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The mass and the width of the Higgs in the strongly interacting Minimal Standard Model

A. DOBADO¹

CERN, CH-1211 Geneva, Switzerland

ABSTRACT

By the use of the $[1, 1]$ Padé approximant to the one-loop amplitude for the $W_L W_L \rightarrow W_L W_L$ scattering in the strongly interacting regime of the Minimal Standard Model we build up a new amplitude for this process which has all the required physical properties such as analyticity, unitarity, absence of ghosts etc. We also show that the $I = J = 0$ partial wave amplitude has a simple pole in the unphysical sheet which can be interpreted as the physical Higgs particle. The mass of this Higgs particle is shown to be finite and in particular under 1.4TeV in the limit λ going to infinity.

¹On leave of absence from Departamento de Física Teórica de la Universidad Complutense de Madrid, 28040 Madrid, Spain

Probably the worst known aspect of the Standard Model (SM) of the electroweak interactions is the one concerning the symmetry breaking of the gauge symmetry $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$, i.e. the Symmetry Breaking Sector (SBS). Amongst the long list of models proposed in the literature to describe how this SBS could be (see [1] for a recent review), the simplest one is the so-called Minimal Standard Model (MSM). In this model, the symmetry breaking is driven by a doublet of complex scalars taking a vacuum expectation value $v \simeq 250 \text{ GeV}$. The model has only one free parameter, namely the self-coupling of the scalars λ and typically predicts the existence of a new particle called the Higgs with a mass which is given at tree level by $m_H^2 = 2\lambda v^2$. Nevertheless, it was pointed out some time ago [2] that if the value of λ is large enough or equivalently if m_H , as defined above, is in the TeV scale, the self-interactions of the scalars become strong. In this case perturbation theory cannot be applied and one could expect the appearance of new states or resonances in the spectrum of the theory. This is referred to in the literature as the Strongly Interacting MSM (SIMSM). One interesting result obtained for the SIMSM is the so-called Equivalence Theorem [3]. This theorem states that, for energies well above the mass of the weak bosons (m_W), the scattering amplitudes for the longitudinal components of these weak bosons (W_L) are the same as the scattering amplitudes corresponding to the Goldstone bosons associated to the symmetry breaking. This result is important because it greatly simplifies the computation of the $W_L - W_L$ elastic scattering amplitudes in the high energy regime.

At the present moment, a lot of work is being done in order to search for the Higgs particle or for setting bounds on its mass (see [4] for a recent review), but up to now the value of m_H has remained essentially unconstrained. In this work, we will deal with the problem of the determination of the $W_L W_L \rightarrow W_L W_L$ scattering amplitudes in the region $m_H \gg 1 \text{ TeV}$ as a way of extracting information about the nature of the physical Higgs in this case. The plan of the paper will be the following; starting with the result for the $W_L - W_L$ elastic scattering amplitude to one loop recently obtained by Dawson and Willenbrok [5] and independently by Veltman and Yndurain [6] in the limit $m_H^2 \gg s \gg m_W^2$ ($\sqrt{s} = E = \text{centre-of-mass energy}$), we will study carefully the analytical and unitary behaviour of this amplitude in the framework of the phenomenological Lagrangian approach followed in [7]. Although the one-loop partial wave amplitudes have the right cut structure in the complex plane, they are unitary only at low energies and hence they cannot be compared with the results of future experiments. On the other hand, the one-loop partial wave amplitudes are polynomials in s except for logarithmic factors, and cannot develop a resonant behaviour i.e., they have no poles in the unphysical sheet. Nevertheless, the appearance of resonances is one of the main features expected in any strongly interacting dynamics. In order to solve these problems we will consider here not the one-loop result directly but the [1, 1] Padé approximant to it as the realistic result to be compared with experiments in the future. This technique has been applied very recently to the $\pi - \pi$ elastic scattering amplitudes and the agreement obtained with the experimental data was quite impressive [8]. On the other hand the Padé approximants have been largely applied in Potential Theory and they have proved to be a good tool in the computation of the energies of discrete spectra (see [9] for instance). This last use of

the Padé approximants will be useful for us here since we are interested in computing the physical Higgs mass. We will show that the [1, 1] Padé approximant to the one-loop result for the partial wave amplitudes is unitary and has a pole in the unphysical sheet of the $l = J = 0$ channel which we claim has to be understood as the physical Higgs particle. The mass of this pole is always under 1.47 TeV and goes to zero (but very slowly) when m_H goes to infinity. In addition, the width of the physical Higgs will be also computed and compared with the naive result obtained at tree level.

As was mentioned before, we will work in the framework of the model introduced in [7]. This model is a phenomenological Lagrangian approach to the dynamics of the SPS of the SM in the same spirit described by Weinberg in [10] for the case of pion dynamics. In fact this approach is much more general than MSM because it can be applied to any theory with a global symmetry breaking pattern $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$, as in the case of the MSM. This pattern is important if we are interested in keeping the ρ parameter close to unity because of the custodial symmetry $SU(2)_{L+R}$ [11]. The one-loop partial wave amplitudes for the $W_L W_L \rightarrow W_L W_L$ scattering were computed also in [7] for the lowest J values ($J = 0, 1$). The general form of these partial wave amplitudes is:

$$a_{IJ}(s) = a_{IJ}^{(0)}(s) + a_{IJ}^{(1)}(s) \quad (1)$$

$$a_{IJ}^{(1)}(s) = s^2 [\Lambda_{IJ}(\nu^2) + i B_{IJ} - C_{IJ} \log(s/\nu^2)]$$

Here ν is an arbitrary renormalization scale but $\Lambda_{IJ}(\nu^2)$ depends on it in such a way that $a_{IJ}(s)$ is ν -independent. For very low energies the approximation $a_{IJ}(s) \simeq a_{IJ}^{(0)}(s)$ applies and it is known sometime in the literature as the Low Energy Theorem. On the other hand it is found that $B_{IJ} = K_{IJ}^2$ and this means

$$\text{Im} a_{IJ}(s) = |a_{IJ}^{(0)}(s)|^2 \quad (2)$$

This last equation can be thought of as some kind of elastic unitarity at low energy. In [7] it was argued that the amplitudes in eq.1 have an acceptable unitarity behaviour for energies under 1.57 TeV . Now, if we want to reproduce the results of reference [5] or [6] (both references agree), the parameters in eq.1 have to be chosen to take the values in the $l = J = 0$ case given by

$$K_{00} = \frac{1}{16\pi\nu^2}$$

$$C_{00} = \frac{50}{36\pi} K_{00}^2$$

$$A_{00}(\nu^2) = \frac{\Delta}{432(4\pi)^3\nu^4} - C_{00} \log(\nu^2/m_H^2) \quad (3)$$

$$\Delta = 297\pi\sqrt{3} - 1673$$

The values of these parameters corresponding to other channels can be read off from [7]. We would like to stress here that in the strongly interacting case we do not expect m_H to

be the mass of any physical particle or resonance but just a free parameter without any other obvious physical meaning.

Let us study now the analytical structure of the one-loop partial wave amplitudes in eq.1. In this formula the variable s is a real number equal to the square of the centre-of-mass energy E . If we want to obtain the proper analytical amplitude promoting s to a complex variable we have to remember that in the computation of the imaginary part of eq.1 in [7] the prescription $\log(-s) = \log(s) - i\pi$ was used. Therefore, the appropriate extension of eq.1 to the whole complex plane is given by

$$a_{IJ}^{(0)}(s) = K_{IJ}s \quad (4)$$

In this formula s is a complex number and $\log(s)$ represents the standard sheet of the analytical logarithm defined as $\log(s) = \log(|s|) + i\arg(s)$ with $-\pi < \arg(s) < \pi$. The function in eq.4 has two cuts (see Fig.1) in the real axis, one from $-\infty$ to zero and another from zero to ∞ . Of course this is the expected analytical structure because we are dealing with partial wave amplitudes corresponding to the elastic scattering of massless particles (we are neglecting m_W all the time). The partial wave amplitudes for the physical process in eq.1 may be obtained by making $s = E^2 + i\epsilon$ in eq.4, as is usually done.

Nevertheless, the amplitudes of the form in eq.4 have two serious drawbacks. First of all, as pointed out above, they are unitary only at low s . On the other hand, they have no poles in any sheet so they cannot really simulate a strongly interacting dynamics because of the absence of resonant behaviour. These two problems may in fact be solved in a simple and elegant way by the use of the Padé approximant of the loop series as was done in [12]. If one considers the $[1, 1]$ Padé approximant of the one-loop result as a reliable approximation to the true $W_L W_L \rightarrow W_L W_L$ partial wave scattering amplitudes instead of the one-loop result itself, then a great improvement in the physical properties of these partial wave amplitudes is obtained. This can be understood if one realizes that the $[1, 1]$ Padé approximant as applied to the one-loop result is nothing but the sum of the infinite terms of the geometrical series generated by the one-loop result. The precise definition of the $[1, 1]$ Padé approximant in our case is given by

$$a_{IJ}^{[1,1]}(s) = \frac{a_{IJ}^{(0)}(s)}{1 - a_{IJ}^{(1)}(s)/a_{IJ}^{(0)}(s)} \quad (5)$$

This partial wave amplitude reproduces the same results as the one in eq.4 up to terms of order s^3 or higher ($a_{IJ}^{[1,1]}(s) = a_{IJ}(s) + O(s^3)$) i.e. where $a_{IJ}(s)$ is reliable. On the other hand, the Padé improved amplitude in eq.5 has the same cuts as the one in eq.4 but is exactly elastic unitary. This last property can be expressed in terms of the discontinuity along the cut on the positive real axis as

$$\lim_{\epsilon \rightarrow 0} [a_{IJ}^{[1,1]}(E^2 + i\epsilon) - a_{IJ}^{[1,1]}(E^2 - i\epsilon)] = 2ia_{IJ}^{[1,1]}(E^2 + i\epsilon)a_{IJ}^{[1,1]}(E^2 - i\epsilon) \quad (6)$$

This equation can be directly checked from eq.4 and eq.5. In fact, eq.6 is a consequence of $\text{Im} a_{IJ}^{[1,1]}(s) = |a_{IJ}^{[1,1]}(s)|^2$ which applies for any value of s . In addition, the Padé improved amplitude has other interesting physical properties that will be analyzed in the following. In order to clarify this last point let us study first under what conditions the partial wave amplitude $a_{IJ}^{[1,1]}(s)$ has poles in the different regions of the complex plane where it is defined. As is well known from the S -matrix theory, poles in the physical sheet of the partial wave amplitudes are physically unacceptable (ghosts). It is not difficult to show that the condition for avoiding such poles in the region close above the positive real axis is given by

$$\gamma = \frac{K_{IJ}^2}{A_{IJ}(m^2) - C_{IJ}} > 0 \quad (7)$$

where m^2 is the solution of the equation

$$m^2 = \frac{K_{IJ}}{A_{IJ}(m^2)} \quad (8)$$

In the case where this last equation has no solutions with $m^2 > 0$ or the corresponding γ is positive, no ghosts appear above the positive real axis. Let us consider now the conditions for having poles in the unphysical sheet of the Padé partial wave amplitudes $a_{IJ}^{[1,1]}(s)$. The unphysical sheet has to be defined as usual as the analytical prolongation of the physical amplitude over the positive real axis into the region under the positive real axis (see Fig.1). The possibility of having poles in this unphysical sheet is interesting because these poles can be interpreted as resonances with the same quantum numbers as the corresponding partial wave amplitudes. In fact the conditions for having these poles are closely related to the no-ghost conditions shown above. More specifically, if we have a solution of eq.8 with $m^2 > 0$ and $\gamma > 0$ then we have a pole in the unphysical sheet of $a_{IJ}^{[1,1]}(s)$ at the point $s_0 = m^2(1 - i\gamma)$. This pole can naturally be interpreted as a physical resonance in the corresponding channel with mass m and width $\Gamma = m\gamma$.

Let us apply the discussion above to the Minimal Standard Model. In particular we will consider the $I = J = 0$ channel because this is the one with the same quantum numbers of the Higgs particle. By the use of eq.3 one can immediately put eq.8 in the following form

$$m^2 = \frac{18(4\pi)^2 v^2}{\Delta/6 - 50 \log(m/m_H)} \quad (9)$$

The meaning of this equation is the following. Provided we can solve it with $m^2 > 0$, this equation gives us the mass of the physical Higgs (m) (defined as the lowest mass resonance in the $I = J = 0$ channel of the $W_L W_L \rightarrow W_L W_L$ scattering amplitude) as a function of m_H . As stressed in the introduction, in the strongly interacting case, m_H has to be considered as a free parameter of the model and not as the mass of any particle or resonance. Solving eq.9 for m_H we get

$$m_H(m) = m \exp\left(\left(\frac{24\pi v}{10m}\right)^2 - \frac{\Delta}{300}\right) \simeq 1.2m \exp\left(56.85\left(\frac{v}{m}\right)^2\right) \quad (10)$$

In Fig.2 a plot of the values m in terms of m_H obtained by inverting this equation is shown. In this figure we see that no solution exists for m if m_H is under a critical value m_{Hc} which is given by

$$\begin{aligned} m_{Hc} &= m_H(m_c) \simeq 5318 \text{ GeV} \\ m_c &= \sqrt{2} \frac{24}{10} \pi v \simeq 2666 \text{ GeV} \end{aligned} \quad (11)$$

For values of m_H larger than m_{Hc} two solutions for m appear starting from the critical value m_c . The upper one grows asymptotically as a linear function of m_H but the second one is a very slowly decreasing function of m_H . This last branch of the solutions is the physically relevant one. To see that, we can compute the γ parameter by the use of eq.3 and eq.7

$$\begin{aligned} \gamma(m) &= \gamma_0(m) \frac{1}{1 - \frac{50}{36\pi} \gamma_0(m)} \\ \gamma_0(m) &= \frac{m^2}{16\pi v^2} \end{aligned} \quad (12)$$

In eq.7 it was implicitly assumed that $|\gamma| < 1$. For $m > m_c$ we have $\gamma(m) < 0$, $|\gamma(m)| > 36\pi/50$ and therefore eq.12 cannot be applied i.e. there is no pole close to the real axis corresponding to the upper branch of m in Fig.2. For $m < m_c$, $\gamma(m) > 0$, but only in the case

$$m^2 < \frac{16\pi v^2}{1 + \frac{50}{36\pi}} \quad (13)$$

or equivalently $m < 5.9v \simeq 1475 \text{ GeV}$, the width became smaller than the mass ($\gamma(m) < 1$) and eq.12 may start to be applied. By increasing m_H in this branch the value of m decreases very slowly and the same happens with the corresponding width being asymptotically $\gamma \simeq \gamma_0$. As far as m^2 and γ are positive along this branch it can be said, from the discussion above, that it corresponds to a true pole in the unphysical sheet of the $I = J = 0$ partial wave amplitude and hence to a physical resonance. By identifying this resonance with the Higgs particle we arrive at the conclusion that for values of m_H greater than 107 eV we have a Higgs particle with a mass around 17 eV or less. Moreover, the mass of the physical Higgs decreases when m_H goes to infinity but very slowly (for instance to get $m \simeq 0.37 \text{ eV}$ a value of m_H of 5×10^{16} is required). Consequently, one result of our analysis of the Padé approximant to the one-loop computation of the $W_L - W_L$ scattering amplitude is that in the limit $m_H \rightarrow \infty$ we always have a physical Higgs with a mass under 1.47 eV . This provides a new kind of bound on the Higgs mass which is of a different nature from others given previously in the literature. It is interesting to compare this result with the naive assumption of considering the m_H parameter as the mass of the Higgs particle itself. This approximation is probably good for small values of m_H , but does not work in the strongly interacting case. In fig.2 we have plotted the curve $m = m_H$ to compare it with the result of our analysis. It is amazing to realize that this curve is very close to the upper branch of our solution i.e., the one without physical meaning. The precise way in which the transition from the weak interacting case ($m \simeq m_H$) to the strongly interacting case (lower branch in fig.1) occurs remains of course unknown.

Concerning the widths of the Higgs in the strongly interacting case, one can read off directly from eq.12:

$$\Gamma(m) = \frac{m^3}{16\pi v^2} \frac{1}{1 - \frac{50m^2}{(24\pi v)^2}} \simeq \frac{m^3}{16\pi v^2} \quad (14)$$

where the last approximation applies when $\Gamma \ll m$ and was already obtained in [12]. This result reduces the tree-level one when the relation $m = m_H$ is assumed by a factor $2/3$.

Finally, we would like to stress that if the general discussion presented in this letter is appropriate for the description of the $W_L - W_L$ elastic scattering, the existence of a Higgs particle with a mass under 1.47 eV becomes a generic prediction of the MSM for "almost any" value of m_H or equivalently of λ . We consider this result as relevant in the sense that this scale of energies will be researchable in the next generation colliders and hence the validity of the MSM could be definitively tested.

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References

- [1] M.S. Chanowitz, Ann. Rev. Nucl. Part. Sci. **38**(1988)323.
- [2] M.J.G. Veltman, Acta Phys. Pol. **B8**(1977)175.
- [3] J.M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. **D10** (1974)1145.
C. V. Vayonakis, Lett. Nuovo Cimento **17**(1976) 383.
- [4] M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**(1985) 379; Phys. Rev. **D36**(1987)1490.
- [5] M.S. Chanowitz and M. Golden, Phys. Rev. Lett. **61**(1988)1053.
- [6] J. F. Gunion et al., Preprint UCD-89-4.
- [7] S. Dawson and S. Willenbrok, Phys. Rev. Lett. **62**(1989)1232.
- [8] M.J.G. Veltman and F.J. Yndurain, Nucl. Phys. **B325**(1989)1.
- [9] A. Dobado and M.J. Herrero, Phys. Lett. **B228**(1989)195.
- [10] A. Dobado, M. J. Herrero and T. N. Truong, Preprint CERN-TH.5555/89.
- [11] J.L. Basdevant, Fortsch. Phys. **20**(1972)283.
- [12] S. Weinberg, Phys. Rev. **D13**(1976)974.
- [13] P. Sikivie et al, Nucl.Phys.**B173**(1980)189.
- [14] A. Dobado, M. J. Herrero and T. N. Truong, Preprint CERN-TH.5124/89.

Fig.1

Fig.2

Figure Captions

Fig. 1: Schematic picture of the cuts, sheets and some points of the $a_{IJ}^{(1,1)}(s)$ partial wave amplitude considered in the text.

Fig. 2: The solid line corresponds to the solutions found for the mass of the physical Higgs m versus the m_H parameter, the dashed line is the $m = m_H$ line and the dotted lines separate different regions of the plot in terms of the value of γ .