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**Transport Equations and QCD Collective Modes in
a Self-Consistent Covariant Background Gauge**

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Abstract:

Following an interpretation of high-temperature instabilities of perturbative QCD, a self-consistent covariant background gauge quantization is proposed. Non-perturbative classically gauge invariant gluon, ghost, and quark equations of motion for coupled one- and two-point functions are derived in binary collision approximation (RPA). A stability analysis is described to determine coloured and colourless collective modes. Preliminary results indicate non-perturbative behaviour of the quark-gluon plasma involving self-consistent classical background fields. It leads to mass generation for gluons as well as for collective modes.

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1. Introduction

Recently experiments of relativistic nuclear collisions have become feasible which are particularly determined to search for the speculative quark-gluon plasma state of matter [1]. Certainly, lattice studies of QCD indicate a rather strong phase transition between high-energy-density hadronic matter and a phase of deconfined colour degrees of freedom. However, a deeper theoretical understanding of the nature of the phase transition is still missing and even of the deconfined phase at high temperatures [2]. The study of dynamical features (such as screening of long-range forces and collective modes), therefore, is quite important. So far it has been guided by the experience with QED plasmas, and one still has to find out the limitations of this analogy.

In order to describe essential non-equilibrium phenomena to be expected in heavy-ion collisions, a transport theory approach seems necessary [3]. It should, however, also be useful to look for collective high-temperature QCD effects, the most relevant of which presumably are still unknown. The purpose of this paper is to report the why and how of deriving self-consistent classically gauge invariant transport equations for QCD. Presently the binary collision approximation is worked out, which eventually can be further simplified by evaluating the equations obtained perturbatively. It extends previous attempts to formulate transport equations for non-abelian gauge theories, which were recently reviewed in ref.[3], where also references to earlier quark-gluon plasma related work can be found. The new results concern the gluon quantum sector in particular, which had not been treated thoroughly before [4].

Given the transport equations from below, QCD transport theory may completely reproduce the one-loop high-temperature field theory results, such as e.g. the worrying instability and gauge fixing dependence of collective plasmon modes [5]-[10] (this is not a complete list of references, cf. also the third of refs.[2]). However, due to its very structure it is not confined to describe a high- T equilibrium system of quarks and gluons, which according to widespread belief is the asymptotically free regime where perturbation theory applies. The transport equations are based on a *canonical quantization of the gauge field using a background gauge fixing supplemented by a self-consistency condition*. It is the latter important, though almost trivial condition which determines the classical background field generated by quantum fluctuations. Such self-consistency has not been taken into account in previous applications of the background field method [5]-[7],[10]-[13]. It may be useful in the wider context of gauge theories, wherever a non-trivial vacuum structure is expected. To what extent the self-consistent canonical quantization proposed here is equivalent to an extension of the effective action method [14] remains to be seen.

As a first application of the transport equations a stability analysis is outlined, corresponding to a self-consistent linear response approach, which can be applied to determine collective modes of the system. Here the distinction is to be stressed and the relation to be discussed between quantum gauge fixing dependent results expected from the study of

an ordinary two-point function (at one-loop order) and gauge fixing independent "physical" results for gauge invariant correlation functions.

Thus, one can proceed to study the dynamics of background field configurations (gauge field condensate) which possibly lead away from the naive perturbative vacuum. The hope is to identify stable/damped collective modes of a quark-gluon plasma. It implies a qualitative change from the usual picture of a hot plasma of weakly interacting quarks and gluons [1,2] analogous to a QED $e^+e^-\gamma$ -plasma close to the Stefan-Boltzmann limit. The essence of such an approach will be the analysis of background fields, i.e. a modified "vacuum", around which a stable perturbation theory can work. Results for the high-temperature QCD plasma will be presented elsewhere.

Following in Section 2, I provide some more detailed remarks motivating these considerations. In Section 3, the comparatively simple mean field transport equations for quarks are reviewed in order to illustrate the main ideas of how to derive gauge invariant transport equations from field operator equations. Furthermore, by comparison, previous attempts for the gluon sector are summarized and their deficiencies pointed out. In Section 4, the new gluon equations are derived based on the self-consistent canonical background gauge quantization and binary collision approximation. A general stability analysis is performed in Section 5, and its relevance for finding collective plasma modes pointed out. In particular, it is discussed how to use the gauge fixing dependent Wigner function equations to study gauge invariant correlation functions. In the final Section 6, I summarize the results obtained so far, next steps to be taken, and, last but not least, mention the attractive features of the transport equations as a model which basically unifies all the physics built into currently fashionable "event generators".² The zeroth order gluon and ghost Wigner functions are calculated in the Appendix, where also a non-perturbative solution for the gluon Wigner function is obtained, which indicates a mechanism of gluon mass generation.

The notation used below follows ref.[3], except that classical and quantum fields will be distinguished by capital and small letters, respectively, wherever necessary. The metric used is $g^{\mu\nu} \equiv \text{diag}(1, -1, -1, -1)$, $a \cdot b \equiv a_\mu b^\mu$, and units are chosen such that $\hbar = c = 1$ unless stated otherwise. The $N^2 - 1$ hermitian generators of $SU(N)$ in the fundamental representation satisfy $\text{Tr } t_a = 0$, $\text{Tr } t_a t_b = \delta_{ab}/2$, and $[t_a, t_b] = if_{abc} t_c$. The adjoint representation matrices are defined by $(T_a)_{bc} \equiv -if_{abc}$ with $\text{Tr } T_a T_b = N\delta_{ab}$. Quite often the distinction between fundamental and adjoint representations of $SU(N)$ will not be maintained explicitly to avoid proliferation of symbols.

2. Motivation

Before becoming more specific about why it seems worthwhile to study gauge invariant transport equations further, it should be recalled that over the years circumstantial evidence is

²Some of the results I reported earlier in refs.[16].

accumulating that that essentially non-perturbative effects may determine the quark-gluon plasma properties even at high temperature:

1. Quite general arguments show that an eventually observable prototype massless ideal gas equation of state, $PV = \frac{1}{3}E$, would not at all imply that the Lagrangian degrees of freedom, quarks and gluons, do interact weakly. Rather, relevant quasi-particles could lead to this behaviour [15].
2. Lattice simulations of full QCD, on the other hand, indicate that even at $T \approx 2T_c$ there are still substantial deviations from the ideal gas behaviour [1],[16(a)].
3. There are severe infra-red divergencies plaguing finite-temperature QCD field theory [2], which are presumably cured by non-perturbatively generating masses for the dangerous long-wavelength static modes. One possibility may be to selectively resum important diagrams from perturbation theory (cf. e.g. ref.[9] and R. F. Alvarez-Estrada in [16(a)], while another is to eliminate the well-behaved massive ($m \sim T$) non-static modes perturbatively and to study the resulting three-dimensional Yang-Mills-Higgs effective theory non-perturbatively (cf. e.g. J. Polonyi in [16(a)]).

Note that typically the methods used in 2,3. to study QCD at high temperatures are tailored to determine *equilibrium properties*. They cannot easily yield information about non-equilibrium relaxation phenomena, real-time scattering processes such as heavy-ion collisions are completely beyond what one could hope for, and also the analysis of collective modes is not straightforward with these tools. These are, however, phenomena accessible to transport theory.

The first important step to investigate small perturbations of a high- T QCD system was taken in ref.[17] setting up a linear response formalism in analogy to QED. Using the "physical" temporal axial or Coulomb gauge, nicely damped plasmon oscillations and ordinary Debye screening were found and, thus, QED-like behaviour of the quark-gluon plasma [18]. However, this approach was later criticized, since despite all efforts the gauge transformation properties of the corresponding response functions remained obscure and, therefore, gauge invariance and physical relevance of their poles cannot be assured [5,10]. The problem resides in the fact that, in order to couple the system to an external perturbation, one has unavoidably been using a gauge-dependent coupling Hamiltonian.

As will be discussed below, the notion of *external perturbation* has to be critically examined for QCD. Physically speaking, we expect only colour singlet excitations to exist outside a quark-gluon plasma, where colour is deconfined. Nevertheless one chooses a coupling Hamiltonian (colour indices are suppressed in the following),

$$H_{ex}(x) = E(x) \cdot \mathcal{E}(x) - B(x) \cdot \mathcal{B}(x), \quad (2.1)$$

describing the interaction between external electric and magnetic fields, $\mathcal{E}$ and $\mathcal{B}$, and the dynamical ones, E and B . On the classical level one can derive this Hamiltonian from a

Lagrangian,

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu} - \mathcal{F}_{\mu\nu})(F^{\mu\nu} - \mathcal{F}^{\mu\nu}), \quad (2.2)$$

where the field strength tensors, F and $\mathcal{F}$, correspond to the above fields as usual. This Lagrangian illustrates the QED-based motivation of eq.(2.1). In that case the linear superposition principle holds and the field strengths are themselves gauge invariant quantities. For a non-abelian gauge theory $\mathcal{L}$ seems questionable when used in quantum theory, i.e. particularly within the path integral formalism. For, if one necessarily fixes the gauge for the quantum fields, then this also restricts the gauge freedom of the external fields, otherwise one could change the relevant action, $\int d^4x \mathcal{L}$, at will. Conversely, if one fixes the gauge for the external fields, this then destroys the full gauge invariance (before gauge fixing) of the quantum theory. Both options contradict the notion of an external physical perturbation, which should be independent of the system to be perturbed.

In any case, the above formulation leads, for example, to the colour electric response formula [17,18],

$$\delta\langle E_i^a(x) \rangle = -i \int d^4x' \langle [E_i^a(x), E_j^b(x')] \rangle^R \mathcal{E}_j^b(x'). \quad (2.3)$$

Evidently the response function determined here by the retarded correlation function of two colour electric fields reflects the dubious gauge transformation properties of $\mathcal{L}$. It transforms covariantly at two different points and certainly represents a gauge dependent quantity. Evaluating it in a "physical" gauge like the temporal axial one, $A_0^a = 0$ ("TAG"), *damped plasma oscillations* were inferred from its pole structure [17,18].

Now, can one trust this result in view of its apparent gauge dependence [19]? Unfortunately there is no firm argument for an affirmative answer at this moment (cf. refs.[8,9] for other attempts to obtain gauge invariant information from the gauge dependent propagator). However, let me remark that the response function, $\langle [E(x), E(x')] \rangle^R$, in the *local limit* $\vec{x} \rightarrow \vec{x}', t \neq t'$ is gauge invariant, even without contracting the colour indices but provided that TAG is applicable. This result is, of course, not sufficient to prove the gauge invariance of the long-wavelength ($\vec{k} \rightarrow 0$) calculations necessary to determine the plasma frequency and damping constants.

Recently difficulties with TAG (see e.g. ref.[20] and references therein), which is actually not compatible with the periodic imaginary-time boundary conditions on the gauge potentials at finite temperature, caused a study using a stationary gauge instead [21], $\partial_0 A^0 = 0$ ("STAG"), which surprisingly yields the same results as the TAG calculation. It should be noted, however, that this calculation does not address the abovementioned basic problems associated with the Lagrangian, eq.(2.2), even if a proper linear response analysis in STAG is performed (as announced in ref.[21]). Furthermore, in order to recover gauge invariance in the local limit (as for TAG) at least, the response function in STAG would have to be something like

$$\langle [E_i^a(x), \{e^{-igA^0(\vec{x}')(t-t')} E_j^b(x') e^{-igA^0(\vec{x})(t-t')}\}^b] \rangle^R,$$

which seems hard to get from a linear response analysis.

Alternatively the effective action method [14] was applied in ref.[5] to describe the high- T plasma, which eventually led to unstable plasma oscillations. To obtain the effective action one begins by defining

$$e^{i\Gamma[A,j]} \equiv \int Da D\bar{c} Dc e^{iS_B}, \quad (2.4)$$

which is a partition function, a being the quantum gauge field, depending parametrically on a classical source j and a classical gauge field A ; S_B denotes the Euclidean action (cf. Section 3. for details of the corresponding Minkowski space Lagrangian),

$$S_B \equiv S_{Y-M}[A+a] + S_{g.f}[A,a] + S_{gh}[\bar{c},c,A,a] + \int d^4x j \cdot a, \quad (2.5)$$

which consists of the usual Yang-Mills, covariant background gauge fixing, and ghost parts, and with a source coupling to the quantum field only. - The main attraction here is the possibility to maintain *gauge invariance with respect to classical gauge transformations* of A in all ensuing calculations [5,14]. - Next, one eliminates the source in favour of the expectation value of $a, \langle a \rangle$, by a Legendre transformation leading to $\Gamma[A, \langle a \rangle]$. Then, the *effective action* is defined,

$$\Gamma_{eff}[A] \equiv \Gamma[A, \langle a \rangle = 0], \quad (2.6)$$

corresponding to a free energy. Gauge invariance and isotropy allow the expansion,

$$\Gamma_{eff}[A] = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left(\epsilon E^2 - \frac{1}{\mu} B^2 + \dots \right) \quad (2.7)$$

$$= -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left(A^\mu(k) \left[k^2 g_{\mu\nu} - k_\mu k_\nu + \Pi_{\mu\nu}(k) \right] A^\nu(-k) + \dots \right). \quad (2.8)$$

Eq.(2.7) exhibits Γ_{eff} as the classical Yang-Mills action modified by a scalar dielectric function, ϵ , and a magnetic permeability function, μ , which describe the influence of quantum fluctuations on the classical background field. Eq.(2.8) equivalently shows it (including only the contributions quadratic in the potentials) in terms of the polarization tensor, $\Pi_{\mu\nu}$. In one-loop approximation the medium effects can be calculated either directly from the Gaussian path integral for Γ_{eff} [5,10] or by applying Feynman rules to obtain $\Pi_{\mu\nu}$ [6,7].

Minimizing the effective action, eq.(2.8),

$$0 = \frac{\delta}{\delta A} \Gamma_{eff}[A], \quad (2.9)$$

yields linearized classical *sourceless Yang-Mills equations* in a medium described by ϵ and μ . They can be solved by ordinary plane waves, $\sim \exp(i(\omega t - \vec{k} \cdot \vec{x}))$, which for longitudinal as well as for transverse modes leads to a dispersion relation,

$$\omega^2(\vec{k}) = \omega_0^2 + \text{const} \cdot \vec{k}^2 + \dots, \quad (2.10)$$

with the plasma frequency $\omega_0^2 \equiv \frac{1}{6} N g^2 T^2$ [2]. However, due to the polarization tensor, ω also obtains an imaginary part,

$$\omega \longrightarrow \omega + i\gamma, \quad \gamma = -\frac{Ng^2T}{24\pi} \left(11 + \frac{1}{4}(1-\xi)^2 \right). \quad (2.11)$$

Whereas the real part is fine and comes out the same in any calculational scheme (see e.g. [3] and references therein), the imaginary part causes trouble for two reasons. First, being negative definite, it describes modes which grow exponentially in time and, therefore, cannot be reasonably interpreted as collective quasi-particles. Secondly, it depends on the quantum gauge fixing parameter ξ , which cannot be for a physical quantity. These results are presently not well understood and spurred numerous attempts to clarify the physical meaning of poles of all kinds of propagators, response functions, etc. (see [2],[10],[17]-[21]).

Now I want to briefly mention some results of the Copenhagen group which suggest an interpretation of the above instability and particularly motivated the derivation of self-consistent gauge invariant transport equations below.

Previous interest in the vacuum structure of QCD at $T=0$ centred around the Savvidy vacuum and modifications thereof. Since a homogeneous background colour-magnetic field, $B^{ai} \equiv \delta^{a3} B^i = \text{const}$, provides a state of lower energy than the "perturbative vacuum" ($B=0$), it was studied as a candidate new groundstate. However, it was soon observed that this state decays through the Nielsen-Olesen unstable mode [22]. It is characterized by the interaction of magnetic dipole moments with the magnetic field, the energy of which is sufficient to put virtual gluon pairs on-shell. A similar instability, besides ordinary decay through charge separation of virtual colour-charged gluon pairs (cf. Schwinger's calculation of vacuum decay in QED [23,24]) was found for a homogeneous background colour-electric field, $E^{ai} \equiv \delta^{a3} E^i = \text{const}$, in which case E decays by working on virtual electric dipole moments [25].

Another crucial interpretation of these instabilities can also be extracted from [25]:

A) Background fields with non-vanishing field-strengths which obey homogeneous Yang-Mills equations simultaneously,

$$F_{\mu\nu} \neq 0 \quad \text{and} \quad D^\mu F_{\mu\nu} = 0, \quad (2.12)$$

require *charges and/or currents at infinity* to support them. In pure Yang-Mills theory this also holds for any classical non-dissipative soliton/energy lump solutions, which do not exist at all [24] without sources.

B) Gauge dependent external sources at infinity are *unphysical*, since admitting quantum fluctuations within the system leads to fluctuations of their colour orientation. Thus, the background fields necessarily decay.

C) The background field method excludes such source fluctuations (at $T \neq 0$ as well), at least in the mean field approximation (cf. Section 4.2). It is obvious in the *sourceless* classical field equation following from eqs.(2.8,2.9) for example.

Concluding from A) - C) I conjecture that the classical background field should be determined self-consistently instead by an equation of the form

$$D^\mu F_{\mu\nu} = j_\nu[\langle aa \rangle, \langle \bar{c}c \rangle, \dots] , \quad (2.13)$$

where the source is a functional of the quantum α -field and ghost two-, ... point functions. Such an equation will be derived in Section 4., eq.(4.24). Finding solutions with a non-trivial background field would presumably cure the instabilities. Note that all the above instabilities are operative even for arbitrarily small background fields. It implies that e.g. thermal fluctuations away from the naive vacuum ($A = 0$) could easily come close to such an unstable configuration and "explode", which again calls for the self-consistent treatment. The problem of gauge fixing dependence will be addressed in Sections 4.2 and 5.

Finally, apart from what has been said above, a short comment seems to be in order concerning the relevance of transport equations as opposed to Green function equations. Whereas time-ordered Green functions are objects which arise naturally in a path integral formulation of field theory, Wightman or Wigner functions seem more convenient in a canonical quantization scheme. The first approach leads to coupled Schwinger-Dyson equations for propagators [24] and the latter to the BBGKY hierarchy for Wigner functions [26]. Preference, however, may be given to the second approach whenever one is interested in a phase space picture and the semiclassical limit is sufficiently accurate to describe the system. It is well known [3] that in the classical limit Wigner functions reduce to phase space distributions obeying Vlasov-Boltzmann type equations.

3. Quark Transport Equations

The quark transport equations were derived and thoroughly discussed before [3,27]. - At the Hartree level, and consistently with the binary collision approximation of Section 4.2, the equations follow from the Dirac field operator equation,

$$(i\gamma^\mu D_\mu - m)\psi = 0 , \quad (3.1)$$

describing the quark field, ψ , interacting with the classical part of the gauge field, A_μ , through standard minimal coupling. A_μ will be determined self-consistently in Section 4.2.

The quark colour four-current can be most conveniently expressed in terms of a Wigner function,

$$j^\mu(x) \equiv t^a(\bar{\psi}(x)\gamma^\mu t^a \psi(x)) = \int d^4p \, t^a \, \text{Tr} \, \gamma^\mu t^a W(x,p) , \quad (3.2)$$

where the trace refers to spinor and colour indices. The gauge covariant quark Wigner function is defined by

$$W(x,p) \equiv \int \frac{d^4y}{(2\pi)^4} e^{-ip \cdot y} \langle \bar{\psi}(x) e^{\frac{1}{2}y \cdot D_\perp} \otimes e^{-\frac{1}{2}y \cdot D_\perp} \psi(x) \rangle , \quad (3.3)$$

where the adjoint covariant derivative, $D^\dagger$, acts as the infinitesimal shift operator to the left and (...) denotes an ensemble average, which has to be further specified in concrete applications. Note that the introduction of covariant derivatives here instead of ordinary ones is what makes W gauge covariant under gauge transformations of the classical field (cf. Section 4.1 for more details),

$$\psi(x) \longrightarrow S(x)\psi(x) , \quad W(x,p) \longrightarrow S(x)W(x,p)S^{-1}(x) . \quad (3.4)$$

Furthermore, in the classical limit, for this unique definition of W only, the momentum variable p can be shown to correspond to the kinetic momentum variable of the resulting phase space distribution [27]. - It is technically important that W can be rewritten,

$$W(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-ip \cdot y} \langle \bar{\psi}(x + \frac{1}{2}y) U(x + \frac{1}{2}y, x) \otimes U(x, x - \frac{1}{2}y) \psi(x - \frac{1}{2}y) \rangle , \quad (3.5)$$

with link operators given by a path-ordered exponential straight-line path integral,

$$U(b,a) \equiv P \exp \left(ig \int_a^b dz^\mu A_\mu(z) \right) , \quad (3.6)$$

where $z \equiv z(s) = a + (b-a)s$, $0 \leq s \leq 1$.

Then, the equation of motion for W follows by inserting eq.(3.1) into eq.(3.5), suitably acting on the right spinor for example, and converting the resulting equation into an equation for W [27]. One obtains:

$$\begin{aligned} & \left(\gamma^\mu p_\mu - m + \frac{1}{2} i \gamma^\mu D_\mu(x) \right) W(x,p) \\ &= \frac{1}{2} ig \partial_p^\nu \gamma^\mu \left(\int_0^1 ds \, \frac{1}{2} (1+s) [e^{-\frac{1}{2}is \partial_p \cdot D(x)} F_{\nu\mu}(x)] W(x,p) \right. \\ & \quad \left. + W(x,p) \int_0^1 ds \, \frac{1}{2} (1-s) [e^{\frac{1}{2}is \partial_p \cdot D(x)} F_{\nu\mu}(x)] \right) , \end{aligned} \quad (3.7)$$

where ∂_p always acts on W and D on the fundamental representation matrices within [...] or to its right. Even though this classically gauge invariant equation does not have the appearance of a transport equation, it can be shown [3,27] that it does contain separately a proper transport equation, which generalizes the classical Vlasov equation by including the non-local quantum effects and spin-dependent terms in particular, and additionally a generalized mass-shell constraint, which again involves non-local quantum and spin-dependent terms.

These equations were studied before in various limiting cases such as the semiclassical Wigner-Kirkwood type expansion based on the approximation " $\hbar \partial_p \cdot D(x) \ll 1$ " and the abelian dominance approximation obtained by restricting the colour degrees of freedom [27] to the maximal abelian subalgebra related to the gauge symmetry group $SU(N)$. Furthermore, it has been shown by a perturbative calculation (see the second of refs.[4]) and by a non-perturbative WKB analysis (cf. refs. in [3]) how the full transport equations describe

particle production/vacuum polarization by the background field. One could also consider the backreaction of dynamical quark fields as a source, eq.(3.2), on a purely classical glue field, i.e. neglecting the proper gluon dynamics described in Section 3. Such a question has recently been asked for scalar QED [28], in which case it already leads to a rather involved initial value problem to be studied numerically.

To compare the above transport theory for quarks with what has been done previously for gluons [4], consider the gluon Wigner function employed there, which in the adjoint representation (see ref.[3] for details) can be written as:

$$\Gamma_{\mu\nu}(x, p) = \int \frac{d^4 y}{(2\pi)^4} e^{-ip \cdot y} \langle : F_{\nu\lambda}^\dagger(x + \frac{y}{2}) \tilde{U}(x + \frac{y}{2}, x) \otimes \tilde{U}(x, x - \frac{y}{2}) F_\mu^\lambda(x - \frac{y}{2}) : \rangle ; \quad (3.8)$$

here the field strength tensor, $F_{\mu\nu} \equiv F_{\mu\nu}[\tilde{A}]$ (an adjoint representation vector here), is determined by the total (classical plus quantum) gauge field potential, $\tilde{A} \equiv A + a$. Similarly, $\tilde{U}$ is defined as U , eq.(3.6), but involving here the total $\tilde{A}$ in the adjoint representation instead of only A in the fundamental representation there. If $\tilde{A}$ transforms in the standard way under classical gauge transformations, then Γ transforms covariantly in analogy to the quark Wigner function, cf. (3.4). Γ can be related to the gluon energy-momentum tensor in much the same way as W to j , eq.(3.2). Furthermore, applying the quadratic Yang-Mills equation for Γ instead of the quadratic Dirac equation for W , it is not any more surprising to find a proper gluon transport equation [3,4] which bears much resemblance to the one for quarks.

There have been, however, several *shortcomings of the previous approach*:

- A) The analogy to quarks (basically changing from fundamental to adjoint representation in constructing the Wigner function) is misleading, since Γ contains contributions from unphysical scalar and longitudinal components of $\tilde{A}$, i.e. gauge degrees of freedom.
- B) Without properly quantizing the gluon field the unphysical contributions cannot be eliminated, the gauge transformation properties of Γ cannot be derived, and the use of the Yang-Mills equation as a basic operator equation cannot be justified (it turns out to be incomplete).
- C) The separation of the mean field part A from the total field $\tilde{A}$ has been done in a rather ad hoc manner, which particularly involved the problem of identifying the colour current carried by the gluon quantum field a and its backreaction on $\tilde{A}$.

Instead of trying to fix these problems one after the other, I show in the following section that they are solved or, rather, avoided by the self-consistent canonical quantization of the gluon field. This will e.g. lead to the definition of a well-behaved Wigner function which is simpler than Γ , eq.(3.8), avoiding the complicated non-local operator structure there, and closer to what one may have naively expected (" $\Gamma \sim \langle aa \rangle$ ").

4. Gluon Transport Equations

4.1 Canonical Quantization in Self-Consistent Covariant Background Gauge

The main point here is to introduce the self-consistent canonical quantization of the gluon field and, then, to show how this leads to the transport equations. Relevant information concerning the canonical background gauge quantization can be found in refs.[11]-[13]. To begin with, consider the *classical Yang-Mills Lagrangian*,

$$\mathcal{L}_{Y-M}[\tilde{A}] \equiv -\frac{1}{4} \tilde{F}_{\mu\nu}^a \tilde{F}^{a\mu\nu}, \quad (4.1)$$

with $\tilde{F}_{\mu\nu} \equiv [\tilde{D}_\mu, \tilde{D}_\nu]/(-ig)$, $\tilde{D}_\mu \equiv D_\mu[\tilde{A}] \equiv \partial_\mu - ig\tilde{A}_\mu$. It is invariant under

$$\tilde{A} \longrightarrow S\tilde{A}S^{-1} + \frac{1}{ig}(\partial S)S^{-1} \approx \tilde{A} + \tilde{D}\lambda, \quad (4.2)$$

where the approximate equality holds for an infinitesimal gauge transformation, $S \equiv \exp ig\lambda$, $\lambda \rightarrow 0$. Next, split $\tilde{A} = A + a$, where a stands for the fluctuating part of the field to be quantized shortly and A for the classical part to be determined self-consistently later on. A is presently considered as a parameter, for which there seems to be no consistent alternative. In particular, it can be interpreted as a *classical collective coordinate* depending on the dynamical fields a .

In order to define the quantum theory properly, the gauge for the a field has to be fixed. It is most convenient to choose the *covariant background gauge fixing* condition,

$$\mathcal{L}_{gf}[A, a] \equiv -\frac{1}{2\xi} (D_\mu^a[A] a^\mu)^2. \quad (4.3)$$

Now, what happens to the gauge transformation (4.2) and the invariance of the Lagrangian, which has become $\mathcal{L}_{Y-M} + \mathcal{L}_{gf}$, in terms of transformations of A and a ? There are exactly two possibilities to realize (4.2):

1. Classical gauge transformations:

$$A \longrightarrow A + D[A]\lambda, \quad a \longrightarrow SaS^{-1}. \quad (4.4)$$

$\mathcal{L}_{Y-M} + \mathcal{L}_{gf}$ is clearly invariant under (4.4). It is worth noting that the fluctuations here transform as an adjoint representation "matter/Higgs" field, this analogy becoming a little closer below (cf. the discussion after eq.(5.1)).

2. Quantum gauge transformations:

$$A \longrightarrow A, \quad a \longrightarrow a + D[A + a]\lambda. \quad (4.5)$$

Here "quantum" refers to the fact that (4.5) becomes operator-valued later on. $\mathcal{L}_{Y-M} + \mathcal{L}_{gf}$ is invariant under (4.5) provided that the gauge transformations are constrained by

$$D_\mu[A]D^\mu[A + a]\lambda = 0, \quad (4.6)$$

which reminds one of the usual covariant gauge condition, $\partial^2 \lambda = 0$. Note that it is tacitly assumed that the finite gauge transformations, $S \equiv \exp i g \lambda$, can be built and correspondingly constrained from the infinitesimal ones (there are subtleties here related to Gribov ambiguities, which are neglected in the context of perturbation theory [24]). The purpose of introducing $\mathcal{L}_{gf}$ was just to place such a constraint on the gauge transformations of the (soon-to-be) quantum field a . In the path integral formulation this then allows us to integrate out the gauge degrees of freedom explicitly [24].

Presently, however, eq.(4.6) becomes an operator equation and, thus, its non-trivial solutions are operators too, which are associated with the *ghost field*, $c \equiv \lambda$. One may interpret eq.(4.6) as the equation of motion for the ghost field c . Correspondingly there should be a part of the action which, upon variation, yields this equation. Introducing a second ghost field, $\bar{c}$, as a Lagrange multiplier field, the ghost Lagrangian is:

$$\mathcal{L}_{gh}[A, a, \bar{c}, c] \equiv -i(D_\mu[A]\bar{c})^a (D^\mu[A+a]c)^a. \quad (4.7)$$

Then, there is an equation for $\bar{c}$ similar to the one for c , cf. below. As can be seen from that equation and from eq.(4.6), also the ghost fields should transform classically, cf. (4.4), according to the adjoint representation. This is sufficient for *classical gauge invariance of the total Lagrangian*,

$$\mathcal{L} \equiv \mathcal{L}_{Y-M} + \mathcal{L}_{gf} + \mathcal{L}_{gh}. \quad (4.8)$$

A crucial step now consists of the observation that the quantum gauge transformations (4.5) can be extended to non-linear BRS supersymmetry transformations as for the covariant gauge [29],

$$\begin{aligned} A_\mu^a &\longrightarrow A_\mu^a, \quad a_\mu^a \longrightarrow a_\mu^a + \eta(D_\mu[A+a]c)^a, \\ c^a &\longrightarrow c^a - \frac{1}{2}\eta g f^{abc}c^b c^c, \quad \bar{c}^a \longrightarrow \bar{c}^a - \frac{1}{\xi}\eta D_\mu^{ab}c^b[A]a^{\mu a}, \end{aligned} \quad (4.9)$$

where η is an infinitesimal global Grassmann parameter ($\eta^2 = -\eta$). For $\mathcal{L}$ to be invariant under (4.9), it is necessary that classically the ghost fields have to be anticommuting Grassmann variables ($c^1 = c, \bar{c}^1 = \bar{c}$). (To show the BRS invariance of $\mathcal{L}$, one has to make use of the Jacobi identity for the structure constants of the $SU(N)$ Lie algebra.) Furthermore, there is a global scale invariance of $\mathcal{L}$ under

$$c \longrightarrow c e^{-\Theta}, \quad \bar{c} \longrightarrow \bar{c} e^{\Theta}. \quad (4.10)$$

According to the invariances under (4.9,4.10) there are conserved Noether currents [12]. The corresponding BRS charge and ghost charge operators play an important role when constructing the physical Hilbert space of the theory [29], which will become relevant for transport theory when studying the ensemble averages defining the Wigner functions (cf. for example the Appendix).

So far, setting up the classical Lagrangian, eq.(4.8), one follows well-known steps. However, in distinction to what has been done before, I keep the *full dependence of $\mathcal{L}$ on the classical A field*. There is a completely classical part in $\mathcal{L}$, $\mathcal{L}_{Y-M}[A]$, which could be dropped, since it does not affect the quantum equations of motion. Then, there are also *terms linear in a* which have always been eliminated before [11]-[13] by assuming eqs.(2.12) for the classical A field and fixing its gauge. Motivated by the considerations in Section 2., I keep the full dependence on A , which will be determined self-consistently instead.

Finally, quantization may proceed in the usual way by postulating canonical equal-time (anti-) commutation relations for all the (ghost) gluon fields $\phi (= c^a, \bar{c}^a, a_\mu^a)$ and their canonically conjugate momenta Π_ϕ :

$$[\Pi_\phi(t, \vec{x}), \phi'(t, \vec{y})]_\pm = -i\delta_{\phi\phi'}\delta^3(\vec{x} - \vec{y}), \quad (4.11)$$

$$[\Pi_\phi(t, \vec{x}), \Pi_{\phi'}(t, \vec{y})]_\pm = 0 = [\phi(t, \vec{x}), \phi'(t, \vec{y})]_\pm. \quad (4.12)$$

Due to the covariant gauge fixing there is no problem with the a_0^a component of the field here, which otherwise could have a zero conjugate momentum:

$$\Pi_{a_0^a} \equiv \frac{\partial \mathcal{L}}{\partial \dot{a}_0^a} = -\frac{1}{\xi}(D_\mu[A]a^\mu)^a, \quad (4.13)$$

$$\Pi_{a_i^a} \equiv \frac{\partial \mathcal{L}}{\partial \dot{a}_i^a} = -F^{a0i}[A+a], \quad (4.14)$$

$$\Pi_{c^a} \equiv \frac{\partial \mathcal{L}}{\partial \dot{c}^a} = +i(D_0[A]\bar{c})^a, \quad (4.15)$$

$$\Pi_{\bar{c}^a} \equiv \frac{\partial \mathcal{L}}{\partial \dot{\bar{c}}^a} = -i(D_0[A+a]c)^a, \quad (4.16)$$

where an additional minus sign in eq.(4.15) is due to the left-differentiation convention with respect to Grassmann variables [29]. Then, the Euler-Lagrange operator equations, obtained with eq.(4.8), are:

$$D_\mu^{ab}[A]D^{b\mu\nu}[A+a]c^\nu = 0 = D_\mu^{ab}[A+a]D^{b\mu\nu}[A]\bar{c}^\nu, \quad (4.17)$$

$$D^{ab\mu\nu}[A+a]F_{\mu\nu}^b[A+a] + \frac{1}{\xi}D_\nu^{ab}[A]D^{b\mu\nu}[A]a^\mu + ig f^{abc}(D_\nu[A]\bar{c})^b c^\nu = 0. \quad (4.18)$$

This complete set of classically gauge invariant operator equations presents the starting point of the following gluon transport theory. Quarks can be trivially included here by inserting their colour current operator, cf. eq.(3.2), into the r.h.s. of eq.(4.18) and including the Dirac eq.(3.1) with $D \equiv D[A+a]$. Note that eq.(4.18) additionally contains gauge fixing and ghost terms as compared to the naive Yang-Mills equation used before [3,4]. Furthermore, there is no quantum gauge covariance of these equations of motion, which is, of course, the essence of gauge fixing.

As is obvious from the above derivation, the benefit from the covariant background gauge fixing is twofold: First it allows quantization to proceed in a rather straightforward way [12,29]. Particularly, the elimination of gauge degrees of freedom can be treated separately from the imposition of the canonical commutation relations. Secondly, it seems a natural procedure to separate the classical field A from the quantum field a beginning with the classical Lagrangian. However, this is achieved at the expense of introducing ghost fields, i.e. a correspondingly enlarged Hilbert space which ultimately has to be projected on the physical subspace [29]. The latter is achieved by requiring the physical states to carry zero BRS and ghost charges; since the latter (operators) generate the global symmetry transformations of the quantized fields corresponding to (4.9) and (4.10), this is the requirement of gauge invariance of the physical states.

4.2 Self-Consistent Classical Field and Gluon/Ghost Transport Equations

Using any of the eqs.(4.17,4.18), the *BBGKY hierarchy* of coupled equations for ensemble averaged products of ghost and gluon field operators can be generated by multiplying the respective equation with appropriate powers of operators before averaging:

$$(\cdot \cdot a^{\dagger m} c^n \cdot \cdot \text{operator eq.} \cdot \cdot) = 0. \quad (4.19)$$

Note that, by time-ordering before taking the ensemble average, one would obtain coupled equations for Feynman propagators instead of Wightman functions. Normal-ordering with respect to the perturbative vacuum is introduced here for later convenience. Furthermore, because of the non-linearity of the operator equations, the above procedure couples n -point to $(n+1)$ -point functions etc., which unavoidably generates an infinite hierarchy.

The simplest approximation to handle the BBGKY hierarchy, in which all n -point function equations decouple from each other, is to restrict the Lagrangian, eq.(4.8), to terms which are at most quadratic in the quantum fields [12]. Correspondingly linearizing the symmetry (4.9) and the equations of motion leads to a mean field approximation here, i.e. an equation for the two-point function including interactions with the classical field only.

In order to include quantum interactions as well, an improved, so-called binary collision approximation (random phase approximation) is to neglect $n \geq 3$ -particle correlations, i.e. to factorize all $n \geq 3$ -point functions completely. This approximation presently excludes correlations between gluons and ghosts; in particular, note that $\langle c \rangle = \langle \bar{c} \rangle = \langle ac \rangle = \langle a\bar{c} \rangle = 0$ for any physical ensemble containing only zero ghost number states. As will be shown in the following, one is left here with coupled non-linear equations for one- and two-point functions only.

Obviously such an approximation scheme can be extended step by step by including (equations for) higher order correlations. It can also be simplified further by considering the resulting equations perturbatively only up to a certain order in the coupling constant, which

presumably corresponds to a loop-wise expansion. Furthermore, one could even imagine to generalize eq.(4.19) into an equation for a suitable generating functional containing all powers of gluon and ghost field operators.

Even within the binary collision approximation, the *classical part of the gluon field*, A , still has to be determined. This is achieved by the selfconsistency condition,

$$\langle \dot{A} \rangle \equiv \langle A + a \rangle = A \quad \text{i.e.} \quad \langle a \rangle = 0, \quad (4.20)$$

which says that whatever classical part (non-vanishing expectation value) the full gluon field, $\dot{A}$, acquires, it should be taken into account by A such that the fluctuations, a , remain purely quantum!

Next, the *gluon Wigner function* (an "adjoint representation" 8×8 matrix with respect to colour $SU(3)$; cf. also eqs.(3.5,3.8)) is defined by

$$G_{\mu\nu}^{ab}(z, p) \equiv \int \frac{d^4 y}{(2\pi)^4} e^{-ip \cdot y} (\cdot \cdot a_{\nu}^b(x + \frac{1}{2}y) \tilde{U}^{ab}(x + \frac{1}{2}y, x) \tilde{U}^{aa'}(x, x - \frac{1}{2}y) a_{\mu}^{a'}(x - \frac{1}{2}y) \cdot \cdot), \quad (4.21)$$

and analogously the *ghost Wigner function*, $G \sim (\cdot \cdot \tilde{c} \tilde{U} \otimes \tilde{U} c \cdot \cdot)$. $\tilde{U}$ is defined as U , eq.(3.6), but involving here the classical field A in the adjoint representation instead of A in the fundamental representation there. Note that the introduction of links, such that $G(z, p) \rightarrow S(z)G(z, p)S^{-1}(z)$ under (4.4), again is essential to obtain classical invariance of the Wigner function equations (it would not be necessary for the classical A -field equation (4.24) below, since the current is a local covariant quantity). Previously we have shown that for any gauge theory the proper transport equations assume the form of generalized Vlasov-Boltzmann type equations if and only if classical invariance is maintained [4,27]. As indicated already after (3.4) above, this is to be expected, since the links are intimately connected to covariant derivatives [27] (cf. e.g. eq.(4.23)) which in turn are related to the kinetic particle momentum.

For ghosts, links involving only the A field are certainly sufficient. For gluons one may want to include the full field, $A + a$, into the link operators which, then, act as covariant parallel transporters not only with respect to classical gauge transformations (4.4) but also with respect to BRS transformations (4.9). However, the a field tied to links does not yet behave in an orderly way. To extend the classically covariant construction of the Wigner function, eq.(4.21), to a BRS covariant object, one has to start with locally covariant quantities, i.e. one is led back for example to eq.(3.8). Thus, one eventually obtains a gauge invariant and, therefore, *gauge fixing independent two-point function*. Nevertheless, it obeys a *gauge fixing dependent equation of motion*, since the operator equation (4.18) to be used for its derivation necessarily has this property. Only the *solution* of the equation for a gauge invariant n -point function should be independent of the gauge. With the precise form of the Wigner function (4.21) presently being fixed according to the requirements of kinetic theory (cf. eqs.(3.2-3.4) and a more detailed discussion in [3,27]), the question of gauge fixing dependence is left at this point and will be taken up again in Section 5.

Applying eq.(4.19) together with the binary collision approximation, one proceeds to derive the relevant one- and two-point function equations. How to convert the two-point function/density matrix equations into Wigner function equations was described in detail before [3,4,27] and the same steps can be followed here. The main task is to calculate first and second order derivatives of the path-ordered link factors from eq.(4.21). For this purpose one employs the following basic formula (as applicable for adjoint representation links $\tilde{U}$):

$$\delta\tilde{U}(b, a) = igA(b) \cdot \delta(1)\tilde{U}(b, a) - ig\tilde{U}(b, a)A(a) \cdot \delta(0) \\ - ig \int_0^1 ds \tilde{U}(b, z(s))F_{\mu\nu}(z(s))\tilde{U}(z(s), a) \frac{dz^\mu(s)}{ds} \delta^\nu(s) . \quad (4.22)$$

Eq.(4.22) describes the variation of a link $\tilde{U}$ due to an infinitesimal variation, $z(s) \rightarrow z(s) + \delta(s)$, $0 \leq s \leq 1$, of the path defining the link (cf. eq.(4.6) for the case of a straight-line path) with the classical field strength tensor, $F_{\mu\nu} \equiv F_{\mu\nu}[A]$, and A in the adjoint representation. It holds for arbitrary paths provided a cohomology condition is satisfied, $\tilde{U}(a, c)\tilde{U}(c, b)\tilde{U}(b, a) = 1$ with arbitrary ordered points a, b, c on the path. As before, by eq.(4.22) one calculates all necessary derivatives [3,27].

Furthermore, to streamline results from taking derivatives, the formula for the covariant shift operator in terms of links is useful [27],

$$e^{-v \cdot D(A(z))O(x)} = \tilde{U}(x, x - y)O(x - y)\tilde{U}(x - y, x) , \quad (4.23)$$

where the covariant derivative and links have to be understood either in fundamental or adjoint representation according to whether O (e.g. $F_{\mu\nu}$) is defined in either one or the other [3], either $O \equiv O_a t_a$ or $O \equiv O_a \tilde{T}_a$.

Applying eq.(4.19) with $l, m, n = 0$ together with eqs.(4.20,4.22) to eq.(4.18) yields the desired *classical A-field equation*,

$$(D^\mu[A]F_{\mu\nu}[A])^\alpha(x) = -gJ_\nu^\alpha(x) \\ \equiv -g \int d^4p \left\{ \text{Tr } T^a(ip_\nu + \frac{1}{2}D_\mu[A])G^\mu(x, p) + iD^{\mu\alpha b}[A] \text{Tr } T^b G_{\mu\nu}(x, p) \right. \\ \left. + i \text{Tr } T^a \{ (-ip_\mu + \frac{1}{2}D_\mu[A])G_\nu^\mu(x, p) - (-ip_\nu + \frac{1}{2}D_\nu[A])G_\mu^\nu(x, p) \} \right\} , \quad (4.24)$$

where T 's are generators and D 's covariant derivatives of the adjoint representation. (To reexpress the r.h.s. in terms of the Wigner functions one could repeat the calculation from Section 4.1 in ref.[27].) Remarkably, the covariantly conserved *gluon colour current*, J_ν^α , follows here from the self-consistent approach automatically. It contains convective as well as polarization contributions. However, in the mean field approximation the r.h.s. of eq.(4.24) would identically vanish, leading back to the unstable situation, eqs.(2.12), and in agreement with the effective action calculation (cf. Section 2.). Thus, only beyond that approximation,

does one observe the expected backreaction of quantum fluctuations on the classical field, i.e. using the binary collision approximation instead. Note that there is no explicit ξ -dependence in eq.(4.24), since by $\langle a \rangle = 0$ the gauge fixing term disappears, and classical invariance of all equations means that one is still free to choose a gauge for the A field.

Similarly, to derive equations for the gluon and ghost Wigner functions, $\tilde{G}_{\mu\nu}$ and G , one uses eq.(4.19) with $l = 1, m = n = 0$ and $l = n = 0, m = 1$, respectively, together with the appropriate operator equation (4.17) and (4.18), the binary collision approximation, and the selfconsistency condition, eq.(4.20). Then, one has to apply eqs.(4.22,4.23) and go through some tedious but straightforward calculation.

Thus, the *ghost Wigner function equation* effectively follows from the equation,

$$(\cdot; \tilde{U} \otimes \tilde{U} D^2[A]c :) = 0 , \quad (4.25)$$

which can be expressed as

$$\tilde{\Phi}_\mu^\mu[G] = 0 . \quad (4.26)$$

Here for any "adjoint representation" $(N^2 - 1) \times (N^2 - 1)$ matrix $M \equiv M(x, p)$,

$$\begin{aligned} \tilde{\Phi}_{\mu\nu}[M] &\equiv (-ip + \frac{\hbar}{2}D)_\mu (-ip + \frac{\hbar}{2}D)_\nu M \\ &\equiv -\frac{\hbar}{4}g(-ip + \frac{\hbar}{2}D)_\mu \partial_p^\sigma \int_0^1 ds \left\{ (1+s)e^{-s\Delta} F_{\sigma\nu} M + (1-s)M e^{s\Delta} F_{\sigma\nu} \right\} + \{\mu \leftrightarrow \nu\} \\ &\quad + \left(\frac{\hbar}{4}g \right)^2 \partial_p^\sigma \partial_p^\tau \int_0^1 ds \int_0^1 ds' \left\{ (1+s)(1-s')e^{-s\Delta} F_{\sigma\mu} M e^{s'\Delta} F_{\tau\nu} \right. \\ &\quad \left. + (1+s)(1+s')\Theta(s-s')e^{-s\Delta} F_{\sigma\mu} e^{-s'\Delta} F_{\tau\nu} M \right. \\ &\quad \left. + (1-s)(1-s')\Theta(s-s')M e^{s\Delta} F_{\sigma\mu} e^{s'\Delta} F_{\tau\nu} \right\} + \{\mu \leftrightarrow \nu\} \\ &\quad + i\frac{\hbar}{4}g \int_0^1 ds \left\{ (1+s)e^{-s\Delta} F_{\mu\nu} M + (1-s)M e^{s\Delta} F_{\nu\mu} \right\} \\ &\quad + \frac{\hbar}{2} \frac{\hbar}{4}g \partial_p^\sigma \int_0^1 ds \left\{ (1+s)^2 e^{-s\Delta} (D_\nu F_{\sigma\mu}) M + (1-s)^2 M e^{s\Delta} (D_\nu F_{\sigma\mu}) \right\} , \end{aligned} \quad (4.27)$$

where $\Delta \equiv \frac{\hbar}{2}i\partial_p \cdot D[A]$, with ∂_p always acting on M (i.e. a Wigner function) and the adjoint representation covariant derivative from Δ on $F \equiv F[A]$ next to its right only.

Despite its complicated appearance, one easily recognizes some distinctive features of eq.(4.26). It contains a proper transport and a generalized mass-shell constraint equation separately, which could be isolated essentially by adding and subtracting the equation and its adjoint. Considering the leading terms, $(-ip + \frac{\hbar}{2}D)^2 = (-p^2 + \frac{\hbar^2}{4}D^2) - i\hbar p \cdot D$, they clearly contain the mass-shell constraint (plus quantum correction) and the covariantly generalized

free-streaming term well known from classical transport equations. Furthermore, terms involving $(-ip + \frac{1}{2}D)^\mu F_{\mu\nu}\partial_\nu^\mu$ obviously generalize the classical Vlasov term, which describes the Lorentz force acting on the charged particles under consideration. All this as well as genuine quantum corrections have been discussed at length in previous abelian and non-abelian cases [27,30]; in particular, the shift operator, $\exp \dots \Delta$, which is present in most of the terms in eqs.(4.26,4.27), once again suggests a Wigner-Kirkwood type semiclassical expansion of all equations in powers of Δ . Note, however, the coupling to the gluon/ghost colour current arising from the non-linear QCD interactions: using eq.(4.24), the (adjoint representation) colour current, $J_\mu \equiv J_\mu^a T^a$, can be inserted instead of $D^\mu F_{\mu\nu}$ appearing in the last terms of eq.(4.27), when the trace, Φ_μ , is taken as in eq.(4.26). Similar current-current type interactions also occur in the gluon case.

In order to derive the gluon Wigner function equation, it is convenient to rewrite the relevant equation following from eq.(4.18) (as described before eq.(4.25)),

$$(\langle a_\nu \tilde{U} \otimes \tilde{U} \{ g_{\mu\sigma} D^2[A] + (\frac{1}{\xi} - 1) D_\mu[A] D_\sigma[A] - 2ig F_{\mu\sigma}[A] + O(a^2) \} a^\sigma \rangle = 0, \quad (4.28)$$

where all terms between a_ν and a^σ are to be understood as adjoint representation matrices. This equation shows the gauge fixing dependent term. Obviously the background Fermi-Feynman gauge, $\xi = 1$, will be most convenient to work with. Furthermore, using the relation $D_\lambda D_\mu a^\lambda = D_\mu D_\lambda a^\lambda - ig[F_{\lambda\mu}, a^\lambda]$, an additional contribution to the (vector) particle magnetic moment coupling term, $\sim F_{\lambda\mu} a^\lambda$, is obtained here which, thus, comes with an anomalous magnetic moment factor two well known for gluons [12,22,25]. Keeping only the $O(a^0)$ terms in $\{ \dots \}$ in eq.(4.28) would correspond to the mean field approximation. Note that no $O(a^1)$ terms survive due to the binary collision approximation together with the selfconsistency condition, i.e. $\langle aaa \rangle \sim \langle a \rangle \langle aa \rangle = 0$. Similarly, $\langle a\bar{c}c \rangle \sim \langle a \rangle \langle \bar{c}c \rangle = 0$. However, there is a genuine scattering term $S \sim \langle aaaa \rangle$ originating from the $O(a^3)$ term in eq.(4.18). It will be considered separately below.

After all, then, the *gluon Wigner function equation* is obtained:

$$g_{\mu\sigma} \Phi_\mu^\dagger [G^{\sigma\nu}] + (\frac{1}{\xi} - 1) \Phi_\mu [G^{\sigma\nu}] - 2i\hbar g_e^{-\Delta} F_{\mu\sigma} G^{\sigma\nu} + S_\mu^\nu [G^{\sigma\theta}] = 0, \quad (4.29)$$

with Φ from eq.(4.27). The scattering term S arises from $(\langle a_\nu \tilde{U} \otimes \tilde{U}(-g) f^{bc\sigma} g f^{bde} a_\mu^\sigma \rangle)$. Factorization according to the binary collision approximation plus selfconsistency condition yields:

$$\begin{aligned} (\langle a_\mu^\dagger a^\sigma a_\sigma^\dagger a_\mu^\sigma \rangle) &= (\langle a_\mu^\dagger a_\sigma^\dagger \rangle) (\langle a^\sigma a_\mu^\sigma \rangle) \\ &+ (\langle a_\mu^\dagger a_\mu^\dagger \rangle) (\langle a^\sigma a_\sigma^\dagger \rangle) + (\langle a_\mu^\dagger a^\sigma \rangle) (\langle a_\sigma^\dagger a_\mu^\sigma \rangle). \end{aligned} \quad (4.30)$$

Using this result, one obtains S in terms of gluon Wigner functions:

$$\begin{aligned} S_{\mu\nu} [G^{\sigma\theta}] &= (\hbar g_e^{-2}) e^{-\Delta} \int d^4k \left\{ \text{Tr} \left(\tilde{T}^\dagger \tilde{T} G_\mu^\sigma(z, k) \right) G_{\sigma\nu}(x, p) \right. \\ &\quad \left. - \text{Tr} \left(\tilde{T}^\dagger G_\sigma^\sigma(z, k) \right) G_{\mu\nu}(x, p) - T^b \text{Tr} \left(T^b G_\mu^\sigma(z, k) \right) G_{\sigma\nu}(x, p) \right\}, \end{aligned} \quad (4.31)$$

where the covariant derivative from $\Delta \equiv \frac{1}{2}i\partial_p \cdot D[A]$ acts only on the first Wigner function in each of the three terms, while ∂_p acts on the second; having $(T^a)_{ab}(T^a)_{bc} = -f_{abc}f_{ade}$, here the definition $(\tilde{T}^\dagger \tilde{T} M)^{ad} \equiv \text{Tr} T^a T^d M$ is used, which is an "adjoint representation" $(N^2 - 1) \times (N^2 - 1)$ matrix. As will become more obvious later, the terms contributing to S are of self-energy type with the last one resembling a tadpole-like contribution.

This completes the derivation of self-consistent covariant background gauge gluon/ghost transport equations in the (non-perturbative) binary collision approximation.

In the Appendix the zeroth order Wigner functions are calculated. A non-perturbative solution of eq.(4.29) for the high- T ($A = 0$)-"vacuum" is also derived there. It shows a *gluon mass generation* ($\Omega^2 = \frac{1}{4}Ng^2T^2 + \dots$) different from the "plasmon" to be discussed in Section 5. Such a mass is necessary in order to render the (otherwise infrared divergent) gluon transport equation finite. Here it presents a first indication that the high- T plasma sustains self-consistent background fields.

It should be remarked, furthermore, that the classical A -field equation (4.24) and the gluon transport equation (4.29) need *renormalization*, since the current in the former and the scattering terms in the latter involve (ultraviolet divergent) four-momentum integrals related to pairs of field operators at coinciding points. That is why normal-ordering is used throughout this paper: The idea is to apply the present formalism first to physical situations close enough to thermal equilibrium. Presumably the ultraviolet divergences are the same as for the $T = 0$ vacuum and can, in principle, be treated as such. At $T \gg 0$ physical quantities are dominated by their "matter parts" [2,17]. Then, neglecting the vacuum contributions is equivalent to normal-ordering and works also for slightly more general than thermal ensembles (cf. [28] and references therein). However, this is not sufficient to cure the QCD infrared divergences observed here as well (see the Appendix).

It is worthwhile to question the *validity of the binary collision approximation* at this point. This approximation is certainly reasonable, if the system is dilute in some sense, which does not necessarily require it to be weakly interacting. In QCD one expects the effective interaction to become strong at a scale of 1fm. At high temperatures, with particle densities growing $\sim T^3$, it could be that one never has a dilute gas situation. On the other hand, if one tries to dilute the system by decreasing T , then the confining forces could prevent just that by bringing virtual particles on-shell. This is not an easy issue to decide. In principle, however, the *relative importance of higher order correlations* can be investigated by non-perturbative lattice calculations.

Finally, note that one- and two-point function equations (4.24), (4.26), and (4.29) form a highly non-linear closed set of non-perturbative equations, the solution of which presents a formidable task. Since eventually one starts looking for solutions using some further approximations, it may be helpful to point out two criteria for selecting the interesting ones: i) a solution with non-vanishing A -field/condensate should have *lower (free) energy* than any

perturbative ($A = 0$) solution; ii) any solution should be *as stable as possible* against arbitrary small fluctuations imposed on it. - It is the latter criterion which, first of all, asks for the stability analysis performed in the following section.

5. Stability Analysis and Collective Modes

As indicated above, a stability analysis, of whatever solution is obtained for the coupled classical field and two-point function equations (4.24), (4.26), and (4.29), is desirable by itself, for example, in order to select one or the other possible Ansatz for the classical A -field. Even though the equations, eq.(4.29) in particular, and their solutions presumably as well, are gauge fixing dependent, an instability (typically growing exponentially in time and with a strength possibly depending on the quantum gauge fixing) would indicate that there should be a better trial state corresponding to a more stable solution.

Such an analysis has recently been done for the particular case of a small perturbation of the perturbative ($A = 0$) "vacuum" at $T \gg 0$ [31]. These authors studied a set of coupled one- and two-point function equations derived in ordinary covariant Fermi-Feynman gauge at $O(g^2)$. They fully reproduced the results of the high-temperature one-loop calculation of the gluon polarization tensor in that gauge [2]. (This calculation is particularly useful, since it shows that the binary collision truncation of the BBGKY hierarchy combined with a lowest order perturbative solution of the transport equations yields the same results as the usual one-loop high- T ~~approximation~~. Anticipating the results of a similar calculation in the present case, it will reproduce the one-loop covariant background gauge polarization tensor obtained before [5,6]. These results, i.e. dispersion relations from the ordinary gluon propagator in covariant (background) gauges indicate the instability of the perturbative vacuum at high T according to the discussion in Section 2.

Before addressing the question of physical collective modes, the stability analysis will be performed here in general. Suppose one has solved eq.(4.24) together with the corresponding gluon and ghost Wigner function equations and assume a small perturbation, δA , on top of that "equilibrium" solution. Then, δA is governed by an equation,

$$\left\{ \left(D^2[A] + \frac{1}{2} g^2 N \int d^4 p \{ W_\mu^\mu + iW \}^{AS} \right) g_{\nu\mu} - D_\nu[A] D_\mu[A] - 2ig F_{\nu\mu}[A] - \frac{3}{2} g^2 N \int d^4 p W_{\nu\mu}^{AS} \right\} \delta A^{\mu\nu} = -g \delta J_\nu^\mu, \quad (5.1)$$

where δJ denotes the change of the current due to induced changes δG , $\delta G_{\sigma\tau}$ of the equilibrium Wigner functions only, i.e. $\delta J_\sigma^\mu \equiv J_\sigma^\mu[\delta G, \delta G_{\sigma\tau}]$ according to eq.(4.24). To arrive at eq.(5.1) one inserts $A + \delta A$, $G + \delta G$, and $G_{\sigma\tau} + \delta G_{\sigma\tau}$ into eq.(4.24), uses the fact that A solves the equation with $J_\sigma^\mu[G, G_{\sigma\tau}]$, and keeps only terms of first order in $\delta \dots$ (*linear response*). The

colour-antisymmetric parts of the gluon and ghost Wigner functions are defined by

$$G_{ab}^{AS} \equiv \frac{1}{N} (T^c)_{ab} \text{Tr } T^c G, \quad (5.2)$$

and correspondingly for the gluon Wigner function. They vanish, if one assumes thermodynamic equilibrium with respect to the perturbative vacuum, in which case the Wigner functions are symmetric, $\sim \delta_{ab}$.

Obviously, eq.(5.1) has some interesting features: The colour-antisymmetric parts of the gluon and ghost Wigner functions, i.e. the quantum fluctuations, provide a *mass* term for the classical collective fluctuation δA . Similarly, there is a *self-energy* term. Note that these two contributions from the quantum fluctuations only occur due to the self-consistent approach of Section 4., which gave rise to the colour current in eq.(4.24) in the first place. Precisely these terms generalize the equation (including the anomalous magnetic moment interaction again) which especially describes the Nielsen-Olesen unstable mode [12,22,25]. A somewhat similar mechanism of gluon mass generation was recently conjectured in ref.[32] involving vortex-like background fields for QCD₃. Note that in the present approach mass generation for (classical) collective modes, however, is rather different from and complementary to that for proper gluons (as discussed in the Appendix).

Finally, eq.(5.1) is classically gauge invariant, since for consistency the classical fluctuation should transform as $\delta A(x) \rightarrow S(x) \delta A(x) S^{-1}(x)$, cf. (4.4). The interesting question arises, whether eq.(5.1) may have even a quantum gauge fixing independent meaning, thus describing a physical collective mode. Could it be (as was stated in the particular case of ref.[32]) that the gauge fixing dependent terms, which are implicitly present here, contribute only longitudinal terms?

To continue the stability analysis, one has to consider the equations for the induced fluctuations δG , $\delta G_{\sigma\tau}$ as well. The procedure is the same as for δA and, in linear response approximation again, eq.(4.26) yields:

$$\begin{aligned} 0 &= \left(\Phi_\mu^\mu + \delta \Phi_\mu^\mu \right) [G + \delta G] \approx \Phi_\mu^\mu[G] + \Phi_\mu^\mu[\delta G] + \delta \Phi_\mu^\mu[G] \\ &= \Phi_\mu^\mu[\delta G] + \int d^4 z \frac{\delta \Phi_\mu^\mu[G]}{\delta A_\lambda^\alpha(z)} \delta A_\lambda^\alpha(z). \end{aligned} \quad (5.3)$$

Thus, δG obeys almost the same equation as G itself. However, it gets an additional source term which is due to the variation of Φ_μ^μ caused by the variation of A . The additional term is rather complicated (and not particularly illuminating) with many contributions arising from variations of the link factors, which are hidden in $\exp \dots \Delta$'s by eq.(4.23). They can be calculated easily using the formula,

$$\begin{aligned} \delta \tilde{U}(b, a) &= ig \int_0^1 ds \frac{dz^\mu}{ds} \tilde{U}(b, z) \delta A_\mu(z) \tilde{U}(z, a) \\ &= ig \tilde{U}(b, x) \int_0^1 ds \frac{dz^\mu}{ds} [e^{-i' D[A]} \delta A_\mu(x)]_{z=x-z} \tilde{U}(x, a), \end{aligned} \quad (5.4)$$

where $z \equiv z(s)$ and $\mathbf{z}$ are points on the path $a \rightarrow b$, and which follows from the infinite product representation of the links, cf. [27].

For practical calculations, one needs some further approximation to be able to work with eq.(5.3) and, similarly, eq.(5.6) below. Note that for the study of fluctuations around the perturbative ($A = 0$) vacuum the variations of the links fortunately do not contribute at all, since they always come with a factor $F_{a\beta}[A = 0] = 0$ from eq.(4.27). In general, one may try to simplify the source term in eq.(5.3) by including consistently only terms up to a certain order in g , which incidentally here is related to a long-wavelength expansion, cf. eq.(4.27).

However, a word of warning also seems to be in order here: looking at the troubling link factors, cf. eq.(3.6), one might be tempted to transform them away by choosing a *Schwinger-Fock type gauge* for the classical field, $dz \cdot A(z) = 0$, along the path under consideration. The way the links actually appear is more complicated, as can best be observed in eq.(4.27). They are coded into $\exp(\mathcal{S}\partial_p \cdot D[A])M(x, p)$. Expanding the exponential, one would like to impose the gauge condition

$$A(x) \cdot \partial_p M(x, p) = 0, \quad (5.5)$$

which superficially might look like an axial gauge, $n(x) \cdot A(x) = 0$. However, this is wrong, since $\partial_p M(x, p)$ actually is not one gauge fixing vector at one space-time point x , but rather presents an infinite set of vectors, $n_p(x)$, parametrized by the momentum variable p . Thus, eq.(5.5) cannot, in general, be fulfilled as a gauge condition on A and consequently the links cannot be gauged away.

Turning to the equation for the fluctuation of the gluon Wigner function, $\delta G_{\sigma\tau}$, one obtains from eqs.(4.29,4.31) and in analogy to the cases of eqs.(5.1,5.3):

$$\begin{aligned} & \int d^4 z \frac{\delta}{\delta A_\lambda^\alpha(z)} \left\{ g_{\mu\sigma} \Phi_\tau^\dagger[G^{\sigma\nu}] + \left(\frac{1}{2} - 1\right) \Phi_{\mu\sigma}^\dagger[G^{\sigma\nu}] - 2i\hbar \frac{g}{e} e^{-\Delta} F_{\mu\sigma} G^{\sigma\nu} + S_\mu^\nu[G^{\alpha\beta}] \right\} \delta A_\lambda^\alpha(z) \\ & + g_{\mu\sigma} \Phi_\tau^\dagger[\delta G^{\sigma\nu}] + \left(\frac{1}{2} - 1\right) \Phi_{\mu\sigma}^\dagger[\delta G^{\sigma\nu}] - 2i\hbar \frac{g}{e} e^{-\Delta} F_{\mu\sigma} \delta G^{\sigma\nu} \\ & + \int d^4 z d^4 k \frac{\delta S_\mu^\nu[G^{\alpha'\beta'}]}{\delta G_{\alpha\beta}(z, k)} \delta G_{\alpha\beta}(z, k) = 0. \end{aligned} \quad (5.6)$$

All the remarks made after eq.(5.3) apply here as well, in particular concerning the large number of terms stemming from the first line in eq.(5.6).

Finally, to complete the general outline of the stability analysis, the following steps still have to be performed in order to arrive at the corresponding dispersion relation.

Eqs.(5.1,5.3,5.6) couple the fluctuations δA , δG , $\delta G_{\sigma\tau}$ to each other linearly and to the "equilibrium" solutions A , G , $G_{\sigma\tau}$. To obtain a closed equation for δA , one eliminates δG in terms of δA using eq.(5.3) and $\delta G_{\sigma\tau}$ in terms of δA using eq.(5.6). Note that the scattering terms in eq.(5.6), giving a non-local contribution in $\delta G_{\sigma\tau}$, generally need some further consideration; however, one may convince oneself that they do not contribute to the

final dispersion relation at $O(g^2)$ in a perturbative evaluation. Retarded boundary conditions have to be applied here always, when inverting the differential operators, since one wants to calculate the effect caused by the fluctuation δA on the rest of the system. In practice this approach is straightforward only in the case of vanishing background field, i.e. close to the perturbative vacuum, and the type of calculation, in principle, is well known from classical plasma physics, cf. also the second of refs.[4] and [31]. The resulting homogeneous second-order partial differential equation for the classical collective mode δA depends functionally on A , G , $G_{\sigma\tau}$ and, in the general case described above, also includes non-local terms. It determines its dispersion relation, i.e. the effective mass and (anti-)damping constant in particular. Away from the perturbative vacuum the problems to be dealt with are analogous to those of calculating ordinary propagators in external fields [24].

As indicated before, there is no general argument that the outcome of the stability analysis should be quantum gauge fixing independent.³ It tells something about the (in-) stability of background field configurations which, due to the approximations made, may not be fully self-consistent. If gauge fixing independence could be achieved, only then, the classical (damped or unstable) collective mode, δA_μ^α , would be a *coloured mode* existing in the deconfined plasma phase. It would come closest to the "plasmon" known from QED plasmas and would be well defined by a gauge invariant dispersion relation, $\omega = \omega(\vec{k})$. In distinction to that, I shortly want to point out an alternative, namely to look for *colourless collective modes* even in the high- T plasma. Both cases will be studied in more detail elsewhere.

Recently some arguments have been put forth that colourless "hadronic" modes persist in the deconfined phase at $T \geq T_c$ [33] (and references therein). Surprisingly little work has been done to study these by any other means than indirect evidence from lattice calculations testing the screening of test charges at high temperature. In the following it is outlined, how one can derive dispersion relations for *gauge invariant correlation functions* using the formalism developed above. Their poles are related to such colourless collective modes of the quark-gluon plasma [19]. They can be excited/observed by suitable "external" hadronic or other probes which couple to the plasma.

For example, consider the simplest scalar gauge invariant two-point function involving only local combinations of gauge field operators,

$$\mathcal{F}(x, x') \equiv \langle : F_{\mu\nu}^\alpha(x) F^{\mu\nu\beta}(x') F^{\beta\sigma\tau}(x') : \rangle, \quad (5.7)$$

which correlates the local action density of the field at two different points. Similarly, by not contracting all the Lorentz indices and forming suitable combinations of such terms, one constructs a correlation function for the energy-momentum tensor (cf. the second of refs. [2]), which is related to macroscopic transport properties of the system.

Since $\mathcal{F}$ is gauge invariant, it can be calculated using any gauge. $\mathcal{F}$ includes terms with up to eight field operators. One has to simplify it, in order to derive a dispersion relation. For

³See, however, the note added at the end of Section 6.

consistency the binary collision approximation (Section 4.2) is applied again, which efficiently decomposes $\mathcal{F}$ into a sum of products of $(: a_a^\dagger a_b^\dagger :)$ -type terms, cf. eq.(4.30). - Note that the binary collision approximation seems to break the gauge invariance of the original $\mathcal{F}$, at least when evaluated perturbatively. If this is the case, part of the attraction of considering gauge invariant correlation functions is lost. Presently, however, no other non-perturbative scheme than the binary collision approximation has been worked out. The possible gauge dependence here, which may result from a bad approximation, has to be clearly distinguished from the apparent one resulting from the operator equations of motion, which was discussed in Section 4.2. The latter cancels for quantities like $\mathcal{F}$ and corresponding dispersion relations, if calculated properly. - One may hope that gauge dependence shows up only through higher order corrections to e.g. the mass of a colourless collective mode, such as for the "plasmon" mass $\sim gT$ calculated to be gauge invariant from the ordinary propagator [2].

One calculates $\mathcal{F}$, having it decomposed (symbolically),

$$\mathcal{F} = (: \partial a \partial a' :)^2 + g^2 (: \partial a \partial a' :)(: a a' :)^2 + (: a a' :)^3 + g^4 (: a a' :)^4, \quad (5.8)$$

by using a non-perturbative solution for $G_{\sigma\tau}$ from eq.(4.29), since by eq.(4.21),

$$(: a_b^\dagger(x) a_a^\dagger(x') :) = \int d^4p e^{ip(x-x')} \tilde{U}^{aa'}(x', x_+) G_{\mu\nu}^{ab'}(x_+, p) \tilde{U}^{bb'}(x_+, x), \quad (5.9)$$

where $x_+ \equiv \frac{1}{2}(x + x')$. In the Appendix a simple such solution is constructed as a preliminary example. Then, two possibilities arise: i) either $\mathcal{F}$, i.e. its Fourier transform with respect to $x - x'$, develops poles, in which case one obtains the colourless collective modes directly; or, ii) it does not, in particular, if the solution for $G_{\sigma\tau}$ is not sufficiently close to the "true" one. In case ii) one has to go one step further, performing a stability analysis for $\mathcal{F}$ itself. - Following the procedure described above, one considers an arbitrary small variation, $\delta\mathcal{F}$, of the "equilibrium" solution $\mathcal{F}$ (keeping $A = 0$ here for simplicity),

$$\delta\mathcal{F}(x, x') = \int d^4z d^4k \frac{\delta\mathcal{F}(x, x')}{\delta G_{\sigma\tau}(z, k)} \delta G_{\sigma\tau}(z, k), \quad (5.10)$$

using eqs.(5.8,5.9). Its equation, then, follows from eq.(5.6), after having eliminated δA_μ^a to obtain a closed equation for $\delta G_{\sigma\tau}$ (symbolically), $L_{\sigma\tau}[\delta G_{\mu\nu}] = 0$,

$$0 = \int d^4z d^4k \frac{\delta\mathcal{F}(x, x')}{\delta G_{\sigma\tau}(z, k)} L_{\sigma\tau}[\delta G_{\mu\nu}](z, k), \quad (5.11)$$

The aim is to convert eq.(5.11) into an equation for $\delta\mathcal{F}$, which yields the dispersion relation for this particular colourless perturbation of the plasma. Again, close to the high- T ($A = 0$)-"vacuum", the calculation can be performed, whereas for general self-consistent background fields, $A \neq 0$, the complexity of the coupled equations makes it very difficult.

Thus, this seems to be roughly as far as one can possibly get, using the stability analysis combined with the binary collision approximation, in order to study the collective behaviour of the (high- T) QCD plasma.

6. Concluding Remarks

To summarize, after discussing some evidence that the high-temperature quark-gluon plasma behaves non-perturbatively, particularly considering the instability of the collective plasmon mode, I presented a tentative interpretation thereof. It is related to the one of the instability of the Savvidy vacuum [12,22,25] and implies that the notion of an *external* colour charge or current cannot be defined for a non-abelian gauge theory. Then, the mean field quark transport equations were shortly reviewed to set the stage for the new development of gluon transport theory. It is based on a canonical covariant background gauge quantization of the gluon field augmented by a self-consistency condition determining the classical/condensate part of the field. The resulting classically gauge invariant transport equations constitute a promising tool to investigate the problems of instability and gauge fixing dependence discussed in Section 1.

Note that classical gauge invariance is indispensable [3,4,27], if one is heading for transport equations which generalize the Vlasov-Boltzmann equation to QCD and which reduce to such type of equations in the classical limit, where perhaps a numerical implementation would have to start. On the other hand, if one tries to solve a full set of coupled quantum equations numerically, which has recently been done for scalar QED [28] under simplifying assumptions about the geometry of the problem, then a completely gauge fixed version of the equations (getting rid of all link factors and the like, see Section 4.2) seems advantageous.

The essential point of the self-consistent background gauge quantization proposed in Section 4.1 is that it allows to separate a classical/condensate part of the gauge field from the quantum fluctuations in a unique way. It preserves a classical local $SU(N)$ gauge invariance of the theory, while the quantum theory is well-defined, i.e. its gauge is fixed as in the usual covariant background gauge [11]-[14]. In distinction to earlier studies, however, where the classical field is externally fixed, the classical field here is determined self-consistently as being generated by the colour-current of quark, gluon, and ghost fields in Section 4.2. The structure of that current strictly follows from the present derivation.

As speculated in the Appendix, the classical invariance might be spontaneously broken. This could help to explain the gluon mass generation found there. It is necessary to cure an infrared divergence in the gluon transport equation at finite temperature (without background fields). However, the internal consistency of such a possibility has to be further investigated.

In order to convert the operator equations following from the canonical quantization procedure into coupled equations for Wightman/Wigner functions, an approximation has to be introduced. Its purpose is to terminate the otherwise infinite (BBGKY) hierarchy of equations. The simplest non-trivial approximation is employed here, i.e. the binary collision (random phase) approximation, which allows to factorize higher order correlation functions. This leads to the set of non-perturbative equations (4.24), (4.26), (4.29), which describe,

in principle, non-equilibrium quark-gluon plasma systems (quarks are easily included as explained in the text) as well as the high- T equilibrium case with or without background fields. The latter serves as a primary testing ground of the formalism, since one can compare with known results from high- T field theory calculations [2].

In particular, the simple non-perturbative solution of the gluon transport equation obtained in the Appendix, yielding a gluon mass, $\Omega^2 = \frac{3}{4}m_d^2 = \frac{1}{4}Ng^2T^2$, deserves further work. One can immediately check, within the present formalism, whether this mass comes out gauge invariantly like m_d . Then, making the general stability analysis of Section 5. practical (similar as in ref.[31]), one can derive the dispersion relation for a fluctuation of the self-consistent background field. This (if gauge invariant) would yield a (physical) coloured "plasmon" mode. It will be interesting to see the effect of the gluon mass on the result of the, by now, straightforward calculation. How does it affect the plasma frequency and damping constant?

Furthermore, in Section 5. the procedure for studying the properties of colourless collective modes is outlined. It should be remarked that the present formalism is conceptually very simple and can be summarized by

$$\text{field operator eqs.} \rightarrow \tilde{L}(C) = 0 \rightarrow L(\delta C) = 0 \rightarrow \text{dispersion relation},$$

where (C) is the correlation function considered, (δC) its fluctuation, and $\tilde{L}, L$ are (integro-) differential operators. However, it can be algebraically rather complex, depending on the particular correlation function and the physical system. Also, the question has to be kept in mind, whether the binary collision approximation is good enough not to destroy the gauge properties of the operator equations and correlation functions. In any case, may be a better, yet still practical approximation scheme can be devised.

Finally, the outstanding problem of renormalization of the transport/Wightman function equations has to be solved. Presently the problem is circumvented by the normal-ordering prescription, which can be applied if the system, as described by its (initial) density matrix, is sufficiently close to a "simple" high- T equilibrium situation.

And, after all, the physically most interesting question as to the role and actual features of the self-consistent classical background fields should receive full attention. Any possible outcome would presumably have consequences for the understanding of the deconfined QCD plasma. For a high- T equilibrium system non-trivial background fields would imply a change of the vacuum as compared to ordinary perturbation theory. They would presumably stabilize the physical collective modes at the same time in the sense of quasi-particles having a finite width. On the other hand, for a small transient quark-gluon plasma, as it is perhaps formed in relativistic nuclear collisions, time-dependent classical fields (which are, anyway, assumed in flux-tube pictures of collision dynamics and particle production [1]) could strongly influence its observable features. However, the relevant time-scales for this to happen are not known.

At last, to advertise the outlined transport theory as a model to describe strongly interacting matter, it is obvious that in order to understand genuine non-equilibrium behavior, such as during the initial stages of high-energy collisions, one needs such an approach. In particular, it contains the essential effect of cascading/parton showering: a) a dynamical background field spontaneously produces quark/gluon pairs giving rise to a spontaneous colour current which backreacts on the field; b) the background field accelerates on-shell charges giving rise to bremsstrahlung which leads to induced pair production, giving rise to ... cf. a), etc. Apart from confinement, which, loosely speaking, needs a separate scale besides, say, the temperature and which would come in through renormalization, this is basically all the physics one tries to build phenomenologically into so-called "event generators". It is obtainable here from QCD. However, this is achieved at the expense that the resulting real-time quantum transport equations are anything but easy to solve for laboratory initial conditions, which are even difficult to formulate.

Note added: While finishing this work, I received the preprint of ref.[35]. There a non-perturbative proof is given that the poles of the ordinary gluon propagator are gauge invariant at zero as well as at finite temperature for a large class of gauges. It includes (fixed) background gauges, but the self-consistent gauge proposed in Section 4.1 belongs to the class of mean field dependent gauges which are explicitly excluded there. Since selfconsistency presumably can be interpreted as a reorganization of perturbation theory, I expect that the proof can be extended to cover this case as well. Then, the stability analysis of Section 5, once made practical in a consistent approximation, yields a physical coloured plasmon besides the colourless excitations suggested there.

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Appendix

The purpose of the Appendix is to calculate the zeroth order ghost and gluon Wigner functions for the *non-interacting thermal ensemble* with $A = 0$. Making use of the latter, also a particular non-perturbative solution to the gluon transport equation, eq.(4.29), will be obtained, pointing towards a mechanism of gluon mass generation.

To begin with, consider the ghost two-point (Wightman) function, which for the non-interacting thermal ensemble is calculated according to

$$\langle c^a c^b \rangle_0 = \text{Tr} (P e^{-\beta(H_n - F)} c^a c^b), \quad (1)$$

where the trace runs over the whole Hilbert space, the projection operator P is inserted to select the physical subspace [12,29,34], and F denotes the corresponding free energy, $F \equiv$

$\text{Tr} \text{Pexp}(-\beta H_0)$, of the noninteracting system. In ref.[34] the projection operator was derived for the covariant gauge in an explicit form appropriate for perturbation theory. It can be given as an exponential of the ghost charge operator. Its effect on ghost fields at finite temperature is to change their boundary conditions from anti-periodic in imaginary time (according to the fermionic quantization, cf. eqs.(4.11,4.12)) to periodic [34].

Thus, for the purpose of the present calculation, ghosts behave like ordinary scalar fields (only in ghost loops an additional fermionic minus sign persists). Using the normal-mode expansion given in ref.[34] (or any other one corresponding to a free massless scalar field), one proceeds directly from the definition of the ghost Wigner function (defined after eq.(4.21):

$$\begin{aligned} G^{ab}(x, p)_0 &\equiv \int \frac{d^4 y}{(2\pi)^4} e^{-ip \cdot y} \langle \bar{c}^b(x + \frac{1}{2}y) c^a(x - \frac{1}{2}y) \rangle_0 \\ &= (2\pi)^{-3} \delta^{ab} \delta(p^2) \left\{ \Theta(p^0) n_B(p^0) + \Theta(-p^0) (n_B(-p^0) + 1) \right\} \\ &= (2\pi)^{-3} \delta^{ab} \delta(p^2) (n_B(|p^0|)) , \end{aligned} \quad (2)$$

where n_B denotes the usual Bose distribution and the last equality holds for the normal-ordered ghost Wigner function only.

The zeroth order gluon Wigner function, cf. eq.(4.21), can be calculated similarly. For the considerations below, however, it will be useful to give the gluons a mass, μ , by hand. Thus, the gluon part of the gauge-fixed (free) Lagrangian, eq.(4.8), is modified by adding a term $+\frac{1}{2}\mu^2 a^a \cdot a^a$, which makes it similar to Stueckelberg's Lagrangian. Using a corresponding normal-mode expansion [24], one obtains:

$$\begin{aligned} G_{\mu\nu}^{ab}(x, p)_0 &\equiv \int \frac{d^4 y}{(2\pi)^4} e^{-ip \cdot y} \langle a_\nu^b(x + \frac{1}{2}y) a_\mu^a(x - \frac{1}{2}y) \rangle_0 \\ &= -(2\pi)^{-3} \delta^{ab} \left(\delta(p^2 - m^2) \frac{p_\mu p_\nu}{\mu^2} + \delta(p^2 - \mu^2) (g_{\mu\nu} - \frac{p_\mu p_\nu}{\mu^2}) \right) (n_B(|p^0|) + \Theta(-p^0)) \\ &= -(2\pi)^{-3} \delta^{ab} \delta(p^2) \left(g_{\mu\nu} + (\xi - 1) \frac{p_\mu p_\nu}{p^2} \right) (n_B(|p^0|) + \Theta(-p^0)) , \end{aligned} \quad (3)$$

where $m^2 \equiv \xi \mu^2$, $\xi \geq 0$, and the last equality holds in the limit $\mu \rightarrow 0$. Note that the scalar gluon contribution in the second of eqs.(3) comes with an unusual minus sign, which is due to the indefinite metric Fock space [12,24]. For the normal-ordered gluon Wigner function to be used below the vacuum contributions $\sim \Theta(-p^0)$ in eqs.(3) have to be omitted.

Turning to the gluon transport equation, only the high- T situation with $A = 0$ will be considered here using the Fermi-Feynman gauge, $\xi = 1$. Then, eq.(4.29) becomes:

$$(-ip + \frac{1}{2}\partial_x)^2 G_{\mu\nu}(x, p) + S_{\mu\nu}[G^{ab}]_{A=0} = 0 . \quad (4)$$

In thermodynamic equilibrium and without background fields the Wigner functions are space-time independent. Thus, using eq.(4.31) and the fact that presently the Wigner functions are diagonal in colour, i.e. there are no colour currents according to eq.(4.24),

$$p^2 G_{\mu\nu}(p) = g^2 N \int d^4 k \left(G_\mu^\sigma(k) G_{\sigma\nu}(p) - G_\sigma^\sigma(k) G_{\mu\nu}(p) \right) , \quad (5)$$

where it should be noted that the last term from eq.(4.31) dropped out in a tadpole-like manner. Next, one may assume the Ansatz $G_{\mu\nu}(p) = -g_{\mu\nu} \mathcal{G}(p)$, which is the case for the zeroth order Wigner function in the ($\xi = 1$)-gauge, cf. eq.(3). It yields

$$(p^2 - \Omega^2[\mathcal{G}]) \mathcal{G}(p) = 0 , \quad (6)$$

where a mass appears which is generated by the gluon self-interaction,

$$\Omega^2[\mathcal{G}] \equiv 3g^2 N \int d^4 k \mathcal{G}(k) . \quad (7)$$

Tracing it back, one observes that Ω^2 originates from the four-gluon vertex; there is no three-gluon vertex contribution here due to the binary collision approximation. Furthermore, note that generally Ω^2 can be expected to be ultraviolet divergent, as mentioned at the end of Section 4.2. This is neglected in the following by applying the normal-ordering prescription and keeping in mind that for $T \gg 0$ the "matter parts" dominate.

One may try to solve eqs.(6,7) perturbatively by expanding,

$$\mathcal{G} = \mathcal{G}_0 + g^2 \mathcal{G}_1 + g^4 \mathcal{G}_2 + \dots ,$$

taking the zeroth order Wigner function for the massless case from eq.(3), inserting into the above equations, and, as usual, compare the coefficients of equal powers of g^2 . However, it does not work. Already $\mathcal{G}_1 \sim \delta(p^2)/p^2$, which exhibits an *infrared divergence* in this equation. - Eq.(6), however, suggests how to cure the problem: it basically has the form of a mass-shell constraint. Thus, assuming that $\mathcal{G}(p)$ corresponds to the zeroth order *massive gluon* Wigner function from eq.(3) with $\xi = 1$ and $\mu^2 = \Omega^2$,

$$\mathcal{G}(p) = +(2\pi)^{-3} \delta(p^2 - \Omega^2) n_B(|p^0|) , \quad (8)$$

one immediately solves eq.(6). Eq.(7), in turn, determines Ω^2 self-consistently, if one inserts eq.(8) on the right hand side.

The relevant integral can be calculated ($n_B(\omega) = [\exp(\omega/T) - 1]^{-1}$),

$$\begin{aligned} I &\equiv 4\pi \int_\Omega d\omega (\omega^2 - \Omega^2)^{\frac{1}{2}} n_B(\omega) \\ &= (2\pi)^3 T^2 \left\{ \frac{1}{12} - \frac{3}{4\pi^2} \bar{\Omega} + \frac{1}{8\pi^2} \bar{\Omega}^2 [1 - \gamma - \ln \frac{\bar{\Omega}}{2\pi}] + O(\bar{\Omega}^3) \right\} , \\ &= (2\pi)^{3/2} T^2 \bar{\Omega}^{1/2} e^{-\bar{\Omega}} \left\{ 1 + O(\bar{\Omega}^{-1}) \right\} , \end{aligned} \quad (9)$$

$$(10)$$

with $\tilde{\Omega} \equiv \Omega/T$, respectively for $\tilde{\Omega} \ll 1$, $\tilde{\Omega} \gg 1$. Some helpful formulae from the second of refs.[18] were employed in the derivation of eq.(9), while eq.(10) is easily derived from the exact representation,

$$I = 4\pi T^2 \tilde{\Omega}^2 \sum_{n=1}^{\infty} (n\tilde{\Omega})^{-1} K_1(n\tilde{\Omega}) ,$$

in terms of modified Bessel functions.

Using these results, eqs.(9,10), one finds Ω^2 from eq.(7) which, then, shows the non-perturbative character of this mass. For $\Omega \ll T$,

$$\left(\frac{\Omega}{T}\right)^2 = \frac{1}{4}g^2 N - \frac{9}{8\pi^2}(g^2 N)^{3/2} + O((g^2 N)^2 \ln g^2 N) , \quad (11)$$

and for $\Omega \gg T$ (i.e. roughly for $\alpha_s = g^2/4\pi \gg 1/N$),

$$\frac{\Omega}{T} = \ln g^2 N + O(\ln \ln g^2 N) . \quad (12)$$

Note that the leading term on the right hand side of eq.(11) is just the one which follows by inserting eq.(8) with $\Omega^2 = 0$ into eq.(7). Furthermore, the value of the dynamical mass Ω^2 found here is close (but not identical) to the electric screening mass, $m_E^2 \equiv \frac{1}{3}N g^2 T^2$ [2]. Since usually the infrared divergences in high- T QCD are believed to be cured by a non-perturbative magnetic screening mass, the above seems a curious result which certainly deserves further study.

So far the gluon mass has been put in by hand and its value determined self-consistently only afterwards. In principle, the mass term should arise dynamically in the Lagrangian (4.8). Therefore, I finally want to speculate, how this may occur. - There is one candidate term contained in the Lagrangian (4.8),

$$\mathcal{L}_4 \equiv -\frac{1}{2}g^2 f^{abc} f^{ade} A_\mu^b A_\mu^d a_\nu^c a_\nu^e , \quad (13)$$

which potentially becomes a gluon mass term. It is part of the original four-gluon interaction and contributes with an opposite sign to the Hamiltonian. Then, using $\langle a_\nu^c a_\nu^e \rangle = N^{-1} \delta_{ce} \langle a_\nu^j a_\nu^j \rangle$, which can only hold approximately in the presence of background fields, one obtains a contribution to the interaction energy,

$$\epsilon = +\frac{1}{2}g^2 A_\mu^i A_\mu^i \langle a_\nu^j a_\nu^j \rangle . \quad (14)$$

It has the correct sign for a vector particle mass term [24], if $\mu^2(x) \equiv -g^2 A_\mu^i(x) A_\mu^i(x) \geq 0$. Thus, the mass is generated by the background field and should be dominated by the space-like components of A_μ .

The same conclusion can be reached by another, more speculative argument. Suppose the background potential points to a globally fixed direction in colour-space. In particular, let

$$A_\mu = \sum_a A_\mu^a t_a \equiv N^{-1/2} A_\mu \sum_a t_a . \quad (15)$$

Such a potential is abelian-like in the sense that the corresponding field strength tensor is $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) \sum_a t_a$. Then, instead of eq.(14), one finds from eq.(13)

$$\epsilon = \frac{1}{2}g^2 A_\mu A_\mu \langle a_\nu^c a_\nu^c \rangle + N^{-1} \sum_{b \neq d} f^{abc} f^{ade} \langle a_\nu^b a_\nu^d \rangle , \quad (16)$$

producing a mass term as above plus additional contributions. The Ansatz of eq.(15) clearly breaks the classical local $SU(N)$ symmetry of Lagrangian (4.8).

Note that eq.(13) can be rewritten generally as

$$\mathcal{L}_4 = -\frac{1}{2}g^2 a_\nu^c (M^2[A])_{ce} a_\nu^e , \quad M^2[A] \equiv A_\mu^b T^b \cdot A^{d\mu} T^d . \quad (17)$$

M^2 is a hermitian matrix which can be diagonalized by a suitable unitary transformation. However, a classical gauge transformation, see (4.4), in general does not work, since it changes $M^2[A]$ in an unsuitable way. A BRS transformation, see (4.9), on the other hand, leaves M^2 invariant and the homogeneous part of the transformation of the a -fields allows to diagonalize it. However, due to the inhomogeneous part, one picks up additional terms. Then, $\mathcal{L}_4$ becomes

$$\mathcal{L}_4 = -\frac{1}{2}g^2 (M^2[A])_{ce} \tilde{a}^c \tilde{a}^e + \dots , \quad (18)$$

where $\tilde{a}_\mu^a$ are the transformed fields, $(M^2)^a$ denote the eigenvalues of the mass matrix, and where the additional terms are not written explicitly. Thus, only under more restricted circumstances, as illustrated above, a simple scalar mass appears. However, little (if anything) is known about the dynamical behaviour of a pseudo-mass term like in eqs.(13,18), which arises self-consistently as described in Section 4.

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