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Electroweak radiative corrections in deep inelastic electron-proton scattering

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Abstract

An overview is given of higher order electroweak effects in neutral and charged current processes in deep inelastic ep scattering at HERA energies in the framework of the Standard Model based on $SU(3)_C \times SU(2)_W \times U(1)$.

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After the choice of three input data all other measurable quantities are predictions according to the structure of the theory. The relations between the experimental quantities, however, are changed when higher order perturbative contributions are included. The electroweak Standard Model as a spontaneously broken gauge theory implies that the theoretical prediction for any observable quantity can (at least in principle) be calculated to an arbitrary order in perturbation theory. In practice the lowest order expressions are supplemented by the one-loop radiative corrections which can be improved by certain higher order leading terms where they tend to become large. For an adequate analysis of experiments the discussion of radiative corrections is an indispensable part:

- Experimental tests of the validity of radiative corrections in various types of reactions and in various kinematical domains are tests of the Standard Model at the quantum level.
- For detecting signals from possible “new physics” the Standard Model predictions have to be known with the adequate accuracy.
- The special situation in ep collisions allows us to probe the structure of the proton by measurements of the quark distributions, thereby testing also the QCD sector of the Standard Model. For this purpose reliable predictions for the electroweak sector are necessary.
- Radiative corrections can be large. This applies in particular to the conventional QED bremsstrahlung corrections. These QED contributions do not contain additional information about the weak interaction part of the complete theory, but they constitute the bulk of the radiative corrections in particular for deep inelastic processes at high momentum transfer. Their proper treatment in the data analysis is of basic importance for testing the QCD and the non-QED electroweak part of the Standard Model.

The radiative correction problem in deep inelastic ep scattering is therefore characterized by the interplay of the strong interaction between the quarks inside the nucleon and the electroweak interactions between the quarks and the incident leptons. The hadronic part as yet cannot be calculated from first principles and has to be treated as an additional experimental input entering the radiative corrections. On the other hand, this hadronic input cannot be extracted from experiments without knowledge of the size of the electroweak radiative corrections.

The present contribution will give an update of the situation of electroweak radiative corrections for neutral and charged current HERA processes. Recent comparisons between independent calculations show that the numerical results are now in agreement within about 1% and are therefore checked against each other with sufficient accuracy.

2 Neutral current deep inelastic scattering

2.1 Lowest order differential cross section

In the quark-parton model and also in the leading log approximation of QCD the fundamental processes may be considered to be the neutral current 4-fermion scattering

1 Introduction

The fundamental process in neutral and charged current deep inelastic lepton-proton scattering is the 4-fermion scattering between the lepton and the quarks inside the nucleon: $lq \rightarrow lq$. In lowest order this process is of pure electroweak origin, in the Standard Model mediated by the exchange of photons and weak Z^0 resp. $W^\pm$ bosons.

The electroweak Standard Model contains, besides fermion masses, quark mixing angles, and the mass of the Higgs scalar, three free parameters in the gauge sector. For a comparison between theory and experiment three independent experimental input data are required for fixing the SU(2) and U(1) gauge coupling constants g_2 , g_1 , and the vacuum expectation value v of the Higgs field. It is, however, more practical to deal with parameters such that each of them has a direct relation to a specific experiment. For deep inelastic scattering a natural choice is given by the electromagnetic fine structure constant α (Thomson scattering), and the masses of the Z and W bosons characterizing the photon, neutral current Z and charged current W exchange. The masses of the Higgs boson and the top quark enter the higher order calculations as additional free parameters unless a direct experimental determination of their values is available.

The existing calculations of electroweak radiative corrections for HERA processes [1-6] follow the lines of the on-shell scheme which treats the masses of the vector bosons M_Z and M_W in a symmetric way: the one-loop amplitudes for 4-fermion processes are expressed in terms of M_Z and M_W (besides α) as the basic quantities defined as the real parts of the pole positions in the Z and W propagators. The Fermi constant G_μ , measured from the muon lifetime, can then be calculated as a constraint

$$G_\mu = G(\alpha, M_Z, M_W, m_t)$$

on the value of M_W after specifying the other quantities. In practice this constraint is used to determine the numerical value for M_W . This removes the dependence of the measurable quantities in deep inelastic scattering from explicit W mass measurements which will not reach the accuracy of the Fermi constant.

processes between the electron and the quark constituents of the proton, as indicated in Figure 1.

Figure 1: Deep inelastic scattering in the parton model

Defining the kinematical variables x, Q^2 by

$$\begin{aligned} Q^2 &= -(p - p')^2, \\ q &= xP \end{aligned} \quad (1)$$

the differential cross section for $ep \rightarrow eX$ in the parton picture is given by

$$\frac{d^2\sigma}{dx dQ^2} \sim \sum_f |\mathcal{M}_f|^2 q_f(x) \quad (2)$$

where $q_f(x)$ denotes the distribution function of the quark with flavor f inside the proton.

For left/right-handed polarized incident electrons the lowest order cross section is explicitly written with help of the dimensionless variable $y = Q^2/xS$, $S = (p + p')^2$ as follows:

$$\frac{d^2\sigma}{dx dQ^2}(e_{L,R}) = \frac{2\pi\alpha^2}{xQ^4} \left[(1 + (1-y)^2) F_2^{L,R} + (1 - (1-y)^2) x F_3^{L,R} \right] \quad (3)$$

The structure functions

$$\begin{aligned} F_2^{L,R} &= \sum_f (xq_f + x\bar{q}_f) \cdot A_f^{L,R} \\ xF_3^{L,R} &= \sum_f (xq_f - x\bar{q}_f) \cdot B_f^{L,R} \end{aligned} \quad (4)$$

contain the quark (q_f) and anti-quark ($\bar{q}_f$) distribution functions as well as coupling constants and propagators corresponding to photon and Z boson exchange:

$$\begin{aligned} A_f^{L,R} &= Q_f^2 + 2Q_e Q_f (v_e \pm a_e) v_f \frac{Q^2}{Q^2 + M_Z^2} + (v_e \pm a_e)^2 (v_f^2 + a_f^2) \left(\frac{Q^2}{Q^2 + M_Z^2} \right)^2 \\ B_f^{L,R} &= \pm 2Q_e Q_f (v_e \pm a_e) a_f \frac{Q^2}{Q^2 + M_Z^2} \pm 2(v_e \pm a_e) v_f a_f \left(\frac{Q^2}{Q^2 + M_Z^2} \right)^2 \end{aligned} \quad (5)$$

According to the $\gamma, \gamma - Z$ interference, and Z contributions the cross section can also be decomposed into

$$d^2\sigma = \sigma_\gamma + \sigma_{\gamma Z} + \sigma_Z. \quad (6)$$

In the range $Q^2 \geq 10^3 \text{ GeV}^2$ the interference and pure Z terms become of non-negligible influence; at $Q^2 \approx 10^4 \text{ GeV}^2$ they are of the same size as the pure photon exchange contribution.

The parametrization of the lowest order cross section in terms of the vector and axial vector coupling constants $v_{e,f}, a_{e,f}$ is quite general and applies to any model where a vector boson with v and a couplings is exchanged together with the photon. In the Standard Model, however, these quantities are not independent. For a quantitative evaluation a well-known ambiguity in the Born level presentation shows up: the coupling constants can either be expressed by means of $(Q_f$ and I_f^3 denote charge and weak isospin of the fermion f)

$$v_f = \frac{I_f^3 - 2s_W^2 Q_f}{2s_W c_W}, \quad a_f = \frac{I_f^3}{2s_W c_W}, \quad (7)$$

with the mixing angle in the minimal model

$$s_W \equiv \sin \theta_W, \quad c_W \equiv \cos \theta_W = M_W/M_Z, \quad (8)$$

or in the following way with help of the Fermi constant:

$$\begin{aligned} a_f &= \left(\frac{\sqrt{2} G_\mu M_Z^2}{4\pi\alpha} \right)^{1/2} I_f^3 \\ v_f &= \left(\frac{\sqrt{2} G_\mu M_Z^2}{4\pi\alpha} \right)^{1/2} (I_f^3 - 2Q_f s_W^2). \end{aligned} \quad (9)$$

The relation between these two parametrizations is established by identifying the lowest order expression for the muon lifetime with the effective Fermi model result:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2}. \quad (10)$$

The numerical results obtained from these two parametrizations are different in general. Of course, after incorporating consistently the weak radiative corrections described next we obtain an unambiguous result.

2.2 Classification of the one-loop corrections

2.2.1 QED corrections

This gauge invariant subclass consists of all photonic virtual and real corrections which are obtained from the underlying Born diagrams by attaching a single photon

line to each charged fermion line in all possible ways (diagrams in Figure 2 together with virtual photon insertions in the external fermion lines). The infra-red singularities are cancelled in the sum of the 2- and 3-body final states after integrating the real photons over their kinematically and experimentally (by cuts) allowed phase space.

Figure 2 : QED corrections to $eq \rightarrow eq$

The QED corrections can be further subdivided into

- QED corrections at the lepton line (Figure 2 a,e,f);
- QED corrections at the quark line (Figure 2 b,g,h);
- the interference of bremsstrahlung from the lepton and the quark line (Figure 2 e-h) and the box diagrams (Figure 2 c,d) which connect the lepton and quark lines by an extra virtual photon.

For the further discussion of the qualitatively different features it is important to note that each of these classes is infra-red finite and gauge invariant.

2.2.2 Weak corrections

The remaining set of the infra-red finite non-QED or weak corrections, schematically depicted in Figure 3, consists of the one-loop corrected diagonal gauge boson propagators, the non-diagonal propagator with $\gamma - Z$ mixing, the vertex corrections after splitting off the photonic terms¹ and the box diagrams with $Z - Z$ and $W - W$ exchange. Being independent of the experimental set-up, these contributions can be absorbed into an effective Born amplitude which is expressed in terms of Q^2 dependent vector and axial vector coupling constants. A comparison between different calculations is straightforward when it is ensured that the same physical input has been used for renormalization and computation of the cross section.

¹these are always combined with the external fermion self-energies.

Figure 3 : Weak corrections to $eq \rightarrow eq$

2.3 Existing calculations

For deep inelastic ep processes with $Q^2 \ll M_Z^2$ the most important electromagnetic corrections to the pure QED contribution (only γ exchange) have been discussed by Mo and Tsai [7] and later by Akhundov et al. [8].

QED corrections arising from real and virtual photons from the lepton line have been calculated in the leading logarithmic approximation by Kripfganz und Möhring [9], Beenakker et al. [10], and Blümlein [11]. The leading log (LL) contribution, together with the fermionic part of the photon and Z self-energies, have already been treated earlier by Consoli and Greco [12]. The question of quark mass singularities according to QED bremsstrahlung from the quark line is discussed in [13] by extending the QCD Altarelli-Parisi equations for the Q^2 evolution of the quark distribution functions by the corresponding collinear QED terms.

Until now two independent calculations of the full $O(\alpha)$ corrections to the neutral current scattering process have been performed: the calculation by Bardin et al. [2,5] based on the on-shell scheme in the unitary gauge, and the calculation by Böhm and Spiesberger [3], also in the on-shell scheme but in the 't Hooft-Feynman gauge. Both calculations make use of analytical and numerical integration methods. All calculations in [2-6] treat the radiated photon as totally inclusive.

2.4 Discussion of the weak (non-QED) corrections

As outlined in the introduction, the concrete calculations of electroweak corrections for HERA processes have been performed by choosing α , M_W , M_Z as definition of a renormalized "physical" input for fixing the corresponding renormalized self-energies

and vertex corrections at the one-loop level. This so-called “on-shell scheme” utilizes the weak mixing angle in the version [14]

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \quad (11)$$

In practice, M_W as well as s_W^2 are determined from M_Z via the corrected form of the relation (10) [14]:

$$\frac{G_\mu}{\sqrt{2}} = \frac{e^2 \alpha}{2s_W^2 c_W^2 M_Z^2} \cdot \frac{1}{1 - \Delta r}. \quad (12)$$

Since the correction term $\Delta r(\alpha, M_Z, M_W, m_t)$ depends also on the Higgs and top masses M_H, m_t , the s_W^2 determined in this way becomes dependent on the presently empirically unknown part of the theory. Note, however, that s_W^2 as derived from (12) for a given Z mass is not the effective mixing angle in the neutral current couplings. The coupling constants get additional contributions by $\gamma - Z$ mixing, vertex corrections, and heavy box diagrams. Also the dependence of the effective mixing angle on m_t and M_H is different from that of s_W^2 .

The incorporation of the non-QED radiative corrections into 4-fermion amplitudes has been discussed in detail in several reports [15,16,17]. Here we concentrate only on the main results.

In a compact notation the weak corrections can be summarized in terms of an effective matrix element for the underlying parton subprocess:

photon exchange amplitude :

The dressed photon amplitude can be written in the following way:

$$M_\gamma = \frac{e^2}{1 - \tilde{\Pi}_\gamma} \cdot \frac{1}{Q^2} \cdot [(Q_e + F_V^e)\gamma_\mu - F_A^e\gamma_\mu\gamma_5] \otimes [(Q_f + F_V^f)\gamma^\mu - F_A^f\gamma^\mu\gamma_5] \quad (13)$$

with Q^2 dependent vertex form factors $F_{V,A}$ and the vacuum polarization $\tilde{\Pi}_\gamma(Q^2)$. Writing it in the denominator corresponds to the summation of the leading log terms from the light fermions in case of large Q^2 . The bosonic content has to be understood as expanded to $O(\alpha)$. The typical size of these contributions at $Q^2 \simeq M_Z^2$ is:

$$\begin{aligned} \tilde{\Pi}_\gamma &\simeq 0.06, \\ F_{V,A} &\simeq 0.001. \end{aligned}$$

Aiming at a precision of 1%, the restriction to the fermionic vacuum polarization is sufficient. For the hadronic part a convenient parametrization has been given by Burkhardt et al. [18] which fits the results obtained from the updated dispersion integral over experimental $e^+e^- \rightarrow \text{hadrons}$ data.

Z exchange amplitude :

The dressed Z amplitude can be expressed with help of Q^2 -dependent form factors ρ and κ in the following way:

$$M_Z = \sqrt{2}M_Z^2 G_\mu \rho \frac{[I_3^f - 2Q_e s_W^2 \kappa_e]\gamma_\mu - I_3^f \gamma_\mu \gamma_5 \otimes [I_3^f - 2Q_f s_W^2 \kappa_f]\gamma^\mu - I_3^f \gamma^\mu \gamma_5}{Q^2 + M_Z^2}. \quad (14)$$

The correction factors ρ, κ contain a universal (i.e. fermion type-independent) part and a fermion-dependent non-universal part:

$$\begin{aligned} \rho &= 1 + (\Delta\rho)_{\text{univ}} + (\Delta\rho)_{\text{non-univ}} \\ \kappa_{e,f} &= 1 + (\Delta\kappa)_{\text{univ}} + (\Delta\kappa)_{\text{non-univ}}. \end{aligned}$$

The universal pieces can become large if the top is heavy, in the leading quadratic term given by

$$(\Delta\rho)_{\text{univ}} \simeq \frac{3G_\mu m_t^2}{8\pi^2 \sqrt{2}} \equiv \Delta\rho \quad (15)$$

and

$$(\Delta\kappa)_{\text{univ}} \simeq \frac{c_W^2}{s_W^2} \Delta\rho \quad (16)$$

yielding

$$\kappa_{e,f} s_W^2 \simeq s_W^2 + c_W^2 \Delta\rho \quad (17)$$

where s_W^2 is the quantity derived from (12).

The non-universal parts of ρ and κ contain such (potentially) large terms only for heavy flavours ($f = b, t$); in all other cases they are practically independent of M_H, m_t and in size smaller than 1%. The form (14) does not yet contain the heavy box diagrams. They depend, besides on Q^2 , also on z , but still have a Born-like structure and can easily be added. Numerically they do not give large corrections. For details we refer to the literature [3,5].

Figure 4: Weak corrections to $d^2\sigma/dx dy$ in %, quark densities from [19] full curves from [5], dashed from [3]; dashed-dotted: from [5] and linearized in α . $M_Z = 93$ GeV, $M_H = 100$ GeV, $m_t = 30$ GeV, $s = 10^5$ GeV 2 .

Figure 4 shows the correction to $d^2\sigma/dx dy$, relative to the Born cross section (3) with coupling constants (7) and s_W^2 from (12), calculated in [5], and for comparison the dashed curve from [3]. The difference between the two results is due to the resummation of $\hat{\Pi}_\gamma$ according to (13), as done in [5], whereas [3] contains $\hat{\Pi}_\gamma$ in linearized form. Linearizing the expressions of [5] to $O(\alpha)$ (dashed-dotted) yields agreement up to $y = 0.7$; but also at higher y the deviations are not bigger than about half a per cent.

We want to conclude this discussion of the non-QED corrections with a practical recipe for obtaining an approximate cross section which incorporates all large corrections and may be sufficient for many purposes. The dominant corrections associated with $\hat{\Pi}_\gamma$ and $\Delta\rho$ (in case of a heavy top) can easily be included in the Born formulae (3)-(5) by the following prescription: replace

$$\alpha \rightarrow \alpha(Q^2) = \frac{\alpha}{1 - \hat{\Pi}_\gamma(Q^2)}$$

and use the coupling constants from (9) with

$$\begin{aligned} G_\mu &\rightarrow G_\mu(1 + \Delta\rho) \\ s_W^2 &\rightarrow s_W^2 + c_W^2 \Delta\rho \end{aligned}$$

where $\Delta\rho$ is given in (15).

2.5 Discussion of the QED corrections

2.5.1 Kinematics

The infra-red singular virtual photon corrections have to be combined with the real photon bremsstrahlung contributions in order to give a finite and physically sensible result. Since the radiative parton subprocess has an additional photon in the final state we have to specify in which variables the corrected cross section is expressed.

From an inclusive measurement of the final electron 4-momentum the variables

$$S = (p + P)^2, \quad Q^2 = -(p - p')^2, \quad U = -(p' - P)^2$$

respectively the dimensionless quantities

$$y = 1 - \frac{U}{S}, \quad x = \frac{Q^2}{S - U}$$

are used for measuring the distributions $d^2\sigma/dx dy$. For the description of the parton subprocess we have the variables

$$x' = \frac{s}{S}, \quad y' = \frac{Q'^2}{S}, \quad s' = (p' + q')^2$$

with

$$s = (p + q)^2, \quad Q'^2 = -(q' - q)^2.$$

Keeping x, y fixed, the inclusive radiatively-corrected cross section is obtained from (full phase space)

$$\frac{d^2\sigma^B}{dx dy} = \int_x^1 dx' \int_0^y dy' \int_{q'}^{q'} ds' \frac{d^2\sigma}{dx dy dx' dy' ds'}. \quad (18)$$

For the more complicated kinematical bounds on s' see e.g. ref [3].

In the classification of 2.2.1 it was emphasized that the subsets of corrections at the lepton and the hadron line as well as the interference between them are gauge invariant and allow a separate treatment.

2.5.2 Radiation from the lepton line

This subset of real photon emission from the electron (Figure 2 e,f) and the leptonic vertex correction (Figure 2 a) constitutes the bulk of all radiative corrections and represents practically the most important contribution: it can amount to a 20 - 50% correction (and even larger at small x), with steep increase in the region of large Q^2 . This behaviour can be understood from the following reasons:

- large logarithmic terms of the form

$$\frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2}$$

are present due to the radiation of photons collinear with the emitting lepton;

- the emission of a very energetic photon shifts the momentum in the propagator of the exchanged γ to a value which is essentially smaller than determined from the energy and momentum of the final electron (hard photon tail).

The second effect is primarily of kinematical nature; it can in principle be reduced in magnitude by applying suitable cuts to the emitted photon. For example, the restriction of the photon energy to $E_\gamma \leq 10$ GeV keeps the corrections below 20% [9].

The results for the relative corrections in % without phase space cuts are displayed in Figure 5: the exact $O(\alpha)$ calculation of [5] together with the LL approximation of [11] in 5a, and the updated results of [3] in 5b. All the various calculations, including also the LL approximation of [10], agree now pretty well at the per cent level.

2.5.3 Radiation from the quark line

The QED corrections associated with the quark line (Figure 2 b,g,h) exhibit a different behaviour: they are essentially smaller in magnitude, typically a few per cent, and are flat functions of Q^2 (Figure 6). The main reason for this difference is the absence of the hard photon tail in the photon propagator, which dominates the radiation from the lepton at large y .

However, another problem now comes up: the occurrence of quark mass singularities of the type

$$\frac{\alpha}{\pi} Q_f^2 \log \frac{Q^2}{m_q^2} \quad (19)$$

which have their origin, quite in analogy to the leptonic case, in the radiation of photons collinear with the emitting quarks. They depend on quark masses which are physically not well understood parameters. Whereas in the collinear radiation from charged leptons the presence of mass singularities is a physical reality (since the electron mass is a physical parameter), quark mass singularities can never be isolated from the corresponding quark distribution functions. Their appearance is therefore

Figure 5a:
Leptonic QED corrections, relative to the γ , γZ , Z cross section of (6).
--- from [5], — LL approximation [11]

Figure 5b: updated results of [3]. Parameters as in Figure 4.

Figure 6a:
Hadronic QED corrections from [5], relative to the γ , γZ , Z cross section of (6).

Figure 6b: Hadronic QED corrections from [3], updated. Parameters as in Figure 4.

an artifact which is related to a naive interpretation of the $q_f(x)$ in (2) as "bare" distribution functions.

In the naive parton model the hadronic part of the QED corrections δ'_h modifies the lowest order cross section with bare parton distributions $q_f^0(x)$ as follows:

$$\frac{d^2\sigma}{dx dQ^2} = \sum_f \frac{d\sigma^f}{dQ^2} \cdot q_f^0(x) (1 + \delta'_h(Q^2)) \quad (20)$$

Any measurement of the bare $q_f^0(x)$ would have to include these QED corrections in the data analysis. On the other hand, it is more natural to define Q^2 -dependent parton distributions

$$q_f(x, Q^2) = q_f^0(x) \cdot (1 + \delta'_h(Q^2))$$

which absorb the singular leading log contributions. If expressed in terms of $q_f(x, Q^2)$, extracted from data at a given value of Q_0^2 , the remnant correction for a process at another Q^2 will result in

$$\frac{d^2\sigma}{dx dQ^2} = \sum_f \frac{d\sigma^f}{dQ^2} \cdot q_f(x, Q_0^2) (1 + \delta'_h(Q^2) - \delta'_h(Q_0^2)) \quad (21)$$

which is now free of the quark mass singularities; instead of (19) the typical leading log correction terms are now of the form

$$\sim \frac{\alpha}{\pi} Q_f^2 \log \frac{Q^2}{Q_0^2}.$$

For the calculation of the cross section (21) parton distributions are required which have been determined without taking into account the QED corrections associated with the quark lines in the data analysis.

Q^2 evolution of the quark distributions:

A more refined treatment of the Q^2 evolution of the parton distribution functions applies the method of the Altarelli-Parisi (A-P) equations, in analogy to QCD, in order to sum the leading log contributions to all orders. In the extensive literature concerning the strong interaction corrections for deep inelastic scattering, it has been shown that the leading logarithmic corrections of QCD can be accounted for by replacing the bare parton distribution functions by Q^2 -dependent ones, where the Q^2 dependence is governed by the A-P equations, and the x dependence has to be taken from experiment.

For semileptonic processes at high Q^2 QED quark mass singularities show up in a similar way as in QCD calculations. The factorization property of these terms

$$\left(\frac{\alpha}{\pi} \log \frac{Q^2}{m^2} \right)^n$$

guarantees that they are process independent and can be absorbed into effective quark distributions as well. The absorbed terms then do not show up as explicit corrections (as would be the case in eq. (20)), but introduce an additional Q^2 dependence of the $q_f(x, Q^2)$ which is of electromagnetic origin.

This dependence can be described by additional electromagnetic terms in the A-P equations:

$$\begin{aligned} \frac{dq_i}{dt}(x, t) = & \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left(\sum_j q_j(y, t) \cdot P_{qi}(y, \frac{x}{y}) + G(y, t) \cdot P_{qi}(\frac{x}{y}) \right) \\ & + \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} q_i(y, t) \cdot P_{q\gamma}^{\gamma}(\frac{x}{y}) \cdot Q_{q_i}^2 \\ & + O(\alpha_s^2) \\ & + O(\alpha_s \cdot \alpha) \end{aligned} \quad (22)$$

for the quark (q_i) and gluon (G) distributions. In one-loop order the transition function $P_{q\gamma}^{\gamma}$ is given by

$$P_{q\gamma}^{\gamma} = \frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(z-1) \quad (23)$$

which differs from the QCD kernels $P_{q_i q_j}$ only by a group factor.

The numerical solution of these equations [13] yields for the QED evolution of the valence quarks $q_V(x, Q^2)$ an increase of 1 - 2% at $Q^2 = 10^4 \text{ GeV}^2$ relative to the experimental input $q_V(x, Q_0^2)$ at $Q_0^2 = 4 \text{ GeV}^2$ [13].

There are of course also other terms in the QED corrections at the quark lines which are not of the leading log behaviour and which depend on the specific process. These can be considered as explicit corrections appearing in the corrected cross section formula (21). In principle they influence the extraction of the quark distributions from experimental data; in practice, however, they are negligible if an experimental accuracy of 1% order is not accessible and a simplified discussion is sufficient.

2.5.4 Interference between lepton - hadron radiation

The separation into leptonic and hadronic parts of the QED corrections is not unique due to the presence of box diagrams (Figure 2 c,d) and interference between leptonic and quarkonic bremsstrahlung in Figure 2 (e,f)-(g,h). In contrast to the previous subset these terms have no analogue in the QCD corrections for deep inelastic scattering.

In the approximation for small fermion masses the box contributions can be cast into the conventional current-current interaction form

$$J_\mu^e \cdot J^{\mu q} \cdot D_{\gamma,z}(Q^2) \cdot F_{\gamma,z}(Q^2, x) \quad (24)$$

with currents $J_\mu^{(q)}$ depending only on the electron (quark) electroweak coupling constants, propagators $D_{\gamma,z}$ which carry the momentum Q^2 of the hard scattering process, and functions $F_{\gamma,z}$ which depend only on the kinematical variables Q^2 and $zS = s$, the CMS energy squared of the parton subprocess. The real photon contributions can be expressed in a similar way. As a consequence, the total interference part is not affected by either lepton or quark masses and is therefore also free of mass singularities. Typically logarithms and dilogarithms of the form

$$\log\left(\frac{Q^2}{zS}\right), \quad Li_2\left(\frac{Q^2}{zS}\right)$$

or with slightly more complicated arguments are the ingredients of the invariant functions $F_{\gamma,z}$.

The non-universality of these terms (the $\gamma\gamma$ and γZ boxes are different in their non-infrared parts) and the explicit dependence also on the variable x forbid an absorption of these interference terms in the quark distribution functions. They have to be taken into account separately and show up as explicit corrections in the cross section.

Quantitatively these interference terms are well behaved: they do not exceed the 5% level in the corrections to the cross section for reasonable values of x , y , and are relatively flat functions of Q^2 (Figure 7). However, for some specific observables like the electron-positron charge asymmetry they can become of sizeable and non-negligible magnitude.

A further problem appearing in the $O(\alpha)$ interference terms should also be mentioned. If the photon is produced at the lepton legs the correct choice for the momentum scale in the parton distributions is obviously $Q'^2 = -(q - q')^2$, whereas for photons produced at the quark legs we would guess $Q^2 = -(p - p')^2$ to be correct. For the interference terms there is no such simple choice. To give the correct answer it would be necessary to analyze the combined $O(\alpha\alpha_s)$ effects on the Q^2 dependence of the structure functions. For practical purposes, however, it is sufficient to know that several choices for the momentum scale in the g_f yield only differences which are numerically negligible.

Figure 7:
The lepton-quark interference corrections relative to the γ , γZ , and Z cross section (6) (from [5])

3 Charged current deep inelastic scattering

3.1 Lowest order cross section

The charged current differential cross section for

$$e + p \rightarrow \nu_e + X$$

with left-handed polarized electrons can be written in two ways:

$$\begin{aligned} \frac{d^2\sigma}{dx dQ^2}(e_L) &= \frac{\pi\alpha^2}{2s_W^4} \cdot \frac{1}{(Q^2 + M_W^2)^2} [u + c + (1-y)^2(\bar{d} + \bar{s})] \\ &= \frac{G_F^2}{\pi} \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 [u + c + (1-y)^2(\bar{d} + \bar{s})]. \end{aligned} \quad (25)$$

The second form has the advantage that it is not changed by large radiative corrections associated with the electromagnetic α and with $\Delta\rho$. The only source for large corrections is the photon radiation collinear to the incoming electron.

3.2 Electroweak one-loop corrections

Complete $O(\alpha)$ electroweak radiative corrections to the charged current process at energy scales comparable to the W mass have been calculated by two different groups [4,6]; QED-like corrections in the LL approximation are also treated in [11].

According to the general classification of the one-loop corrections one could interpret all diagrams with an additional real or virtual photon as displayed in Figure 8 (in addition to the self energies of the external charged fermions) as “QED” corrections. In contrast to the neutral current process, however, such a subset is not gauge invariant. Moreover the appearance of the non-Abelian γWW vertex in the real and virtual photonic diagrams, not present within the conventional QED but typical for a non-Abelian gauge theory, indicates that the usual classification convenient for NC processes is not very sensible for the charged current case.

Figure 8: Real and virtual photon corrections to the CC parton process

A gauge invariant separation is only possible for the infrared- and mass-singular parts

of the photonic corrections. If quark masses are used for regularization of the collinear photon-quark singularities, the initial (f) and final (f') quarks appear in different combinations:

$$Q_f^2 \log \frac{Q^2}{m_f^2}, \quad Q_{f'}^2 \log \frac{Q^2}{m_{f'}^2}, \quad Q_e Q_f \log \frac{Q^2}{m_e m_f}.$$

As shown in [6], all terms of the virtual contributions which contain the final quark masses cancel against the real photon contributions where the photon is collinear with the final quark, according to the KNL-theorem [20]. The only left-over singularities are the same universal parts as in the NC case. Those terms, depending on the initial quark masses, can therefore be absorbed in the universal parton densities $q_f(x, Q^2)$ as was discussed in the previous section. The numerical results of Figure 9, however, contain the mass terms explicitly and use the density functions of [19] as input.

Figure 9:
Total electroweak corrections to the CC cross section (from [6]) for electrons (–) and positrons (+)

Since the separation of a QED-like subclass, in principle possible for the singular terms, is not unique we prefer to discuss the electroweak corrections to the CC cross section as total corrections. Figure 9 displays the total electroweak corrections in per cent relative to the Born cross section (25) in the G_μ parametrization. They are dominated by the radiation from the electron leg. Differently from NC scattering

there is no hard photon tail for $y \rightarrow 1$ since the cross section is a smooth function of Q^2 . The sizeable negative corrections for large x are due to the restriction of the photon phase space to the soft-photon domain; they are qualitatively the same as in the NC case.

4 Conclusions

Radiative corrections in deep inelastic scattering processes are in general large at HERA energies.

In neutral current scattering, by far the largest part consists of electromagnetic corrections (real and virtual bremsstrahlung) associated with the lepton line. Radiation from the quarks can be treated to its largest extent in terms of a QED-induced Q^2 evolution of the parton distribution functions absorbing the leading logarithmic terms with quark mass singularities. These singular terms are universal and appear in the charged current process as well. The left-over non-universal parts from quark radiation and lepton-quark interference contribute at the level of a few per cent.

Among the corrections not of the bremsstrahlung type the dominant contributions enter in terms of the running electromagnetic $\alpha(Q^2)$ in the photon exchange amplitude and the $\Delta\rho$ term with the m_t^2 increase in the Z exchange diagram. The charged current amplitude, if expressed in terms of G_μ , is not affected by these types of corrections. The other corrections are typically of order 1% or below.

In the charged current scattering process the conventional separation into QED and weak corrections is less sensible since such a separation is neither unique nor gauge invariant. For the inclusive cross section the corrections are smaller than in the NC case for large y ; for small y and large x they are similar to the NC corrections, both dominated by soft-photon radiation.

In view of the large $O(\alpha)$ corrections the higher order QED contributions $\geq O(\alpha^2)$ play a non-negligible role. Their most important part can be obtained by an exponentiation of the leading soft photon corrections and leads to an additional positive contribution in the (inclusive) cross section [5].

A further contribution to the NC cross section is expected in the form of a radiative tail from the elastic peak. This elastic tail has not been discussed here since it is not of the deep-inelastic type. For totally inclusive measurements of the final lepton only, however, both contributions cannot be distinguished. A calculation of the elastic tail in the HERA range has been performed in [21]. For $x < 0.1$ and $y > 0.7$ it becomes of non-negligible influence.

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