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Effective Lagrangian Approach to QCD Phase Transitions

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ABSTRACT

We use an effective low-energy Lagrangian for broken chiral and scale invariance to discuss the condensation and confinement of quarks and of gluons in QCD. The Skyrme model convinces us that quark condensation and confinement are identical, and we argue that well-defined glueball states exist only when gluons condense. Previous calculations with chiral Lagrangians have indicated that the quark transition at the finite temperature T_q is second order if gluonic degrees of freedom can be neglected, and we argue that the gluonic transition at the finite temperature T_g is generically first-order. The scaling properties of the effective Lagrangian tell us that $T_q \leq T_g$. We argue that T_q can be below T_g in $SU(2)$ gauge theory, whereas $T_q = T_g$ and both quark and gluon transitions are simultaneous and first-order in QCD with its $SU(3)$ gauge group. These results agree qualitatively with lattice results. We speculate that the divergent Hagedorn spectrum may drive the joint quark/gluon transition at some temperature below the Hagedorn temperature.

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The phase transition between the quark-gluon phase at high temperatures T and/or baryonic chemical potential μ_B is not susceptible to direct analysis via perturbative QCD, which can only be used reliably at very high T and/or μ_B . Lattice Monte Carlo calculations [1] give useful numerical results in the region of the phase transition, but need to be supplemented by other sources of physical intuition. This can be provided in part by analytical arguments bearing on the order of the phase transitions, and in part by phenomenological models that interpret aspects of the lattice Monte Carlo data. Another tool that can be used to gain intuition about the QCD phase transitions is the effective Lagrangian describing hadrons at low momenta.

There are two important non-zero local order parameters in the hadron phase of QCD, namely the quark-antiquark condensate $\langle 0 | \bar{q}q | 0 \rangle$ which breaks chiral symmetry, and the gluon condensate $\langle 0 | F_{\mu\nu} F^{\mu\nu} | 0 \rangle$ which breaks scale invariance. We will discuss now the disappearances of these order parameters at high T or μ_B might be correlated with quark or gluon deconfinement. Hadron dynamics at low energies can be described by an effective field theory Lagrangian which embodies the approximate scale and chiral symmetries of QCD [2], and describes the implications of the formation of these condensates. These include baryons as topological solitons [3], as well as pseudoscalar mesons and a flavour-singlet scalar gluonium state as elementary excitations.

Finite-temperature corrections calculated [4,5] using a chiral Lagrangian [6] have been used to discuss the disappearance of the quark condensate $\langle 0 | \bar{q}q | 0 \rangle$ at high T , and chiral Lagrangians [6] have also been used to discuss pion and kaon condensation at high μ_B [7]. However, the effective Lagrangian approach has not yet been fully utilized. In particular, we do not know how of any applications of broken scale invariant Lagrangians [2] at T or $\mu_B \neq 0$. These can be used to estimate the nature of the gluon decondensation transition and the temperature at which it occurs, and also to cast light on the relation between condensation and confinement for quarks and gluons.

After introducing the effective Lagrangian for scale and chiral invariance [2], we first argue on the basis of the Skyrme model [3] that the quark confinement and condensation temperatures should be identified (T_q), and on the basis of the effective scale Lagrangian that gluon confinement and condensation should be identified (T_g). The scaling properties of the effective

Lagrangian tell us that $T_q \leq T_g$. We argue that the gluon transition is first order, whereas the order of the chiral quark phase transition depends whether $T_q < T_g$. If so, which we argue is more likely for $SU(2)$ colour, the quark transition is second order. However, if $T_q = T_g$ as we argue is more likely for $SU(3)$ colour, then the quark transition is also first order. These results are consistent with lattice Monte Carlo simulation for gauge theories [8]. Finally, we speculate how a Hagedorn spectrum of states [9] may drive the joint quark/gluon transition at some temperature below the Hagedorn temperature.

In order to discuss the interpretation of the quark and gluon transitions, and their relation, we need an effective hadron Lagrangian which incorporates both the chiral quark condensate $\langle 0 | \bar{q}q | 0 \rangle$ and the gluon condensate $\langle 0 | F_{\mu\nu} F^{\mu\nu} | 0 \rangle$. The effective chiral Lagrangian [6] is

$$\mathcal{L}_c = \frac{f_\pi^2}{16} \text{Tr} [\partial_\mu U \partial^\mu U^\dagger] - c \text{Tr} [m_q (U + U^\dagger)] - \frac{m_0^2}{2} \phi_0^2 + \dots \quad (1)$$

where $U(x) \equiv \exp \left(\frac{i \sum_{a=1}^8 \lambda_a \phi_a(x)}{f_\pi} \right)$; $f_\pi \simeq 93 \text{ MeV}$ is the pion decay constant and the ϕ_i ($i = 0, 1, \dots, 8$) are fields for the nine pseudo-Goldstone pseudoscalar mesons. The pseudoscalar meson masses originate in the second and third terms in (1): $m_{\phi_i}^2 = \frac{c}{f_\pi^2} m_q$ where m_q is the quark mass matrix: $m_q = \text{diag}(m_u, m_d, m_s)$, c is a constant with dimensions of mass cubed, and $m_0^2 = 0(\frac{1}{N_c})$ is an extra $U(1)$ -breaking mass term for the ninth pseudoscalar meson ϕ_0 . The dots represent terms of higher order in field derivatives ∂_μ , chiral symmetry breaking m_q , and $\frac{1}{N_c}$, whose precise form is not essential for our discussion. The rôle of the quark condensate is played by

$$\langle 0 | \bar{q}q | 0 \rangle = c < 0 | (U + U^\dagger) | 0 \rangle \quad (2)$$

Finite-temperature corrections have been calculated [4,5] for the chiral Lagrangian (1), and it has been shown that

$$\langle 0 | \bar{q}q | 0 \rangle = c < 0 | \bar{q}q | 0 \rangle_0 \left(1 - \frac{N_f^2 - 1}{N_f} \frac{T^2}{12 f_\pi^2} + \dots \right) \quad (3)$$

in the limit $m_q \rightarrow 0$, where N_f ($= 2$ or 3 in our case) is the number of light quark flavours. Since $f_\pi \propto \sqrt{N_c}$, Eq. (3) is highly suggestive of a second-order chiral phase transition at a

temperature

$$T_c \sim \frac{f_\pi}{\sqrt{N_f}} \propto \sqrt{\frac{N_c}{N_f}} \quad (4)$$

The next-to-leading terms indicated by dots in Eq. (3) have also been computed [5], and are consistent with (4) at leading order in N_c .

We believe that the chiral phase temperature T_c should be identified with the quark confinement temperature T_q . Our strongest motivation for this belief is the Skyrme model [3], which is exact in the twin limits of chiral symmetry and $N_c \rightarrow \infty$, and has recently gained strong new support [10] from the spin structure of the proton as revealed by the EMC [11]. According to the Skyrme model, baryons are topological solitons or defects in the chiral order parameter $<0| \bar{q}q |0>$ (2). In this picture, baryons can only exist if $<0| \bar{q}q |0> \neq 0$. Indeed, if one looks at the chiral soliton equations for $m_q = 0$, since the only non-zero mass parameter that they contain is f_π , clearly the baryon mass and radius scale as

$$m_B \propto f_\pi \quad \text{and} \quad r_B \propto \frac{1}{f_\pi} \quad (5)$$

Since the PCAC and current algebra relation

$$F_\pi^2 m_\pi^2 = (m_\pi + m_d) <0| \bar{q}q |0> \quad (6)$$

is expected to hold at small m_q also when $T \neq 0$, $f_\pi \rightarrow 0$ when $<0| \bar{q}q |0> \rightarrow 0$. Equation (5) therefore tells us that $m_B \rightarrow 0$ and $r_B \rightarrow \infty$ when $<0| \bar{q}q |0> \rightarrow 0$ at T_c [12] and there is no Skyrminion when $T > T_c$. There may be heavier bound states with baryon number, but the Skyrminion is surely the most important, being the lightest when $T \rightarrow T_c$. The fact that the Skyrminion does not exist when $T > T_c$ is very suggestive that quarks become unconfined at T_c . The fact that $r_B \rightarrow \infty$ at the same temperature is also highly suggestive that quarks become deconfined at T_c . Therefore we identify the chiral phase transition temperature T_c with the quark confinement temperature T_q .

We now turn our attention to gluon condensation and confinement, using as our tool an effective Lagrangian for broken scale invariance [2]. This involves introducing the dilatation

charge $D = \partial^\mu S_\mu$, where S_μ is the scale current and $[D, \Phi] = i(x_\mu \partial^\mu + d\Phi)\Phi$ for any field Φ , where $d\Phi$ is the scaling dimension. This effective Lagrangian is constructed on the basis of (1) by introducing a dimension-1 singlet scalar field χ , which is used to rescale the other terms in (1) so that they all have the same scaling dimensions as the corresponding terms in the quark-gluon Lagrangian. The kinetic terms should have scaling dimension 4, the chiral symmetry breaking terms should have dimension 3 (cf $\bar{q}q$) and the mass term for the ϕ_0 , which is related to the QCD topological susceptibility, should have scaling dimension 4. Thus we have

$$\mathcal{L}_g = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \frac{f_\pi^2}{16} \left(\frac{\chi}{\chi_0} \right)^4 \text{Tr} \left[\partial_\mu U^\dagger \partial^\mu U \right] - c \left(\frac{\chi}{\chi_0} \right)^3 \text{Tr} \left[m_q (U + U^\dagger) \right] - \frac{m_0^2}{2} \phi_0^2 \left(\frac{\chi}{\chi_0} \right)^4 + \dots \quad (7)$$

The representation of U in terms of the ϕ_a requires U to have zero scaling dimension, and we imposed the correct c -number dimensionality by introducing $\chi_0 \equiv <0| \chi |0>$. To take correct account of the QCD trace anomaly $\phi_\mu^\mu = \frac{d}{2g} F_{\mu\nu} F^{\mu\nu}$, one subtracts from the effective Lagrangian (7) an effective potential [13]

$$V_g = B \chi^4 \ln \left(-\frac{\chi}{c \chi_0} \right) \quad (8)$$

which has its minimum when $\chi = \chi_0 \equiv <0| \chi |0>$.

Small oscillations about this asymmetric minimum can be interpreted as a massive scalar gluonium (glueball) particle state. Clearly a similar particle interpretation cannot be maintained for small oscillations about $\chi = 0$, where $V_g'' = 0$ and the extremum of the potential is unstable. In general, the particle interpretation is not regained at finite temperature or chemical potential. For example, a naive computation using $\mathcal{L}_g - V_g$ of finite temperature corrections to the effective potential gives at large T

$$\delta V_g(T) = + \left(\frac{1}{2} \right) B \chi^2 \ln \left(\frac{\chi^{c/4}}{\chi_0} \right) T^2 + O(T^4) + O(m_q \chi^3) \quad (9)$$

This expansion is only valid when $\chi < T$, or more accurately when $\frac{\partial^2 V}{\partial \chi^2} < T^2$. When this condition is satisfied, the state $\chi = 0$ becomes the lowest-energy state for sufficiently large T . However, the potential is not quadratic in χ , and hence does not admit a particle interpretation

when expanded around $\chi = 0$. (When the expansion (9) is not valid, clearly there is no particle interpretation in the neighbourhood of $\chi = 0$.) We shall argue later that other massive states must be taken into account when computing the finite-temperature corrections to the effective potential for the glueball field, but it will remain true that no particle interpretation is possible when expanding around $\chi = 0$. We conclude that the gluon (de)condensation temperature at which $\chi = 0$ is also the gluon (de)confinement temperature T_g .

We now address the question of the order of the gluon phase transition. We first neglect the chiral symmetry breaking mass terms in (7). The remaining zero-temperature potential V_g (8) is very flat, and we expect any additional potential due to finite-temperature or chemical potential corrections to look like $\chi^n : n < 4$ at small χ , in which case the phase transition to $\chi = 0$ will be first order. For simplicity, we will for now analyze corrections to the zero-temperature potential of the general form

$$V(\chi, T \text{ or } \mu_B) = V_g(\chi) + c(T \text{ or } \mu_B)\chi^n : n < 4 \quad (10)$$

where the *a priori* unknown coefficient $c(T \text{ or } \mu_B)$ is expected to be a monotonically increasing function of both μ_B and T considered separately. The following analysis is only indicative, but similar results (though perhaps not expressible in such a simple analytic form) would be found for other forms of the correction to $V_g(\chi)$ in (10). The general behaviour of the effective potential as μ_B or T increases is shown in Fig. 1. The $\chi \neq 0$ vacuum is energetically favoured for small $T(\mu)$ until $V = 0$ for the same non-zero value of χ at which $V' = 0$, namely

$$\chi_C = \chi_0 e^{-\frac{n}{4-n}B} \quad (11)$$

which occurs at the critical temperature $T = T_C(\mu_B = \mu_C)$ shown in Fig. 1 where

$$c(T_C \text{ or } \mu_C) = \frac{1}{4-n} \chi_0^{4-n} e^{-\frac{n}{4-n}B} \quad (12)$$

In the solutions (11) and (12), and again in (14) and (15) below, we have neglected the T^4 terms in (9), which may differ at $\chi \simeq 0$ and $\chi \simeq \chi_0$ but do not affect our qualitative results.

The phase transition may take place when $T = T_C$ ($\mu = \mu_C$) when the $\chi \neq 0$ states cease to be a global minimum, or the system may supercool in the gluonic vacuum with $\chi = 0$ which is still a local minimum for any $T(\mu_B) : c > 0$, or it may superheat to a higher temperature $T \leq T_X$. The latter is the temperature at which there ceases to be even a local minimum at $\chi \neq 0$. This occurs at $T_X(\mu_X) : V' = V'' = 0$ for the same $\chi \neq 0$ given by

$$\chi_X = \chi_0 e^{-\frac{1}{4-n}} \quad (13)$$

where

$$c(T_X \text{ or } \mu_X) = \frac{4}{n(4-n)} \chi_0^{4-n} \frac{B}{c} \quad (14)$$

which is also indicated in Fig. 1. When $T > T_X$ there is not even a metastable phase with $\chi \neq 0$, and the system can only exist in the $\chi = 0$ phase, in which we believe there are free gluons for the reasons discussed earlier. Whether the gluon transition between the $\chi \neq 0$ and $\chi = 0$ phases occurs at $T_g \simeq 0$, in which case $\chi \simeq \chi_0$, or at $T_g \simeq T_C$, in which case $\chi \simeq \chi_C$, or at $T_g \simeq T_X$, in which case $\chi \simeq \chi_X$, or somewhere in between, it is always first order.

We now address the relationship between the quark transition at T_q and the gluonic transition at T_g . It is apparent from Eq. (7) that when the gluonic degrees of freedom are taken into account we should identify

$$< 0 | \bar{q}q | 0 > = c \left(\frac{\chi}{\chi_0} \right)^3 < 0 | (U + U^\dagger) | 0 > \quad (15)$$

instead of (2). The quark condensate may therefore vanish either (a) because $< 0 | (U + U^\dagger) | 0 > \rightarrow 0$, which was the possibility analyzed in Refs. [4, 5] and reviewed earlier, or (b) because $\chi \rightarrow 0$. In case (a), which is illustrated in Fig. 2a, $T_q < T_g$ and the quark transition is presumably second order as expected in the simple chiral model (1). In case (b), which is illustrated in Fig. 2b, $T_q = T_g$ and the quark transition becomes first order, just like the gluon transition discussed above.

We cannot calculate either T_q or T_g reliably, and hence cannot say with certainty which of these two possibilities is realized for conventional QCD with $N_c = 3$ colours and $N_f = 3$

light flavours. However, we do expect (4) that $T_q \sim \sqrt{\frac{N_c}{N_f}}$, whereas $T_g \sim N_c^0$, and hence can anticipate that case (a) would be realized for a sufficiently large number of flavours if N_c is kept fixed, whereas case (b) would be realized for a sufficiently large number of colours if N_f is kept fixed. These predictions could in principle be confronted with lattice QCD calculations, that can be performed with any desired values of N_c and N_f . Lattice simulations [8] indeed indicate that case (b) is realized when $N_c = N_f = 3$, since the chiral condensate drops suddenly to zero at the phase transition temperature. However, just below this temperature $< 0 \leq qg \mid 0 >$ seems already to have fallen gradually to about $\frac{1}{4}$ of its value at $T = 0$ [8,14]. This indicates that one is not far from case (a), and one might be able to reach it by considering $\frac{N_c}{N_f} < 1$, e.g., $N_c = 2$ and $N_f \geq 3$ or $N_c = 3$, $N_f \geq 4$. If $T_C < T_q < T_X$, it might also be possible to realize case (a) by superheating in the glueball phase. This could even occur in physically interesting situations such as heavy ion collisions or the collapse of a supernova core to form a neutron or quark star.

We would like to speculate finally on the mechanism that drives the gluon transition. In the case of the quark transition, the chiral symmetry associated with light quarks might justify a calculation of T_q using just the light pseudoscalar meson degrees of freedom (case (a) above), but there is no justification for neglecting massive degrees of freedom when calculating T_g . The scalar glueball is just one of an infinite number of hadrons whose masses are $\propto \Lambda_{QCD}$. All of these should *a priori* be included in calculating the finite-temperature effective potential. In the limit of chiral symmetry, the χ field couples to other hadrons essentially as a dilaton:

$$< 0 \mid \chi \mid 0 > = g_{\chi hh} = m_h \quad (16)$$

Here we see that all hadron masses vanish when $< 0 \mid \chi \mid 0 > \rightarrow 0$, suggesting that this does indeed correspond to a deconfined phase. If we sum over all hadron contributions to the effective χ potential assuming a phenomenological Hagedorn spectrum [9] with density $n(m) \sim m^{-a} \exp(bm)$, it is well known that a singularity may occur as $T \rightarrow T_H = \frac{1}{b}$, the Hagedorn temperature [15], although this singularity can be avoided if hard core interactions or excluded volumes are assumed [16]. Specifically, we find in the limit $T \gg \chi$ that the

pressure

$$P = \frac{cT^{\frac{5}{2}}}{(2\pi)^{\frac{1}{2}}} \left(\frac{TT_H}{T_H - T} \right)^{\frac{1}{2}-a} \Gamma\left(\frac{5}{2} - a, \frac{y}{2}\right) : y = 12\chi \frac{(T_H - T)}{TT_H} \quad (17)$$

If we choose $a = \frac{3}{2}$ as favoured in some old hadron models, then there is indeed a singularity in (17) as $T \rightarrow T_H$. This means that the $\chi = 0$ unconfined phase becomes energetically favoured at some temperature below T_H . Although Eq. (17) is not of the simple form $c\chi^n$ as in Eq. (10), it can be approximated for small χ by Eq. (10) with $n = 1$. As argued earlier, the hadron $\rightarrow$ quark/gluon phase transition will then take place at some temperature $T_g \leq T_X$: perhaps $T_g > T_C$ (superheating), $= T_C$, or $< T_C$ (supercooling), but in any case $T_g < T_H$. A sketch of the possible behaviour of the free energy in the hadron and quark/gluon phases is shown in Fig. 1/three, where we have added a bag constant $\frac{1}{4}B\chi_0^4$ in the quark/gluon phase. Since the hadron line is concave and the quark/gluon line is convex, they can either miss each other completely, kiss each other once, or cross each other twice. Of these, only the first and last options are generic, and the first one is phenomenologically unacceptable, so we focus on the possibility of two crossings at temperatures T_C and T_C' . In fact, the hadron line is not complete and terminates at some temperature T_X because of the instability discussed above. If $T_X < T_C$, there is again no crossing in the allowed region of the hadron line, which is again phenomenologically unsatisfactory, but on the basis of the previous analysis we in fact expect $T_X > T_C$ as suggested in Fig. 1. The preferred situation is $T_C < T_X < T_C'$, as emphasized in Fig. 3, in which case the hadron $\rightarrow$ quark/gluon transition as T increases and quark/gluon $\rightarrow$ hadron transition as T decreases are both satisfactory. The case $T_X > T_C'$ is problematic because it would correspond to a return to the hadron phase at high temperatures, and an end to the pressure curve in a stable phase which is not acceptable thermodynamically.

The acceptable case $T_C < T_X < T_C' < T_H$ does indeed occur for some generic and otherwise acceptable range of the unknown parameters, and we plan to report shortly on this [17], as well as on the applications to heavy ion collisions, supernova collapse and the early Universe of the ideas discussed in this paper.

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FIGURE CAPTIONS

- 1) Sketch of the likely behaviour of the effective potential for the gluon condensate parameter χ as the temperature T is increased (the behaviour for increasing chemical potential μ_B may be similar). At T_C there are two competing vacua, one with $\chi = 0$ and one with $\chi = \chi_C \neq 0$. At T_X there ceases even to be a metastable state with $\chi \neq 0$.
- 2) Sketches of the possible behaviours of the gluon order parameter $\langle 0 | \chi | 0 \rangle$ and of the quark-antiquark condensate $\langle 0 | \bar{q} | 0 \rangle$. In case (a), $T_q < T_g$ and the quark transition is second order. In case (b), $T_q = T_g$ and the quark transition is (weakly) first order. The dotted line indicates how $\langle 0 | \bar{q}q | 0 \rangle$ would have behaved in the absence of other degrees of freedom.
- 3) Sketch of the desirable behaviours of the free energy in the hadron and quark/gluon phases. The hadron curve is concave, because of the approach to a singularity at the Hagedorn temperature T_H , and would cross the convex quark/gluon curve at two temperatures T_C and $T_{C'}$, were it not that the hadron curve terminates at $T_X < T_{C'}$, as illustrated in Fig. 1.

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- Fig. 1 -

a)

b)

- Fig. 2 -

- Fig. 3 -