1: Bosen star mass in units of M_{Pl}^2/m (solid line) and particle number in units of M_{Pl}^2/m^2 (broken line) as a function of $\phi_0(0)$. The charge e is given in units of $M_{Pl}/(\sqrt{8\pi}m)$. No quartic self-coupling is present. Also the lines going through the maxima of mass and particle number are drawn.

FIGURE 1