ACCSIM – A PROGRAM TO SIMULATE THE ACCUMULATION OF INTENSE PROTON BEAMS

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Abstract. ACCSIM is a multiparticle 3-D tracking code initially written to simulate multiturn injection. It treats simultaneously, but to a certain degree of approximation, transverse and longitudinal space charge forces, lattice perturbations simulated by thin multipoles, and energy loss, Coulomb and nuclear scattering in charge exchange foils or internal targets. Output includes particle distributions in phase space and real space, loss statistics and figures of merit for the retained ensemble. The program is described in detail and examples given of its use for existing and proposed machines.

INTRODUCTION

The injection process for a storage ring or synchrotron should generate particle distributions which are stable during accumulation and subsequent acceleration while minimizing beam loss and residual activation. The process is affected by, inter alia, details of the incoming beam emittance and beam line optics, the ring lattice and acceptance, space charge forces, and the angular scatter and energy loss in a charge exchange medium. The latter are especially important for the 100 μA TRIUMF KAON Factory proposal for which 3×10^{11} protons/bucket would be accumulated over ~10^4 turns from a cyclotron injector. Several existing codes performed first- and second-order 3-D tracking for single particles, others included space charge in transverse or longitudinal planes, but none included all features pertinent to multiturn injection. ACCSIM was therefore written assuming that a somewhat less exact description of unstable behaviour would be acceptable and that problems revealed would be studied analytically or with more specialized codes.

The program uses a combination of tracking and Monte Carlo simulation to follow an ensemble of “superparticles”. Multiturn injection is modelled by adding new particles to the ensemble at intervals during the injection cycle. The injected particles are generated from a choice of incoming beam distributions and injection painting schemes.

RUN PREPARATION

The term “node” is used to refer to those locations in the lattice at which particle co-ordinates are updated. These include all rf cavities, thin multipoles, apertures, etc. The
lattice design program DIMAD\textsuperscript{2} produces a file containing first-order node-to-node transfer matrices and other parameters used by ACCSIM. If momentum-dependent (chromatic) tracking is required, additional matrices and closed orbits are calculated for energies above and below the reference energy and are fit by quadratic functions. The rf system is specified by the number and placement of cavities and the amplitude and relative phase of up to 2 harmonics. Multipoles are treated as “kicks” and are specified by their order \((n=1,2,3\ldots)\), whether skew or normal, and by the strength parameter \(K_nL = (L/Bp)(\partial^n B_y/\partial x^n)\).

Beam injection may be treated as either a single turn transfer from a pre-injector or continuous injection of particles drawn from a uniformly populated ellipse or from a binomial distribution. Either the rms or the limiting emittance is specified together with the “Twiss” parameters \(\alpha\) and \(\beta\). The parameters adjusted during injection are ramped in a piecewise linear manner and include the position (horizontal, vertical, energy and rf phase) of the injected beam, the position of the closed orbit, and the cavity voltage. It is intended to add field and rf adjustments to simulate acceleration.

Stripping foils or targets, assumed rectangular, are specified by the atomic number, atomic weight, density, nuclear and Coulomb scattering coefficients of the material, and by their horizontal and vertical boundaries. Apertures, e.g. collimators, may be elliptical or rectangular.

### TRACKING

Co-ordinates similar to TRANSPORT are used: \(x, x', y, y', \phi, \delta\), where \(x\) and \(y\) are measured with respect to the reference trajectory at the centre of the vacuum chamber, \(\phi\) is rf phase, and \(\delta = (p - p_s)/p_s\) is measured with respect to the synchronous momentum \(p_s\). Co-ordinates are tracked from one node to the next by applying the relevant DIMAD matrix together with any other impulses. For example, the co-ordinates \(x_1, x'_1, y_1, y'_1, \phi_1, \delta_1(\Delta E_1)\) at a cavity (node 1) would be transformed to the next node by

\[
\begin{align*}
\Delta E_2 &= \Delta E_1 + \frac{V_{rf}}{N_e} \sin \phi_1 + f_R V_{sc}'(\phi) \\
\delta_2 &= \frac{\Delta E_2}{\beta_s^2 E_s} \\
x_2 &= R_{11} x_1 + R_{12} x'_1 + R_{16} \delta_2; \quad x'_2 = R_{21} x_1 + R_{22} x'_1 + R_{26} \delta_2 \\
y_2 &= R_{33} y_1 + R_{34} y'_1; \quad y'_2 = R_{43} y_1 + R_{44} y'_1 \\
\phi_2 &= \phi_1 + \frac{2\pi h}{L_T} \left[ R_{51} x_1 + R_{52} x'_1 + \left( R_{56} - \frac{L_R}{\gamma_s^2} \right) \delta_2 \right],
\end{align*}
\]  

where \(V_{rf}/N_e\) is voltage/cavity, \(h\) the harmonic number, \(V_{sc}'(\phi)\) the longitudinal space-charge potential, \(L_T\) the machine length, \(f_R = L_R/L_T\) the fractional section length, and \(R\) the node-to-node transfer matrix. For the moment the synchronous phase \(\phi_s = 0\). \(V_{sc}'(\phi)\) is computed from a smoothed numerical differentiation of the linear density \(\lambda(\phi)\). Skew quadrupoles or multipoles at node 1 further modify \(x'_1\) and \(y'_1\). The transverse phase space co-ordinates \(x_b = x - \eta(\delta p/p) - x_{co}\) (and similarly \(x'_b, y_b\) and \(y'_b\)) are computed
Particles undergoing transverse betatron oscillations execute a Lissajous trajectory in x-y space. Trajectories for particles with the same $\epsilon_x$ and $\epsilon_y$ but with different relative phases of oscillation fill a rectangle [Fig. 1(a)]. To compute space charge forces the particles' amplitudes are binned into a 10x10 array. Each bin is associated with a density distribution of rectangular outline, Fig. 1(b), formed by the Lissajous trajectories. The space charge potential of each rectangle is computed semi-analytically, as is the central particle density, and the contributions summed. A least squares fit to a multipole expansion gives the coefficients, to dodecapole, needed to calculate the tune shifts for individual particles. This is then used to adjust the turn-to-turn phase advance. The effect on amplitudes is second order and not included at this stage. The image effects of an elliptical beam pipe can be included. The computed G-factor is the summed central particle density normalized by the average; this is only meaningful if the beam is not hollow.

The space charge calculation assumes that the $\epsilon_x$, $\epsilon_y$ distribution is quasistationary with strength modulated by the linear density $\lambda(\phi)$ along the bunch, and that dispersion and momentum spread are sufficiently small that beam size is largely determined by betatron amplitude. It is not valid for beams executing coherent coupled x-y oscillations.

Foils, Targets and Apertures

At the appropriate nodes a test is made to see whether a particle's co-ordinates lie within structural boundaries. Particles outside an aperture are tagged as lost, removed
from the ensemble, and their parameters stored. At a foil or target the energy loss is calculated using the Landau formalism; at present this is truncated to $\Delta E \leq 3.5$ keV, underestimating 0.1% of the interactions in the proposed TRIUMF foil. The foil or target surface is divided into bins and the $\Delta E$/bin accumulated. This may then be used for radiation damage or temperature estimations; note that for very thin foils much of the energy escapes as photons or energetic electrons. The mean number $m$ of Coulomb scattering events/traversal is calculated from the foil thickness and total cross section and an angular kick applied using a Monte-Carlo routine with appropriate scattering distribution. If $m > 20$ Molière theory is used; if $m \leq 20$ a plural scattering approach is followed. It is assumed that nuclear interactions scatter particles outside the acceptance. For thin targets this loss is computed at the end of the calculation from the product of total foil interactions and the event probability.

**OUTPUT**

An intermediate output may be produced at regular intervals. At the end of run a file containing the parameters of lost and stored beam is generated for subsequent analysis or a continuation run. A summary table gives several different specifications of emittance, the fraction of beam hitting collimators, the average number of foil traversals per proton and other statistics. The foil-hit data may be folded with analytic descriptions of interaction processes to better describe beam halo where statistics are poor. Figure 2 illustrates the tabular and graphics output generated at run end and periodically during the calculation.

The code has been developed in the VAX/VMS environment and can be used in either interactive or batch mode. Array capacities can be varied and are presently set to $10^4$ particles to provide efficient operation without significant "paging" overhead. A typical run accumulating 2000 particles over 20,000 turns takes about 0.5 cpu-h without transverse space charge and 3 cpu-h with transverse space charge on a VAX8650.

**TESTS AND EXAMPLES OF USE**

Single-particle and ensemble tests of the matrix tracking scheme have shown correct Liouvillian behaviour over $\sim 10^4$ turns. Off-momentum matrix interpolation compares well with second-order tracking in DIMAD. The foil scattering algorithms show emittance growth rates for an ensemble repeatedly traversing a foil that are in line with analytic predictions. Space charge calculations with elliptical distributions exhibit long-term near-stationary behaviour under space charge, and the calculated forces agree with those of LONGID. The motion of an individual particle calculated by the two programs can differ; this arises from the coarser step size and fewer macro-particles in ACCSIM. The analytically tractable K-V and parabolic distributions have been used to check transverse space charge calculations. The difference in $G$-factor is 0.4% and 5%, respectively, and the
maximum tune shift is \( \leq 3\% \) and \( \leq 8\% \). The error in the fitted potential is \(< 5\% \) everywhere within the beam.

The transverse space charge subroutines had been used earlier to interpret various measured amplitude distributions in the CERN PS Booster and very good agreement had been found with experimental measurements of tune shift.\(^7\) The program has recently been used to explain experimental results at the CERN PS\(^8\); also tune shifts computed by ACCSIM fit reasonably into the available tune space.\(^9\) Simulations of beam accumulation and beam loss at the Los Alamos Proton Storage Ring agree reasonably well with measurements. A promising application of the code is in detailed comparison with diagnostic signals.

The program is currently being used to study accumulation for the proposed European Hadron Facility\(^10\) and the TRIUMF KAON Factory. In general the final ensembles are only slightly affected by inclusion of space charge and chromaticity, exceptions being beams hollow in longitudinal phase space or beams subject to \(x-y\) coupling. However, the motion of individual particles can show significant changes in, say, the number of foil traversals when their individual tune values satisfy a resonance condition \(mQ_x + nQ_y + \ell Q_z = \pi\). Painting schemes involving significant coherent behaviour should be carefully examined.

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REFERENCES

FIGURE 2. Example of tabular output and scatterplots (at halfway point and end of injection) for TRIUMF KAON Factory A Ring with weak skew quadrupole and transverse space charge. \( N_m \) is the number of superparticles accumulated, \( N_p \) the number of protons, \( \text{Hits} \) the total number of foil traversals, \( \text{avg} \) the average hits/proton. \( X_{co} \) and \( Y_{co} \) are the closed orbit position for synchronous energy \( T_{sync} \), \( Y_0 \) and \( D_0 \) the center of the injected beam, \( E_x \) and \( E_y \) the emittance (\( \pi \) mm-mrad) for 99% of stored beam, \( B_f \) the bunching factor \( \bar{\lambda}(\phi)/\bar{\lambda} \), \( G_f \) a measure of peak to average density in \( x-y \) space, \( D_{Qx} \) and \( D_{Qy} \) the maximum space charge tune shifts.