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On the Particle Multiplicity in High-Energy e^+e^- Collisions

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ABSTRACT

We propose a new model for calculating the growth of particle multiplicity in high-energy e^+e^- collisions, based on the idea that near-mass-shell ("cool") partons produced by conventional perturbative QCD showering break chiral symmetry spontaneously and independently when they convert non-perturbatively into hadrons. Since pseudoscalar mesons are associated with inhomogeneities in the chiral condensate fields, we hypothesize that their multiplicity is proportional to the number of links in a triangulation using the different "cool" partons as vertices, thus extending a previous suggestion that the multiplicity of baryons is proportional to the number of simplices in the triangulation. We find that the numbers of links in Υ decays and in the e^+e^- continuum depend only on the numbers of "cool" partons, and the number of links increases faster than the number of partons. As a result, the charged particle multiplicity increases faster than the perturbative parton multiplicity. We present numerical results that are in good agreement with current e^+e^- data for charged particle multiplicity, and make predictions for the SLC and LEP as functions of energy and event sphericity.

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Underlying all modern models of particle production in high-energy hard collisions is the showering mechanism, whereby the initial highly-virtual (“hard”) partons split and radiate quanta perturbatively until they are close to mass-shell (“cool”).^[1] At this point, some non-perturbative mechanism takes over to convert the “cool” partons into hadrons.^[2] This could be via string formation and the creation of quark-antiquark or diquark-antidiquark pairs, as in the Lund model.^[3] Or it could be via the formation of independent clusters and their subsequent hadronization.^[4] The hadronization is an essentially non-perturbative process, whereby a transition takes place between the quark/gluon and hadron phases of QCD. In our view, this crucial point is not adequately taken into account by the hadronization models currently in use. Two local order parameters form in the hadronic phase: a chiral condensate $\langle 0 | \bar{q} L q R | 0 \rangle \equiv U$ and a gluon condensate $\langle 0 | \frac{g}{4} G_{\mu\nu} G^{\mu\nu} | 0 \rangle \equiv \chi^4$. Local fluctuations in U and χ are associated with pseudoscalar mesons and scalar glueballs, respectively.^[5] In addition, the U field can exhibit topological defects that correspond to baryons.^[6] We believe that useful intuition, and perhaps eventually a calculational scheme, can be obtained by considering how these order parameters freeze out. Two of us have already discussed,^[7] within this philosophy, the production of baryons in high-energy e^+e^- collisions, using an approach familiar from the study of the early Universe, where mismatches between the random orientations of an order parameter in independent spatial domains give rise to production of monopoles and other topological defects.^[8]

We assume^[7] that to each “cool” parton there corresponds a domain with a common randomly-chosen orientation of the chiral order parameter U in the internal space. Mismatches between the orientations can give rise to topological defects that we interpret^[7] as baryons. Consider a triangulation of space with “cool” partons serving as vertices. According to this topological baryon production model, the number of baryons is, on the average, just $1/16$ of the number of simplices in such a triangulation.* Pseudoscalar mesons can be associated with

* The number quoted is exact for flavour $SU(2)$, we expect that the number for flavour

topologically-trivial spatial fluctuations in the chiral order parameter U . We are therefore led to expect that the probability of pseudoscalar meson production depends on the relative orientations of U in adjacent domains. Since these orientations are assumed to be random and independent, we hypothesize that the average probability of pseudoscalar meson production will be proportional to the number of links in the triangulation.

In this paper we set up and apply this model for the production of charged particles in high-energy e^+e^- collisions. We find that the number of links per “cool” parton increases with the number of partons, but is essentially independent of whether they originate from Υ decay or in the e^+e^- continuum. An argument is presented to the effect that baryon production does not make a big correction to the total charged particle multiplicity estimated from the link multiplicity, which increases more rapidly than the perturbative parton multiplicity. Normalizing the charged particle multiplicity to the experimental data at $\sqrt{s} = 30 \text{ GeV}$,^[9,10] we make successful calculations of the multiplicity in the e^+e^- continuum from $\sqrt{s} = 10 \text{ GeV}$ to $\sqrt{s} = 91 \text{ GeV}$, and also compare with experimental data in Υ decays. Predictions are also given for the charged particle multiplicity at the higher energies of LEP. We also discuss likely deviations from a Poisson multiplicity distribution, and make predictions for the dependence of the multiplicity distribution on the event sphericity.

Most of the stable hadrons produced in high energy e^+e^- collisions are pseudoscalar mesons, whilst the rest are baryons. Many of the pseudoscalar mesons are the decay products of more massive states, especially vector mesons and spin- $\frac{3}{2}$ decuplet baryons. Here we explore the assumption that the general trends of the total hadron multiplicity can be modelled without taking into account the detailed dynamics of these resonances. Since hadronization is a soft process, one could hope that particle formation is described by a low-energy effective Lagrangian, once the chiral condensate field $U(x)$ is formed. In the case

$SU(3)$ does not differ greatly.

of the dominant pseudoscalar mesons this should be a phenomenological chiral Lagrangian^[6]

$$\mathcal{L} = \frac{f_\pi^2}{16} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) - c \text{Tr} [m_q (U + U^\dagger)] + \mathcal{O}((\partial_\mu U)^4) \quad (1)$$

where $f_\pi \simeq 93 \text{ MeV}$ is the pion decay constant, $U = \exp(i \sum_{i=0}^8 \lambda_i \phi^i / 2f_\pi)$, $\{\phi^i\} = \{\pi, K, \bar{K}, \eta^8, \eta^0\}$ is the 3×3 chiral field matrix, and $m_q = \text{diag}(m_u, m_d, m_s)$ is the quark mass matrix. If desired, the chiral Lagrangian (1) could be extended to include vector and/or scalar mesons^[6] but we shall not do so here. Extra fields do not need to be introduced for baryons, because they can be interpreted as topological defects in a spatially inhomogeneous U field.^[6] Isolated pseudoscalar mesons correspond to plane-wave solutions of the classical equations of motion associated with the chiral Lagrangian (1). As such, they correspond to topologically-trivial spatial inhomogeneities in the chiral field U , much like spin waves in a ferromagnet.

The near-mass-shell “cool” partons are produced by the conventional perturbative QCD showering mechanism occurring in the early stages of the formation of hadronic final states in high-energy collisions.^[1,4] In our model, each “cool” parton gives rise to an independent chiral domain with a random orientation of the chiral field U . Previously we used simple combinatorics to estimate the probability of a mismatch in the orientation of the chiral field U in different domains, leading to a topological defect interpreted as a baryon. According to this picture, in the case of $SU(2)$ flavour, the number of baryons is

$$n_B = p_B \cdot n_T \quad : \quad p_B = \frac{1}{16} \quad (2)$$

where n_T is the number of tetrahedra in a triangulation of three-space. Using the random lattice construction of Ref. 11 we found^[7] that n_T/n_D (where n_D is the number of domains) was almost independent of the initial state, but

depended significantly only on n_D , increasing as n_D increased towards an asymptotic limit:^[11]

$$n_T/n_D \xrightarrow{n_D \rightarrow \infty} \frac{24}{35} \pi^2 \quad (3)$$

leading to an asymptotic yield of about 0.4 baryons per “cool” parton.^{*}

The natural extension of this model to pseudoscalar mesons is to suggest that their production rate depends only on the relative orientations ΔU of the chiral fields in different domains, assuming that the spatial fluctuations within domains are negligible. In principle, the mean pseudoscalar meson multiplicity $\langle n_p \rangle$ could depend simultaneously on the relative orientations in several different domains, but we assume that it only depends on the differences $\Delta_{ij} U$ in orientations between 2 “adjacent” domains (i, j) :

$$\langle n_p \rangle = \sum_{(i,j)} f(\Delta_{ij} U) \quad (4)$$

Here “adjacency” is defined by the same random lattice triangulation algorithm^[11] as we used previously. Since the orientations of the chiral field U are independent in different domains, the mean pseudoscalar meson multiplicity n_p should be proportional to the number of links in this triangulation:

$$\langle n_p \rangle = b n_L \quad (5)$$

where b is a constant we cannot calculate, but will fit from existing e^+e^- data.

As mentioned above, there could in principle be corrections to the above formula due to local inhomogeneities within the domains. Such corrections would

^{*} We note in passing that this model explains qualitatively the softer z distribution observed^[12] in deep inelastic neutrino interactions which have baryons in the final state.

Baryon production is favoured in events with more final-state partons, such as the high-sphericity e^+e^- events discussed in Ref. 7, or deep inelastic events in which gluon bremsstrahlung has occurred, giving a softer z distribution.

presumably be proportional to n_D . We believe that they are a secondary effect and accordingly we shall neglect them: this seemingly crude approximation turns out to give a rather accurate description of the data. In addition, we should in principle make a correction to (5) proportional to the number of baryons and hence to n_T . This is because spatial inhomogeneities $\Delta_{ij}U$ cannot be simultaneously topologically trivial and therefore contribute to n_p , and part of a topological defect interpreted as a baryon. Therefore we subtract from n_L in (5) those links that contribute to the baryon multiplicity. Such links define the boundary of a tetrahedron corresponding to a baryon-antibaryon pair, with a probability $p_B = 1/16$. The resulting correction is therefore:

$$\langle \delta n_p \rangle = -b \cdot 4 \cdot p_B \cdot n_T \quad (6)$$

On the other hand, most multiplicity data are for all charged particles n_c including both baryons and pseudoscalar mesons. Since there are equal numbers of baryons and antibaryons, and since half of each are charged, we use eq. (2) and apply a second correction $+n_B$ to get the total mean multiplicity:

$$\langle n_c \rangle = \langle n_p \rangle + \langle \delta n_p \rangle + \langle n_B \rangle = bn_L + (-4b + 1)p_B n_T \quad : \quad p_B = \frac{1}{16} \quad (7)$$

We shall see later that b is close to $1/4$ for charged particles, so the correction terms $\propto n_T$ almost vanish, and we will neglect them hereafter.[†]

Before fitting experimental data with the formula (7) and making predictions for the future, we first justify our claim that n_L is essentially just a function of n_D . Figure 1 shows the mean values of n_L for different numbers of domains n_D found by the random lattice algorithm of Ref. 11 applied to perturbative parton shower configurations in the same way as in Ref. 7, using as initial states the Υ

[†] Since there is a small probability that each produced baryon has spin-3/2, and may therefore decay into a spin-1/2 baryon plus a charged pseudoscalar meson, the second parenthesized correction factor in (7) should be somewhat larger than unity. But, as we see below, the best value of b is about $1/3.83$, so the cancellation persists.

(three gluons, $\sqrt{s} = 9.46$ MeV), the e^+e^- continuum ($\bar{q}q$, $\sqrt{s} = 30, 91$ and 200 GeV) and an imaginary toponium (three gluons, $\sqrt{s} = 100$ GeV). We see that to a good approximation the $\langle n_L \rangle$ lie approximately on a universal curve, and that the $\langle n_L \rangle / n_D$ increase with n_D in a manner well parametrized in the present energy range by

$$\langle n_L \rangle \simeq 6.4n_D - 33.6 \quad (8)$$

In the random lattice approach^[11]

$$\langle n_L \rangle / n_D \xrightarrow{n_D \rightarrow \infty} \left(1 + \frac{24}{35} \pi^2 \right) \quad (9)$$

in the limit of a very large number of truly randomly located domains. We fit $\langle n_c \rangle = b \langle n_L \rangle$ initially to the e^+e^- data at $\sqrt{s} = 30$ GeV.^[6,10] We find $\langle n_L \rangle_{30 \text{ GeV}} = 47.6$ and hence

$$b \simeq \frac{1}{3.83} \quad (10)$$

Figure 2 compares our $\langle n_c \rangle = b \langle n_L \rangle$ curve with the available data^[(6,14)] from Υ decay and the e^+e^- continuum from $\sqrt{s} = 10$ to 91 GeV.^[14] We make no apologies for the quality of the agreement of our model with experiment,[†] and have some confidence in the extrapolation of our curve to higher SLC/LEP energies. We note that because of the rise in $\langle n_L \rangle / n_D$ shown in fig. 1, we predict that $\langle n_c \rangle$ increases faster than the number of partons in this energy range, and hence faster than in conventional QCD cluster models.^[(1,2,3,4)]

So far we have made no cut on the length of the links. However, as for the sizes of the tetrahedra leading to baryons, it is plausible to argue that one should make a cut on the length of the links of order m_π^{-1} . Therefore, we show in figure 2 as a dotted line predictions made assuming that $\langle n_c \rangle = \hat{b} \langle \hat{n}_L \rangle$, where $\langle \hat{n}_L \rangle$ is the number of links shorter than $m_\pi^{-1} \sim 1.5$ fermi. The corresponding

[†] In particular, the ALEPH data^[(14)] became available after our calculations were completed.

constant of proportionality was determined from $\langle n_c \rangle$ at $\sqrt{s} = 30 \text{ GeV}^{[9]}$ to be $\tilde{b} = 1/2.25$. Again, the agreement of the prediction with the trend of the data for $10 \text{ GeV} < \sqrt{s} < 60 \text{ GeV}$ is very good, and the prediction for SLC/LEP energies do not differ greatly from the solid line which was obtained without cutting on the lengths of the links.

Our model also predicts a rise in the total charged multiplicity in $\epsilon^+\epsilon^-$ decay, compared with the $\epsilon^+\epsilon^-$ continuum, which is less than that we predicted previously for the baryon multiplicity, but more than that found in the data.⁽¹³⁾ The modest success of the model in this case may be due to the large number of partons in a small volume in $\epsilon^+\epsilon^-$ decay, which could give correlations between the hadronizations of different partons. It is not clear, however, whether one can obtain a consistent picture of the baryon and pion multiplicities in $\epsilon^+\epsilon^-$ decays. This point requires further study.

It is possible to make predictions for the mean charged particle multiplicity in certain classes of events without introducing extra assumptions. For example, if one selects events whose sphericity S at the parton level is large (small), $\langle n_L \rangle$ and hence $\langle n_c \rangle$ are larger (smaller) than for events with no sphericity cut. This effect is illustrated in the Table, where we show $\langle n_c \rangle$ for events with $S > (<) 0.15$ and $\sqrt{s} = 10, 30, 91, 200 \text{ GeV}$. It is in principle possible to make analogous selections and predictions by making cuts in acoplanarity.

In contrast, making predictions for the shape of the charged multiplicity distribution does require an extra assumption. This is because it involves a convolution of the probability distribution for n_L with the probability distribution for n_p at a given value of n_L . We cannot calculate the latter, so we consider an illustrative model, according to which the charged multiplicity associated with each link has a Poisson distribution with mean value b . The resulting charged multiplicity distributions for $\sqrt{s} = 10, 30, 91$ and 200 GeV are shown in figure 3. Again, the solid-line distributions are calculated using all links, whereas the dotted-line distributions are calculated using links of length < 1.5 fermi. We note

that the total multiplicity distribution is approximately Poisson at low energies and becomes progressively broader than Poisson at high energy.

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TABLE

The dependence of n_c on sphericity cuts

$\sqrt{s}$ (GeV)	All events	Parton sphericity $s > 0.1$	Parton sphericity $s < 0.15$
10	6.1	8.4	5.3
30	12.4	17.8	11.3
91	21.9	31.5	20.5
200	30.8	44.9	29.3

FIGURE CAPTIONS

- 1) The dependence of the number of links n_L on the number of "cool" parton domains n_D , for events generated by $\Upsilon \rightarrow 3$ gluon decay at $\sqrt{s} = 9.46$ GeV, $e^+e^- \rightarrow \bar{q}q$ at $\sqrt{s} = 30, 91, 200$ GeV, and an imaginary toponium $\rightarrow$ three gluon decay at $\sqrt{s} = 100$ GeV. Also shown is the approximate universal curve (8) and the asymptotic limit (9) for a truly random configuration of partons.
- 2) Data ^(10,11) on $\langle n_c \rangle$ at different energies compared with the prediction $\langle n_c \rangle = b \langle n_L \rangle$ calculated without any cut on the link length (dot-dash line) and links < 1.5 fermi long (dashed line). Also shown is a perturbative QCD prediction (dotted line). Phenomenological fit of Ref. 10 is shown as a solid line.
- 3) Charged particle multiplicity distributions at $\sqrt{s} = 10, 30, 91$ and 200 GeV. The solid (dashed) lines are for links without (with) a length cut-off of 1.5 fermi. Also shown are the corresponding Poisson distributions (dotted lines).

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- Fig. 1 -

- Fig. 2 -

- Fig. 3 -